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Analysis of luminous flux transfer through a conical ring-core light guide

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Analysis of luminous flux transfer through a conical ring-core light guide is presented. Three optical guides of this kind are the main elements of an original instrument for measurement of the luminance distribution in the field of view constructed by the authors. It was found that in the case of the output surface perpendicular to the symmetry axis of the cone having vertical angles greater than 50° the luminous flux is not transmitted through this kind of light guide. The conical output surface with great vertical angle considerably improves the process of transfering the luminous flux through a conical ring-core light guide. Replacing a flat outlet surface of the light guide by a conical one makes the transferred luminous flux more uniform to some extent.

Keywords: luminous flux, radiative transfer, photometry.

1. Introduction

The measuring instrument for the assessment of the luminance distribution in the field of view developed by the authors [1], [2] requires the use of ring-core light guides in the form of cones with appropriately selected vertical angle and numerical aperture. The properties of transferring the luminous flux by such a light guide are generally not known [3], [4]. Hence the need to carry out an analysis of the physical phenomena occurring in such an element.

In the discussion below the following simplifying assumptions have been made:

1. The following dimensions of a light guide of a step-index profile are known (Fig. 1): the length l, the diameter of the outlet front d, half of the vertical angle of the cone α, refractive indices of the core n_1 and cladding n_2.
2. The source of light illuminating from a great distance the inlet surface of the light guide is of small dimensions and it produces constant intensity of lighting E in the plane perpendicular to the direction of propagation.
3. The symmetry axis of the light guide coincides with the z axis of the rectangular system of coordinates with its origin at 0.
4. The dimensions of the core and the cladding are many times greater than the wavelength. The values of the refractive indices n_1 and n_2 are constant (also as

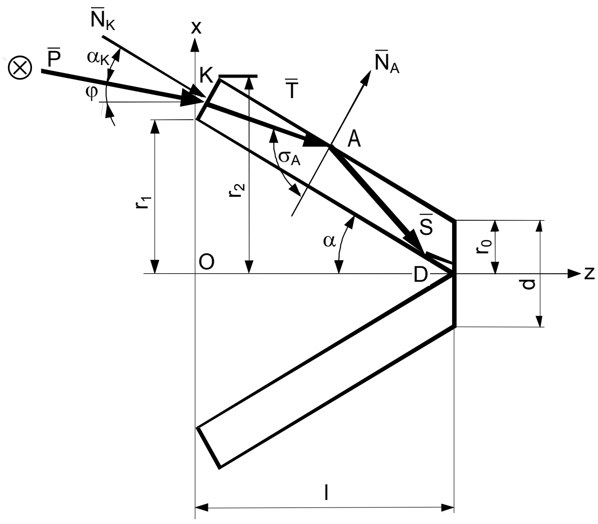

Fig. 1. Idea of the conical light guide with cylindrical core.

functions of wavelength). The losses caused by reflection between the core and the cladding and the losses caused by absorption of the luminous flux in the core of the light guide are neglected. The luminous flux penetrating from the core to the cladding is absorbed or leaves the guide through the side surface.

5. The elementary luminous flux is represented by a vector. The element of the surface interacting with this vector is flat both when the luminous flux passes through the core-cladding boundary and when the flux is reflected.

6. The elementary luminous fluxes, leaving the light guide are summed.

7. Because of the waveguide symmetry with respect to the plane *yz*, it is enough to analyse only the operation of half of the inlet surface.

2. Entrance of the luminous flux into the core

The inlet surface of the light guide was divided into k zones (rings) of equal width. The width s_{z1} of a single zone, measured in the plane xy, is equal to (Fig. 2):

$$s_{z1} = \frac{r_2 - r_1}{k_Z} = \frac{r_o \cos^2 \alpha}{k_Z} \tag{1}$$

where: $r_1 = l \tan \alpha$ – inner radius of the cone base, $r_2 = r_1 + r_o \cos^2 \alpha$ – outer radius of the cone base, k_Z – number of zones, α – half of the vertical angle of the conical light guide. The real width of the zone s_{z2} will be greater:

$$s_{z2} = \frac{s_{z1}}{\cos \alpha} = \frac{r_o \cos \alpha}{k}. \tag{2}$$

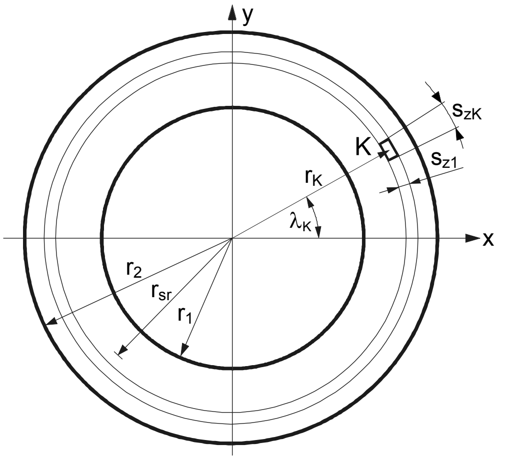

Fig. 2. Projection of the input surface of a light guide on the plane *xy*.

