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Analysis of the beam quality of a multi-Gaussian Schell-model vortex beam in atmospheric turbulence

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Analytical formulas of the angular width and propagation factor of a multi-Gaussian Schell-model vortex (MGSMV) beam through atmospheric turbulence are derived on the basis of the extended Huygens–Fresnel integral and second-order moments of Wigner distribution function. Evolution properties of the angular width and propagation factor of MGSMV beams propagating in atmospheric turbulence are investigated numerically. The results show that a multi-Gaussian Schell-model beam is more affected by atmospheric turbulence than a MGSMV beam, which will be useful for the practical application of the MGSMV beam.

Keywords: multi-Gaussian Schell-model vortex beams, propagation factor, turbulent atmosphere.

1. Introduction

Vortex beams with spiralling wavefronts and orbital angular momentum, have been widely used in applications such as high-resolution fluorescence microscopy [1], trapping of small particles and optical manipulation [2–5], quantum information [6, 7] and optical communications [8–10], *etc*. More attention has been paid to the research of vortex beams from both theoretical and applicative aspects [11–14]. SINGH and CHOWDHURY studied the propagation of the trajectory and shape of the noncanonical optical vortices by experiment [14]. GBUR and TYSON reported the propagation of vortex beams through weak-to-strong atmospheric turbulence with the method of multiple phase screen simulations and found that under certain conditions, the topological charge could act as information carrier for optical communications [13]. ZHU *et al*. presented a robust method to probe the topological charge of a vortex beam by use of dynamic angular double slits [15, 16]. The intensity distribution and the polarization of a vortex beam have been investigated [17]. Up to now, there have been a few papers devoted to the beam quality of vortex beams in turbulent atmosphere [18–20]. The research about the

Bessel–Gaussian beams and Gaussian Schell-model vortex beams have shown that vortex beams are better than non-vortex beams [18]. LI *et al.* performed the propagation properties of the partially coherent flat-topped vortex beams, and concluded that a beam with vortex is better than a beam without vortex in reducing the interference of atmospheric turbulence [20].

Recently, a novel vortex beam, *i.e.* the multi-Gaussian Schell-model vortex (MGSMV) beam, the focused beam profile of which could be formed by modulating the beam parameters, has been introduced [21, 22]. The purpose of the paper is to study the beam quality of a MGSMV beam in atmosphere. In this paper, analytical expression of the angular width and propagation factor of a MGSMV beam propagating in atmospheric turbulence are given and the dependences of the beam quality on the source and the turbulence properties are also discussed.

2. Theoretical formulation

The initial field distribution of a beam with optical vortex is expressed as [23]

$$U(\mathbf{s}, z=0) = u(\mathbf{s})\left[s_x + i\,\mathrm{sgn}(l)s_y\right]^{|l|} \tag{1}$$

where $\mathbf{s} \equiv (s_x, s_y)$ is a two-dimensional position vector, $U(\mathbf{s})$ stands for the profile of background beam envelope, sgn(·) means the sign function, and l denotes the topological charge, which indicates the vorticity of the beam.

It is assumed that $u(\mathbf{s})$ adopts a multi-Gaussian Schell-model (MGSM) form, and the cross-spectral density (CSD) for a MGSMV beam at $z = 0$ is given as [21]

$$W^{(0)}(\mathbf{s}_1, \mathbf{s}_2, 0) = \frac{1}{C_0}\left[s_{1x}s_{2x} + s_{1y}s_{2y} - i\,\mathrm{sgn}(l)s_{1x}s_{2y} - i\,\mathrm{sgn}(l)s_{2x}s_{1y}\right]^l$$
$$\times \exp\left(-\frac{\mathbf{s}_1^2 + \mathbf{s}_2^2}{w_0^2}\right)\sum_{m=1}^{N}\frac{(-1)^{m-1}}{m}\binom{N}{m}\exp\left[-\frac{(\mathbf{s}_1 - \mathbf{s}_2)^2}{2m\delta^2}\right] \tag{2}$$

where $\mathbf{s}_1$ and $\mathbf{s}_2$ specify two different points in the $z = 0$ plane, and

$$C_0 = \sum_{m=1}^{N}\frac{(-1)^{m-1}}{N}\binom{N}{m}$$

denotes the normalization factor, $\binom{N}{m}$ specifies a binomial coefficient, w_0 is the beam width, and δ is the transverse coherence width. In the following, the parameter l is restricted to be ±1.

