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# Telephoto combination of holographic optical elements for polychromatic light

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Telephoto dublet system has been considered as a combination of separated positive and negative holographic optical elements. It is shown that in this case chromatic aberration is generated when the telephoto lens works at the wavelengths of light different from the one used for construction. The analytical conditions for achromatization of the system of two separated holographic optical elements are discussed.

## 1. Introduction

The usage of holographic optical elements as lenses has been proposed by several authors [1]–[3]. Telephoto lens is defined as a lens, the length of which from the front vertex to the back focal plane is shorter than its focal length. The purpose of this lens is to produce larger image of a distant object because of focal length that is usually 20% to 50% longer than the overall length of the objective [4], [5]. This can be achieved by using a combination of a converging front component separated by a suitable distance from a divergent rear component. In reality, the divergent element is an enlarging lens, therefore the effective focal length of the objective is long and given approximately by the equation

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2} \tag{1}$$

where $f_1$ and $f_2$ are the focal lengths of the positive and negative elements, respectively, and $d$ is the separation of the latters.

It will be convenient to characterize the telephoto lens by its magnification known as the telephoto magnification and defined as the ratio of the effective focal length to the back focal length

$$M_T = f/z_I \tag{2}$$

with the two distances $f$ and $z_I$ being depicted in Fig. 1. The illustration is drawn in Cartesian coordinate system whose $z$-axis coincides with the axis of the optical system. The telephoto magnification is usually equal to $M_T \approx 2$ ($M_T \geqslant 2$), but is never greater than $M_T = 3$. It is difficult to design the telephoto lens of greater values of magnification because of the field curvature and distortion which increase proportionally to telephoto magnification. In this paper, we present an attempt to

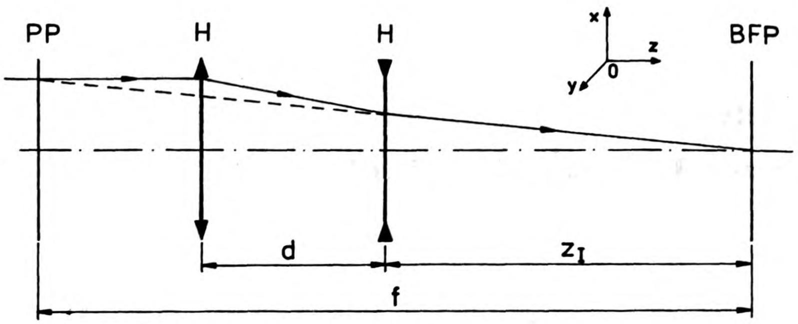

Fig. 1. Telephoto lens system characterized by a great effective focal length. Its $H_1$, $H_2$ are the optical elements: positive and negative, respectively. *PP* is the principal plane, *BFP* is the back focal plane of the system

design holographic telephoto lens in the first approximation. It seems that the positive holographic optical element used for this purpose could be recorded on a spherical substrate in order to satisfy the Abbe's sine condition.

## 2. Holographic components and their dispersive power

The recording geometries for the two components of a telephoto lens are shown in Fig. 2. The first one (Fig. 2a) is the positive element produced by interference of one plane and one converging spherical wave front, while the second one (Fig. 2b) – by two converging spherical wave fronts. For white light the longitudinal chromatic aberration of these holographic elements has to be analysed and the conditions for achromatic combination considered. The optical powers of the two optical elements recorded with the geometry shown in Fig. 2 are:

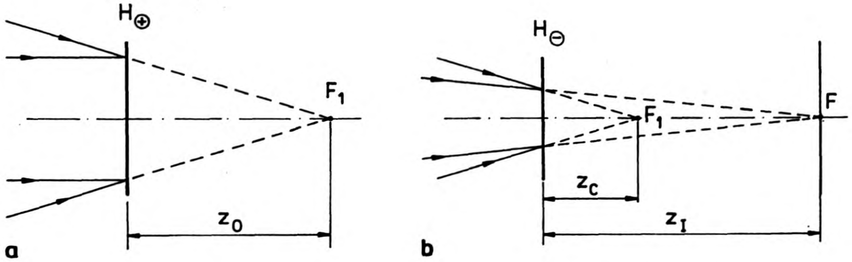

Fig. 2. Recording geometry of the two holographic elements: a – $H_+$, b – negative $H_-$

$$\frac{1}{f_1} = \frac{\mu}{z_O}$$

and (3)

$$\frac{1}{f_2} = \mu\left(\frac{1}{z_I} - \frac{1}{z_C}\right)$$

respectively, where $z_C = f_1 - d$ and $\mu = \lambda/\lambda_0$ is the ratio of the reconstructing wavelength to the recording one.

