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The polarization and coherence behavior of the flat-topped array beam through non-Kolmogorov turbulence

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In the present paper, investigation on the polarization and coherence fluctuations of a flat-topped array beam in non-Kolmogorov atmospheric optics links has been presented. For this purpose, the spectral degree of polarization and coherence at the receiver plane is analytically formulated via the extended Huygens–Fresnel integral and the unified theory of polarization and coherence. The influences of the laser beam parameters and the power law exponent that describes the non-Kolmogorov spectrum of the statistical propagation properties of a partially coherent flat-topped array laser beam has been studied in detail. For the employed parameters, it can be concluded that the increase in the structure constant of turbulence (which is equivalent to the increase in turbulence strength) leads to a fast reduction in the spectral degree of coherence. Moreover, when the power law exponent is 3.1, the spectral degree of coherence exhibits a minimum value in comparison with the Kolmogorov atmospheric turbulence.

Keywords: non-Kolmogorov turbulence, flat-topped array laser beam, polarization, coherence, intensity distribution.

1. Introduction

In recent years, the propagation of laser beams in turbulence as the free-space optical (FSO) communications has achieved a wide study due to the undeniable privileges of this communication system, which are supplying high-speed, large-capacity wireless network [1, 2]. Free-space optical system performance is severely degraded significantly due to the destructive turbulence effect on the optical wave propagation behavior through the beam propagation path. A great deal of attention has been paid by statistical optics scientists to the subject of the turbulence's effects on the propagation behavior of laser beams in various random media [1–5]. Traditionally, Kolmogorov turbulence spectrum model is widely used to predict the degradation of the optical wave propagation through atmosphere [3–5]. However, it is known that the atmosphere does not always behave in line with the Kolmogorov spectrum, but can also assume the non-Kolmogorov spectrum as well [6–10]. The new theoretical spectrum model known as non-Kolmogorov power spectrum by using the power law exponent α instead of a constant standard exponent value, 11/3, and a generalized amplitude factor instead of a constant value 0.033 has been suggested [10]. In FSO communication through the turbulent atmosphere, the propagation of laser beams and their stochastic parameters inevitably come upon. The beams' stochastic parameters are the average intensity, the beam width, the M^2 factor, the scintillation index, the degree of coherence and polarization, which are affected by turbulence factors and must be investigated to clarify the destructive effects of turbulence on the laser beams [1]. Analyzing the impact of non-Kolmogorov nature of the spectrum on beams propagation is worthy when one is dealing with optical communication. So far, based on the non-Kolmogorov power spectrum, considerable number of interesting investigations have been presented, such as second-order statistics of stochastic laser beams [11], beam propagation factor of beams [12–17], the scintillation behavior of laser beams [18], the average spreading, the field correlation, the angle of arrival fluctuations of laser beams and so on [19–23].

As it is mentioned above, the turbulent atmosphere can corrupt the FSO performance and, thus, finding a good solution to overcome the wrecking effect of turbulence and consequently attaining the higher feature of the communication factors in FSO systems is a delightful topic which has attracted theoretical and experimental researcher's attention [24–27]. Using the appropriate laser beam profile is one of the important strategies for improving the efficiency of the data modulation process in FSO communication systems. The flat topped laser beam has attracted the researchers' attention due to unique features of this intensity distribution. The most noticeable feature of flat-topped laser beams which is a uniform intensity distribution in the beam cross-sectional profiles is providing much better data modulation compared with conventional Gaussian laser beams; it has been found that a flat-topped beam has advantages over a Gaussian beam for overcoming turbulence-induced degradation [24, 25, 28]. In the literature, various techniques have been employed in optical communication systems to reduce the destructive effects of turbulence, the one of the significant methods is using the array laser beam instead of the single beam. The laser array beams have been commonly examined due to their advantages such as compactness, efficiency and reliability in high power systems. There is also a substantial interest in finding the benefits of using such arrays in free space optical communication links [29, 30].

Considering the importance of the non-Kolmogorov model as well as the effects of laser beams in the form of an array to compensate the destructive effects of atmospheric turbulence, and moreover, with a review of recent studies in this field it can be concluded that the investigation of the propagation properties of a partially coherent flat-topped (PCFT) array laser beams through non-Kolmogorov atmospheric turbulence have not been investigated in detail. Therefore, our motivation is to understand the effects of source and non-Kolmogorov atmospheric turbulence factors on the polarization and the coherence behavior of the mentioned beam by considering the non-Kolmogorov atmospheric turbulence. The current paper fulfils this specific mission. To achieve these goals, an analytical expression for cross-spectral density matrix elements (hence, average intensity) is calculated by using the extended Huygens–Fresnel integral. Then, based on the unified theory of polarization and coherence, the analytical expression for the spectral degree of polarization and the coherence of the PCFT array laser beam propagating through non-Kolmogorov atmospheric turbulence have been calculated.