Using the extended Huygens–Fresnel principle [24, 25], the CSD of a MGSMV beam in turbulence is written as

$$W(\boldsymbol{\rho}, \boldsymbol{\rho}_d, z) = \left(\frac{k}{2\pi z}\right)^2 \iint W^{(0)}(\mathbf{s}, \mathbf{s}_d, 0) \times \exp\left[\frac{ik}{z}(\boldsymbol{\rho} - \mathbf{s})(\boldsymbol{\rho}_d - \mathbf{s}_d) - H(\boldsymbol{\rho}_d, \mathbf{s}_d, z)\right] d^2\mathbf{s}\, d^2\mathbf{s}_d \tag{3}$$

in which

$$\mathbf{s} = \frac{\mathbf{s}_1 + \mathbf{s}_2}{2} \tag{4a}$$

$$\mathbf{s}_d = \mathbf{s}_1 - \mathbf{s}_2 \tag{4b}$$

$$\boldsymbol{\rho} = \frac{\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2}{2} \tag{4c}$$

$$\boldsymbol{\rho}_d = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2 \tag{4d}$$

and

$$W^{(0)}(\mathbf{s}, \mathbf{s}_d, 0) = W^{(0)}(\mathbf{s}_1, \mathbf{s}_2, 0) = W^{(0)}\left(\mathbf{s} + \frac{\mathbf{s}_d}{2}, \mathbf{s} - \frac{\mathbf{s}_d}{2}, 0\right) \tag{5}$$

The parameter $H(\boldsymbol{\rho}_d, \mathbf{s}_d, z)$ denotes the intensity of the turbulence and is defined as [25, 26]

$$H(\boldsymbol{\rho}_d, \mathbf{s}_d, z) = 4\pi^2 k^2 z \int_0^1 d\xi \int_0^\infty \left[1 - J_0\left(\kappa\left|\mathbf{s}_d \xi + (1 - \xi)\boldsymbol{\rho}_d\right|\right)\right] \Phi_n(\kappa)\kappa d\kappa \tag{6}$$

where $J_0(\cdot)$ denotes the Bessel function of zero order, and $\Phi_n(\kappa)$ is the power spectrum of the refractive index fluctuations.

The CSD of MGSMV beams through turbulence in Eq. (3) can be expressed as another form [26, 27]

$$W(\boldsymbol{\rho}, \boldsymbol{\rho}_d, z) = \left(\frac{1}{2\pi}\right)^2 \iint W\left(\mathbf{s}', \boldsymbol{\rho}_d + \frac{z}{k}\boldsymbol{\kappa}_d, 0\right) \times \exp\left[-i\boldsymbol{\rho} \cdot \boldsymbol{\kappa}_d + i\mathbf{s}' \cdot \boldsymbol{\kappa}_d - H(\boldsymbol{\rho}_d, \mathbf{s}_d, z)\right] d^2\mathbf{s}'\, d^2\boldsymbol{\kappa}_d \tag{7}$$

The Wigner distribution function (WDF) of MGSMV beams for atmospheric propagation is expressed as [25, 26, 28]

$$h(\boldsymbol{\rho}, \boldsymbol{\theta}, z) = \left(\frac{k}{2\pi}\right)^2 \iint W(\boldsymbol{\rho}, \boldsymbol{\rho}_d, z) \exp(-ik\boldsymbol{\theta} \cdot \boldsymbol{\rho}_d)\, d^2\boldsymbol{\rho}_d \tag{8}$$

where the vector $\boldsymbol{\theta} = (\theta_x, \theta_y)$ denotes an angle of propagation.

