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Reciprocity relations for light wave scattered by a particulate collection

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Two Fourier relations of light waves scattered by a random-distributed particulate medium have been investigated. We find that the scattered field and the particulate collection satisfy two Fourier relations, *i.e.* the spectral density is directly proportional to a Fourier transform of a convolution of correlation coefficient of each particle and correlation coefficient of distribution function of the whole collection, and the spectral degree of coherence is directly proportional to a Fourier transform of a convolution of strength of the scattering potential of each particle and strength of the distribution function of the whole collection. To illustrate these relations, behaviors of the far-field generated by Gaussian-correlated particles with Gaussian-correlated distributions have been discussed.

Keywords: light scattering, particulate collection, spectral density, spectral degree of coherence.

1. Introduction

As one of important methods to determine the structural information of an unknown object, the weak scattering theory is always a topic that has attracted much attention. To properly describe the characteristic of the scattering medium, a lot of models of scattering media were constructed, for example, the quasi-homogeneous medium [1–6], the anisotropic medium [7–9], the semisoft boundary medium [10–13], and the particulate medium [14–19]. It has been shown that there is some important structural information which can be obtained by measuring scattered field [20]. This phenomenon may provide a way to determine structural characteristic of an unknown object from the measurement of scattered field (see, for examples, [21–27]).

When we discuss light scattering, a model of quasi-homogeneous medium is usually considered. It is well-known that there are special Fourier relations between distribution of scattered field and characteristic of the medium, which is known as reciprocity relations [28]. These relations attracted much attention because they could provide available ways to measure structural information of scatterer. For example, XINYUE DU and DAOMU ZHAO discussed the reciprocity relations of an anisotropic medium [29], and

JIMING YU and JIA LI discussed the reciprocity relations of two incident beams which are generated by Young's pinhole [30]. Recently, the scattering behaviors of light wave from a particulate medium were discussed extensively. For example, the spectral degree of coherence of light wave on scattering from a particulate medium was discussed, and the particle-related coherence changes and the distribution-related coherence changes were investigated [31]; the scattering behavior of light wave from a mixed collection composed by different types was discussed, and it is shown that both the distribution characteristic of random-distributed particles and the location of determinate-distributed particles play a role in the behaviors of the far-zone scattered field [32]. In this manuscript, based on the random-distributed identical particles collection, the reciprocity relations between the scattered field and the particulate collection will be studied. To illustration this relations, an example to illustrate behaviors of light waves scattered from Gaussian-correlated particles with Gaussian-correlated distributions will be discussed.

2. Theory

As shown in Fig. 1, assume that a spatially coherent plane light wave, propagating in a direction of $\mathbf{s}_0$, is incident on a collection of particles. To analyze the statistical properties of incident field, we employ the cross-spectral density function located in two position vectors $\mathbf{r}'_1$ and $\mathbf{r}'_2$, which is defined as [33]

$$W^{(i)}(\mathbf{r}'_1, \mathbf{r}'_2, \mathbf{s}_0, \omega) = \langle U^{(i)*}(\mathbf{r}'_1, \mathbf{s}_0, \omega) U^{(i)}(\mathbf{r}'_2, \mathbf{s}_0, \omega)\rangle \tag{1}$$

where $*$ denotes complex conjugate, and $\langle\cdot\rangle$ denotes the ensemble average, with $U^{(i)}$ being the incident field, *i.e.*

$$U^{(i)}(\mathbf{r}', \mathbf{s}_0, \omega) = a(\omega)\exp(ik\mathbf{s}_0 \cdot \mathbf{r}') \tag{2}$$

where $a(\omega)$ is a random amplitude, and $k = \omega/c$ is wave number.

