O P E R A T I O N S R E S E A R C H A N D D E C I S I O N S

No. 3 2019
DOI: 10.5277/ord190301

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DECISION-MAKER'S PREFERENCES FOR MODELING MULTIPLE OBJECTIVE STOCHASTIC LINEAR PROGRAMMING PROBLEMS

A method has been suggested which solves a multiobjective stochastic linear programming problem with normal multivariate distributions in accordance with the minimum-risk criterion. The approach to the problem uses the concept of satisfaction functions for the explicit integration of the preferences of the decision-maker for different achievement level of each objective. Thereafter, a nonlinear deterministic equivalent problem is formulated and solved by the bisection method. Numerical examples with two and three objectives are given for illustration. The solutions obtained by this method are compared with the solutions given by other approaches.

Keywords: *multiobjective programming, stochastic programming, nonlinear programming, satisfaction function*

1. Introduction

Multiobjective stochastic linear programming (MOSLP) is an appropriate tool to model concrete, real-life problems in several domains. Such a class of problems includes water use planning [10, 13], mineral blending [21], manufacturing systems in production planning situation [18], investment and energy resources planning [31, 35] and multi-product batch plant design [36] to mention a few. Among the applications of MOSLP in portfolio selection, we can mention the recent works of Shing and Nagasawa [27], Ogryczak [26], Ben Abdelaziz et al. [11], Aouni et al. [4], Boswarva and Aouni [12]. Despite the purely mathematical nature of many works in this field [2, 7–9, 14–16, 28], several technical methods for solving MOSLP problems have been developed.

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Goicoechea et al. [19] develop a method called the probabilistic trade-off development method (PROTRADE). It treats a problem with general distributions for random coefficients of linear objectives but its use requires the assessment of multi-attribute utility function which limits its area of application. Stancu-Minasian [29] offers a sequential method for solving the multiple minimum risk problem. For contexts of practical scenarios on objectives and some constraints, Teghem et al. [32] propose the STRANGE method where uncertainty in the constraints is taken into account by a recourse approach. For multiple criteria framework with a discrete number of states of nature, Klein et al. [20] develop an interactive method with recourse using a two-stage mathematical programming model. Urli and Nadeau [34] propose a scenarios approach where the probabilities of scenarios are incompletely specified. Muñoz and Ruiz [25] present an interactive algorithm for stochastic multiobjective problems with continuous random variables. This method combines the concept of probability efficiency for stochastic problems with the reference point philosophy for deterministic multiobjective problems. The decision-maker expresses her/his references by dividing the variation range of each objective into intervals, and by setting the desired probability for each objective to achieve values belonging to each interval. Luque et al. [22] suggest the synchronous reference point-based interactive method for a class of MOSLP problems where only the objective functions are random. There are also some methods designed for multiobjective stochastic integer linear programming (MOSILP) problems. The reader may refer, for example, to Teghem et al. [33], Abbas and Bellahcene [1], Amrouche and Moulay [3], Chaabane and Mebrek [17] for further research in this field.

In both theoretical or applied works, the issues including randomness are usually transformed into deterministic problems. There are five criteria for such transformation: expected value, minimum variance, expected value standard deviation, maximum probability or minimum risk and Kataoka. The first three criteria are often used in applied works but they are not very risky. However, the application of minimum risk and Kataoka criteria requires the collaboration of the decision-maker who has to fix an aspiration level for each stochastic objective. For instance, this is the situation when the expected value and the expected value-standard deviation of the objective function are considered not to be a good measure of criteria. Therefore, modelling with minimal risk and Kataoka criteria is of great interest to the scientific management. These criteria offer good solutions in terms of probability.

Wishing to explore this area and make a modest contribution, we focus on a MOSLP problem with normal multivariate distributions where the minimum risk criterion is used for transformation to deterministic. First, satisfaction functions [23] are introduced to explicitly integrate the decision-maker's preferences for different achievement level of each objective. Thereafter, a nonlinear programming problem is formulated and an efficient solution method based on the bisection method [5] developed to obtain its optimal solution.