Here, we focus on the effects of some source factors (such as laser wavelength λ, beam width w_0, *etc.*) and some atmospheric turbulence parameters (such as the structure constant of the turbulent atmosphere C_n^2, the power law exponent α, *etc.*) on the propagation behavior of PCFT array beam in the typical FSO communication link. The derivation of required formulas, simulations and, consequently, analyzing the results have been presented respectively in the next sections.

2. Propagation of partially coherent flat-topped array beam through non-Kolmogorov atmospheric turbulence

Using the appropriate laser beam profile is one of the important methods for dominating detrimental effects of turbulence on the optical communication link. A kind of beam profile which has been proven that is less affected by turbulence is a partially coherent flat-topped array beam. In the present paper, the propagation properties of PCFT array laser beam in a non-Kolmogorov turbulent are investigated. For this purpose, we assume that an array laser beam consists of n_t equal PCFT beamlets (namely, with equal power and source size), which are located symmetrically on a ring with a radius d and in the center of the ring (see Fig. 1).

The PCFT array laser beam has been assumed at the source plane. The laser beams propagate along the z axis in the Cartesian coordinate system. The optical field of the

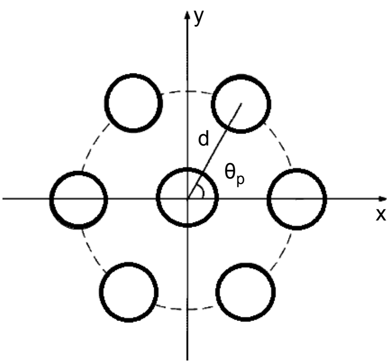

Fig. 1. The schematic layout of mentioned array beam.

flat-topped array laser beam by considering each beam's position in the source plane can be expressed as [29, 30]:

$$E_{\mathrm{p}}(\mathbf{r}', z=0) = \sum_{n=1}^{N} \frac{(-1)^{n-1}}{N} \binom{N}{n} \exp\left\{ \frac{-n\left[(x'-a_{\mathrm{p}})^2 + (y'-b_{\mathrm{p}})^2\right]}{w_0^2} \right\} \tag{1}$$

where w_0 is the beam waist of the Gaussian beams which make the source flat-topped beamlets, $\mathbf{r}' = (x', y')$ is the position vector of a point in the receiver plane, and indicates a two-dimensional transverse vector vertical to the direction of propagation, $\binom{N}{n}$ denotes the binomial coefficient, N is the order of the flatness, and (a_p, b_p) is the center of the p-th beamlet located at the source plane. The cross-spectral density in the space-frequency domain of the beam at the source plane ($z = 0$), can be defined as [25, 26, 29, 30]:

$$W_{ij}^0(\mathbf{r}_1', \mathbf{r}_2'; z=0) = \langle E(\mathbf{r}_1'; z=0)\, E^*(\mathbf{r}_2'; z=0)\, g_{ij}(|\mathbf{r}_1' - \mathbf{r}_2'|)\rangle \tag{2}$$

where $i, j = (x, y)$. The asterisk stands for the complex conjugate. The angle bracket represents the average taken over the ensemble of realizations of the electric field. In the sense of the coherence theory in the space-frequency domain, $g_{ij}(\mathbf{r}_1' - \mathbf{r}_2', 0)$ is the degree of the spatial coherence and may be interpreted as a correlation function at the points $\mathbf{r}_1'$ and $\mathbf{r}_2'$ at the source plane, and can be defined as [25, 26]

$$g_{ij}(\mathbf{r}_1' - \mathbf{r}_2', 0) = \frac{1}{N} \sum_{t=1}^{N} B_{ij} \exp\left[\frac{-t(\mathbf{r}_1' - \mathbf{r}_2')^2}{2\delta_{ij}^2} \right], \quad i, j = (x, y) \tag{3}$$

In the above equation, the complex degree of spatial coherence is considered as the summation of some Gaussian functions, which result in a Gaussian function with the maximum amount of unity. The δ_{ij} is the correlation length, a scale of distance in which two electric fields correlate with each other (effective width of the spectral degree of coherence); B_{ij} is the constant.