Assume that the scattering medium is random-distributed random particles, *i.e.* the scattering potential of each particle is a random function and the location of each par-

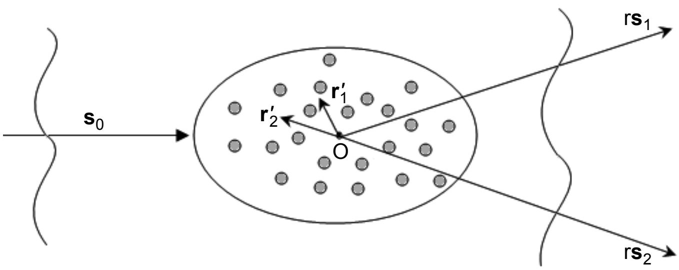

Fig. 1. Illustration of the notations.

ticle in the collection is also a random function. In the situation, characteristics of collection should be described by its correlation function, *i.e.* [24]

$$C_F(\mathbf{r}_1', \mathbf{r}_2', \omega) = \langle F^*(\mathbf{r}_1', \omega) F(\mathbf{r}_2', \omega) \rangle \tag{3}$$

where

$$F(\mathbf{r}', \omega) = \sum_n f(\mathbf{r}' - \mathbf{r}_n', \omega) \tag{4}$$

denotes scattering potential of collection with $\mathbf{r}_n'$ being the location vectors of the particles, and n denotes the sum of particles [19].

If the refractive index only slightly differs from unity, the magnitude of the scattered field may be smaller enough than the incident one. In this case, the scattering process could be analyzed within the first-order Born approximation [34]. Assume that the ensemble averages of that over the Fourier transform of particle's scattering potential and of that over the Fourier transform of the distribution function are independent. Then the far-zone cross-spectral density function at two location vectors $r\mathbf{s}_1$ and $r\mathbf{s}_2$ can be expressed as [31]

$$\begin{aligned} W^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = {} & \frac{S^{(i)}(\omega)}{r^2} \tilde{C}_f\Big(-k(\mathbf{s}_1 - \mathbf{s}_0), k(\mathbf{s}_2 - \mathbf{s}_0), \omega\Big) \\ & \times \tilde{C}_n\Big(-k(\mathbf{s}_1 - \mathbf{s}_0), k(\mathbf{s}_2 - \mathbf{s}_0), \omega\Big) \end{aligned} \tag{5}$$

where $S^{(i)}(\omega)$ is the spectrum of the incident field, and

$$\tilde{C}_f(\mathbf{K}_1, \mathbf{K}_2, \omega) = \iint_D C_f(\mathbf{r}_1', \mathbf{r}_2', \omega) \exp\Big[-i(\mathbf{K}_1 \cdot \mathbf{r}_1' + \mathbf{K}_2 \cdot \mathbf{r}_2')\Big] \mathrm{d}^3 r_1' \, \mathrm{d}^3 r_2' \tag{6}$$

and

$$\tilde{C}_n(\mathbf{K}_1, \mathbf{K}_2, \omega) = \iint_D C_n(\mathbf{r}_1', \mathbf{r}_2', \omega) \exp\Big[-i(\mathbf{K}_1 \cdot \mathbf{r}_1' + \mathbf{K}_2 \cdot \mathbf{r}_2')\Big] \mathrm{d}^3 r_1' \, \mathrm{d}^3 r_2' \tag{7}$$

are two spatially Fourier transforms of C_f and C_n, respectively, with

$$C_f(\mathbf{r}_1', \mathbf{r}_2', \omega) = \langle f^*(\mathbf{r}_1', \omega) f(\mathbf{r}_2', \omega) \rangle \tag{8}$$

and

$$C_n(\mathbf{r}_1', \mathbf{r}_2', \omega) = \langle \sum_m \sum_n \delta^*(\mathbf{r}_1' - \mathbf{r}_m', \omega) \delta(\mathbf{r}_2' - \mathbf{r}_n', \omega) \rangle \tag{9}$$

When the two unit vectors of scattering directions are the same (*i.e.*, $\mathbf{s}_1 = \mathbf{s}_2 = \mathbf{s}$), the spectral density in the far field can be given by the expression

$$S^{(s)}(r\mathbf{s}, \mathbf{s}_0, \omega) = \frac{S^{(i)}(\omega)}{r^2}\tilde{C}_f\Big(-k(\mathbf{s}-\mathbf{s}_0), k(\mathbf{s}-\mathbf{s}_0), \omega\Big)\tilde{C}_n\Big(-k(\mathbf{s}-\mathbf{s}_0), k(\mathbf{s}-\mathbf{s}_0), \omega\Big) \tag{10}$$