The mean radius r_{sr} of all the zones is equal to

$$r_{sr} = \frac{r_1 + r_2}{2} = r_1 + \frac{r_o}{2}\cos^2\alpha.$$

Half of the circumference of the circle with the radius r_{sr} was divided into such a number of *m* elements that the length of the elementary field did not differ greatly from its width

$$m = \mathrm{INT}\left(\frac{\pi r_{sr}}{s_{z2}} + 1\right). \tag{3}$$

It is convient to present the division of the front surface into elements in the polar coordinate system.

The radius r_k of the *k*-th zone is equal to:

$$r_K = r_1 + \frac{s_{z1}}{2} + (i-1)\,s_{z1} \qquad \text{for } i = 1 \text{ to } k, \tag{4}$$

$$\lambda_K = (j-1)s_{zK} + \frac{s_{zK}}{2}, \qquad \text{for } j = 1 \text{ to } m \tag{5}$$

where $s_{zK} = \pi/m$ denotes increment of the angle λ_K corresponding to the mean length of the elementary surface.

The coordinates of the point *K*, being the centre of the elementary surface in the system of the rectangular coordinates, are determined from the dependences:

$$x_K = r_K\cos\lambda_K, \quad y_K = r_K\sin\lambda_K, \quad z_K = (r_K - r_1)\tan\alpha. \tag{6}$$

The field of the elementary surface ΔS_K on which the point K is situated, is equal to

$$\Delta S_K = s_{z2} r_K s_{zK}. \tag{7}$$

The elementary luminous flux $\Delta \Phi_K$ hits the elementary surface at the angle α_K. This angle can be determined when the vectors of the incident flux **P** and the normal $\mathbf{N_K}$ to the element of the inlet surface are known: $\mathbf{p}(-\sin\varphi,\ 0,\ \cos\varphi)$ – unit vector, $\mathbf{N_K}(-\sin\alpha\cos\lambda_K,\ -\sin\alpha\sin\lambda_K,\ \cos\alpha)$ – unit vector, normal to the surface at the point K.

The cosine of the angle α_K between these vectors is equal to:

$$\cos\alpha_K = \cos(\mathbf{p}, \mathbf{N_K}) = \sin\varphi\sin\alpha\cos\lambda_K + \cos\varphi\cos\alpha. \tag{8}$$

Thus, the elementary luminous flux $\Delta\Phi_K = |\mathbf{P}|$, incident on the elementary surface is:

$$\Delta\Phi_K = E\cos\alpha_K\Delta S_K. \tag{9}$$

If the illuminance E is given in luxes and the field of the elementary surface in square metres, then the luminous flux will be determined in lumens.

Knowing the angle α_K, at which the luminous flux hits the inlet surface of the light guide, we can determine the refractive angle β_K. Since $\sin\beta_K = (\sin\alpha_K)/n_1$ hence $\beta_K = \arcsin[(\sin\alpha_K)/n_1]$.

Now, we can determine Fresnel's coefficient of reflection:

$$\rho_K = \frac{1}{2}\left[\frac{\sin^2(\alpha_K-\beta_K)}{\sin^2(\alpha_K+\beta_K)} + \frac{\tan^2(\alpha_K-\beta_K)}{\tan^2(\alpha_K+\beta_K)}\right]. \tag{10}$$

The elementary luminous flux $\Delta\Phi_T$, which will enter the core of the light guide is equal to:

$$\Delta\Phi_T = (1-\rho_K)\Delta\Phi_K. \tag{11}$$

3. Propagation of the luminous flux in the light guide

The vector equation of the refractive beam [2] has the form:

$$\mathbf{T} = \left(\frac{\mathbf{P}}{n_1} - \frac{|\mathbf{P}|\cos\alpha_K}{n_1}\cdot\mathbf{N_K} + |\mathbf{P}|\cos\beta_K\cdot\mathbf{N_K}\right)(1-\rho_K).$$

Thus the components of the vector **T**, representing the elementary luminous flux after refraction are as follows:

$$
\begin{aligned}
T_x &= \left[\frac{P_x}{n_1} - \left(\frac{|\mathbf{P}|\cos\alpha_K}{n_1} - |\mathbf{P}|\cos\beta_K\right)(-\sin\alpha\cos\lambda_K)\right](1-\rho_K),\\
T_y &= \left[-\left(\frac{|\mathbf{P}|\cos\alpha_K}{n_1} - |\mathbf{P}|\cos\beta_K\right)(-\sin\alpha\cos\lambda_K)\right](1-\rho_K),\\
T_z &= \left[\frac{P_z}{n_1} - \left(\frac{|\mathbf{P}|\cos\alpha_K}{n_1} - |\mathbf{P}|\cos\beta_K\right)\cos\alpha\right](1-\rho_K).
\end{aligned} \tag{12}
$$