2. Problem statement

Let us consider the multiobjective stochastic linear programming problem formulated as:

$$\begin{gathered} \min(C_1^t x, C_2^t x, \ldots, C_q^t x) \\ \text{subject to} \\ Ax \le b,\ x \ge 0 \end{gathered} \tag{1}$$

where x is an n-dimensional decision variable column vector, A is an $m \times n$ coefficient matrix and b an m-dimensional column vector. The set $S = \left\{ x \in \mathbb{R}^n \mid Ax \le b, x \ge 0 \right\}$ is nonempty and compact in $\mathbb{R}^n$. We assume that each vector C_k has a multivariate normal distribution with mean $\overline{C_k}$ and covariance matrix V_k.

Substituting the minimisation of the stochastic objective functions for the maximisation of the probability that each objective is less than a certain permissible level u_k leads us to so-called multiple minimum risk problem with levels $u_1, u_2, \ldots, u_q$.

$$\begin{gathered} \max P_1(x, u_1) = Pr\left[C_1^t x \le u_1 \right] \\ \max P_2(x, u_2) = Pr\left[C_2^t x \le u_2 \right] \\ \vdots \\ \max P_q(x, u_q) = Pr\left[C_q^t x \le u_q \right] \\ \text{subject to} \\ Ax \le b,\ x \ge 0 \end{gathered} \tag{2}$$

Definition 1 [30]. $x^* \in S$ is an $u_1, u_2, \ldots, u_q$ minimum-risk solution for problem (1) if it is Pareto optimal to the problem (2).

Since each component C_{kj} of C_k occurs according to a normal distribution, $C_k x$ is also normally distributed with mean $\overline{C_k} x$ and variance $x^t V_k x.$ Thereby, each objective function in (2) is rewritten as follows:

$$\Pr\left[C_k^t x \le u_k \right] = \Pr\left[\frac{C_k^t x - \overline{C_k} x}{\sqrt{x^t V_k x}} \le \frac{u_k - \overline{C_k} x}{\sqrt{x^t V_k x}} \right] = \Phi\left(\frac{u_k - \overline{C_k} x}{\sqrt{x^t V_k x}} \right)$$

where $\Phi(\cdot)$ is the distribution function of the standard normal distribution.

From this transformation, problem (2) is naturally reduced to the following deterministic multiobjective programming problem:

$$\max P_k(x,u_k) = \Phi\left(\frac{u_k - \overline{C_k^t}x}{\sqrt{x^t V_k x}}\right), \quad k = 1,\dots,q$$
$$\text{subject to} \tag{3}$$
$$Ax \le b, x \ge 0$$

whose Pareto optimal solutions are defined according to the following well-known definition (see, for example [24].

Definition 2. $x^* \in S$ is Pareto optimal for problem (3) if and only if there does not exist another $x \in S$ such that $P_k(x,u_k) \ge P_k(x^*, u_k)$ for all $k = 1,\dots,q$ and $P_k(x,u_k) > P_k(x^*, u_k)$ for at least one k.

Since Φ is an increasing function, a possible way to find efficient solutions to problem (3) is to solve the nonlinear multiobjective fractional problem whose objective functions are $f_k(x) = (u_k - \overline{C_k^t}x)/\sqrt{x^t V_k x}$, $k = 1,\dots,q$. However, finding efficient solutions for such problems is complicated enough because of the square root. Therefore, to overcome this difficulty, we use satisfaction functions to introduce explicitly the decision-maker's preferences for different achievement level of each objective and formulate a satisfaction model that can be easily solved by the bisection method.

3. The satisfaction model

We assume that the decision-maker can fix achievement probabilities (target values) $P_{k\min}$ and $P_{k\max}$ for each objective function $P_k(x,u_k)$. These probabilities are used to construct the satisfaction functions μ_k as follows:

$$\mu_k\left(P_k(x,u_k)\right) = \begin{cases} 1 & \text{if } P_k(x,u_k) \ge P_{k\max} \\ g_k\left(P_k(x,u_k)\right) & \text{if } P_{k\min} \le P_k(x,u_k) \le P_{k\max} \\ 0 & \text{if } P_k(x,u_k) \le P_{k\min} \end{cases}$$

where g_k is a monotonously increasing function of $P_k(x,u_k)$.

