Coupled fuzzy logic and experimental design application for simulation of a coal classifier in an industrial environment

Hamid Khoshdast, Ali Soflaeian, Vahideh Shojaei

Department of Mining Engineering, Higher Education Complex of Zarand, Zarand, Iran

Corresponding author: khoshdast_hamid@yahoo.com (Hamid Khoshdast)

Abstract: Design of experiments (DOE) is an effective method providing useful information about the interaction of operating variables and the way the total system works by using statistical analyses. However, its industrial application is limited because it is almost difficult to maintain variables in DOE matrix at desired constant levels in industrial environment. Thus, this paper aims to present a new mixed modeling method which is a combination of fuzzy logic and design of experiments methods to overcome such practical limitations. The method first uses a fuzzy model which is trained by practical data gathered from industry to predict DOE response corresponding to each run in DOE matrix. Then, a statistical parametric model is constructed for the prediction of process response to any change of operating parameters under real industrial conditions. The proposed mixed method was successfully validated by using data obtained from a coal hydraulic classifier at Zarand Coal Washing Plant (Kerman, Iran). The method also seems to be a promising tool for modeling all devices and processes in real industrial environment and allows researchers to benefit from all the advantages of experimental design and fuzzy logic methods simultaneously.

1. Introduction

Design of experiments (DOE) is a systematic, rigorous method to determine the relationship between factors affecting a process and the output of that process. In other words, it is used to find cause-and-effect relationships. This method helps investigators to solve their comparative, screening, modeling or optimizing problems, and to ensure drawing valid, defensible, and supportable conclusions with minimal expenditure of experimental runs, time, and money. Nowadays, the usefulness of DOE in industrial investigation is well established. These studies consist of new product design and formulation, manufacturing process development, process improvement, and development of predictive models so that the process affected minimally by external sources of variability as a robust process (Montgomery, 2013). A well-designed experiment is important because the results and conclusions drawn from the experiment are extremely dependent on the employed data collection method; and this is why DOE is confronted in some industrial application. General methodology of DOE is to define constants' levels for each input variable in the form of a tabulated structure using statistical approaches, whereas it is actually very difficult, in some cases impossible, to keep operating parameters not only with limited fluctuations but also in a constant predefined level. This is where fuzzy logic approach can be successfully applied.

Fuzzy modeling is one of the most powerful techniques to estimate input–output relation in complex systems. Fuzzy logic actually considers the process as a black box and correlate input parameters to output response(s) using a set of user-supplied human language rules using real data after a training stage. The fuzzy systems then convert these rules to their mathematical equivalents to represent the way process behaves in the actual environment with acceptable accuracy (Rahmanian et al., 2011). Other advantages of fuzzy logic are its simplicity and flexibility, capability of handling problems with

imprecise and incomplete multivariate data, the ability of modeling nonlinear functions of arbitrary complexity, and easy interpretation. Another advantage of fuzzy modeling lies in the training phase where it is possible to establish rules even in the absence of data relying instead on expert knowledge to constitute the rule base (Yager and Zadeh, 1992; Harris, 2000; Nguyen and Walker, 2006).

Combined application of fuzzy logic and design of experiments has been considered by different researchers. These studies can be summarized as follows:

- Using fuzzy logic as a decision-making tool for selecting the appropriate experimental designs: Olivero et al. (1993) developed a fuzzy-based system to assist investigators choosing suitable experimental designs for their research projects in an interactive session.
- Screening input variables of fuzzy model using design of experiments: Some processes may have many input variables and, hence, identification of the most important variables (screening) is often crucial (Dean and Lewis, 2006). In this regard, some investigators first used DOE method for decomposing the complex input-output relationship to determine the significant factors. The effective parameters were then used as inputs of fuzzy modeling (Tseng et al., 2016).
- Fuzzy modeling of results obtained from designed experiments: Due to extreme complexity and nonlinearity of some processes, fuzzy logic method has been used to simulate response of such processes investigated by design of experiments (Yang et al., 2006; Azadeh et al., 2011; Rahmanian et al., 2011; Hametner and Jakubek, 2013).

The last two series of studies are limited to batch investigations. Therefore, the aim of this paper is to introduce a novel combined application of fuzzy logic and design of experiments methods for simulating real industrial processes. The proposed modeling approach was also verified by simulating a coal hydraulic classifier in an industrial environment using actual data.

2. Investigation stages

Our study is started with collecting performance data from a coal hydraulic classifier at Zarand Coal Washing Plant (Kerman, Iran). Then, a fuzzy model was developed to predict the classifier's cut size as process response. Afterwards, an experimental design was constructed on the basis of variation range of operating parameters and cut size was predicted by using the developed fuzzy model. Finally, a phenomenological model was developed by statistical analysis of DOE results and was verified by actual data. Fig. 1 shows the sequence of these studies.

2.1. Industrial investigations

The experimental data was measured from the samples collected from a hydraulic classifier at Zarand Coal Washing Plant (Zarand, Iran). Following an 8-month sampling program, 140 representative samples were collected. For preparing each representative sample, 5 sub-samples were collected from feed and products' streams in each operating shift and then mixed and divided to obtain appropriate weight of the sample for particle size analysis. Other parameters including feed pulp flowrate and solid content were also measured during the sample collection program.

