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SHADOW BANKING AND LIFE INSURANCE POLICYHOLDER PROTECTION

This paper develops a life insurance liability valuation model that integrates the balance-sheet insurer loans with the shadow banking entrusted loans in a premature default risk environment. It is shown that the life insurance policyholder significantly benefits from the entrusted loan activities in a less likely premature default risk environment. The policyholder protection is increased in accord with a high guaranteed interest rate, particularly when the life insurance company has ample access to entrusted loans. The policyholder protection is also significantly increased by a high participation level when the life insurance company 'shrinks away' from accessing entrusted loans. Overall, the authors concluded that shadow banking entrusted loans help policyholder protection.

Keywords: entrusted loan, guaranteed rate, participation rate, barrier option

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1. INTRODUCTION

The subprime-related 2007/2008 global financial crisis raised fundamental issues about the role of shadow banking activities. An important fact is that these activities have grown outside of the monitoring byeeingregulators. Adrian and Shin (2010) argued that the financial crisis can be viewed as a liquidity crisis that originated in shadow banking activities. As was also pointed out by Brennan et al. (2013), insurance companies try to match maturity profiles of assets and liabilities, however several insurance failures arose from concentrations in balance-sheet and off-balance-sheet

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illiquid assets matched by liabilities that accelerated in a time of stress.[1] Yet, Pozsar et al. (2013) suggested that increased liquidity standards for insurance companies will likely enhance the returns to shadow banking activities. There has been considerable research effort towards modelling life insurance liabilities for the purpose of analysing interest rate guarantees and participation distribution schemes in a profit-sharing life insurance policy (Chen and Suchanecki, 2007). However, little attention has been paid to the effects of shadow banking activities on the liabilities of a life insurance company. This can be justified based on an insurance asset-liability matching argument in the spirit of Collier et al. (2015): insurance companies are the new shadow banks.

In the presented paper, the authors construct a contingent claim model along the lines of Grosen and Jørgensen (2002), for the valuation of the equity and the liability of a life insurance company (or a life insurer, an insurer, or a company informally in this model). Their main contribution is to explicitly consider default risk in a contingent claim model where default can occur at any time before the maturity date, i.e. a simple knock-out barrier option feature is added to the different components of the life insurance contract. The principal advantage of that approach is the explicit treatment of knock-out uncertainty which has played a prominent role in discussions of intermediary behaviour. This, however, omits two aspects of the behaviour of life insurers. First, it is assumed that asset (i.e. loan) and life insurance contract markets are perfectly competitive so that quantity-setting is the relevant behavioural mode in both markets. This assumption is not applicable to loan markets since such markets are virtually always concentrated where life insurance companies set rates and face random loan levels. The effect of loan quantity-setting behaviour is that liquidity considerations are ignored. Second, the approach ignores the shadow banking activities (e.g. entrusted loans) incurred in liquidity operations.

In light of previous work, the purpose of this paper was to develop a knock-out barrier option model to evaluate the equity and liabilities of a life insurance company which additionally operates shadow banking entrusted loans. Only one type of life insurance policy, the profit-sharing policy as specified by Briys and de Varenne (1994), was considered here, , in which the policyholder is entitled to a guaranteed interest rate and a profit-sharing participation ratio of the company's net financial revenues regulated by the insurance authority. The results of this paper show how guaranteed rate, profit-sharing participation ratio, entrusted loan, and interest rate conditions jointly determine the optimal insurer interest margin, namely the spread

[1] Below is a partial list of insurance companies that have been taken over by state insurance departments: Medical Savings Insurance Company in 2009, Universal Life Insurance Company in 2010, Golden State Mutual Life Insurance Company in 2011, Standard Life Insurance Company of Indiana in 2012, Executive Life Insurance Company of New York in 2013, and See Change Health Insurance Company (National Organization of Life & Health Insurance Guaranty Associations, 2015).

between the loan rate and the guaranteed rate. The life insurance liabilities evaluated at the optimal margin are derived and analysed.