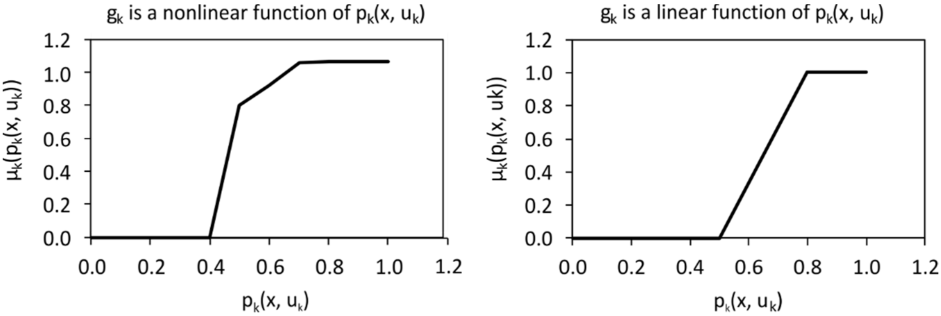

Fig. 1. Shape of the satisfaction function

Through the satisfaction functions, the decision-maker may explicitly express his/her preferences with regards to the deviations associated with the target values fixed for each objective. This satisfaction function means that the decision-maker is entirely satisfied when the objective function $P_k(x, u_k)$ is more than $P_{k\max}$, partially satisfied if $P_k(x, u_k)$ is between $P_{k\min}$ and $P_{k\max}$ but she/he is not satisfied if $P_k(x, u_k)$ is less than $P_{k\min}$. The satisfaction functions μ_k are strictly increasing and continuous with respect to $P_k(x, u_k)$.

Using these satisfaction functions, we formulate the following model which maximizes the smaller degree of satisfaction:

$$\begin{gathered} \max \min \mu_k\left(P_k(x, u_k)\right), \quad k = 1, \ldots, q \\ \text{subject to} \\ Ax \le b,\ x \ge 0 \end{gathered} \tag{4}$$

Problem (4) is referred to as the Chebyshev problem. Then, according to the general result in Miettinen [24], at least one of the optimal solutions of problem (4) is Pareto optimal for problem (3) and if problem (4) has a unique optimal solution, then; it is automatically Pareto optimal to problem (3).

Setting $h = \min \mu_k(P_k(x, u_k))$ be the minimum value of all the functions $\mu_k(P_k(x, u_k))$, for $k = 1, \ldots, q$, problem (4) is reformulated as

$$\begin{gathered} \max h \\ \text{subject to} \\ \mu_k\left(P_k(x, u_k)\right) \ge h, \quad k = 1, \ldots, q \\ Ax \le b \\ x \ge 0,\ 0 \le h \le 1 \end{gathered} \tag{5}$$

The first constraints in (5) can be rewritten as:

$$\begin{aligned}\mu_k\left(P_k(x,u_k)\right)\geq h &\Leftrightarrow g_k\left(P_k(x,u_k)\right)\geq h\\ &\Leftrightarrow P_k(x,u_k)\geq g_k^{-1}(h)\\ &\Leftrightarrow \Phi\left(\frac{u_k-\overline{C_k^t}x}{\sqrt{x^tV_kx}}\right)\geq g_k^{-1}(h)\\ &\Leftrightarrow \overline{C_k^t}x+\Phi^{-1}\left[g_k^{-1}(h)\right]\sqrt{x^tV_kx}\leq u_k\end{aligned}$$

Thus, problem (5) is reduced to the following non-linear problem:

$$\begin{gathered}\max h\\ \text{subject to}\\ \overline{C_k^t}x+\Phi^{-1}\left[g_k^{-1}(h)\right]\sqrt{x^tV_kx}\leq u_k,\quad k=1,\dots,q\\ Ax\leq b\\ x\geq 0,\ 0\leq h\leq 1\end{gathered}\tag{6}$$

For simplicity, we use linear satisfaction functions of the form

$$\mu_k\left(P_k(x,u_k)\right)\begin{cases}1 & \text{if } P_k(x,u_k)\geq P_{k\max}\\ \dfrac{P_k(x,u_k)-P_{k\min}}{P_{k\max}-P_{k\min}} & \text{if } P_{k\min}\leq P_k(x,u_k)\leq P_{k\max}\\ 0 & \text{if } P_k(x,u_k)\leq P_{k\min}\end{cases}$$

In this case, the numbers $g_k^{-1}(h)$ are determined as follows:
Let $y\in[0,1]$ an arbitrary value of $\mu_k\left(P_k(x,u_k)\right)$, then;

$$\mu_k\left(P_k(x,u_k)\right)=y\Leftrightarrow g_k\left(P_k(x,u_k)\right)=y\Leftrightarrow P_k(x,u_k)=g_k^{-1}(y)$$