Actual cut size values were obtained by plotting the partition coefficient curve for each sample. The effect of operating parameters including feed particle size (characteristic size and imperfection coefficient), pulp flowrate and solid content on cut size was evaluated by plotting the variation of cut size vs. each parameter and then, the fitting line was used to interpret the effect thereof.

2.2. Development of fuzzy model

Fuzzy systems commonly have four components: (1) a fuzzification process; (2) a rule base; (3) an inference engine, and (4) a defuzzification process. Fuzzification is a process that classifies numerical measurements into fuzzy sets. The rule base contains the ''if … then'' rules that embody linguistic reasoning. An inference engine applies the rule base to the fuzzy sets to obtain a fuzzy outcome. A defuzzification procedure converts the fuzzy outcome to a crisp one (Abkhoshk et al., 2010).
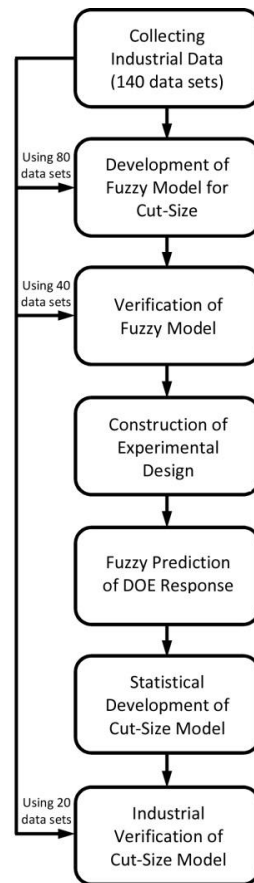
Fig. 1. Stages considered for developing the proposed new modeling approach

A) Fuzzification of input and output variables: This phase in the design of the fuzzy expert system consists of assigning a degree of membership to each variable. Different membership functions can be used, depending on the type of application. This study used the triangular fuzzy sets because (1) a small amount of data is needed to define the membership function, (2) easy of modification of parameters (modal values) of membership functions on the basis of measured values of the input-output of a system, (3) the possibility of obtaining input-output mapping of a model which is a hypersurface consisting of linear segments, and (4) triangular membership function means the condition of a partition of unity (it means that the sum of membership grades for each value x amounts to 1) is easily satisfied (Sivanandam et al., 2007; Abkhoshk et al., 2010). Based on industrial investigations, the data used in fuzzy models was arranged in the form of four input parameters covering the characteristic size (d_{80}) and the imperfection coefficient (I), the feed pulp flowrate (Q) and the solid content (X), and the output parameter which is the cut size (d_{50}) as process response parameter. Figs. 2 and 3 present the shape and range of each membership function for input and output variables, respectively.

B) If–then rule statements: The next stage in designing the fuzzy expert system is the creation of a rule base with rules linking up the input and the output variable(s) and specified, in our case, as multi-input/single-output rules. The fuzzy if–then rule (R_i) takes the following general form: R_i: If X is A_i and Y is B_i,… then Z is C_i,…,i=1…n; where X, Y and Z are linguistic variables. Using fuzzy set generalizes the information used to describe the behavior of the system (Abkhoshk et al., 2010). The formation of fuzzy rules in this study is based on the data collected from an industrial hydraulic classifier.

C) Design of an inference engine: The inference engine used to relate the outcomes of the rule base to membership function values for the output variable quality groups is that typically encountered in the Mamdani controller (Mamdani and Assilian, 1999).