Substituting Eq. (7) into (8), the formula of the WDF for the MGSMV beam for turbulent propagation is given as

$$h(\boldsymbol{\rho}, \boldsymbol{\theta}, z) = \frac{w_0^2 k^2}{16\pi^3} \frac{1}{C_0} \sum_{m=1}^{N} \frac{(-1)^{m-1}}{m} \binom{N}{m}$$
$$\times \iint \exp\left[-\frac{w_0^2 \boldsymbol{\kappa}_\mathrm{d}^2}{8} - ik\boldsymbol{\theta} \cdot \boldsymbol{\rho}_\mathrm{d} - i\boldsymbol{\rho} \cdot \boldsymbol{\kappa}_\mathrm{d} - \left(\frac{1}{2w_0^2} + \frac{1}{2m\delta^2}\right)\mathbf{s}_\mathrm{d}^2 - H(\boldsymbol{\rho}_\mathrm{d}, \mathbf{s}_\mathrm{d}, z)\right]$$
$$\times \frac{w_0^2}{2}\left[1 - \frac{w_0^2 \boldsymbol{\kappa}_\mathrm{d}^2}{8} - \frac{\mathbf{s}_\mathrm{d}^2}{4} \pm \frac{w_0^2}{4}(s_{\mathrm{d}y}\kappa_{\mathrm{d}x} - s_{\mathrm{d}x}\kappa_{\mathrm{d}y})\right]\mathrm{d}^2\boldsymbol{\kappa}_\mathrm{d}\,\mathrm{d}^2\boldsymbol{\rho}_\mathrm{d} \tag{9}$$

The moments of order $n_1 + n_2 + m_1 + m_2$ of the WDF for a three-dimensional beam is expressed as [25, 26]

$$\langle \rho_x^{n_1} \rho_y^{n_2} \theta_x^{m_1} \theta_y^{m_2} \rangle = \frac{1}{P} \iint \rho_x^{n_1} \rho_y^{n_2} \theta_x^{m_1} \theta_y^{m_2} h(\boldsymbol{\rho}, \boldsymbol{\theta}, z)\,\mathrm{d}^2\boldsymbol{\rho}\,\mathrm{d}^2\boldsymbol{\theta} \tag{10}$$

in which

$$P = \iint h(\boldsymbol{\rho}, \boldsymbol{\theta}, z)\,\mathrm{d}^2\boldsymbol{\rho}\,\mathrm{d}^2\boldsymbol{\theta} \tag{11}$$

By using the second moments of WDF, the propagation factor of laser beams is given as [19, 26]

$$M^2(z) = k\left(\langle \boldsymbol{\rho}^2 \rangle \langle \boldsymbol{\theta}^2 \rangle - \langle \boldsymbol{\rho} \cdot \boldsymbol{\theta} \rangle^2\right)^{1/2} \tag{12}$$

where

$$\langle \boldsymbol{\rho}^2 \rangle = \langle \rho_x^2 \rangle + \langle \rho_y^2 \rangle \tag{13}$$

$$\langle \boldsymbol{\theta}^2 \rangle = \langle \theta_x^2 \rangle + \langle \theta_y^2 \rangle \tag{14}$$

$$\langle \boldsymbol{\rho} \cdot \boldsymbol{\theta} \rangle = \langle \rho_x \theta_x \rangle + \langle \rho_y \theta_y \rangle \tag{15}$$

Substituting Eq. (9) in Eq. (10), we get

$$P = \frac{1}{C_0} \sum_{m=1}^{N} \frac{(-1)^{m-1}}{m} \binom{N}{m} \frac{\pi w_0^4}{4} \tag{16}$$

$$\langle \boldsymbol{\rho}^2 \rangle = \frac{1}{C_0} \sum_{m=1}^{N} \frac{(-1)^{m-1}}{m} \binom{N}{m} \left(w_0^2 + \frac{4z^2}{w_0^2 k^2} + \frac{2z^2}{m\delta^2 k^2} \right) + \frac{4z^2}{k^2} T \tag{17}$$

$$\langle \boldsymbol{\theta}^2 \rangle = \frac{1}{C_0} \sum_{m=1}^{N} \frac{(-1)^{m-1}}{m} \binom{N}{m} \left(\frac{4}{w_0^2 k^2} + \frac{2}{m\delta^2 k^2} \right) + \frac{12}{k^2} T \tag{18}$$

$$\langle \boldsymbol{\rho} \cdot \boldsymbol{\theta} \rangle = \frac{1}{C_0} \sum_{m=1}^{N} \frac{(-1)^{m-1}}{m} \binom{N}{m} \left(\frac{4z}{w_0^2 k^2} + \frac{2z}{m\delta^2 k^2} \right) + \frac{6z^2}{k^2} T \tag{19}$$

where

$$T = \frac{\pi^2 k^2 z}{3} \int_0^\infty \kappa^3 \Phi_n(\kappa) \mathrm{d}\kappa \tag{20}$$

is used to characterize the contribution of turbulent atmosphere, and it has the meaning of the strength of turbulent atmosphere [29].