3.1. Case A

The vector **T**, running from the point K, meets the outer surface of the conical light guide at the point A (Fig. 1). In order to determine the position of this point the system of equations must be solved:

i) $\dfrac{x_A - x_K}{T_x} = \dfrac{y_A - y_K}{T_y} = \dfrac{z_A - z_K}{T_z}$ – equations for the straight line passing through the point K and parallel to the vector **T**,

ii) $x^2 + y^2 + \left[(l-z)\tan\alpha + r_o\right]^2$ – equation of the cone (outer surface).

As a result of substitution a quadratic equation is obtained:

$$
z_A^2\left(\frac{T_x^2}{T_z^2} + \frac{T_y^2}{T_z^2} - \tan^2\alpha\right) + z_A \cdot 2\left[(r_o + r_1)\tan\alpha + \left(x_K - \frac{T_x}{T_z}z_K\right)\frac{T_x}{T_z} + \left(y_K - \frac{T_y}{T_z}z_K\right)\frac{T_y}{T_z}\right]
$$

$$
+ \left(x_K - \frac{T_x}{T_z}z_K\right)^2 + \left(y_K - \frac{T_x}{T_z}z_K\right)^2 - (r_o + r_1)^2 = 0 \tag{13}
$$

from which z_A is determined, and next the other coordinates of the point A:

$$
\begin{aligned}
x_A &= \frac{T_x}{T_z}(z_A - z_K) + x_K,\\
y_A &= \frac{T_y}{T_z}(z_A - z_K) + y_K.
\end{aligned} \tag{14}
$$

Point A lies in the plane $z = z_A$, at the angular distance λ_A with respect to the axis x

$$\lambda_A = \arctan \frac{y_A}{x_A}. \tag{15}$$

The vector $\mathbf{N_A}$ (unit) normal to the outer cone surface at the point A is as follows:

$$\mathbf{N_A}(\cos\lambda_A \cos\alpha,\ \sin\lambda_A \cos\alpha,\ \sin\alpha).$$

On this basis we can determine the angle of incidence σ_A of the beam $\mathbf{T}$ on the boundary of the cone:

$$\cos\sigma_A = \cos(\mathbf{T}, \mathbf{N_A}) = \frac{T_x \cos\lambda_A \cos\alpha + T_y \sin\lambda_A \cos\alpha + T_z \sin\alpha}{\sqrt{T_x^2 + T_y^2 + T_z^2}}.$$

Knowing the angle σ_A we can judge from the following condition whether at the point under consideration the total internal reflection of the luminous flux occurs:

$$\frac{n_1}{n_2} \sin\sigma_A > 1. \tag{16}$$

If condition (16) is not satisfied, the luminous flux penetrates into the cladding and (according to the assumptions) is absorbed; otherwise, the light beam $\mathbf{S}$ is reflected. The vector equation of the reflected luminous flux has the form [2]:

$$\mathbf{S} = \mathbf{T} - 2\mathbf{N_A}|\mathbf{T}|\cos\sigma_A.$$

The components of the vector $\mathbf{S}$ are as follows:

$$\begin{aligned} S_x &= T_x - 2|\mathbf{T}|\cos\sigma_A \cos\lambda_A \cos\alpha, \\ S_y &= T_y - 2|\mathbf{T}|\cos\sigma_A \sin\lambda_A \cos\alpha, \\ S_z &= T_z - 2|\mathbf{T}|\cos\sigma_A \sin\alpha. \end{aligned} \tag{17}$$

The value of the reflected luminous flux $|\mathbf{S}|$ is equal to the incident flux $|\mathbf{T}|$ since the reflection is without loss.

Equations of the straight line passing through the point A and parallel to the vector $\mathbf{S}$

$$\frac{x_D - x_A}{S_x} = \frac{y_D - y_A}{S_y} = \frac{z_D - z_A}{S_z}$$

with the equation of the cone, describing the inner surface of the light guide core:

$$x_D^2 + y_D^2 = (l - z_D)^2 \tan^2\alpha,$$

form a system of equations, whose solution is reduced to quadratic equation:

$$z_D^2\left(\frac{S_x^2}{S_z^2} + \frac{S_y^2}{S_z^2} - \tan^2\alpha\right) + z_D \cdot 2\left[l\tan^2\alpha + \left(x_A - \frac{S_x}{S_z}z_A\right)\frac{S_x}{S_z} + \left(y_A - \frac{S_y}{S_z}z_A\right)\frac{S_y}{S_z}\right]$$

$$+ \left(x_A - \frac{S_x}{S_z}z_A\right)^2 + \left(y_A - \frac{S_y}{S_z}z_A\right)^2 - l^2\tan^2\alpha = 0. \qquad (18)$$