D) Defuzzification process: Several defuzzification methods have been presented in literature among. In this study, the center-of-area (or center-of-gravity) method was applied as defuzzification method. The center-of-gravity formula is one of the most frequently referred methods in literature (Sivanandam et al., 2007; Leekwijck and Kerre, 1999).
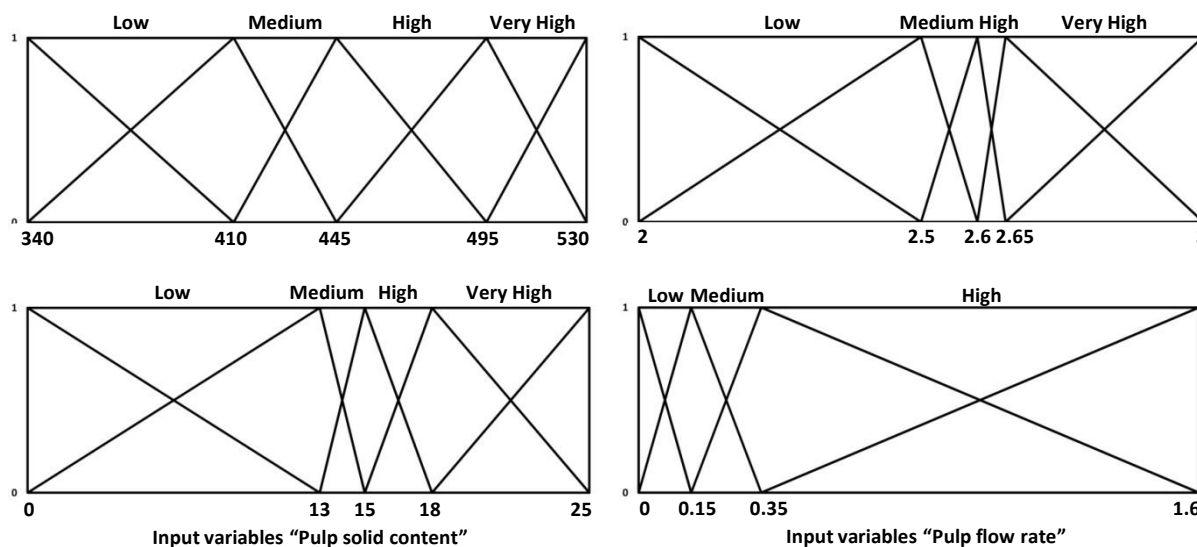
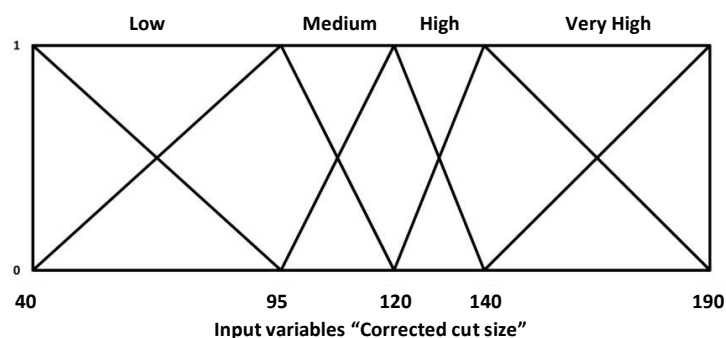Fig. 2. Membership functions of input variables used in this study

Fig. 3. Membership function of output response used in this study

2.3. Experimental design construction

Different types of experimental designs are now available. Choosing a suitable design depends on the objectives of the investigation and the number of factors involved in the study. Since the main objective of this study is to develop a parametric model for prediction of the cut size, the response surface methodology (RSM) was selected. Using response surface analysis, we run a series of full factorial experiments and map the response to generate mathematical equations that describe how factors affect the response(s). Additionally, RSM allows us to estimate interaction and even quadratic effects, and consequently gives us an idea of the (local) shape of the response surface we are investigating (Montgomery, 2013).

In this study, central composite design (CCD) method was applied as experimental design strategy to investigate the performance of classifier. Central composite design is a useful strategy for building a second order (quadratic) model of the response variable with no need to use a complete three-level factorial experiment. Central composite design structurally consists of three distinct sets of experimental runs (Cornell, 1990):

1. A factorial design for the factors studied, each having two levels;
2. A set of center points, experimental runs whose values of each factor are the medians of the values used in the factorial portion. This point is often replicated in order to improve the precision of the experiment;
3. A set of axial points, experimental runs identical to the center points except for one factor, which will take on values both below and above the median of the two factorial levels, and typically both outside their range.

The levels of interest for each factor were tried to be selected based on the variation ranges of input and output variables measured during industrial investigations. The experimental design matrix shown in Table 1 consists of a two-level full factorial design with 16 fact points ($2^4 = 16$), 6 center points and 8 axial points.

Table 1. Experimental design matrix and results for the response surface of cut size prediction

Run	Coded variables				Actual variables				Response (d_{50})	
	A (d_{80})	B (*I*)	C (*X*)	D (*Q*)	A (d_{80})	B (*I*)	C (*X*)	D (*Q*)	Experimental*	Predicted
1	+2	0	0	0	530	2.5	17.5	0.52	115	122.77
2	0	0	0	0	430	2.5	17.5	0.52	81.2	81.24
3	0	+2	0	0	430	3	17.5	0.52	115	116.91
4	+1	+1	+1	-1	480	2.75	21	0.29	115	115.10
5	0	0	0	+2	430	2.5	17.5	0.98	81.2	82.48
6	0	0	0	0	430	2.5	17.5	0.52	81.2	81.24
7	0	0	-2	0	430	2.5	10.5	0.52	138	139.91
8	-1	+1	-1	-1	380	2.75	14	0.29	115	98.83
9	0	0	0	0	430	2.5	17.5	0.52	81.2	81.24
10	0	0	0	-2	430	2.5	17.5	0.06	104	104.67
11	+1	-2	+1	-1	480	2.25	21	0.29	153	152.31
12	-1	-2	-1	-1	380	2.25	14	0.29	91.2	88.69
13	+1	-2	+1	+1	480	2.25	21	0.75	115	115.73
14	-1	+1	+1	-1	380	2.75	21	0.29	115	130.16
15	0	0	0	0	430	2.5	17.5	0.52	81.2	81.24
16	+1	-1	-1	-1	480	2.25	14	0.29	152	154.65
17	-2	0	0	0	330	2.5	17.5	0.52	81.2	83.09
18	-1	-1	-1	+1	380	2.25	14	0.75	94.1	94.76
19	0	-2	0	0	430	2	17.5	0.52	115	135.66
20	-1	+1	-1	+1	380	2.75	14	0.75	115	113.23
21	+1	-1	-1	+1	480	2.25	14	0.75	152	149.50
22	+1	+1	+1	+1	480	2.75	21	0.75	115	86.84
23	+1	+1	-1	+1	480	2.75	14	0.75	115	115.68
24	0	0	0	0	430	2.5	17.5	0.52	81.2	81.24
25	-1	+1	+1	+1	380	2.75	21	0.75	115	113.12
26	0	0	+2	0	430	2.5	24.5	0.52	115	137.46
27	-1	-1	+1	-1	380	2.25	21	0.29	115	115.09
28	0	0	0	0	430	2.5	17.5	0.52	81.2	81.24
29	+1	+1	-1	-1	480	2.75	14	0.29	115	112.50
30	-1	-1	+1	+1	380	2.25	21	0.75	115	89.72