Substituting Eqs. (17)–(19) into Eq. (12), we get

$$\begin{aligned} M^2(z) = k \Bigg\{ & \left[\frac{1}{C_0} \sum_{m=1}^{N} \frac{(-1)^{m-1}}{m} \binom{N}{m} \left(w_0^2 + \frac{4z^2}{w_0^2 k^2} + \frac{2z^2}{m\delta^2 k^2} \right) + \frac{4z^2}{k^2} T \right] \\ & \times \left[\frac{1}{C_0} \sum_{m=1}^{N} \frac{(-1)^{m-1}}{m} \binom{N}{m} \left(\frac{4}{w_0^2 k^2} + \frac{2}{m\delta^2 k^2} \right) + \frac{12}{k^2} T \right] \\ & - \left[\frac{1}{C_0} \sum_{m=1}^{N} \frac{(-1)^{m-1}}{m} \binom{N}{m} \left(\frac{4z}{w_0^2 k^2} + \frac{2z}{m\delta^2 k^2} \right) + \frac{6z}{k^2} T \right]^2 \Bigg\}^{1/2} \end{aligned} \tag{21}$$

Equation (21) can be reduced to the case of a GSMV beam in atmosphere when $N = 1$, which is in accordance with Eq. (25) of Ref. [19].

If $T = 0$, Eq. (21) is easily reduced to the result of the MGSMV beam through free space

$$M^2(z) = \left[\frac{1}{C_0} \sum_{m=1}^{N} \frac{(-1)^{m-1}}{m} \binom{N}{m} \left(4 + \frac{2w_0^2}{m\delta^2} \right) \right]^{1/2} \tag{22}$$

When $l = 0$, we get the expression of propagation factor of a MGSM beam through atmospheric turbulence

$$M^2(z) = k\left\{\left[\frac{1}{C_0}\sum_{m=1}^{N}\frac{(-1)^{m-1}}{m}\binom{N}{m}\left(\frac{w_0^2}{2}+\frac{2z^2}{w_0^2k^2}+\frac{2z^2}{m\delta^2k^2}\right)+\frac{4z^2}{k^2}T\right]\right.$$
$$\times\left[\frac{1}{C_0}\sum_{m=1}^{N}\frac{(-1)^{m-1}}{m}\binom{N}{m}\left(\frac{1}{w_0^2}+\frac{1}{m\delta^2}\right)\frac{2}{k^2}+\frac{12}{k^2}T\right]$$
$$\left.-\left[\frac{1}{C_0}\sum_{m=1}^{N}\frac{(-1)^{m-1}}{m}\binom{N}{m}\left(\frac{1}{w_0^2}+\frac{1}{m\delta^2}\right)\frac{2z}{k^2}+\frac{6z}{k^2}T\right]^2\right\}^{1/2} \tag{23}$$

Equation (23) can be reduced to the case of a GSM beam in atmosphere if $N = 1$, which is in accordance with Eq. (26) of Ref. [19].

The angular width of a MGSMV beam in atmospheric turbulence is written as [27, 30]

$$\boldsymbol{\theta}(z) = \left(\langle|\boldsymbol{\theta}-\langle\boldsymbol{\theta}\rangle|^2\rangle\right)^{1/2} = \left(\langle\boldsymbol{\theta}^2\rangle\right)^{1/2}$$
$$= \left[\frac{1}{C_0}\sum_{m=1}^{N}\frac{(-1)^{m-1}}{m}\binom{N}{m}\left(\frac{4}{w_0^2k^2}+\frac{2}{m\delta^2k^2}\right)+\frac{12}{k^2}T\right]^{1/2} \tag{24}$$

For $l = 0$, we get the expression of angular width of a MGSM beam through atmospheric turbulence

$$\boldsymbol{\theta}(z) = \left[\frac{1}{C_0}\sum_{m=1}^{N}\frac{(-1)^{m-1}}{m}\binom{N}{m}\left(\frac{2}{w_0^2k^2}+\frac{2}{m\delta^2k^2}\right)+\frac{12}{k^2}T\right]^{1/2} \tag{25}$$