On the other hand,

$$g_k\left(P_k(x,u_k)\right)=y\Leftrightarrow\frac{P_k(x,u_k)-P_{k\min}}{P_{k\max}-P_{k\min}}=y\Leftrightarrow P_k(x,u_k)=y(P_{k\max}-P_{k\min})+P_{k\min}$$

This implies that

$$g_k^{-1}(y)=P_k(x,u_k)=y(P_{k\max}-P_{k\min})+P_{k\min}$$

In particular, $g_k^{-1}(h)=P_k(x,u_k)=h(P_{k\max}-P_{k\min})+P_{k\min}$.

4. The proposed solution method

Problem (6) is non-convex, it is, in general, difficult to solve it directly. However, if the value of *h* is fixed in the interval $\left[0,1\right]$, solving problem (6) is equivalent to determining a feasible solution x in the set

$$D_h = \left\{ x \in \mathbb{R}_+^n \left| \overline{C_k^t} x + \Phi^{-1}\left[g_k^{-1}(h) \right] \sqrt{x^t V_k x} \le u_k, \quad k = 1, ..., q, \ Ax \le b, \ x \ge 0 \right. \right\}$$

The numbers $\Phi^{-1}\left[g_k^{-1}(h) \right]$ are assumed to be positive for a fixed value of the variable *h* in the interval $\left[0,1\right]$ in order to guarantee the achievement of the objective functions with probabilities at least equal to 1/2. So, an immediate consequence of this assumption is that the constraint functions $f_k(x,h) = -u_k + \overline{C_k^t} x + \Phi^{-1}\left[g_k^{-1}(h) \right] \sqrt{x^t V_k x}$ are convex with respect to x and $0.5 < P_{k\,\min} < P_{k\,\max} < 1$.

Proposition 1. If $0 < h_1 < h_2 < 1$, then $D_{h_1} \supset D_{h_2}$.

Proof: From the increasing of g_k and $0 < h_1 < h_2 < 1$, it holds that $g_k^{-1}(h_1) \le g_k^{-1}(h_2)$ and $\Phi^{-1}\left[g_k^{-1}(h_1) \right] \le \Phi^{-1}\left[g_k^{-1}(h_2) \right]$. This means that $f_k(x,h_1) \le f_k(x,h_2)$ for any $x \in S$. It results that $D_{h_1} \supset D_{h_2}$ for any h_1, h_2 such that $0 < h_1 < h_2 < 1$.

From Proposition 1, we can solve problem (6) using the following algorithm which exploits the bisection method:

Step 1. Set $h_a = 0$, $h_b = 1$, $h_0 = h_a$, ε a small and positive value.
Step 2. Set $h_t = (h_a + h_b)/2$.
Step 3. Evaluate $\Phi^{-1}\left[g_k^{-1}(h_t) \right]$.
Step 4. Solve the system in D_h.
Step 5. If the feasible solution does not exist, set $h_b = h_t$ and return to Step 2;
If the feasible solution exists, set $h_a = h_t$ and return to Step 2;
If the feasible solution exists and $\left| h_t - h_{t-1} \right| \le \varepsilon$, x_t is the optimal solution of problem (6).

Illustrative example

Let us consider the following bi-objective problem:

$$\max P_1(x,u_1) = \Pr\left[C_1^t x \geq 20\ 000 \right]$$
$$\max P_2(x,u_2) = \Pr\left[C_2^t x \leq 3000 \right]$$
$$\text{subject to}$$
$$1.5x_1 + 7.5x_2 \leq 240$$
$$-2.5x_1 + 2.5x_2 \leq 105$$
$$17.5x_1 - 10x_2 \leq 295$$
$$x_1 \geq 0,\ x_2 \geq 0$$

where C_1 and C_2 follow normal distributions with means $\overline{C_1}$ = (920, 210) and $\overline{C_2}$ = (117, 55) and variances

$$V_1 = \begin{pmatrix} 4 & 2.5 \\ 2.5 & 9 \end{pmatrix}, \quad V_2 = \begin{pmatrix} 5.2 & -0.3 \\ -0.3 & 7 \end{pmatrix}$$