* Determined by fuzzy model

3.   Results and discussion

3.1. Development and verification of fuzzy model

Fig. 2 shows four inputs to the fuzzy models including the characteristic size (d_{80}) and imperfection coefficient (*I*), pulp flowrate (*Q*) and solid content (*X*). These inputs ranges are [340, 530], [2, 3], [0, 1.6] and [0, 25], respectively. For the input variables d_{80}, *I* and *X*, four membership functions L, M, H and VH are used. Whereas, for input *Q* three membership functions L, M and H were used due to its limited data distribution. As shown in Fig. 2, the maximal pulp flowrate for the corresponding membership

function is considered to be 1.6, while at the maximum level in the experimental design, it is estimated to be about 1. During industrial measurements, only two flowrate data points were obtained with values more than 1 and about 1.6, which according to the claims of process engineers were irrational and due to error. Thus, these two points were not considered for the pilot project. Fig. 3 shows that the output range, i.e. cut size (d_{50}), is [40, 190] with four membership functions L, M, H and VH. Based on multiplicity rule, the initial numbers of rules should be equal to the product of the multiplication of the number of membership functions, i.e. 192 (4×4×4×3) (Tseng et al., 2016). After eliminating overlapping rules and multi-step optimization of rules by trial and error, the number of rules was reduced to 62 with sufficient accuracy. Precision evaluations showed that the accuracy of the program varied by less than 2% by decreasing the number of codes (such that the determination coefficient decreased from 91.68% to 90.03% for 192 and 92 rules, respectively), which is acceptable due to the dramatic decrease in the number of rules. Finally, a decision from the combinations of input membership functions to output membership functions was made by data set.

One important advantage of fuzzy modeling lies in the training phase where it is possible to establish rules to constitute the rule base, even in the absence of data, by relying on expert knowledge. Training the fuzzy program using experimental data is also considered to minimize the prediction error. The fuzzy logic program developed in this study was trained using 80 data sets gathered during industrial investigation. After training, 40 data sets were used to validate the accuracy of the model for the estimation of cut size. To assess the accuracy of the fuzzy logic model, determination coefficient (R^2) was calculated as follows (Razavi Parizi, 2010):

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}} \tag{1}$$

where SS_{res} and SS_{tot} are residual and total sum of squares, respectively,

$$SS_{res} = \sum_i (d_{50, exp, i} - d_{50, model, i})^2 \tag{2}$$

$$SS_{tot} = \sum_i (d_{50, exp, i} - \overline{d}_{50, exp})^2 \tag{3}$$

Measured cut size values are plotted against those values predicted by fuzzy model as shown in Fig. 4. As seen, determination coefficient, which quantify the degree of agreement between experimental observations, and numerically calculated values was found to be about 90%. The result shows that predicted values obtained from fuzzy model is in a very good agreement with the measured cut size values in industrial environment.
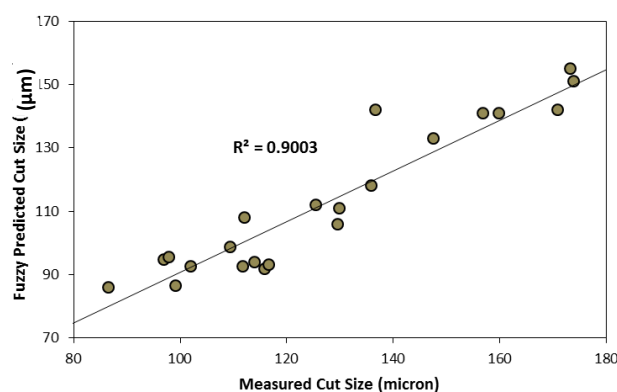

Fig. 4. Comparison between the fuzzy predicted and measured values for the cut size