The study arrived at three main results. First, shadow banking entrusted loans help the life insurance company to increase its equity return, but deteriorate its life insurance liabilities, in particular when the company is facing a low risk of a premature default. This result implies, for example, that in 2014 the Chinese regulatory authorities introduced stricter rules about shadow loans to protect insurance policyholders. As a result, in the first two months of this year, bank loans rose to 77% of total financing, up from 64% a year earlier (Collier et al., 2015). The authors argue that it looks like shadow finance is on the wane due to the shadow banking regulation in the life insurance industry, and thus the policyholder protection is increased. Second, the required guaranteed interest rate of the life insurance policy significantly encourages the life insurance company to increase its equity return when the company gets less involved in shadow banking entrusted loan activities, but significantly diminishes the life insurance liabilities when the company gets more involved in the entrusted loan activities. The regulatory guaranteed interest rates as such are not guaranteed to produce greater safety for the life insurance company when the company has ample access to shadow loans. Third, the participation level of the life insurance company discourages the company from increasing its equity return and enhances the life insurance liabilities, in particular when the company becomes less involved in the entrusted loan activities. The regulatory participation rate as such is also not guaranteed to produce greater safety for the life insurance company when the company shrinks away from accessing the shadow loans. This paper contributes to the debate on life insurance regulation in financial intermediation by adding to the role played by shadow banking.

The remainder of this paper is organised as follows. Section 2 discusses the related literature. Section 3 lays out the basic model of a life insurance company's equity value and liability. Section 4 characterises the optimal policy loan rate and develops the comparative static properties of the model. Section 5 presents a numerical analysis to explain the intuition of the comparative static results. The final section concludes the paper.

2. RELATED LITERATURE

The authors' theory of insurer liability management is related to a fair valuation strand of the literature. Briys and de Varenne (1994) constructed a contingent model to evaluate the equity of a life insurance company where the liability consists only of the policyholder's payments. The authors concluded that policyholders benefit from a guaranteed interest rate and a percentage (a participation level) of the performance of the company's asset portfolio, contributing to insurance stability.

Grosen and Jørgensen (2002) took the model of Briys and de Varenne (1994) as the point of departure. A contribution of their paper is to explicitly consider the risk of a premature default to the valuation of a life insurance contract. More specifically, the stakeholders' claims will change from plain vanilla options (Briys and de Varenne, 1994) to more exotic option types with features similar to financial knockout barriers option (Grosen and Jørgensen, 2002). This extension opens up for a wide range of interesting analyses in relation to the issues discussed in the life insurance liability literature.

Chen and Suchanecki (2007) developed a contingent claim model along the lines of Briys and de Varenne (1994) and Grosen and Jørgensen (2002), for the valuation of a life insurance company's equity and liability. Chen and Suchanecki (2007) argued that the Grosen and Jørgensen (2002) modelled the insolvency risk, which does not reflect the reality well, based on an important assumption that default and liquidation are considered as equivalent events, and therefore extended their model to study the effect of bankruptcy procedure on the valuation of the life insurance company's liabilities. Their study was realised using a standard and cumulative Parisian barrier option framework.

The three related papers above focus on an analysis of the participating life insurance contracts with a built-in minimum interest rate guarantee in a contingent claim framework. However, this fair pricing approach generally only works under the assumption of perfectly competitive markets and does not consider the insurer's spread behaviour. As mentioned in the introduction, the effect of spread behaviour is that liquidity consideration, which has also played a prominent role in discussions of intermediary behaviour, is suggested to be integrated with the analysis of life insurance contracts, in particular when shadow banking activities are emphasised. While the authors also take a contingent claim approach to integrate insurer spread behaviour with the market valuation of the equity and the liabilities in a life insurance company, the focus on the entrusted loan management aspects of shadow banking takes this analysis in an alternative direction of life insurance contract literature.

3. THE VALUATION MODEL

The present setting is closest to the one described in Grosen and Jørgensen (2002), whose description this study partially adopted. A life insurance company whose planning horizon extends over a given time interval $t \in [0,1]$ is considered. Time $t=1$ can be considered as the time to maturity of a single cohort of life insurance policies issued at $t=0$, and as the time at which the life insurance company is subject to a comprehensive on-site audit by the insurance authority. At time $t=0$, the life insurance company acquires an asset portfolio consisting of risky loans A and default-free liquid assets B, and finances this portfolio with the premiums of life insurance cohort L and paid-in capital K. The proportion of initial assets $A+B$ financed by equity capital is given by $(1-\alpha)$, where $0<\alpha<1$ is a leverage variable. The initial balance sheet is given in Table 1. The portfolio of assets $A+B$ during the period horizon is assumed to be totally invested in the financial markets.