The coordinate z_D of the point D, determined in this way allows the other coordinates to be calculated:

$$\begin{aligned} x_D &= \frac{S_x}{S_z}(z_D - z_A) + x_A, \\ y_D &= \frac{S_y}{S_z}(z_D - z_A) + y_A. \end{aligned} \qquad (19)$$

If the coordinate z_D satisfies the inequality $z_D < l$ (where l is the length of the light guide), then the flux **S** falls on the inner surface of the light guide core, and in the opposite case – on the outlet surface of the guide. At the point D we must also determine the angle of incidence σ_D and make sure that the total internal reflection occurs. For this purpose the components of the normal vector $\mathbf{N_D}$ at the point D of the inner surface of the cone are determined $\mathbf{N_D}(-\cos\lambda_D\cos\alpha, -\sin\lambda_D\cos\alpha, -\sin\alpha)$ and, similarly as Eq. (15), $\lambda_D = \arctan(y_D/x_D)$.

The angle of incidence σ_D of the flux at the point D can be determined from the dependence:

$$\cos\sigma_D = \cos(\mathbf{S}, \mathbf{N_D}) = \frac{-S_x\cos\lambda_D\cos\alpha - S_y\sin\lambda_D\cos\alpha - S_z\sin\alpha}{\sqrt{S_x^2 + S_y^2 + S_z^2}}. \qquad (20)$$

Next, the condition of the reflection of the luminous flux from inner surface of the core is checked:

$$\frac{n_1}{n_2}\sin\sigma_D > 1.$$

If the above condition is satisfied, the elementary flux **S** will be reflected again from the cladding surface. In the opposite case it will be lost.

3.2. Case B

In the case of large values of the inclination angles φ of the incident ray **P**, the ray **T** (after refraction) may hit the point B of the inner surface of the core (Fig. 3). On the basis of the system of equations:

$$\frac{x_B - x_K}{T_x} = \frac{y_B - y_K}{T_y} = \frac{z_B - z_K}{T_z},$$

$$x_B^2 + y_B^2 = (l - z_B)^2 \tan^2\alpha$$

we can write the quadratic equation:

$$z_B^2\left(\frac{T_x^2}{T_z^2} + \frac{T_y^2}{T_z^2} - \tan^2\alpha\right) + z_B \cdot 2\left[l\tan^2\alpha + \left(x_K - \frac{T_x}{T_z}z_K\right)\frac{T_x}{T_z} + \left(y_K - \frac{T_y}{T_z}z_K\right)\frac{T_y}{T_z}\right]$$
$$+ \left(x_K - \frac{T_x}{T_z}z_K\right)^2 + \left(y_K - \frac{T_y}{T_z}z_K\right)^2 - l^2\tan^2\alpha = 0 \tag{21}$$

from which the coordinate z_B of the point B is determined and next the other coordinates:

$$x_B = \frac{T_x}{T_z}(z_B - z_K) + x_K, \quad y_B = \frac{T_y}{T_z}(z_B - z_K) + y_K.$$

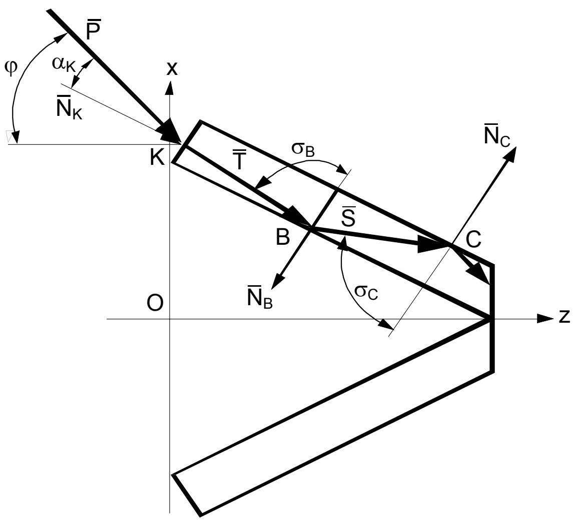

Fig. 3. Path of a light ray for great incidence angle φ.