3. Numerical calculations and analyses

For comparison, the normalized propagation factor ($M^2(z)/M^2(0)$) is used, and the Von Kármán spectrum is chosen and its analytical expression is as follows [31]:

$$\Phi_n(\kappa) \approx 0.033\,C_n^2\,\frac{\exp\left[-(\kappa^2/\kappa_m^2)\right]}{(\kappa^2+\kappa_0^2)^{11/6}}, \qquad 0 \le \kappa \le \infty \tag{26}$$

where $\kappa_0 = 1/L_0$ with L_0 being the outer scale of a turbulence, $\kappa_m = 5.92/l_0$ with l_0 being the inner scale of a turbulence, and C_n^2 is structure constant of a turbulence. For convenience of analysis, the calculation parameters are given as follows: $\delta = 1$ cm, $w_0 = 2$ cm, $L_0 = 10$ m, $l_0 = 0.01$ m, $\lambda = 632.8$ nm, $C_n^2 = 10^{-14}$ m$^{-2/3}$, $N = 2$, other parameters are stated in the relevant figures.

The normalized angular widths of a MGSMV beam and a MGSM beam for propagation in turbulent atmosphere are presented in Fig. 1. It is found that the normalized angular width of a MGSMV beam and a MGSM beam increase with the propagation distance, and the normalized angular width of a MGSM beam spreads faster than that of a MGSMV beam.

The normalized angular width of a MGSMV beam propagating in atmospheric turbulence with different beam order N and transverse coherence δ are depicted in Fig. 2. For a certain distance, the normalized angular width decreases as the beam order N

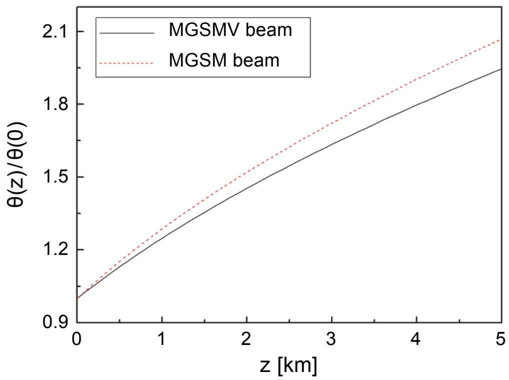

Fig. 1. Normalized angular widths of a MGSMV beam and a MGSM beam propagating in turbulent atmosphere.

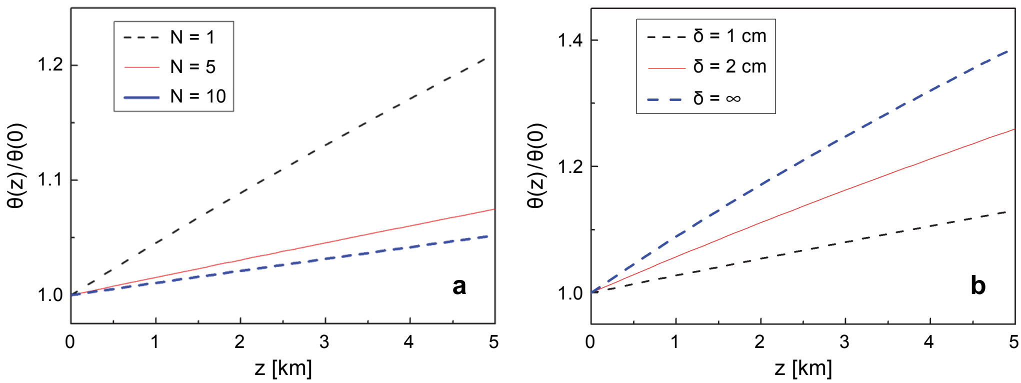

Fig. 2. Normalized angular width for a MGSMV beam for propagation in atmospheric turbulence with different beam order N (**a**), and spatial coherence δ (**b**).

becomes larger or the transverse coherence becomes smaller. The interaction of turbulence with the normalized angular width is greater for a MGSMV beam as the beam order N becomes smaller or the transverse coherence δ becomes larger.