3.2. Statistical analysis of fuzzy results

Analysis of Variance (ANOVA) was performed to test significance of studied parameters. Significance of each effect was determined by *p*-values. "Prob > F" values of less than 0.05 (for 95% confidence interval) indicate that effects are significant. Additionally, the wide range of variation in response factor (i.e. 81 to 153 μm) predicted by fuzzy model (Table 1) suggests that cut size is strongly influenced by the selected process variables (Clarke and Kempson, 1997). ANOVA results showed that except solid

content, all other parameters have significant effects on cut size; however, the effect of solid content was also considered in analysis. In addition, the significant interaction effects were d_{80}-I, d_{80}-X and X-Q. For better understanding of the effects of operating parameters on cut size, main effect plots derived from ANOVA results were compared with industrial data and are shown in Fig. 5. Regarding the experimental plots (Fig. 5 left), although a relatively increasing trend is observed for cut size-d_{80} relationship, finding a clear trend for other plots is actually impossible. In addition, for a given value of operating parameters in all plots, cut size changes in a wide range; therefore, drawing fitting lines, incompatibly a linear type, to monitor general variation trends of cut size relative to operating parameters does not lead to reliable conclusion. Instead, statistical plots of main effects (Fig. 5 right) show real relationship between operating parameters and cut size as well as their nonlinear behaviors.

In the case of cut size-pulp flowrate plot, for example, it is observed that cut size decreases by increasing the flowrate. Theoretically, it is expected that cut size is directly proportional to flowrate such that cut size increases as flowrate is increased due to increments in the rising force of overflow stream to product launder (Khoshdast et al., 2014). Therefore, this unusual behavior can doubtlessly be attributed to the interactive relationship between flowrate and other parameters, i.e. an effect that cannot be understood from experimental plots. Thus, direct interpretation of experimental data may lead to wrong conclusions. In addition, Fig. 4 actually includes 40 data points whereas fewer points can be observed. This is due to the overlapping of some data points while their operating conditions are different. This indicates that there is an interaction effect between operating parameters which results in similar cut size values. Similar interpretation can be presented for experimental plots in Fig. 5. Such interaction effects can only be recognized using a well experimental design as indicated by ANOVA results and warnings in the main effect plots in Fig. 5.

The three-dimensional (3D) response surface plots of the dependent variable as a function of two independent variables varying within their experimental ranges while maintaining all other variables at fixed (center) levels can provide information on their relationships and can be helpful in understanding both main and interaction effects of these two independent variables (Liu and Chiou, 2005). Each RSM plot represents an infinite number of combination of two tested variables with other two maintained at their respective zero level. Therefore, in order to gain better understanding of the effects of the independent variables and their interactions on the dependent variable, 3D response surface plots for the measured responses were constructed based on the quadratic model. Since the quadratic model in this study had four independent variables, two variables were held constant at their center level for each plot. The influence of four different process variables on the response factor is visualized in the 3D response surface plots in Fig. 6. Fig. 6 clearly shows interaction effects between operating parameters. Therefore, reliable interpretation of industrial data should be done using combination of experimental, main effect and interaction plots. However, such interpretations are not the subject of this paper.

3.3. DOE-based model development and verification

As proved by statistical analysis of fuzzy results, cut size is a function of feed characteristic size (d_{80}), imperfection coefficient (I), solid content (X) and pulp flowrate (Q); so, it can be expressed that:

$$d_{50} = f(d_{80}, I, X, Q) \tag{4}$$

Therefore, by response surface methodology, a quadratic polynomial equation was developed to predict the cur size as a function of independent variables involving in their interactions (Cornell, 1990; Clarke and Kempson, 1997). In general, the experimental data obtained from the designed experiments is analyzed by response surface regression procedure using the following quadratic polynomial equation (Montgomery, 2013):

$$y = b_0 + \sum b_i x_i + \sum b_{ii} x_i^2 + \sum b_{ij} x_i x_j + \varepsilon \tag{5}$$