On the other hand, the spectral degree of coherence in the far field is defined as

$$\mu^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = \frac{W^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega)}{\sqrt{S^{(s)}(r\mathbf{s}_1, \mathbf{s}_0, \omega)}\sqrt{S^{(s)}(r\mathbf{s}_2, \mathbf{s}_0, \omega)}} \tag{11}$$

Then on employing Eqs. (5) and (10), the spectral degree of coherence defined by Eq. (11) can be rewritten as

$$\begin{aligned} &\mu^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) \\ &= \frac{\tilde{C}_f\Big(-k(\mathbf{s}_1-\mathbf{s}_0), k(\mathbf{s}_2-\mathbf{s}_0), \omega\Big)}{\sqrt{\tilde{C}_f\Big(-k(\mathbf{s}_1-\mathbf{s}_0), k(\mathbf{s}_1-\mathbf{s}_0), \omega\Big)}\sqrt{\tilde{C}_f\Big(-k(\mathbf{s}_2-\mathbf{s}_0), k(\mathbf{s}_2-\mathbf{s}_0), \omega\Big)}} \\ &\times \frac{\tilde{C}_n\Big(-k(\mathbf{s}_1-\mathbf{s}_0), k(\mathbf{s}_2-\mathbf{s}_0), \omega\Big)}{\sqrt{\tilde{C}_n\Big(-k(\mathbf{s}_1-\mathbf{s}_0), k(\mathbf{s}_1-\mathbf{s}_0), \omega\Big)}\sqrt{\tilde{C}_n\Big(-k(\mathbf{s}_2-\mathbf{s}_0), k(\mathbf{s}_2-\mathbf{s}_0), \omega\Big)}} \end{aligned} \tag{12}$$

In the following discussion, we will discuss Fourier relations between distributions of scattered field and properties of particulate collection. Hypothesize that all of particles in the collection are quasi-homogeneous, *i.e.*, its strength of the scattering potential $S_f(\mathbf{r})$ is a “slow” function of $\mathbf{r}$, whereas the normalized correlation coefficient of the scattering potential $\mu_f(\mathbf{r}')$ is a “fast” function of $\mathbf{r}'$ with $\mathbf{r}' = \mathbf{r}_2 - \mathbf{r}_1$ [28]. In this case, the correlation of scattering potential of the collection has a form [31]

$$C_f(\mathbf{r}'_1, \mathbf{r}'_2, \omega) = S_f\left(\frac{\mathbf{r}'_1+\mathbf{r}'_2}{2}, \omega\right)\mu_f(\mathbf{r}'_2-\mathbf{r}'_1, \omega) \tag{13}$$

where S_f and μ_f denote strength and normalized correlation coefficient of scattering potential of a particle, respectively. Moreover, assume that distribution functions of particles in the collection are also quasi-homogeneous, *i.e.*

$$C_n(\mathbf{r}'_1, \mathbf{r}'_2, \omega) = S_n\left(\frac{\mathbf{r}'_1+\mathbf{r}'_2}{2}, \omega\right)\mu_n(\mathbf{r}'_2-\mathbf{r}'_1, \omega) \tag{14}$$

where S_n and μ_n denote strength and normalized correlation coefficient of distribution function, respectively.

Substituting Eq. (13) into Eq. (6), after manipulating integration, one obtains that

$$\tilde{C}_f(\mathbf{K}_1, \mathbf{K}_2, \omega) = \tilde{S}_f(\mathbf{K}_1 + \mathbf{K}_2, \omega)\,\tilde{\mu}_f\left(\frac{\mathbf{K}_2 - \mathbf{K}_1}{2}, \omega\right) \tag{15}$$

where

$$\tilde{S}_f(\mathbf{K}, \omega) = \int_D S_f(\mathbf{r}, \omega)\exp(-i\mathbf{K}\cdot\mathbf{r})\mathrm{d}^3 r \tag{16}$$

and

$$\tilde{\mu}_f(\mathbf{K}', \omega) = \int_D \mu_f(\mathbf{r}', \omega)\exp(-i\mathbf{K}'\cdot\mathbf{r}')\mathrm{d}^3 r' \tag{17}$$

denote the Fourier transform of S_f and μ_f, respectively. Similarly, substituting Eq. (14) into Eq. (7), the Fourier transform of distribution function is given as