As we know, the properties of the optical elements vary with wavelength of the light beam being transformed by these elements. For example, the longitudinal aberration is the variation of the image position with wavelength. It is well known that for conventional lenses the index of refraction of glasses causes that the light of short wavelengths is refracted more strongly at the surface of a lens than the light of long wavelengths, whereas for holographic lenses the long wavelengths are diffracted more strongly than the short ones. In our case, the dispersion caused by the variation of the diffraction angle with wavelength results in a change of optical power of holographic optical elements in the following way

$$\Delta\left(\frac{1}{f_1}\right) = \frac{\Delta\mu}{z_O}$$

and (4)

$$\Delta\left(\frac{1}{f_2}\right) = \Delta\mu\left(\frac{1}{z_I} - \frac{1}{z_C}\right)$$

where $\Delta\mu = \Delta\lambda/\lambda_O$. The ratio of the change of the optical power caused by light dispersion or reconstructed wave fronts to the mean optical power is the dispersive power of the holographic optical element. Thus, we have

$$f_1\Delta\left(\frac{1}{f_1}\right) = f_2\Delta\left(\frac{1}{f_2}\right) = \frac{\Delta\lambda}{\lambda}. \tag{5}$$

$\lambda$ is the mean wavelength in the spectral range of reconstructing light used. The dispersion of a holographic optical element has been obtained by differentiating Eqs. (3) with respect to $\mu$, and by substituting the recording ($\lambda_0$) and reconstructing ($\lambda$) wavelengths for of the $\mu$-ratio. Then, the reciprocal of the dispersive power is given by

$$V = \frac{1/f}{\Delta(1/f)} = \frac{\lambda}{\Delta\lambda}, \tag{6}$$

which determines a HOE couterpart of Abbe's number.

## 3. Achromatic combination

The positions and powers of the two components of telephoto system are usually determined by the desired length of the system from the front vertex to back focal plane, and by the separation between the two components. The length from the front vertex of the positive lens to the final image plane of the system is called the telephoto effect and in most cases lies between 0.75 and 0.85 fraction of the effective focal length. However, the total focal length of the telephoto lens also depends on the separation between the two components, and it can be readily shown that the power of the negative component reaches a minimum when it is placed midway between the positive component and the final image plane (which is the back focal plane of the system). This is an optimal condition that leads to a rather long system which is quite acceptable in small systems, for example, in motion picture cameras.

If polychromatic light is emitted from an object point, the chromatic aberration causes the light of different wavelengths to be brought to focus at different locations. Therefore, an image suffering from this aberration can be in a sharp focus for one wavelength only. Consequently, the overall image will be a superposition of much out of focus images of different colours and different sizes. The chromatic aberration may be reduced or minimized by combining positive and negative elements. Usually, it is minimized by restricting the spectral width of light passing through the optical system. The holographic lens illuminated by white light, travelling along the direction of reference wave front, disperses the various wavelengths over a range of diffraction angles. Therefore, the negative component of the system should be a compensating element which produces an equal but opposite dispersion. As a result, the image is observed at the proper location and will appear to be achromatic one. It is quite possible to obtain an analytical solution using the method applicable to the two components satisfying achromatic condition at the determined separation. The total optical power of two separate components for the telephoto lens is expressed by Eq. (1). It is well known that a change of reconstructing wavelength will result in a change of optical power. Substituting Eq. (3) to Eq. (1), the effective focal lengths of the telephoto lens may be written in the form

$$f=\frac{Cz_O\lambda_O^2}{\lambda[\lambda_O(C+z_O)-\lambda d]}$$

where $C^{-1}=\frac{1}{z_I}-\frac{1}{z_C}$. Differentiation of the effective focal length with respect to the wavelength $\lambda$ gives

$$\frac{\partial f(\lambda)}{\partial\lambda}=-\frac{Cz_O\lambda_O^2[(z_O+C)\lambda_O-2\lambda d]}{\lambda^2[(z_O+C)\lambda_O-\lambda d]^2}. \tag{7}$$