Using the extended Huygens–Fresnel principle, the cross-spectral density of the array beam in the observed plane can be determined by using the cross-spectral density in the source plane ($z = 0$), $W_{ij}^0(\mathbf{r}_1', \mathbf{r}_2', 0)$, as follows [1, 25, 26, 29, 30]:

$$W_{ij}^0(\mathbf{r}_1', \mathbf{r}_2', z; \omega) = \left(\frac{k}{2\pi z}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}^2\mathbf{r}_1' \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}^2\mathbf{r}_2'\, W_{ij}^0(\mathbf{r}_1', \mathbf{r}_2', z=0; \omega)$$

$$\times \exp\left\{ \frac{ik}{2z}(\mathbf{r}_1 - \mathbf{r}_1')^2 - \frac{ik}{2z}(\mathbf{r}_2 - \mathbf{r}_2')^2 \right\} \langle \exp[\psi(\mathbf{r}_1, \mathbf{r}_1', z) + \psi^*(\mathbf{r}_2, \mathbf{r}_2', z)]\rangle \tag{4}$$

where $\mathbf{r}_1 = (x_1, y_1)$ and $\mathbf{r}_2 = (x_2, y_2)$ denote the transverse coordinate at the receiving plane, ψ is a random part of the phase factor representing the effect of turbulence on the propagation of a spherical wave, $k = \omega/c$ is the wave number. The last term in the integrand of Eq. (4) for non-Kolmogorov atmospheric turbulence can be expressed as [1, 25, 26, 29, 30]:

$$\langle \exp[\psi(\mathbf{r}_1, \mathbf{r}_1', z) + \psi^*(\mathbf{r}_2, \mathbf{r}_2', z)]\rangle \cong \exp\left[-\frac{(\mathbf{r}_1-\mathbf{r}_2)^2 + (\mathbf{r}_1-\mathbf{r}_2)(\mathbf{r}_1'-\mathbf{r}_2') + (\mathbf{r}_1'-\mathbf{r}_2')^2}{\rho_0^2}\right] \tag{5}$$

where ρ_0 is the coherence length of the wave in non-Kolmogorov turbulent medium; ρ_0 is dependent on the power law exponent α and can be given as [28]:

$$\rho_0 = \left\{\frac{2^{-\alpha}\alpha\Gamma(\alpha-1)\Gamma(-\alpha/2)\sin[(\alpha-3)\pi/2]}{(\alpha-1)\Gamma(\alpha/2)}k^2\tilde{C}_n^2 L\right\}^{\frac{-1}{\alpha-2}}, \quad (3<\alpha<4) \tag{6}$$

where $\tilde{C}_n^2$ is the structure constant for the non-Kolmogorov turbulent medium. When $\alpha = 11/3$, the spatial power spectrum is yield to Kolmogorov spectrum. Equivalent structure constant $\tilde{C}_n^2$ can be written in terms of the structure constant in Kolmogorov turbulent medium C_n^2 as [28]:

$$\tilde{C}_n^2 = C_n^2\frac{0.5\Gamma(\alpha)(2\pi)^{-11/6+\alpha/2}(\lambda L)^{11/6-\alpha/2}}{\Gamma(1-\alpha/2)[\Gamma(\alpha/2)]^2\Gamma(\alpha-1)\cos(\alpha\pi/2)\sin(\alpha\pi/4)} \tag{7}$$

Substituting Eqs. (3), (5)–(7) into Eq. (4) and calculating the related integral, can lead up to:

$$W_{ij}(\mathbf{r}_1, \mathbf{r}_2, z; \omega) = \left(\frac{k}{2\pi z}\right)^2\sum_{q=1}^{n_t}\sum_{p=1}^{n_t}\sum_{t=1}^{N}\sum_{m=1}^{N}\sum_{n=1}^{N}A_iA_jB_{ij}\frac{(-1)^{n+m-2}}{N^3}\binom{N}{n}\binom{N}{m}$$
$$\times\frac{4\pi^2}{4\alpha_1\alpha_2-\alpha_7^2}\exp(\alpha_8+\alpha_9)\exp\left[-\frac{\alpha_2(\alpha_3^2+\alpha_4^2)+\alpha_1(\alpha_5^2+\alpha_6^2)-\alpha_7(\alpha_3\alpha_5+\alpha_4\alpha_6)}{4\alpha_1\alpha_2-\alpha_7^2}\right] \tag{8}$$

where

$$\alpha_1 = \frac{-n}{w_0^2} + \frac{ik}{2z} - \frac{t}{2\delta_{ij}^2} - \frac{1}{\rho_0^2} \tag{9a}$$

$$\alpha_2 = \frac{-m}{w_0^2} - \frac{ik}{2z} - \frac{t}{2\delta_{ij}^2} - \frac{1}{\rho_0^2} \tag{9b}$$