$$\tilde{C}_n(\mathbf{K}_1, \mathbf{K}_2, \omega) = \tilde{S}_n(\mathbf{K}_1 + \mathbf{K}_2, \omega)\,\tilde{\mu}_n\left(\frac{\mathbf{K}_2 - \mathbf{K}_1}{2}, \omega\right) \tag{18}$$

where $\tilde{S}_n$ and $\tilde{\mu}_n$ denote the Fourier transform of S_n and μ_n, respectively. Substituting Eqs. (15) and (18) into Eq. (10), one gets the spectral density in the far field as

$$S^{(s)}(r\mathbf{s}, \mathbf{s}_0, \omega) = \frac{S^{(i)}(\omega)}{r^2}\tilde{S}_f(0, \omega)\,\tilde{S}_n(0, \omega)\,\tilde{\mu}_f\Big(k(\mathbf{s} - \mathbf{s}_0), \omega\Big)\,\tilde{\mu}_n\Big(k(\mathbf{s} - \mathbf{s}_0), \omega\Big) \tag{19}$$

Next, on employing Eqs. (15) and (18), and after the rearrangement, the spectral degree of coherence given by Eq. (12) can be rewritten as

$$\mu^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = \frac{\tilde{S}_f\Big(k(\mathbf{s}_2 - \mathbf{s}_1), \omega\Big)\,\tilde{\mu}_f\left(k\left(\frac{\mathbf{s}_1 + \mathbf{s}_2}{2} - \mathbf{s}_0\right), \omega\right)}{\tilde{S}_f(0, \omega)\sqrt{\tilde{\mu}_f\Big(k(\mathbf{s}_1 - \mathbf{s}_0), \omega\Big)}\sqrt{\tilde{\mu}_f\Big(k(\mathbf{s}_2 - \mathbf{s}_0), \omega\Big)}} \times \frac{\tilde{S}_n\Big(k(\mathbf{s}_2 - \mathbf{s}_1), \omega\Big)\,\tilde{\mu}_n\left(k\left(\frac{\mathbf{s}_1 + \mathbf{s}_2}{2} - \mathbf{s}_0\right), \omega\right)}{\tilde{S}_n(0, \omega)\sqrt{\tilde{\mu}_n\Big(k(\mathbf{s}_1 - \mathbf{s}_0), \omega\Big)}\sqrt{\tilde{\mu}_n\Big(k(\mathbf{s}_2 - \mathbf{s}_0), \omega\Big)}} \tag{20}$$

For a quasi-homogeneous medium, $\tilde{\mu}_n$ should be a slow function of $k\mathbf{s}$. In this case, one can obtain the approximation relation [33], *i.e.*

$$\tilde{\mu}_i\Big(k(\mathbf{s}_1 - \mathbf{s}_0), \omega\Big) \approx \tilde{\mu}_i\Big(k(\mathbf{s}_2 - \mathbf{s}_0), \omega\Big) \approx \tilde{\mu}_i\left(k\left(\frac{\mathbf{s}_1 + \mathbf{s}_2}{2} - \mathbf{s}_0\right), \omega\right) \tag{21}$$

By employing the approximation relationship given by Eq. (21), the spectral degree of coherence can be obtained, *i.e.*

$$\mu^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = \frac{\tilde{S}_f\Big(k(\mathbf{s}_2 - \mathbf{s}_1), \omega\Big)}{\tilde{S}_f(0, \omega)} \frac{\tilde{S}_n\Big(k(\mathbf{s}_2 - \mathbf{s}_1), \omega\Big)}{\tilde{S}_n(0, \omega)} \tag{22}$$