If $z_D < l$, the angle of incidence σ_B must be determined in order to judge whether the internal reflections occur. The normal vector $\mathbf{N_B}$ at the point B has the components $\mathbf{N_B}(-\cos\lambda_B\cos\alpha, -\sin\lambda_B\cos\alpha, -\sin\alpha)$ and $\lambda_B = \arctan(y_B/x_B)$, thus

$$\cos\sigma_B = \frac{-T_x\cos\lambda_B\cos\alpha - T_y\sin\lambda_B\cos\alpha - T_z\sin\alpha}{\sqrt{T_x^2 + T_y^2 + T_z^2}}. \tag{22}$$

If the inequality $\dfrac{n_1}{n_2}\sin\sigma_B > 1$ is satisfied, the elementary flux is reflected at the point B.

The components of the vector of reflected $\mathbf{S}$ are as follows:

$$S_x = T_x - 2|\mathbf{T}|\cos\sigma_B\cos\lambda_B\cos\alpha,$$

$$S_y = T_y - 2|\mathbf{T}|\cos\sigma_B\sin\lambda_B\cos\alpha,$$

$$S_z = T_z - 2|\mathbf{T}|\cos\sigma_B\sin\alpha$$

with the vector $\mathbf{S}$ coincident with the straight line described by the equations:

$$\frac{x_C - x_B}{S_x} = \frac{y_C - y_B}{S_y} = \frac{z_C - z_B}{S_z}$$

which intersects the outer surface of the core $x_C^2 + y_C^2 = [(l - z_C)\tan\alpha + r_0]^2$, at the point C.

The solution of the above system of equations consists in determining the elements of the quadratic equation:

$$z_C^2\left(\frac{S_x^2}{S_z^2} + \frac{S_y^2}{S_z^2} - \tan^2\alpha\right) + z_C\cdot 2\left[(r_0 + r_1)\tan\alpha + \left(x_B - \frac{S_x}{S_z}z_B\right)\frac{S_x}{S_z} + \left(y_B - \frac{S_y}{S_z}z_B\right)\frac{S_y}{S_z}\right]$$

$$+\left(x_B - \frac{S_x}{S_z}z_B\right)^2 + \left(y_B - \frac{S_y}{S_z}z_B\right)^2 - (r_0 + r_1)^2 = 0. \tag{23}$$

Knowing the coordinate z_C we can calculate the other coordinates of the point C from the dependence:

$$x_C = \frac{S_x}{S_z}(z_C - z_B) + x_B, \qquad y_C = \frac{S_y}{S_z}(z_C - z_B) + y_B.$$

If the coordinate z_C satisfies the inequality $z_C < l$, the angle of incidence σ_C of the ray $\mathbf{S}$ at the point C is determined from the following relation:

$$\cos\sigma_C = \cos(\mathbf{S}, \mathbf{N_C}) = \frac{S_x \cos\lambda_C \cos\alpha + S_y \sin\lambda_C \cos\alpha + S_z \sin\alpha}{\sqrt{S_x^2 + S_y^2 + S_z^2}} \tag{24}$$

where: $\lambda_C = \arctan(y_C/x_C)$ – angular distance of the coordinate x of the point C in the plane $z = z_C$, $\mathbf{N_C}(\cos\lambda_C \cos\alpha,\ \sin\lambda_C \cos\alpha,\ \sin\alpha)$ – unit vector normal to the surface of the cone at point C.

If the inequality $\frac{n_1}{n_2}\sin\sigma_C > 1$ is satisfied, the total internal reflection occurs and the elementary luminous flux **S** under dismission becomes reflected again.

In this way, the phenomenon of reflection of the elementary luminous flux can occur many times along the light guide.

3.3. Case C

If the angle of incidence φ of the elementary light beam **T** is close to the angle α of the cone generator (Fig. 1) or coincides with it, then the luminous flux, after refraction, can hit directly the outlet surface of the light guide (Fig. 4). In this case, the coordinate z_B of the point B lies outside the exit surface of the cone ($z_B > l$), and the light beam hits this surface at the point W.

The conical light guide output surface can be hit by fluxes running directly from the point K (Fig. 4) and by fluxes refracted at arbitrary points (A, D, B, C) considered above, as well as at other points of repeated reflections. The components of the incident vector (**T** or **S**) enable the angle of incidence β_w to be determined:

$$\cos\beta_W = \frac{T_z}{\sqrt{T_x^2 + T_y^2 + T_z^2}}, \quad \text{or} \quad \cos\beta_W = \frac{S_z}{\sqrt{S_x^2 + S_y^2 + S_z^2}}. \tag{25}$$

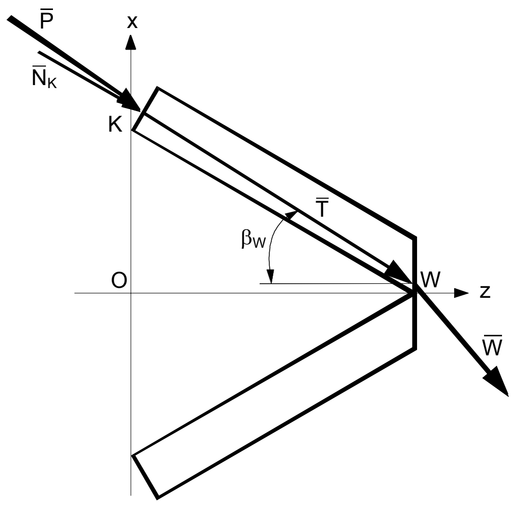

Fig. 4. Path of light beam when $\varphi \approx \alpha$.