where y is the predicted response, b_0 the constant coefficient, b_i the linear coefficients, b_{ii} the quadratic coefficients, b_{ij} the interaction coefficients, x_i and x_j are coded (or actual) values of the independent process variables, and ε is the residual error.
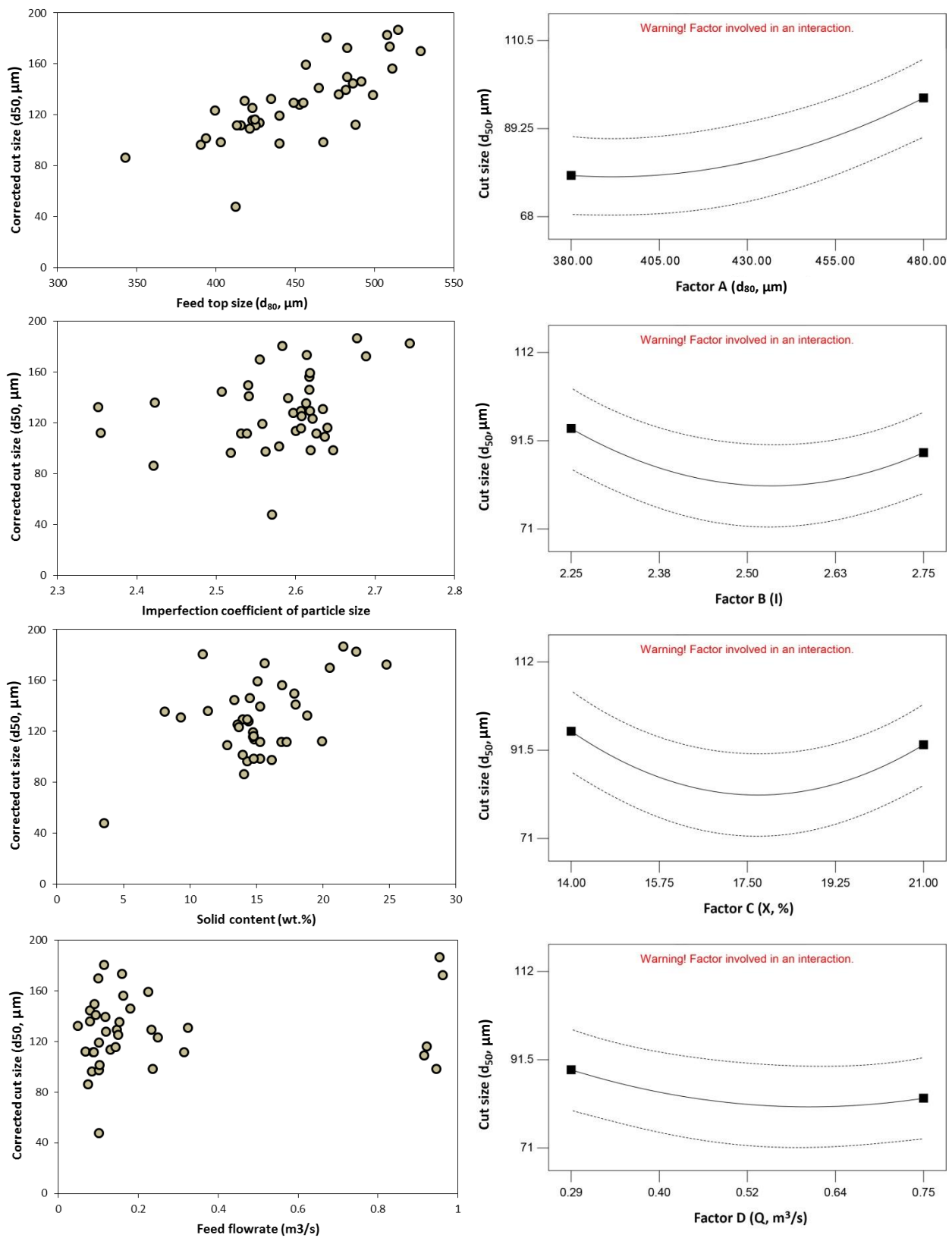
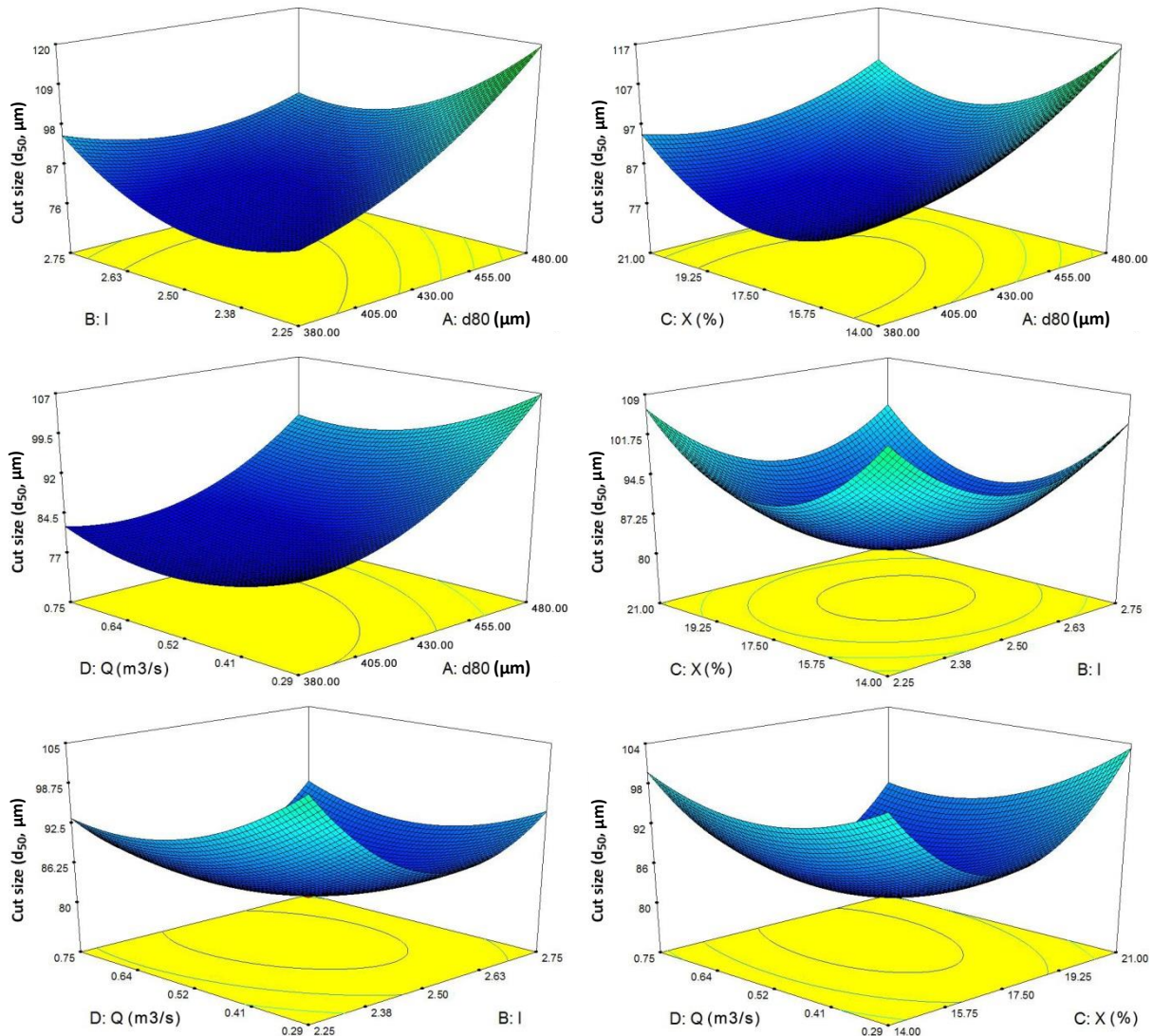Fig. 5. Effects of operating parameters on cut size: experimental (left) and statistical plots (right)