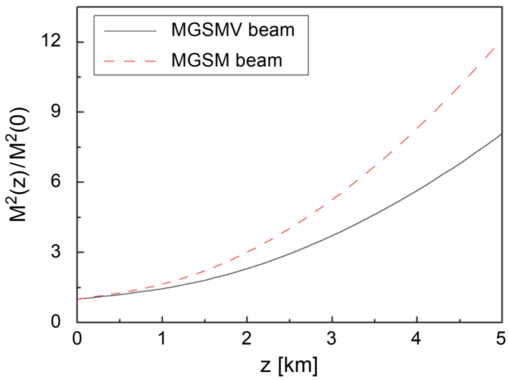

Fig. 3. Normalized propagation factors of a MGSMV beam and a MGSM beam propagating in atmospheric turbulence.

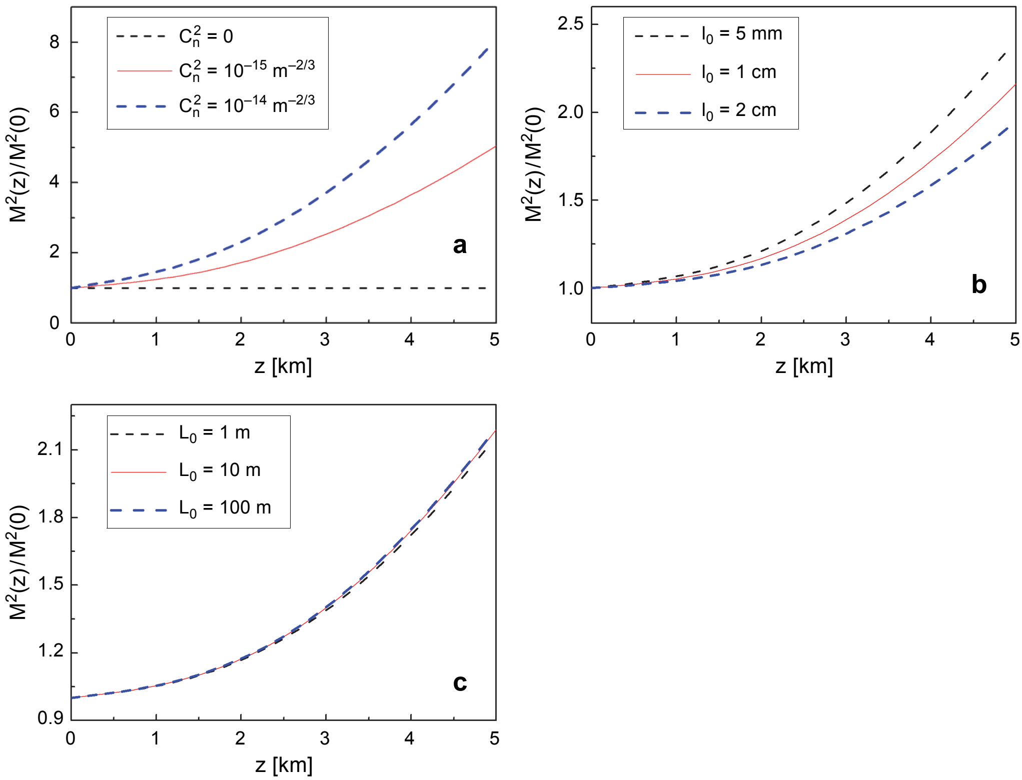

Fig. 4. Normalized propagation factor for a MGSMV beam propagating in atmospheric turbulence with different structure constants C_n^2 (**a**), inner scale l_0 (**b**), and outer scale L_0 (**c**).

The normalized propagation factors for a MGSM beam and a MGSMV beam for atmospheric propagation are given in Fig. 3. We can see from the figure that the normalized propagation factors of a MGSMV beam and MGSM beam increase with the propagation distance, and the normalized propagation factor of a MGSM beam spreads faster than that of a MGSMV beam, that is, the effect on the MGSMV beam is smaller than that of the MGSM beam, and thus the beam quality is better. It may be physically interpreted by the fact that the MGSMV beam contains screw wavefront dislocations, which may have the stronger ability to reduce the effect of atmospheric turbulence than the MGSM beam.