Table 1
The bank's initial balance sheet

Assets		**Liabilities and equity**	
Loans	A	Life insurance cohort	$L=\alpha(A+B)$
Liquid assets	B	Equity	$K=(1-\alpha)(A+B)$
Total	$A+B$	Total	$A+B$

Source: self-created.

The company's loans in the model, for simplicity, belong to a single homogenous class of fixed-rate claims that mature at $t=1$. The demand for loans is governed by a downward-sloping demand function, $A(R_A)$, where $R_A>0$ is the loan rate chosen by the company. The assumption $\partial A/\partial R_A<0$ implies that the company exercises some monopoly power in its loan market. In addition to loans, the company can also hold a number of B liquid assets on its balance sheet during the period. These assets earn the security-market interest rate $R_S>0$.

The initial asset portfolio $A+B$ is financed partly by a life insurance policy. The authors followed Briys and de Varenne (1994) and structured a type of profit-share life insurance policy as follows. The regulation of life insurance contracts, and in particular of participating policies, includes (i) bounds on the interest rate guarantee $R>0$, which is usually less than the security-market interest rate ($R_S>R$),[2] and (ii) the participation rate $0<\delta<1$ in the annual return of the company's asset portfolio. The participation rate can be viewed as making up for the difference between the guaranteed rate and the security-market interest rate and embodying the required risk premium by policyholders holding the contracts.

In addition to balance-sheet life insurance activities, the company is also involved in shadow banking activities with a special emphasis on entrusted loans in this model. There are entrusted loans M made by a firm (trustor) in the non-financial sector that is run by the life insurance company (trustee) for legal reasons, but with the company indemnified from the credit risk of the borrowing firm by the trustor

[2] As pointed out by Eling and Holder (2013), the guaranteed interest rate is proportional to the current average market interest rate at contract inception, typically 60% of the 10-year rolling average of government bond yields. The authors refer to it as the 60% rule.

(Elliott et al., 2015).[3] The fund is then lent through the company to the borrowing firm at a specified interest rate R_A as instructed by the trustor.[4] As a trustee, the company helps collect the principal with interest $(1+R_A)M$ from the borrowing firm on the behalf of the trustor. Therefore, the company is not providing the capital for the shadow banking project, but it charges a commission mM for the service as well as the credit risk burden if the repayments to the trustor from the borrowing firm are insured by the company – this is the case that the study focused on. The life insurance company's shadow banking entrusted loan operation is given in Figure 1.

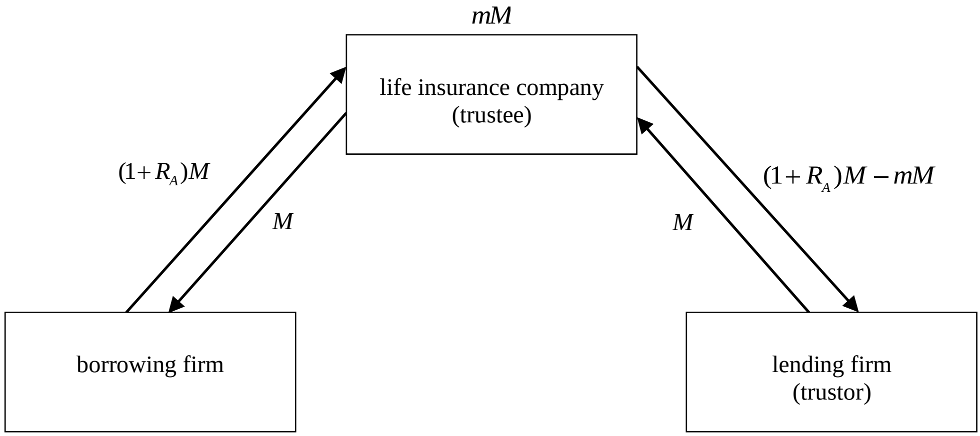


Fig. 1. The life insurance company's shadow banking entrusted loan operation

Source: self-created.