$$\alpha_3 = -\frac{ikx_1}{z} - \frac{x_1-x_2}{\rho_0^2} + \frac{2na_p}{w_0^2} \tag{9c}$$

$$\alpha_4 = -\frac{iky_1}{z} - \frac{y_1 - y_2}{\rho_0^2} + \frac{2nb_\mathrm{p}}{w_0^2} \tag{9d}$$

$$\alpha_5 = \frac{ikx_2}{z} + \frac{x_1 - x_2}{\rho_0^2} + \frac{2ma_\mathrm{q}}{w_0^2} \tag{9e}$$

$$\alpha_6 = \frac{iky_2}{z} + \frac{y_1 - y_2}{\rho_0^2} + \frac{2mb_\mathrm{q}}{w_0^2} \tag{9f}$$

$$\alpha_7 = \frac{2}{\rho_0^2} + \frac{t}{\delta_{ij}^2} \tag{9g}$$

$$\alpha_8 = \frac{ik(x_1^2 - x_2^2)}{z} - \frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{\rho_0^2} \tag{9h}$$

$$\alpha_9 = -\frac{n(a_\mathrm{p}^2 + b_\mathrm{p}^2) + m(a_\mathrm{q}^2 + b_\mathrm{q}^2)}{w_0^2} \tag{9i}$$

Investigation continued by formulating the propagation laws governing the propagation behavior of the average intensity of PCFT laser array beam fields in non-Kolmogorov turbulence. The average intensity (spectral density) distribution at the receiver plane can be calculated by the cross-spectral density matrix elements $W_{ij}(\mathbf{r}_1, \mathbf{r}_2, z; \lambda)$ as follows [1, 25, 26]:

$$\langle I(\mathbf{r}, z; \omega)\rangle = \mathrm{Tr}[W(\mathbf{r}, \mathbf{r}, z; \omega)] \tag{10}$$

where Tr denotes the trace of the cross-spectral density matrix.

The spectral degree of polarization based on the unified theory of coherence and polarization at the point $(\mathbf{r}, z)$ is given by the formula [31]

$$P(\mathbf{r}, z; \omega) = \sqrt{1 - \frac{4\mathrm{Det}[\mathbf{W}(\mathbf{r}, \mathbf{r}, z; \omega)]}{\{\mathrm{Tr}[\mathbf{W}(\mathbf{r}, \mathbf{r}, z; \omega)]\}^2}} \tag{11}$$

where Det denotes the determinant of the matrix $\mathbf{W}(\mathbf{r}, \mathbf{r}, z; \omega)$.

As it is well known, the spectral degree of coherence can be accounted with the help of 2×2 cross-spectral density matrix. The coherence properties of the beams can also be derived from the spectral degree of coherence of the electric field at a pair of field points $(\mathbf{r}_1, z)$ and $(\mathbf{r}_2, z)$ is given by [31]

$$\mu(\mathbf{r}_1, \mathbf{r}_2, z; \omega) = \frac{\mathrm{Tr}[\mathbf{W}(\mathbf{r}_1, \mathbf{r}_2, z; \omega)]}{\sqrt{\mathrm{Tr}[\mathbf{W}(\mathbf{r}_1, \mathbf{r}_1, z; \omega)]}\sqrt{\mathrm{Tr}[\mathbf{W}(\mathbf{r}_2, \mathbf{r}_2, z; \omega)]}} \tag{12}$$

Equations (8)–(12) serve as the basic formula for studying the changes in the spectral degree of polarization and coherence. The results of investigation based on the derived formula have been presented with the help of graphs in the next section.

3. Numerical calculations and analysis

As it is mentioned in the last section, the analytical formula for describing the intensity distribution, the spectral degree of polarization and the spectral degree of coherence of partially coherent flat-topped array beam propagating through atmospheric turbulence based on the elements of cross-spectral density matrix and the unified theory of coherence and polarization have been derived. The parameters of the source and atmospheric turbulence are taken to be fixed, except the parameter which is altered for investigation: $A_x = 0.5$, $A_y = 1.5$, $\lambda = 1550$ nm, $w_0 = 0.01$ m, $\delta_{ii} = \delta_{jj} = \delta_{ij} = \delta_{ji} = w_0/5$, $B_{ii} = B_{jj} = 1$, $\alpha = 3.3$, $B_{ij} = B_{ji}^* = 0.2\exp(-i\pi/3)$, $N = 3$, and $C_n^2 = 1 \times 10^{-13}$.