If $n_1 \sin\beta_W > 1$, the inside reflection does not occur and the output angle α_W of the ray from the light guide is $\alpha_W = \arcsin(n_1 \sin\beta_W)$.

On the output surface there will take place Fresnel's reflection which is defined by the reflection coefficient ρ_W

$$\rho_W = \frac{1}{2}\left[\frac{\sin^2(\alpha_W - \beta_W)}{\sin^2(\alpha_W + \beta_W)} + \frac{\tan^2(\alpha_W - \beta_W)}{\tan^2(\alpha_W + \beta_W)}\right]. \tag{26}$$

The elementary luminous flux is as follows:

$$\Delta\Phi_W = |\mathbf{W}| = (1-\rho_W)\Delta\Phi_T = (1-\rho_K)(1-\rho_W)\,\Delta S_K\,E\cos\alpha_K. \tag{27}$$

Summing up all elementary luminous fluxes $\Delta\Phi_W$, we can determine the value of the luminous flux Φ_φ incident from the direction φ, which has passed through the light guide

$$\Phi_\varphi = 2\sum\Delta\Phi_W. \tag{28}$$

The sum obtained must be doubled, since only half of the inlet surface is analysed (item 7 of the adopted simplifying assumptions).

4. Results of calculations

In the calculations the following data were assumed: $l = 20$ mm – length of the conical cylindrical core light guide, $d = 4$ mm – diameter of the outlet surface of the light guide, $n_1 = 1.5400$ – refractive index of the core, $n_2 = 1.5181$ – refractive index of the cladding, $E = 1000$ lx – intensity of light.

The angle of inclination of the generator of the cone with respect to the symmetry axis of the light guide was changed in a step-like mode beginning from $\alpha_0 = 15°$ to 45°, every 5°. The entrance surface of the conical ring-core light guide was divided into $k = 14$ zones so that the width of the zone s_{z2} was from 0.101 mm at the angle 45° to 0.134 mm for the angle $\alpha_0 = 15°$, and the number of elementary light beams was from 794 948 to 239 232, respectively. The calculation results of the value of the luminous flux Φ_φ as a function of the angle φ is shown in Fig. 5. At the angle $\alpha_0 = 50°$ the luminous flux does not pass trough the light guide.

Computer simulation of a conical, ring-core light guide with steady inclination of the generator $\alpha_0 = 30°$ and changing diameter of the outlet front was also carried out. The diameter d_o do was changed step-like from 0.5 to 10 mm. The calculation results are shown in Fig. 6.

As follows from Fig. 5, at a high value of the angle α_0 (above 35°), the angle of incidence β_W of elementary beams on the output surface of the light guide is rather

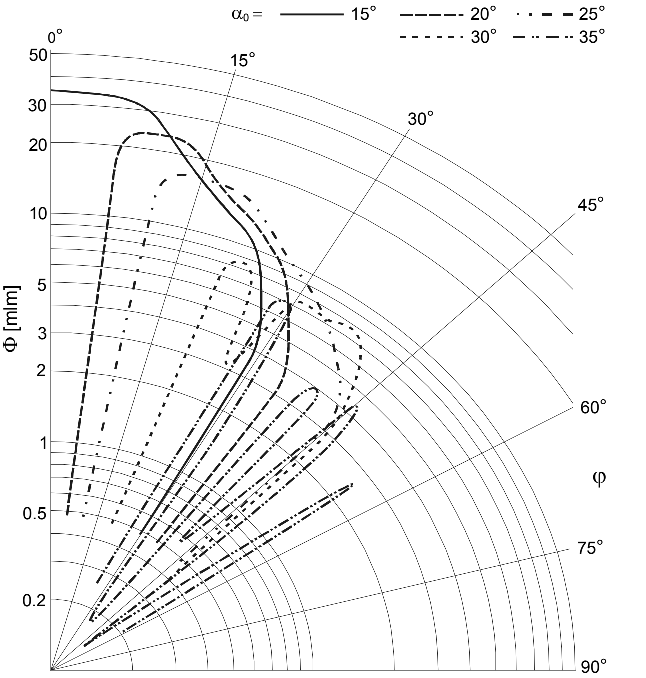

Fig. 5. Luminous flux Φ_{φ} as a function of the incidence angle φ for varying angle α_0 for flat output surface.