respectively, $P_{1\min} = 0.95,\ P_{1\max} = 0.98,\ P_{2\min} = 0.85,\ P_{2\max} = 0.96,\ \varepsilon = 0.0003.$

$$\mu_1\left(P_1(x,u_1)\right) = \begin{cases} 1 & \text{if } P_1(x,u_1) \geq 0.98 \\ \dfrac{P_1(x,u_1) - 0.95}{0.03} & \text{if } 0.95 \leq P_1(x,u_1) \leq 0.98 \\ 0 & \text{if } P_1(x,u_1) \leq 0.95 \end{cases}$$

$$\mu_2\left(P_2(x,u_2)\right) = \begin{cases} 1 & \text{if } P_2(x,u_2) \geq 0.96 \\ \dfrac{P_2(x,u_2) - 0.85}{0.11} & \text{if } 0.85 \leq P_2(x,u_2) \leq 0.96 \\ 0 & \text{if } P_2(x,u_2) \leq 0.85 \end{cases}$$

Knowing that $P_1(x,u_1) = \Pr\left[C_1^t x \geq u_1 \right] = \Pr\left[-C_1^t x \leq -u_1 \right]$, the deterministic problem of the form (6) to be solved is:

$$\max h$$
$$\text{subject to}$$
$$920x_1 + 120x_2 - \Phi^{-1}(0.03h + 0.95)\sqrt{4x_1^2 + 5x_1x_2 + 9x_2^2} \geq 20\ 000$$
$$117x_1 + 55x_2 + \Phi^{-1}(0.11h + 0.85)\sqrt{5.2x_1^2 - 0.6x_1x_2 + 7x_2^2} \leq 3000$$

$$1.5x_1 + 7.5x_2 \le 240$$
$$-2.5x_1 + 2.5x_2 \le 105$$
$$17.5x_1 - 10x_2 \le 295$$
$$x_1 \ge 0,\ x_2 \ge 0,\ 0 \le h \le 1$$

For solving the nonlinear system in D_h, we have used the LINGO software. The optimal solution is $(x_1, x_2) = (20.43475, 6.260814)$.

By applying the method of Bellahcene and Marthon [6], we find $(x_1, x_2) = (20.4253, 6.24423)$ and

$$P_1(x, u_1) = \Pr\left[C_1^t x \ge 20\ 000 \right] = 0.98875$$

$$P_2(x, u_2) = \Pr\left[C_2^t x \le 3000 \right] = 0.99097$$

Let now consider the example given by Bellahcene and Marthon [6]:

$$\max \Pr\left[C_1^t x \ge 82\ 830 \right]$$
$$\max \Pr\left[C_2^t x \le 5280 \right]$$
$$\max \Pr\left[C_3^t x \le 16\ 100 \right]$$
$$\text{subject to}$$
$$2.5x_1 + 7.5x_2 \le 240$$
$$0.125x_1 + 0.125x_2 \le 5$$
$$17.5x_1 + 10x_2 \le 595$$
$$x_1 \ge 0,\ x_2 \ge 0$$

where, C_1, C_2 and C_3 are random vectors that follow normal distributions with respective means $\overline{C_1} = (\overline{C_{11}}, \overline{C_{12}}) = (2350, 1600)$, $\overline{C_2} = (\overline{C_{21}}, \overline{C_{22}}) = (120, 100)$, $\overline{C_3} = (\overline{C_{31}}, \overline{C_{32}}) = (430, 300)$ and positive definite variance-covariance matrices:

$$V_1 = \begin{pmatrix} 33.64 & 0.98 \\ 0.98 & 5.29 \end{pmatrix}, \qquad V_2 = \begin{pmatrix} 44.89 & -0.76 \\ -0.76 & 3.61 \end{pmatrix}, \qquad V_3 = \begin{pmatrix} 6.76 & 0 \\ 0 & 1.44 \end{pmatrix}$$

We find that, the optimal solution of the deterministic problem is $(x_1, x_2) = (26, 14)$. Its corresponding probabilities of achieving goals are given by:

$$\Pr\left[C_1^t x \geq 82\ 830\right] = 0.99, \quad \Pr\left[C_2^t x \leq 5280\right] = 0.99, \quad \Pr\left[C_3^t x \leq 16\ 100\right] = 0.99$$

The application of the bisection method with $P_{1\min} = 0.96$, $P_{1\max} = 0.99$, $P_{2\min} = 0.85$, $P_{2\max} = 0.99$, $P_{3\min} = 0.95$, $P_{3\max} = 0.99$ gives us $(x_1, x_2) = (25.9998,\ 14.0011)$.