Figure 2 displays the schematic of a communication link through atmospheric turbulence, which contains the transmitter (PCFT array beam), propagation medium (atmosphere) and receiver. In the next step, we are interested in determining the key role of source and turbulence parameters on the intensity distribution, the beam width and M^2-factor of PCFT array beam. Due to the importance of propagation behavior of the intensity distribution along z-axis for analyzing the propagation behavior of PCFT ar-

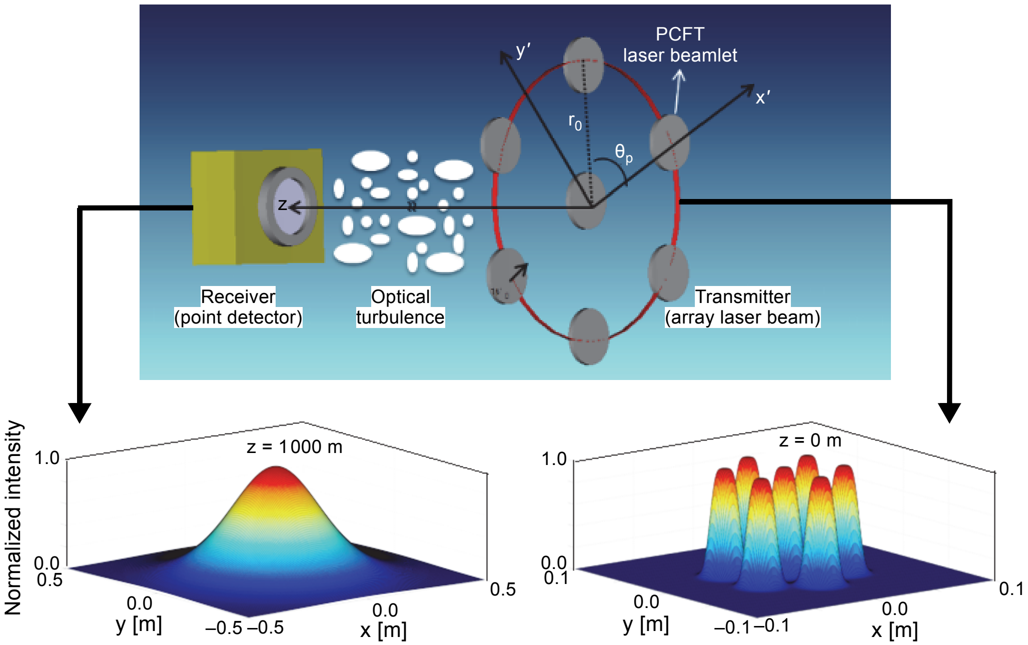

Fig. 2. The schematic layout of communication link which contains the transmitter (PCFT array beam), propagation medium (atmosphere) and receiver, along with intensity distributions in the transceiver.