Figures 4**a**–4**c** present the normalized propagation factor of a MGSMV beam in atmospheric turbulence with different structure constant C_n^2, inner scale l_0, and outer scale L_0, respectively. Obviously, the normalized propagation factor for a MGSMV beam becomes larger when the structure constant is larger or the inner scale is smaller, which indicates that under these conditions, the beam quality is worse. It can also be found that the outer scale L_0 has little effect on the beam.

To analyze the relationship between the propagation factor of a MGSMV beam and its beam parameters, the normalized propagation factors for a MGSMV beam for prop-

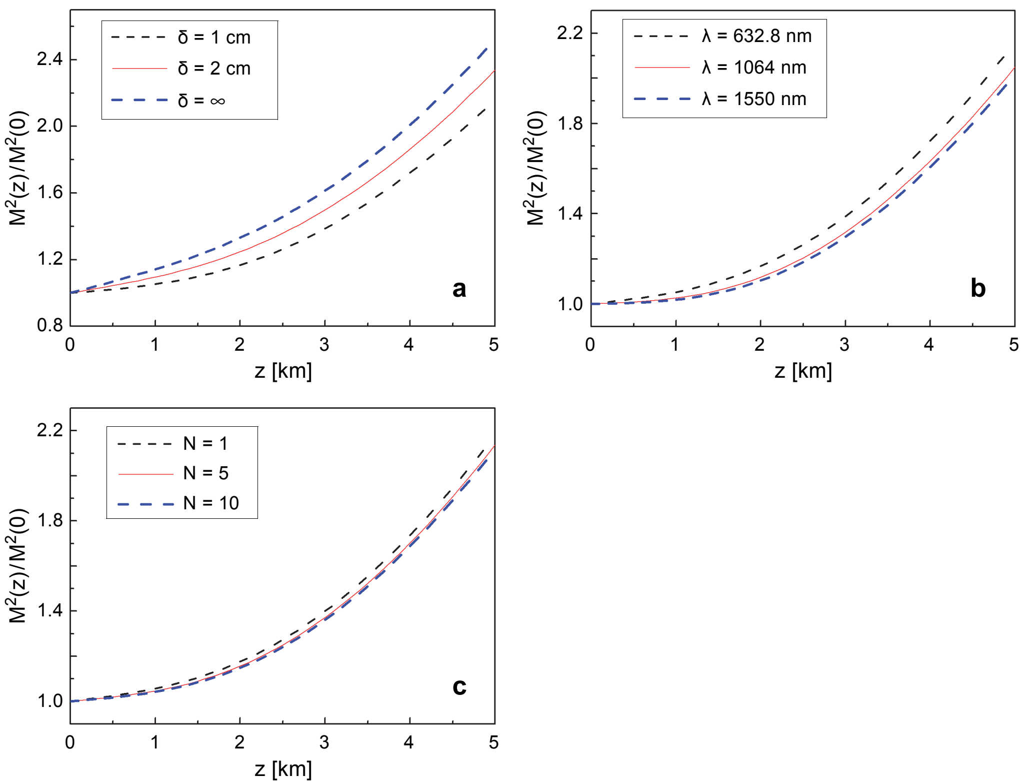

Fig. 5. Normalized propagation factor for a MGSMV beam propagating in turbulent atmosphere with different transverse coherence δ (**a**), the wavelength λ (**b**), and the beam order N (**c**).

agation with different transverse coherence δ, wavelength λ, beam order N are depicted in Fig. 5**a**–5**c**, respectively. It is found from Fig. 5 that the normalized propagation factor for a MGSMV beam spreads fast in turbulence as the transverse coherence δ is larger, or the wavelength λ, the beam order N is smaller. Therefore, it is vital to select appropriate beam parameters of a MGSMV beam for practical applications.

4. Conclusion

In summary, analytical formulas of the angular width and propagation factor of a MGSMV beam propagating in turbulent atmosphere are derived. The influence of turbulent atmosphere on the beam quality has been investigated. Within the range of parameters examined, it is shown that the MGSMV beam is better for the optical communications than the MGSM beam. In addition, as the structure constant C_n^2, and the transverse coherence δ are smaller, or the inner scale l_0, and the wavelength λ, are larger, the beam quality is better. It is expected that this work will be useful for free space optical communications.

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