5. Conclusion

A decision making method for solving a MOSLP problem where several probabilities are minimised is proposed. The problem is reformulated into a deterministic multi-objective problem introducing chance constraints based on the stochastic programming approach. The resulting goals are quantified by eliciting the corresponding satisfaction functions for permissible levels. The satisfactory solution is easily obtained by applying the bisection method. The usefulness and the simplicity of the developed method are shown through its application to problems with two and three objectives functions. In order to test the efficiency of this method, we compare it with the Bellahcene and Marthon method in terms of probability. We find that the two methods produce the same solutions. The unique difference between these two methods is that in the Bellahcene and Marthon method we first find the optimal solution, and then we find the probabilities of achieving goals. In the bisection method, we first fix the desired probabilities of achieving goals, and then we find the optimal solution. As to future work, we are working on determining which method is speeder.

References

[1] ABBAS M., BELLAHCENE F., *Cutting plane method for multiple objective stochastic integer linear programming*, Eur. J. Oper. Res., 2006, 168, 967–984.
[2] ADEYEFA A., LUHANDJULA M., *Multiobjective stochastic linear programming: an overview*, Amer. J. Oper. Res., 2011, 1 (4), 203–213.
[3] AMROUCHE S., MOULAÏ M., *Multi-objective stochastic integer linear programming with fixed recourse*, Int. J. Multicr. Dec. Making, 2012, 2 (4), 355–378.
[4] AOUNI B., COLAPINTO C., LA TORRE D., *Solving stochastic multi-attribute portfolio selection through the goal programming model*, J. Fin. Dec. Making, 2010, 6 (2), 17–30.
[5] BAZARAA M., SHERALI H., SHETTY C., *Theory and Algorithms*, 2nd Ed., Wiley, New York 1993.
[6] BELLAHCENE F., MARTHON P., *A compromise solution method for the multiobjective minimum risk problem*, Oper. Res., May 2019, DOI 10.1007/ s12351-019-00493-1.
[7] BEN ABDELAZIZ F., *L'efficacité en programmation multi-objectifs stochastique*, Ph.D. Thesis, Université de Laval, Québec 1992.
[8] BEN ABDELAZIZ F., LAND P., NADEAU R., *Distributional unanimity multiobjective stochastic linear programming*, [In:] J. Climaco (Ed.), *Multicriteria Analysis*, Proc. 11th Int. Conf. MCDM, Springer-Verlag, Berlin 1997, 225–236.

[9] BEN ABDELAZIZ F., LANG P., NADEAU R., *Dominance and efficiency in multicriteria decision under uncertainty*, Theory Dec., 1999, 47 (3), 191–211.

[10] BEN ABDELAZIZ F., MEJRI S., *Application of goal programming in a multi-objective reservoir operation model in Tunisia*, Eur. J. Oper. Res., 2001, 133, 352–361.

[11] BEN ABDELAZIZ F., AOUNI B., EL FAYEDH R., *Multi-objective stochastic programming for portfolio selection*, Eur. J. Oper. Res., 2007, 177 (3), 1811–1823.

[12] BOSWARVA I., AOUNI B., *Different probability distributions for portfolio selection in the chance constrained compromise programming model*, Inf. Syst. Oper. Res. J., 2012, 50 (3), 140–146.

[13] BRAVO M., GONZALEZ I., *Applying stochastic goal programming. A case study on water use planning*, Eur. J. Oper. Res., 2009, 196 (39), 1123–1129.

[14] CABALLERO R., CERDA E., MUÑOZ M.M., REY L., *Relations among several efficiency concepts in stochastic multiple objective programming*, [In:] Y.Y. Haimes, R. Steuer (Eds.), *Research and Practice in Multiple Criteria Decision Making*, Lecture Notes in Economics and Mathematical Systems, 487, Springer-Verlag/Kluwer, 2000, 57, 58.

[15] CABALLERO R., CERDA E., DEL MAR M., REY L., *Efficient solution concepts and their relations in stochastic multiobjective programming*, J. Opt. Theory Appl., 2001, 110 (1), 53–74.