Fig. 6. The response surface plot showing interaction effects of operating parameters on cut size

The values of the coefficients were calculated using Design-Expert Software v.7.1.5 (DX7). The best fitted model equation was obtained as:

$$d_{50} = 243.14 + 1.46d_{80} - 328.14I - 14.73X + 143.78Q - 1.49d_{80}I - 0.06d_{80}X - 0.35d_{80}Q$$
$$+ 1.41IX + 36.17IQ - 9.76XQ + 0.004d_{80}^2 + 180.16I^2 + 1.17X^2 + 58.28Q^2 \tag{6}$$

As seen, the model also includes both insignificant main and interaction parameters to yield the highest prediction accuracy (Montgomery, 2013). Model Eq. (6) was used to evaluate the influence of the process variables on overflow cut size. Analysis of Variance (ANOVA) was performed to test significance of the model. ANOVA results of the quadratic regression model (Eq. (6)) suggested that the model was highly significant as was evident from Fisher's F-test (F model = 83.75) with a very low probability value (p-value < 0.0001).

Normal probability plot of the residuals is an important diagnostic tool to detect and explain systematic departure from the assumption that errors are normally distributed and independent from each other and that the error variance is homogeneous (Yetilmezsoy et al., 2009). Normal probability of the residuals for cut size is shown in Fig. 7(a) and reveals almost no serious violation of the assumptions underlying the analyses, which confirms normality assumptions and independence of the residuals. All of the above considerations indicated adequacy of the developed relationship. A high value of R^2 (0.9957) indicates high dependence and correlation between the measured and predicted values of the response. Cut size values predicted by Eq. (6) are given in Table 1. Moreover, a closely high value of the

adjusted correlation coefficient (Adj R^2 = 0.9839) also shows high significance of the model; also, total variation of about 98% for cut size was attributed to the independent variables and only about 2% of total variation could not be explained by this model. This fact was also confirmed by the predicted versus observed values plot for cut size shown in Fig. 7(b). The Pred R^2 was 0.8279, implying that it could explain variability in predicting new observations. This was in reasonable agreement with the Adj R^2 of 0.9839. Adeq precision measures signal to noise ratio; a ratio of greater than 4 is desirable. In this investigation, the ratio was 27.55, which indicated an adequate signal. Therefore, the model could be used to navigate the design space.
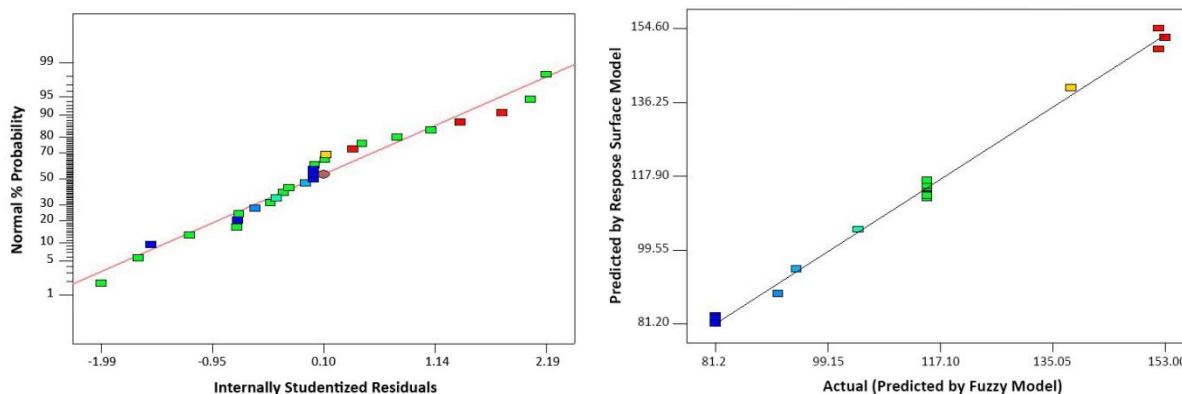