Next, let us investigate the relations between the scattered field and the collection of particles. After some rearrangements, one finds spectral density denoted by Eq. (19) and spectral degree of coherence denoted by Eq. (22) can be presented as

$$S^{(s)}(r\mathbf{s}, \mathbf{s}_0, \omega) = \frac{S^{(i)}(\omega)}{r^2}\tilde{S}(0, \omega)\,\tilde{\mu}\Big(k(\mathbf{s} - \mathbf{s}_0), \omega\Big) \tag{23}$$

and

$$\mu^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = \frac{1}{\tilde{S}(0, \omega)}\tilde{S}\Big(k(\mathbf{s}_2 - \mathbf{s}_1), \omega\Big) \tag{24}$$

where $\tilde{\mu}$ is the Fourier transform of μ, with

$$\mu(\mathbf{r}_2' - \mathbf{r}_1', \omega) = \mu_f(\mathbf{r}_2' - \mathbf{r}_1', \omega) \otimes \mu_n(\mathbf{r}_2' - \mathbf{r}_1', \omega) \tag{25}$$

and $\tilde{S}$ is the Fourier transform of S, with

$$S\left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}, \omega\right) = S_f\left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}, \omega\right) \otimes S_n\left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}, \omega\right) \tag{26}$$

where $\otimes$ represents the convolution. Based on Eqs. (23) and (24), when light waves are scattered from a particulate collection, two new reciprocity relations can be found, which may be expressed as:

1) Spectral density produced by light waves incident on random-distributed random particles is directly proportional to the Fourier transform of a convolution of correlation coefficient of each particle and correlation coefficient of distribution function of whole collection;

2) Spectral degree of coherence produced by light waves incident on random-distributed random particles is directly proportional to the Fourier transform of a convolution of strength of scattering potential of each particle and strength of distribution function of whole collection.

3. Numerical results

As an example, let us further suppose that strength function and correlation coefficient, *i.e.* Eqs. (13) and (14), are Gaussian-centered [31], *i.e.*

$$S_f(\mathbf{r}, \omega) = A\exp\left(-\frac{\mathbf{r}^2}{2\sigma_{S_f}^2}\right) \tag{27a}$$

$$\mu_f(\mathbf{r}', \omega) = \exp\left(-\frac{\mathbf{r}'^2}{2\sigma_{\mu_f}^2}\right) \tag{27b}$$

and

$$S_n(\mathbf{r}, \omega) = B\exp\left(-\frac{\mathbf{r}^2}{2\sigma_{S_n}^2}\right) \tag{28a}$$

$$\mu_n(\mathbf{r}', \omega) = \exp\left(-\frac{\mathbf{r}'^2}{2\sigma_{\mu_n}^2}\right) \tag{28b}$$

where σ_{S_f} and σ_{μ_f} are effective width and effective correlation width of scattering potential of each particle, respectively, and σ_{S_n} and σ_{μ_n} are effective width and effective correlation width of distribution function, respectively. Substituting Eqs. (27) and (28) first into Eqs. (25) and (26), and then into Eqs. (23) and (24), after manipulating the Fourier transform, one finds the far-zone spectral density and far-zone spectral degree of coherence as

$$S^{(s)}(r\mathbf{s}, \mathbf{s}_0, \omega) = \frac{S^{(i)}(\omega)}{r^2}AB(2\pi)^6\sigma_{S_f}^3\sigma_{S_n}^3\sigma_{\mu_f}^3\sigma_{\mu_n}^3\exp\left[-\frac{(\sigma_{\mu_f}^2+\sigma_{\mu_n}^2)k^2}{2}(\mathbf{s}-\mathbf{s}_0)^2\right] \tag{29}$$

$$\mu^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = \exp\left[-\frac{(\sigma_{S_f}^2+\sigma_{S_n}^2)k^2}{2}(\mathbf{s}_2-\mathbf{s}_1)^2\right] \tag{30}$$

In what follows, some necessarily numerical results relating to scattered spectral density and scattered spectral degree of coherence will be presented to further illustrate the reciprocity relations. Figure 2 presents the normalized spectral density *vs*. the scattering angle θ. It should be noted that the scattering angle is the angle between the incident direction $\mathbf{s}_0$ and the scattering direction $\mathbf{s}$. Figure 2**a** plots spectral density with three different effective correlation widths of distribution functions, and Fig. 2**b** plots spectral density with three different effective correlation widths of scattering potential of particles. It is shown from Fig. 2 that both effective correlation width of scattering potential of particles and effective correlation width of distribution function of collection may affect the distribution of the far field. Figure 3 presents the degree of the