The life insurance company's objective is to set R_A to maximise the market value of a barrier option function defined in terms of profits, subject to the company's initial balance sheet. A direct implication of this framework is that equity will be priced as a form of a down-and-out call (*DOC*) option. The market value of the life insurance company's underlying assets in the life insurance and shadow banking activities follows a geometric Brownian motion of the form:

$$dV = \mu V dt + \sigma V dW, \tag{1}$$

[3] Note that the trustee in Elliott et al. (2015) is assumed to be a bank. However, the authors argue that this assumption is applicable to the case of a life insurance company since both are financial firms permitted to operate shadow banking activities by the regulatory authorities.

[4] For the sake of simplicity, the entrusted loan interest rate set by the trustor is assumed to be equal to the loan rate set by the life insurance company.

where

$$V = (1+R_A)A + (1+R_A)M$$

and where V is the expected repayment value from the asset portfolio with the instantaneous drift μ and the instantaneous volatility σ. W is a standard Wiener process. The first term V can be identified as the expected repayments from the balance-sheet risky loans, and the second term can be identified as the expected repayments from the off-balance-sheet entrusted loans. The equity position is a hybrid position and its value during the period is given by:[5]

$$S = DOC(V,Z) - \delta DOC(\alpha V, Z) = (SC - DIC) - \delta(SC_\alpha - DIC_\alpha), \qquad (2)$$

where:

$$SC = VN(a_1) - Ze^{-(R_S - R)}N(a_2)\ ,$$

$$DIC = V\left(\frac{H}{V}\right)^{2\eta} N(b_1) - Ze^{-(R_S - R)}\left(\frac{H}{V}\right)^{2\eta-2} N(b_2)\ ,$$

$$Z = \alpha(A+B)e^R - (1+R_S)B + (1+R_A)M - mM\ ,\ 0 < m < 1,$$

$$H = \beta Z\ ,\ 0 < \beta < 1,$$

$$\eta = \frac{R_S - R}{\sigma^2} + \frac{1}{2},$$

$$a_1 = \frac{1}{\sigma}\left(\ln\frac{V}{Z} + (R_S - R) + \frac{\sigma^2}{2}\right),\ a_2 = a_1 - \sigma\ ,$$

$$b_1 = \frac{1}{\sigma}\left(\ln\frac{H^2}{VZ} + (R_S - R) + \frac{\sigma^2}{2}\right),\ b_2 = b_1 - \sigma\ ,$$

$$SC_\alpha = \alpha VN(c_1) - Ze^{-(R_S - R)}N(c_2)\ ,$$

$$DIC_\alpha = \alpha V\left(\frac{H}{\alpha V}\right)^{2\eta} N(d_1) - Ze^{-(R_S - R)}\left(\frac{H}{\alpha V}\right)^{2\eta-2} N(d_2),$$

[5] A down-and-out call (.. DOC ..) option in general includes three terms: a standard call (SC) option, a down-and-in call (DIC) option, and a rebate received by the equity holder if the barrier is reached before expiration (Merton, 1973). This model follows Brockman and Turtle (2003) and ignores the rebate term in the authors' analysis.

$$c_1 = \frac{1}{\sigma}\left(\ln\frac{\alpha V}{Z} + (R_S - R) + \frac{\sigma^2}{2} \right),\ c_2 = c_1 - \sigma\,,$$

$$d_1 = \frac{1}{\sigma}\left(\ln\frac{H^2}{\alpha VZ} + (R_S - R) + \frac{\sigma^2}{2} \right),\ d_2 = d_1 - \sigma\,,$$

$$0 < \delta < 1.$$

In Equation (2), the strike price of the SC option is the value of the company's net-obligation payments *Z*. The first term on the right-hand side of *Z* can be interpreted as the guaranteed payoff to policyholders, the second term can be interpreted as the repayments from the company's liquid-asset investment, the third term can be interpreted as the entrusted loan repayments to the trustors, and the last term can be treated as the commission for the entrusted loan services. $(R_S - R) > 0$ is the default-free discounted rate. $H = \beta Z$ with the condition of $0 < \beta < 1$ is defined as the value of the company's assets that triggers bankruptcy (this is the barrier or knock-out value of the company) and β is specified as the barrier-to-debt ratio. $N(\cdot)$ is the cumulative density function of the standard normal distribution. δ is the regulatory participation level.