Fig. 7. Normal Probability plot of the residuals for cut size model (a) and Relation between observed and predicted cut size values (b)

The prediction model Eq. (6) was validated by a series of industrial data collected from the studied classifier. Fig. 8 shows the cut size values predicted by RSM model versus those measured in the plant. As seen, predicted results are in acceptable agreement with actual values with determination coefficient of 0.8328. Model precision depends on various factors. Since the DOE response values are determined by fuzzy model, the factors involved in fuzzy modeling process play a key role in model's accuracy. Fuzzy models are developed based on actual data and hence, the number of data used for training fuzzy rules is important. In addition, the precision of the sampling process during industrial measurements is of vital importance. Therefore, it is strongly recommended that sampling is done by engineers rather than operators. The accuracy of experimental data directly affects the definition of the range of data which is required for development of fuzzy rules. Unusual dispersion of the collected data, for example seen for pulp flowrate data, may make ranging process difficult. Therefore, triangular functioning approach was applied in this study, due to its simplicity; additionally, it can approximate most non-triangular functions. However, as mentioned earlier, there are different types of membership functions which can be used for shaping data domains. Some investigators have also used gbel function with smooth edges; in this study, gbel approach was not accepted because its functioning overlap may interfere with statistical model when predicting interactions between operating parameters.

Another key problem comes from naturally varying properties of the coal fed to the plant. Furthermore, many coal washing plants are fed by coal material received from different mines. The feed of Zarand Coal Washing Plant, for example, is supplied from over 10 different coal mines. For these reasons, some coal washing plants inevitably suffer from nonhomogeneous feed problem. A few mathematical equations are developed to describe the behavior of particles in hydraulic classifiers which all are based on general stokes and/or newton laws (Gupta et al., 2006; Gui et al., 2010; Tripathy et al., 2015; Wills and Finch, 2015). However, these models show low efficiency when being used for coal classifiers due to varying properties of coal particles. Therefore, some investigators have tried to develop models which can be calibrated for in-situ operating conditions the classifier working with. For example, Masliyah proposed a mathematical model in which cut size is applied on both sides of model equation (Khoshdast et al., 2012); the cut size is estimated by a reciprocating fitting solution using experimental data to minimize errors. A recently developed KAAJ model employs calibration constants so that the model can be excluded to the working conditions of classifier after a fitting calibration with a series of experimental data. However, the accuracy of these models is still low (below 80%)

(Khoshdast, 2014). In addition, these models can only describe the effect of parameters involved in model construction and thus, the influence of special conditions is missed. As mentioned previously, fuzzy model can estimate the input-output relation in the bases of pure response of cut size to operating parameters using a black box approach and therefore, the effect of special conditions would be inherently involved in the final model. For example, the studied classifier at Zarand Coal Washing Plant is equipped with a baffle plate to moderate the turbulency of sorting chamber. In practice, none of the mathematical models are able to assess the baffle effect on cut size. Moreover, there are other errors caused by sampling vessel, operator, analyses etc., which may not be taken into account in the parametric models. Meanwhile, using fuzzy method accompanied by experimental design facilitates the assessment of any potential interaction effect between dependent operating parameters.
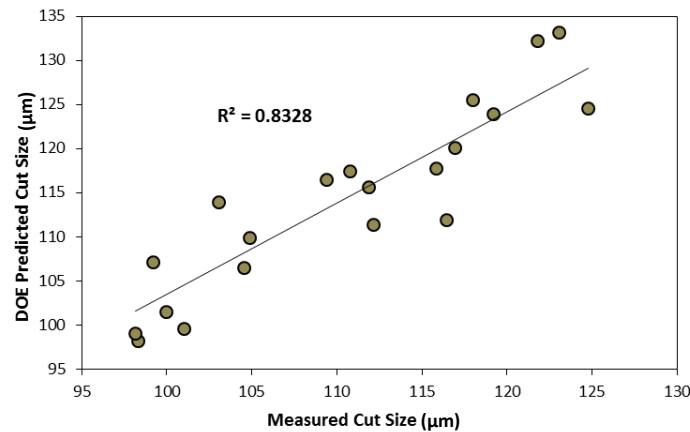

Fig. 8. Comparison between the DOE predicted and experimental values for the cut size

4. Conclusions

In this study, a mixed modeling approach was used to develop a phenomenological model for the prediction of cut size of an industrial hydraulic classifier. The model employs a combination of fuzzy logic and experimental design methods to evaluate the effects of operating parameters on overflow product's cut size of the studied classifier at Zarand Coal Washing Plant. Model validation using actual data showed that model can successfully be employed in industrial environment. The modeling method seems to be promising for simulating all industrial devices and can also be used for the development of prediction equations for evaluating potential relations between input variables to the device and process response thereof. In this paper, triangular function was used due to its simplicity and easy implementation in a computer program; however, it is not continuously differentiable. Thus, the evaluation of other membership functions may also be useful for the optimization of modelling efficiency.

Acknowledgments

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