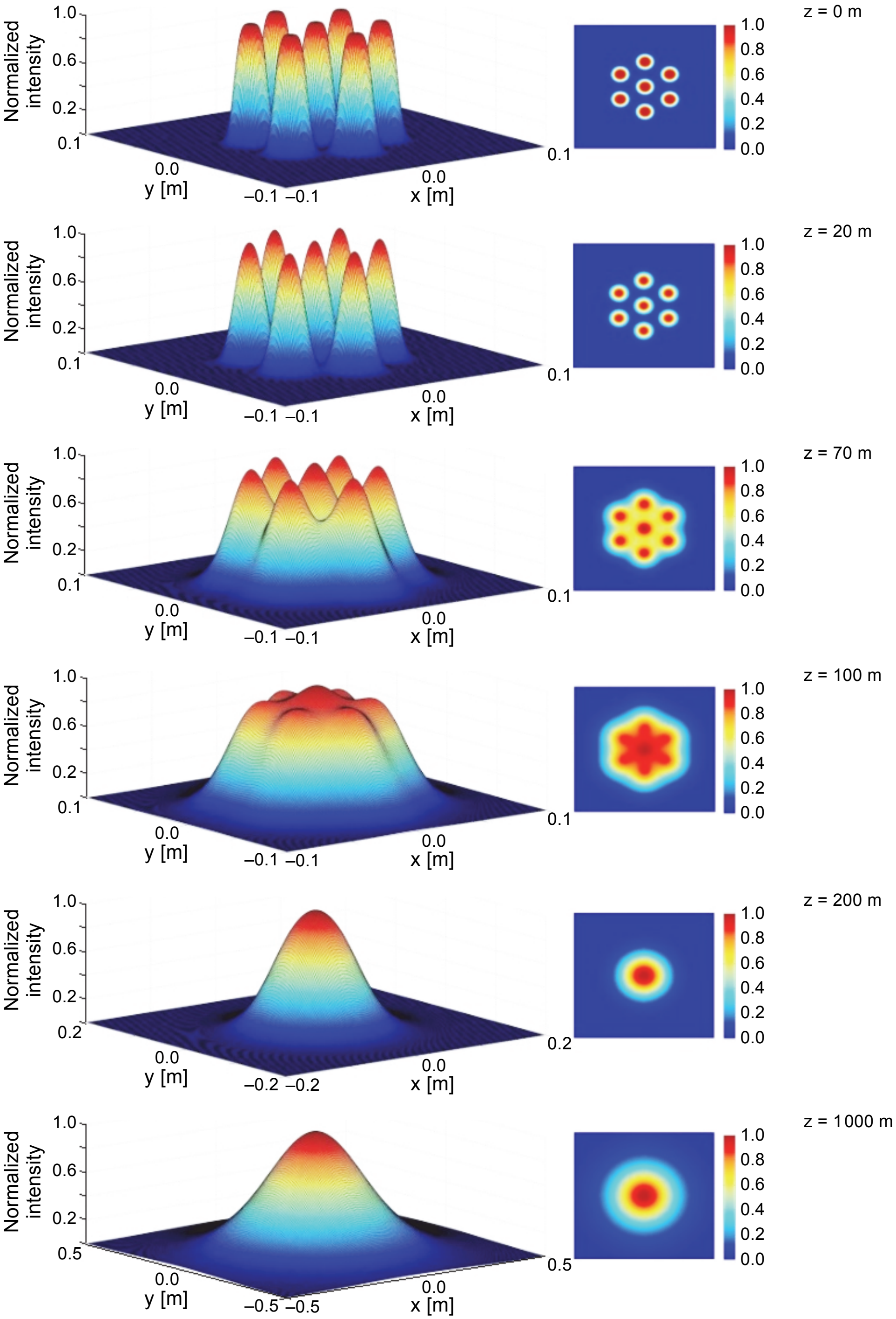

Fig. 3. Normalized 3D distribution of average intensity and corresponding contour-plot of PCFT array laser beam propagating through atmospheric turbulence at different propagation path lengths.

ray beam, Fig. 3 shows 3D intensity distribution of the PCFT array beam, at several propagation distances in atmospheric turbulence. As it is shown, the flat-topped beam will gradually spread and change its shape to the Gaussian beam as the propagation distance *z* increases. After sufficiently large distance, these Gaussian beams get wider by propagating through atmosphere and they overlap together and make up a single Gaussian beam.

The effect of power law exponent α on the propagation behavior of PCFT array beam is illustrated in Figs. 4 and 5. The investigation of the spectral degree of coherence *vs*. *x* (*x* is the distance from the center of the central beam) shows that higher power law exponent value α yields smaller amount of the spectral degree of coherence in the specific point. For the highest value of the power law exponent, the spectral degree of coherence quickly reduces. The oscillatory behavior of polarization can appear in different amount of power law exponent α through propagation path length. By exploring effects of the structure constant of turbulence C_n^2 on the spectral degree of coherence of PCFT array beam through turbulent atmosphere in Fig. 6, it can be concluded that the increase in the structure constant of turbulence C_n^2 (which is equivalent to the increase in turbulence strength) leads to fast reduction in the degree of spectral degree of coherence. Also, the Kolmogorov spectrum's effect is shown in the mentioned figure. In Fig. 7, the fluctuation of the spectral degree of polarization through propa-

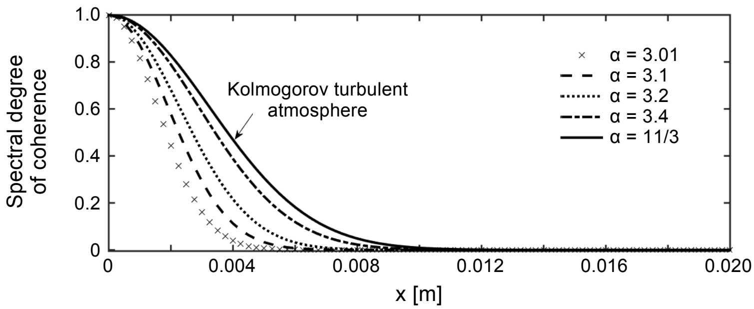

Fig. 4. The spectral degree of coherence of PCFT array laser beam propagating through atmospheric turbulence at different exponent parameter α.