Equity in Equation (2) is a portfolio of two DOCs. The first term $DOC(V, Z)$ is the limited-liability barrier option, including that SC is recognised as the expected company value and the present value of the debt payment using the call option view, and DIC, the down-and-in call option, lay dormant until a barrier is breached. Shareholders of the company have the option to walk away if things go wrong. The DIC provides its holders with a call option, which offers protection to policyholders by allowing them to "call in their chips" before asset values deteriorate further. The second term $\delta DOC(\alpha V, Z)$ corresponds to a short position. In that position, shareholders have written a call to policyholders by introducing a contractual asset-based participation clause. The equity value of the life insurance company is thus made of a long barrier call position and a short barrier call option, the latter being weighted by the participation coefficient δ. It is easily observed that barrier options are a wider class than call options, because as H approaches zero in Equation (2), both the DIC and DIC_α terms vanish, and one arrives at the usual call option price that captures the company's equity.

As far as liabilities are concerned, the value of the company's liabilities based on the specification of Equation (2) as of the time $t = 1$ is given by:

$$Lia = \left\lfloor Ze^{-(R_S - R)} - PUT(V, Z) + DIC \right\rfloor + \delta DOC\left(\alpha V, Z\right), \tag{3}$$

where

$$PUT(V,Z) = Ze^{-(R_S - R)}N(-a_2) - VN(-a_1)\ .$$

The first two terms on the right-hand side of the term $[\cdot]$ in Equation (3) represent the Merton (1974) value of debt. The first term is the discounted value of the payments to policyholders and the trustor net of shadow banking service charges. The second term is the put option or the value of the fair (re)insurance needed in order to make insurance policies and shadow banking payments to the trustor risk--free. The term DIC demonstrates that both the policyholders and the trustor would cash in on this option if they were able to jointly seize the assets of the company when the company's assets dropped to *H*. As the barrier increases, debt behaves more like equity and equity converges to zero. More specifically, the first term on the right-hand side represents the value of a risky policy without participation. The second term on the right-hand side is a barrier call option on the α fraction of the company with the exercise price *Z*. Thus, the liabilities are made up of a long position on a barrier call on a risk-free payoff, a short position on a put on default and a short position on a barrier call on financial revenues.

4. SOLUTION AND RESULTS

Partially differentiating Equation (2) with respect to R_A, the first-order condition is given by:

$$\frac{\partial S}{\partial R_A} = \frac{\partial DOC(V,Z)}{\partial R_A} - \delta \frac{\partial DOC(\alpha V, Z)}{\partial R_A} = 0. \tag{4}$$

A sufficient condition for an optimum is $\partial^2 S / \partial R_A^2 < 0$. The first term of Equation (4) can be identified as the marginal equity value of the loan rate in the long barrier call position, while the second term can be identified as that in the short barrier call position. The optimal loan rate (and thus the optimal insurer interest margin) is chosen for the equity return maximisation where both the marginal values are equal. One can further substitute the optimal loan rate to obtain the company's liabilities in Equation (3) staying on the maximisation of optimisation.

Having examined the solution to the company's optimisation problem, the authors considered the effects on the company's liabilities from changes in the entrusted loans, the required guaranteed interest rate, and the participation level. Differentiation of Equation (3) evaluated at the optimal loan rate with respect to *M*, *R*, and δ yields:

$$\frac{dLia}{dP} = \frac{\partial Lia}{\partial P} + \frac{\partial Lia}{\partial R_A}\frac{\partial R_A}{\partial P}, \tag{5}$$

where:

$$\frac{\partial R_A}{\partial P} = -\frac{\partial^2 S}{\partial R_A \partial P} / \frac{\partial^2 S}{\partial R_A^2},$$

$$P = M,\ R, \text{ or } \delta .$$

Investigating changes in the option value to small changes in parameter variables is essential for life insurance liability management. To that end, the authors computed the comparative static results of Equation (5). In general, the added complexity of path-dependent options does not always lead to clear-cut results, but one can certainly speak of derived results for reasonable parameter levels corresponding roughly to the life insurance company with invested loans and entrusted loans. In the next section, the authors assess the comparative static results by conducting the numerical analysis.

5. NUMERICAL RESULTS

This section first presents the values of the baseline parameters and the endogenous variable. Second, the authors report and discuss the valuations in relation to the barrier options. Third, for each result, the procedure used to obtain it is discussed. Finally, the authors show the comparative static results and provide the intuition for each main finding.