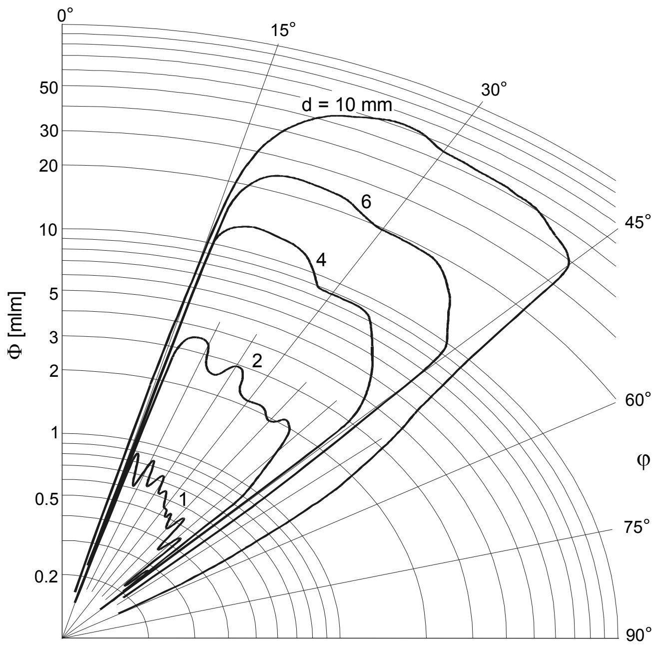

Fig. 6. Luminous flux Φ_{φ} as a function of the incidence angle φ for varying diameter d_o.

high and many beams are subjected to complete inner reflection. These elementary light beams do not leave the light guide.

In order to reduce the probability of complete inner reflection on the outlet surface of the light guide that surface in the form of a cone was considered (Fig. 7). In this way, the angles of incidence β_W of the beams hitting the outlet surface become considerably reduced and the probability of complete inner reflection is smaller. However, for this outlet shape, the vector **W** leaving the outlet surface, may hit it again. This was not taken into consideration.

The coordinates of the point W can be determined solving the system of equations: $(x_W - x)/S_x = (y_W - y)/S_y = (z_W - z)/S_z$ – equations defining the straight line passing through the point with the coordinates x, y, z, parallel to the vector **S**, $(x_W^2 + y_W^2)\tan^2\alpha_o = (z_W - l)^2$ – equation of the outlet surface of the light guide. Thus, we must calculate the roots of the quadratic equation

$$z_W^2\left[\left(\frac{S_x^2}{S_z^2} + \frac{S_y^2}{S_z^2}\right)\tan^2\alpha_o + 1\right] + z_W \cdot 2\left\{\left[\frac{S_x}{S_z}\left(x - \frac{S_x}{S_z}z\right) + \frac{S_y}{S_z}\left(y - \frac{S_y}{S_z}z\right)\right]\tan^2\alpha_o + l\right\}$$

$$+\left[\left(\frac{S_x}{S_z}z - x\right)^2 + \left(\frac{S_y}{S_z}z - y\right)^2\right]\tan^2\alpha_o - l^2 = 0. \tag{29}$$

Knowing the coordinate z_W the other coordinates can be determined from the dependence:

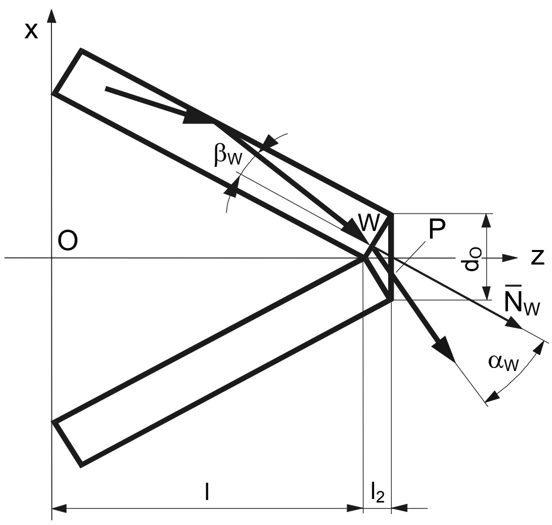

Fig. 7. Escape of an elementary luminous flux from the conical output surface.

$$x_W = \frac{S_x}{S_z}(z_W - z) + x, \quad y_W = \frac{S_y}{S_z}(z_W - z) + y. \tag{30}$$

At the point W we define the unit vector $\mathbf{N_W}$ normal to the outlet surface of the light guide: $\mathbf{N_W}(-\cos\alpha_o\cos\lambda_W, -\sin\alpha_o\cos\lambda_W, \cos\alpha_o)$ where $\lambda_W = \arctan(y_W/x_W)$.