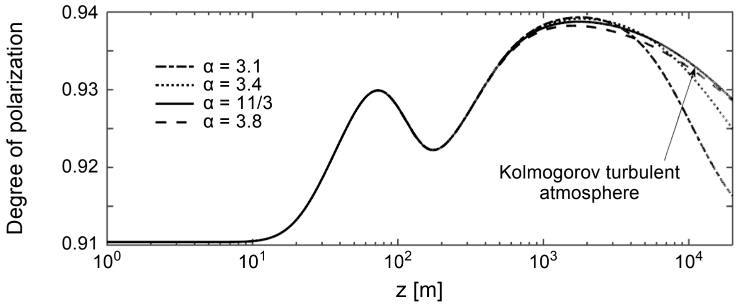

Fig. 5. The degree of polarization of PCFT array laser beam propagating through atmospheric turbulence at different exponent parameter α.

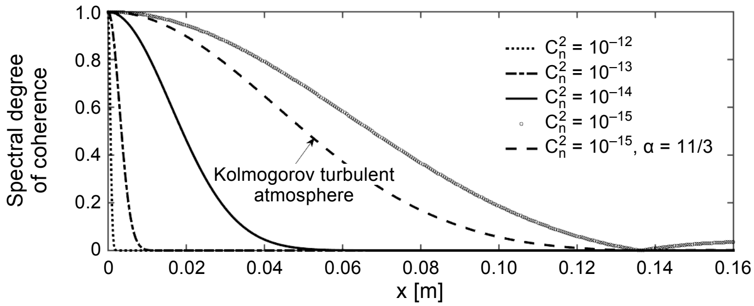

Fig. 6. The spectral degree of coherence of PCFT array laser beam propagating through atmospheric turbulence at different structure constant of turbulence C_n^2.

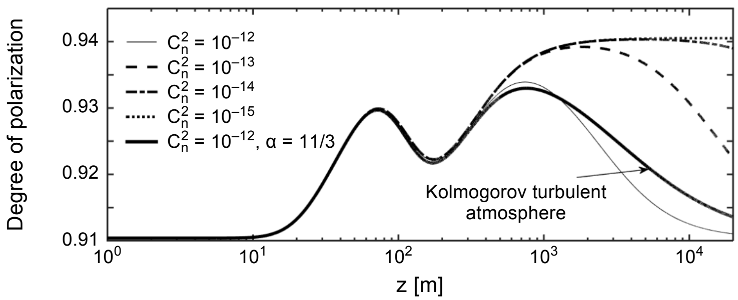

Fig. 7. The degree of polarization of PCFT array laser beam propagating through atmospheric turbulence at different structure constant of turbulence C_n^2.

gation path represents the significant impact of the strength of turbulence on this factor under the circumstances of the different non-Kolmogorov and Kolmogorov turbulent atmosphere. The quick fluctuation of the spectral degree of polarization for the large value of structure constant of turbulence C_n^2 is shown in Fig. 7 as well as the spectral degree of coherence.

Investigation on the influence of laser wavelength λ on propagation behavior of PCFT array beam has been shown in Figs. 8 and 9. This study has clearly demonstrated

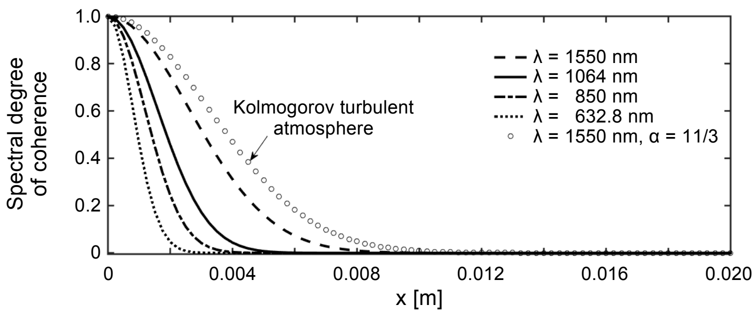

Fig. 8. The spectral degree of coherence of PCFT array laser beam propagating through atmospheric turbulence at different wavelength λ.

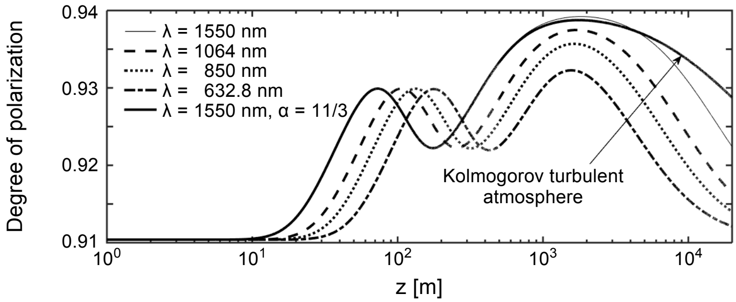

Fig. 9. The degree of polarization of PCFT array laser beam propagating through atmospheric turbulence at different wavelength λ.