In the following exemplary analysis, it was assumed that the levels of the parameters, unless otherwise indicated, are $\alpha = 0.90$, $R_S = 4.00\%$, $\sigma = 0.30$, $\beta = 0.60$, $B = 200$, and $m = 0.10\%$. Let demand for loans (R_A (%), A) change from (5.80, 33.9) to (6.40, 27) due to its downward-sloping condition. The values of the parameters are explained as follows; (i) $\alpha = 0.90$ is due to Briys and de Varenne (1994) who assumed $0.70 \le \alpha \le 0.99$ in their numerical analysis for life insurance regulation; (ii) the condition of $R_A > R_S = 4.00\%$ demonstrates that R_A, the loan rate with compensation risk, is in general greater than R_S, the risk-free security market interest rate; (iii) $\sigma = 0.30$ is also due to Briys and de Varenne (1994), assuming $0.10 \le \sigma \le 0.50$ in their numerical analysis; (iv) $\beta = 0.60$ used in this numerical analysis is based on an empirical finding of Brockman and Turtle (2003) that the mean value of the barrier is 0.6920 with a corresponding standard deviation of 0.2259; (v) $B = 200$ indicates that the life insurance company holds a high percentage of risk-free assets in its earning-asset portfolio, for example, 86.96%

when $A = 30$; this evidence is from M Financial Group (2015).[6] Finally, (vi) $m = 0.10\%$ is a constant commission rate for the entrusted loan services.

Table 2
Validation of hitting the barrier before the expiration date in the objective function

	$(R_A$ (%), $A)$						
M	(5.80, 33.9)	(5.90, 33.8)	(6.00, 33.5)	(6.10, 33)	(6.20, 32)	(6.30, 30)	(6.40, 27)
	DIC (10^{-6})						
30	0.4537	0.4456	0.4246	0.3921	0.3339	0.2380	0.1356
32	0.6790	0.6680	0.6394	0.5949	0.5145	0.3794	0.2293
34	0.9831	0.9685	0.9306	0.8716	0.7640	0.5800	0.3687
36	1.3824	1.3637	1.3149	1.2385	1.0984	0.8553	0.5680
38	1.8947	1.8711	1.8096	1.7132	1.5351	1.2219	0.8428
40	2.5384	2.5093	2.4333	2.3139	2.0923	1.6981	1.2105
	DIC_α (10^{-6})						
30	2.7973	2.7502	2.6272	2.4363	2.0921	1.5182	0.8904
32	4.0960	4.0333	3.8694	3.6141	3.1496	2.3600	1.4642
34	5.8117	5.7305	5.5183	5.1864	4.5778	3.5262	2.2955
36	8.0217	7.9192	7.6508	7.2298	6.4528	5.0909	3.4542
38	10.8060	10.6792	10.3471	9.8245	8.8545	7.1333	5.0166
40	14.2462	14.0923	13.6888	13.0524	11.8652	9.7359	7.0645

Note: unless otherwise indicated, $\alpha = 0.90$, $R_S = 4.00\%$, $\sigma = 0.30$, $\beta = 0.60$, $B = 200$, $m = 0.1\%$, $R = 3.00\%$, and $\delta = 0.85$

Source: self-created.

As mentioned previously, a significant weakness of the path-independent approach is that it evaluates stakeholders' claims only at the maturity date. The results obtained from Table 1 correct this weakness. Let the amount of entrusted loans increase from 30 to 40. When $R = 3.00\%$ and $\delta = 0.85$, the authors present evidence that both DIC in the long barrier call position and DIC_α in the short barrier call position are consistently positive in sign. By ignoring the existence of barriers, both the long and short standard call options are overvalued. The result is understood because the stakeholders' claims have changed from plain vanilla options to more exotic types of options with features with the so-called knockout barrier option. Specifically, policyholders will be compensated in the event of premature closure at the hitting time β. The authors' barrier presentation for the equity valuation of a life insurance company is largely supported by Grosen and Jørgensen (2002).

[6] M Financial Group (2015) reports that the portfolios of top 30 life insurance companies in the United States include a well-diversified mix of 85.70% risk-free assets (for example, bonds and mortgages) and 14.30% risky assets (for example, policy loans and stocks).