The angle between the vectors $\mathbf{S}$ and $\mathbf{N_W}$ is the angle of incidence of the beam on the output surface of the light guide (β_W):

$$\cos\beta_W = \frac{-S_x\sin\alpha_o\cos\lambda_W - S_y\sin\alpha_o\sin\lambda_W + S_z\cos\alpha_o}{\sqrt{S_x^2 + S_y^2 + S_z^2}}.$$

If the inequality $n_1\sin\beta_W < 1$ is not satisfied, there takes place the total internal reflection on the output surface of the light guide.

Next, the angle α_W of the escape of the elementary light beam from the light guide $\alpha_W = \arcsin(n_1\sin\beta_W)$ and the coefficient ρ_W Fresnel's reflections occurring on the outlet surface (relation (26)) are determined. The components of vector $\mathbf{W}$ leaving the light guide are defined by the dependences:

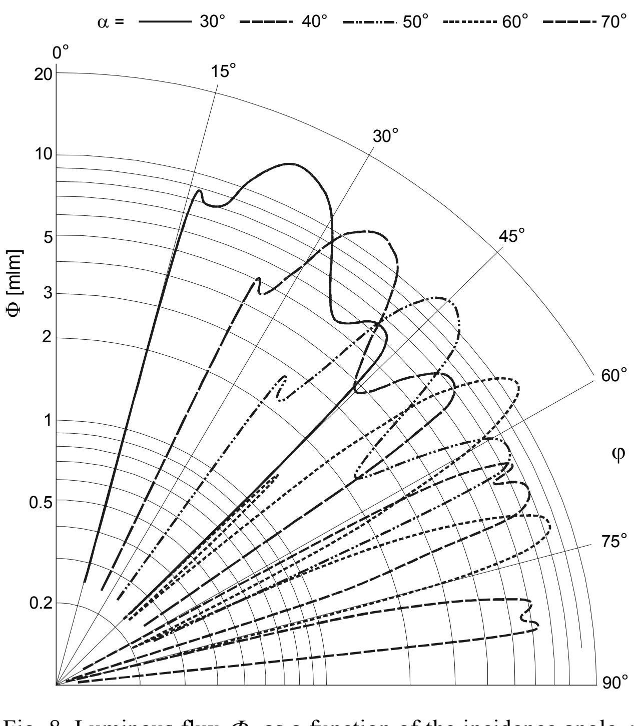

Fig. 8. Luminous flux Φ_φ as a function of the incidence angle φ for varying angle α for conical output surface.

$$W_x = \left[n_1 S_x + |\mathbf{S}|(n_1 \cos\beta_W - \cos\alpha_W)\sin\alpha_o \cos\lambda_W\right](1 - \rho_w),$$

$$W_y = \left[n_1 S_y + |\mathbf{S}|(n_1 \cos\beta_W - \cos\alpha_W)\sin\alpha_o \cos\lambda_W\right](1 - \rho_W),$$

$$W_z = \left[n_1 S_z - |\mathbf{S}|(n_1 \cos\beta_W - \cos\alpha_W)\cos\alpha_o\right](1 - \rho_W).$$

This vector intersects the outlet surface at the point P (Fig. 7) with the coordinates:

$$x_P = \frac{W_x}{W_z}(l + l_2 - z_W) + x_W, \qquad y_P = \frac{W_y}{W_z}(l + l_2 - z_W) + y_W.$$

Point P must lie on the surface of the circle with the diameter d_o, which is a condition of the escape of elementary luminous flux from the light guide:

$$\sqrt{x_P^2 + y_P^2} \le \frac{d_o}{2}.$$

The dependences (27) and (28) in this case are also valid.

The results of calculations of the values of the luminous flux Φ_φ, passing through the light guide with conical output surface, as a function of the angle of incidence φ are presented in Fig. 8.

5. Conclusions

This work can be sumarized as follows:

1. In the case of the outlet surface perpendicular to the symmetry axis of the cone having greater vertical angles than 50° the luminous flux does not pass the analysed light guide (Fig. 5). The output of conical shape with great vertical angle (90° – α) considerably improves the transfer of the luminous flux through a conical ring-core light guide (Fig. 8).

2. The luminous flux passing through the light guide increases with an the increase of the outlet diameter d since the outlet surface is greater.

3. Although a constant value lighting (1000 lx) in the plane perpendicular to the direction of the course of the luminous flux incident on the light guide has been assumed, the value of the luminous flux leaving the light guide is not steady and depends on the incidence angle φ. At greater vertical angles ($\alpha \ge 40°$) the value of the luminous flux decreases to 10% of the maximal value. Replacing a flat outlet surface of the light guide by a conical one makes the transferred luminous flux more uniform to some extent.

4. Comparing the results of calculations displayed in Figs. 5 and 8 shows that for conical entrance surface it is possible to use conical light guides with greater vertical angles.

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