that, as the wavelength increases, the spectral degree of coherence of considered beam slowly decreases in comparison with small wavelength. Figure shows that in the specific wavelength, the spectral degree of coherence in the Kolmogorov turbulence condition reduces slower than the non-Kolmogorov turbulence while the amount of the power law exponent α is smaller than 11/3 ($\alpha = 11/3$ is corresponding to the Kolmogorov turbulence). Also, the degree of polarization has been investigated in Fig. 9. By investigating oscillatory behavior of the degree of polarization for different wavelength, it

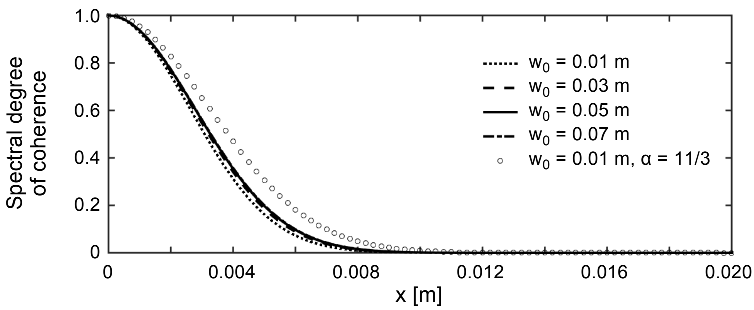

Fig. 10. The spectral degree of coherence of PCFT array laser beam propagating through atmospheric turbulence by considering different beam waist w_0.

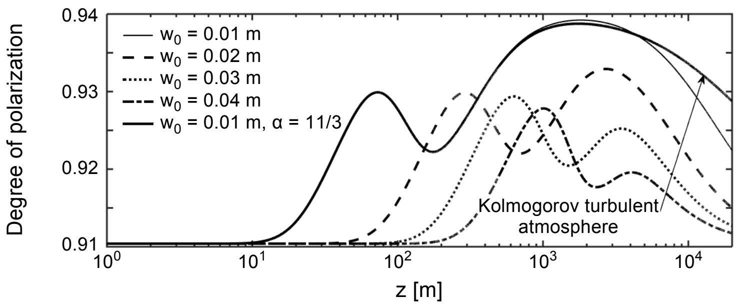

Fig. 11. The degree of polarization of PCFT array laser beam propagating through atmospheric turbulence by considering different beam waist w_0.

can be concluded that by increasing wavelength, the degree of polarization reaches the maximum value in shorter propagation path length. Furthermore, the Kolmogorov turbulence condition can be compared with the non-Kolmogorov turbulence due to the degree of polarization, which shows that the Kolmogorov turbulence fluctuates slower.

In Figs. 10 and 11, the effect of change in the beam waist w_0 on the spectral degree of coherence and the polarization of PCFT array beam have been demonstrated. It can be inferred that by increasing the beam waist, the spectral degree of coherence decreases slower. Investigation of the beam waist's effect on the polarization of PCFT array beam in the Fig. 11 has shown that the increase in the beam waist causes the degree of polarization to fluctuate at the long propagation length.

4. Conclusion

In summary, general analytical expressions for cross-spectral density and consequently the analytical formula for describing the spectral degree of coherence and the degree of polarization of the PCFT array beam have been derived. The spectral degree of coherence and polarization of the PCFT array beam propagating in turbulent atmosphere with different parameters of turbulence and laser beam have been investigated. For the highest value of the power law exponent α the spectral degree of coherence quickly reduces. It can be concluded that the increase in the structure constant of turbulence C_n^2 (which is equivalent to the increase in turbulence strength) leads to fast reduction in the degree of spectral degree of coherence and the quick fluctuation of the spectral degree of polarization. As the wavelength increases, dispersion and diffraction caused by eddies and turbulence decrease, and therefore the laser beam becomes less sensitive. Consequently, by investigating oscillatory behavior of the degree of polarization for different wavelength, it can be concluded that by increasing the wavelength, the degree of polarization reaches the maximum value in shorter propagation path length. Also, the spectral degree of coherence of considered beam slowly decreases in comparison with small wavelength. In the specific wavelength, the spectral degree of coherence in the Kolmogorov turbulence condition reduces slower than the non-Kolmogorov turbulence (while the amount of the power law exponent α is smaller than 11/3). By studying the effect of the change in the beam waist w_0 on the laser beam, it can be concluded that by increasing the beam waist w_0 the diffraction decreases and also leads to a smaller divergence angle. Consequently, it can be inferred that by increasing the beam waist, the spectral degree of coherence decreases slower.

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