Table 3
Responsiveness of life insurance liabilities to shadow banking entrusted loans

	$(R_A$ (%), $A)$						
M	(5.80, 33.9)	(5.90, 33.8)	(6.00, 33.5)	(6.10, 33)	(6.20, 32)	(6.30, 30)	(6.40, 27)
	$\partial R_A / \partial M$ (‰)						
30→32	-	0.6296	1.0557	0.5995	0.5258	0.9960	-
32→34	-	0.6020	0.9863	0.5548	0.4823	0.9119	-
34→36	-	0.5754	0.9195	0.5117	0.4402	0.8293	-
36→38	-	0.5501	0.8565	0.4710	0.4003	0.7500	-
38→40	-	0.5264	0.7979	0.4332	0.3631	0.6753	-
	$dLia / dM$: total effect						
30→32	-	0.9860	0.9846	0.9852	0.9834	0.9695	-
32→34	-	0.9872	0.9860	0.9866	0.9851	0.9725	-
34→36	-	0.9883	0.9872	0.9879	0.9867	0.9753	-
36→38	-	0.9893	0.9883	0.9891	0.9881	0.9779	-
38→40	-	0.9902	0.9894	0.9901	0.9893	0.9803	-

Notes: unless otherwise indicated, $\alpha = 0.90$, $R_S = 4.00\%$, $\sigma = 0.30$, $\beta = 0.60$, $B = 200$, $m = 0.1\%$, $R = 3.00\%$, and $\delta = 0.85$. The shaded areas represent the corresponding values with an approximate optimal loan rate of 5.90%. The direct effect $(\partial Lia / \partial M)$ is positive in sign, while the indirect effect $(\partial Lia / \partial R_A)(\partial R_A / \partial M)$ is negative in sign. The indirect effect is insufficient to offset the direct effect.

Source: self-created.

First, the study considers the impacts on the optimal loan rate and further on the liabilities of the life insurance company from increases in the entrusted loans. The results of Equation (5) where $P = M$ observed from Table 3 are stated in the following proposition.

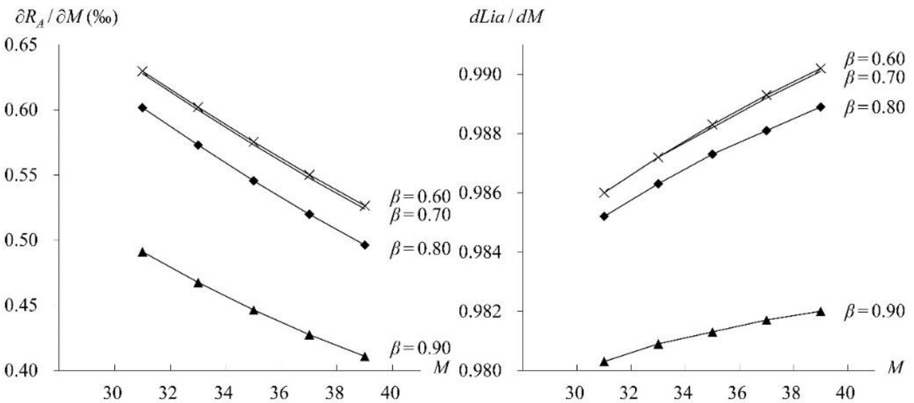
Proposition 1. The shadow banking entrusted loan involvement increases the loan rate and the liabilities of the life insurance company.

As the company increases the entrusted loan, it must now provide a return to a larger risk base. One way the company may attempt to augment its total returns is by shifting its investments to the liquid-asset market and away from the loan. If loan demand is relatively rate-elastic, a lesser loan is possible at an increased loan rate. Accordingly, it may be argued that entrusted loans and loans are substitutes in the earning-asset portfolio of the company. The earnings from the margin typically account for a significant portion of company profits. As a result, $\partial R_A / \partial M > 0$ can be explained that an increase in the entrusted loan leads to increasing the profit of the life insurance company. This result is supported by the report by Collier et al. (2015).[7]

[7] The Chinese insurance industry broke the record in 2014. Premium income for 2014 increased at the highest rate since the 2008 financial crisis, specifically, growing at 17.5% year-over-year to Rbm 2.0

An explanation of the impact on the company's liabilities from increases in the entrusted loan involvement is possible in terms of the direct and indirect effects. The direct effect is positive because an increase in the entrusted loan increases the barrier call on the risk-free payoff, the put on the default, and the barrier call option on financial revenues, and then enhances the liabilities, holding the optimal loan rate constant. The indirect effect is negative. As mentioned previously, the sign $\partial R_A / \partial M$ is positive. Further, an increase in the loan rate decreases the company's liabilities since the risky asset of the loan held by the company is decreased. An increase in the entrusted loan, thus, decreases the company's liabilities through the loan rate-setting mechanism. The negative indirect effect is insufficient to offset the positive direct effect to give an overall positive response of the company's liabilities to an increase in the entrusted loan. Overall, the authors concluded that, as the entrusted loan increases, the life insurance liabilities are increased due to the company's increased risky investments based on a liquidity argument. This finding is consistent with Collier et al. (2015).