[16] CABALLERO R., CERDA E., MUÑOZ M.M., REY L., *Stochastic approach versus multiobjective approach for obtaining efficient solutions in stochastic multiobjective programming problems*, Eur. J. Oper. Res., 2004, 158, 633–638.

[17] CHAABANE D., MEBREK F., *Optimization of a linear function over the set of stochastic efficient solutions*, Comp. Manage. Sci., 2014, 11 (1), 157–178.

[18] FAZLOLLAHTABAR H., MAHDAVI I., *Applying stochastic programming for optimizing production time and cost in an automated manufacturing system*, Int. Conf. Computers and Industrial Engineering, Troyes 2009, 1226–1230.

[19] GOICOECHEA A., DUKSTEIN L., BULFIN R.L., *Multiobjective stochastic programming, the PROTRADE method*, Oper. Res. Soc. Amer., Miami Beach 1976.

[20] KLEIN G., MOSKOWITZ H., RAVINDRAN A., *Interactive multiobjective optimization under uncertainty*, Manage. Sci., 1990, 36 (1), 58–75.

[21] KUMRAL M., *Application of chance-constrained programming based on multiobjective simulated annealing to solve mineral blending problem*, Eng. Opt., 2003, 35 (6), 661–673.

[22] LUQUE M., RUIZ F., CABELLO J.M., *A synchronous reference point-based interactive method for stochastic multiobjective programming*, OR Spectrum, 2012, 34, 763–784.

[23] MARTEL J.M., AOUNI B., *Incorporating the decision-makers preferences in the goal-programming model*, Journal of the Operational Research Society, 1990, 41 (12), 1121–1132.

[24] MIETTINEN K.M., *Nonlinear Multiobjective Optimization*, Kluwer's International Series, 1999.

[25] MUÑOZ M.M., RUIZ F., *ISTMO, an interval reference point-based method for stochastic multiobjective programming problems*, Eur. J. Oper. Res., 2009, 197, 25–35.

[26] OGRYCZAK W., *Multiple criteria linear programming model for portfolio selection*, Ann. Oper. Res., 2000, 97, 143–162.

[27] SHING C., NAGASAWA H., *Interactive decision system in stochastic multi-objective portfolio selection*, Int. J. Prod. Econ., 1999, 60, 187–193.

[28] SLOWINSKI R., TEGHEM T., *Stochastic versus fuzzy approaches to multiobjective mathematical programming under uncertainty*, Kluwer, Boston 1990.

[29] STANCU-MINASIAN I.M., *On the problem of multiple minimum risk. I. The case of two objective functions, II. The case of r ($r > 2$) objective functions*, Stud. Cerc. Mat., 1976, 28 (5), 617–623, ibidem 28 (6), 723–734 (in Romanian).

[30] STANCU-MINASIAN I.M., TIGAN S., *The vectorial minimum risk problem*, Proc. Coll. Approximation and Optimization, Cluj-Napoca, 1984, 321–328.

[31] TEGHEM J., KUNSCH P., *Application of multiobjective stochastic linear programming to power systems planning*, Eng. Costs Prod. Econ., 1985, 9 (13), 83–89.
[32] TEGHEM J., DUFRANE D., THAUVOYE M., KUNSCH P.L., *STRANGE, an interactive method for multiobjective stochastic linear programming under uncertainty*, Eur. J. Oper. Res., 1986, 26 (1), 65–82.
[33] TEGHEM J., *STRANGE-MOMIX. An interactive method for mixed integer linear programming*, [In:] R. Slowinski, J. Teghem (Eds.), *Stochastic Versus Fuzzy Apprroaches to Multiobjective Mathematical Programming Under Uncertainty*, Kluwer, Dordrecht 1990, 101–115.
[34] URLI B., NADEAU R., *PROMISE/scenarios: An interactive method for multiobjective stochastic linear programming under partial uncertainty*, Eur. J. Oper. Res., 2004, 155, 361–372.
[35] VAHIDINASAB V., JADID S., *Stochastic multiobjective self-scheduling of a power producer in joint energy and reserves markets*, Elect. Power Syst. Res., 2010, 80 (7), 760–769.
[36] WANG Z., JIA X.P., SHI L., *Optimization of multi-product batch plant design under uncertainty with environmental considerations*, Clean Techn. Environ. Pol., 2009, 12 (3), 273–282.

Received 25 March 2019
Accepted 23 November 2019