If the derivative $\frac{\partial f(\lambda)}{\partial\lambda}=0$, then the focal length is independent of the wavelength of light used to produce images in the telephoto system. Whence

$$(C+z_O)\lambda_O-2\lambda d=0,$$

and the separation of the two components equals half the sum of their focal lengths

$$d=\frac{1}{2}(f_1+f_2). \tag{8}$$

But the second component of the telephoto system is a negative lens, therefore to obtain the telephoto effect it must be assured that $f_1 > |f_2|$ because of $f_2 < 0$.

If the principal planes of the telephoto lens do not coincide with each other for different wavelengths, then the images formed in the final image plane will not be free from the chromatic aberration, even when the condition (8) is satisfied. In general, the wave fronts corresponding to different wavelengths will have greater or smaller mean curvature than the curvature of Gaussian reference sphere $U'$ (see Fig. 3).

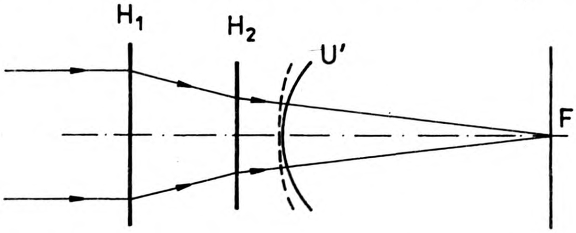

Fig. 3. Aberrated wave front introduced by a longitudinal displacement of the principal plane for other wavelengths

Perfect achromatism would exist if it were possible to cause wave fronts corresponding to different wavelengths to coincide absolutely with $U'$. We must therefore consider the expressions (3), and analyse the dependence of the image distance $z_I$ on the wavelength of reconstructing light. By differentiating the distances described by Eq. (3) with respect to the wavelength $\lambda$, we have:

$$\begin{aligned} \frac{\partial f_1(\lambda)}{\partial \lambda} &= -f_1^2 \frac{\partial}{\partial \lambda}\left(\frac{1}{f_1}\right), \\ \frac{\partial f_2(\lambda)}{\partial \lambda} &= -f_2^2 \frac{\partial}{\partial \lambda}\left(\frac{1}{f_2}\right), \\ \frac{\partial z_C}{\partial \lambda} = \frac{\partial f_1(\lambda)}{\partial \lambda} &= -\frac{f_1}{\lambda}. \end{aligned} \tag{9}$$

On the other hand, the derivative

$$\frac{\partial}{\partial \lambda}\left(\frac{1}{z_I}\right) = -\frac{1}{z_C^2}\frac{\partial z_C}{\partial \lambda} + \frac{1}{\lambda f_2} = \frac{1}{\lambda}\left(\frac{f_1}{z_C^2} + \frac{1}{f_2}\right).$$

If the derivative tends to zero, then the images formed in the back focal plane of telephoto system should be achromatic for all wavelengths of light within the bandwidth considered . Therefore, the achromatization condition for these two components of the system may be described by the equation

$$f_1 f_2 = -z_C^2. \tag{10}$$

It is evident that the focal lengths of the two components of the system have to be of opposite signs. From this point onward , such a configuration of separated doublet composed of positive and negative optical elements is free from longitudinal chromatic aberration.

## 4. Concluding remarks

In this paper, we have discussed the achromatic combination of two holographic optical elements constituting a telephoto lens. The aberrations were induced as a result of the change in wavelengths of the light used in the stage of holographic elements construction and reconstruction, respectively. If no attempt is made to compensate for these aberrations, the deviations from the Gaussian reference sphere can be quite severe, as presented in Fig. 3. We have shown that these aberrations can be reduced for a given change of wavelengths in a system containing a separated doublet of positive and negative holographic optical elements.

### References

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