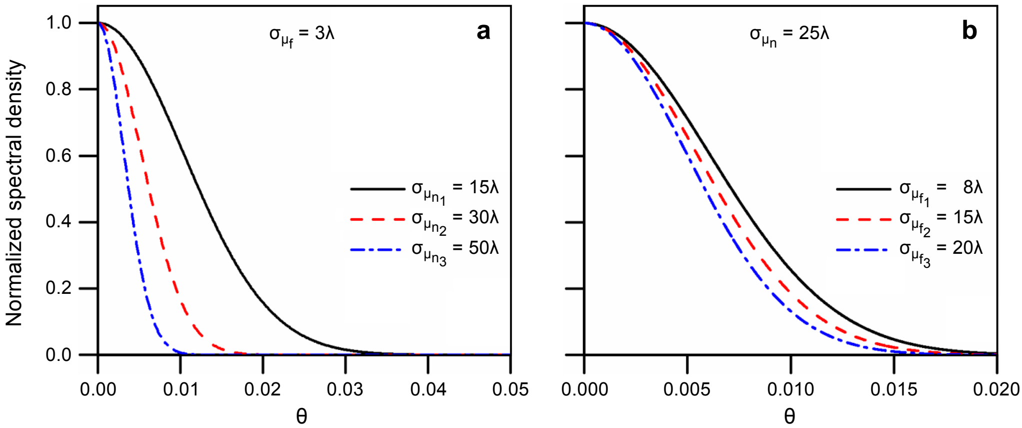

Fig. 2. Normalized spectral density generated by light waves scattered from three different collections with three different effective correlation widths of distribution functions (**a**) and three different effective correlation widths of scattering potentials of particles (**b**).

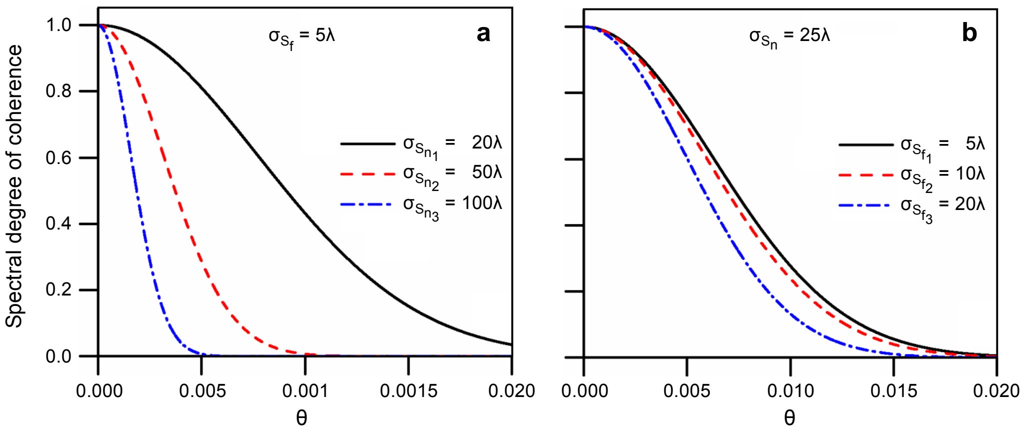

Fig. 3. Spectral degree of coherence generated by light waves scattered from three different collections with three different effective widths of distribution functions (**a**) and three different effective widths of scattering potentials of particles (**b**).

coherence in the far field. Figures 3**a** and 3**b** plot spectral degree of coherence with different effective widths of distribution functions and that with three different effective widths of scattering potentials of particles, respectively. It is shown from Figs. 3**a** and 3**b** that the spectral degree of coherence is affected by effective width of scattering potential of particles and by effective width of distribution function of collection.

4. Conclusions

In conclusion, new reciprocity relations relating to light waves incident on random-distributed random particles were discussed. We show that the spectral density in the far field is affected by correlation coefficient of scattering potential of each particle and correlation coefficient of particle's distribution function, while the spectral degree of coherence in the far field is affected by strength of scattering potential of particles and strength of particle's distribution function. These results may provide a simple way to determine the structural characteristics of a particulate medium from the measurements of the scattered field. Specifically, one can determine the density information of the scattering potentials of a particulate medium from the measurement of the spectral degree of coherence of the scattered field, and one can determine the correlation information of the scattering potentials of a particulate medium from the measurement of the spectral density of the scattered field.

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