Next, the results of Equation (5), where $P = M$ at various levels of knockout barrier obtained from Figure 2, are stated in the following proposition.



Notes: unless otherwise indicated, $\alpha = 0.90$, $R_S = 4.00\%$, $\sigma = 0.30$, $B = 200$, $m = 0.1\%$, $R = 3.00\%$, and $\delta = 0.85$. The values at various levels of the barrier are computed based on an approximate optimal loan rate of 5.90%.

Fig. 2. Responsiveness of life insurance liabilities to shadow banking entrusted loans at various levels of the barrier

Source: self-created.

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trillion. The authors expect the industry's net profit to more than double, rising 106.4% year-over-year to Rmb204.7 billion in 2014 (Collier et al., 2015).

Proposition 2. As the barrier is raised, the positive effect of the shadow banking entrusted loan on the loan rate is decreased, and that on the life insurance liabilities is decreased as well.

At a given level of the barrier, an increase in the entrusted loan increases the optimal loan rate (and thus the company's profit), and further increases the company's life insurance liabilities, as mentioned earlier. In addition, as the barrier is raised, the terms of SC and SC_α are not affected, while the terms of DIC and DIC_α are increased. As usual, the sign of the difference between DIC and δDIC_α in Equations (2) and (3) is indeterminate. However, the increased value DIC is much higher than the increased value of αDIC_α because of $DIC > DIC_\alpha$ and $0 < \delta < 1$. As the policyholder protection is increased due to an increase in the barrier, a transfer of wealth from equity holders to policyholders takes place, implying better protection of policyholders, which results in decreasing the company's equity return and liabilities. The latter result is consistent with evidence that this mechanism is shown to reduce life insurance liabilities of the company, and hence implicitly reduces the insolvency risk of the issued contracts (Grosen and Jørgensen, 2002).

Table 4
Responsiveness of life insurance liabilities to the required guaranteed interest rate

	$(R_A$ (%), $A)$						
R (%)	(5.80, 33.9)	(5.90, 33.8)	(6.00, 33.5)	(6.10, 33)	(6.20, 32)	(6.30, 30)	(6.40, 27)
	$\partial R_A / \partial R$ (%)						
2.25→2.50	-	0.2835	0.8477	0.5652	0.5631	1.1091	-
2.50→2.75	-	0.2901	0.8639	0.5757	0.5736	1.1309	-
2.75→3.00	-	0.2960	0.8786	0.5853	0.5832	1.1510	-
3.00→3.25	-	0.3014	0.8916	0.5938	0.5918	1.1691	-
3.25→3.50	-	0.3060	0.9030	0.6013	0.5993	1.1853	-
	$dLia / dR$: total effect						
2.25→2.50	-	0.4842	0.4641	0.4568	0.4241	0.2563	-
2.50→2.75	-	0.4930	0.4725	0.4652	0.4318	0.2607	-
2.75→3.00	-	0.5019	0.4811	0.4736	0.4398	0.2655	-
3.00→3.25	-	0.5108	0.4897	0.4822	0.4479	0.2707	-
3.25→3.50	-	0.5198	0.4984	0.4908	0.4561	0.2763	-

Notes: unless otherwise indicated, $\alpha = 0.90$, $R_S = 4.00\%$, $\sigma = 0.30$, $\beta = 0.60$, $B = 200$, $m = 0.1\%$, $M = 36$, and $\delta = 0.85$. The shaded areas represent the corresponding values with an approximate optimal loan rate of 5.90%. The direct effect $(\partial Lia / \partial R)$ is positive. The indirect effect $(\partial Lia / \partial R_A)(\partial R_A / \partial R)$ is negative. The indirect effect is insufficient to offset the direct effect.

Source: self-created.

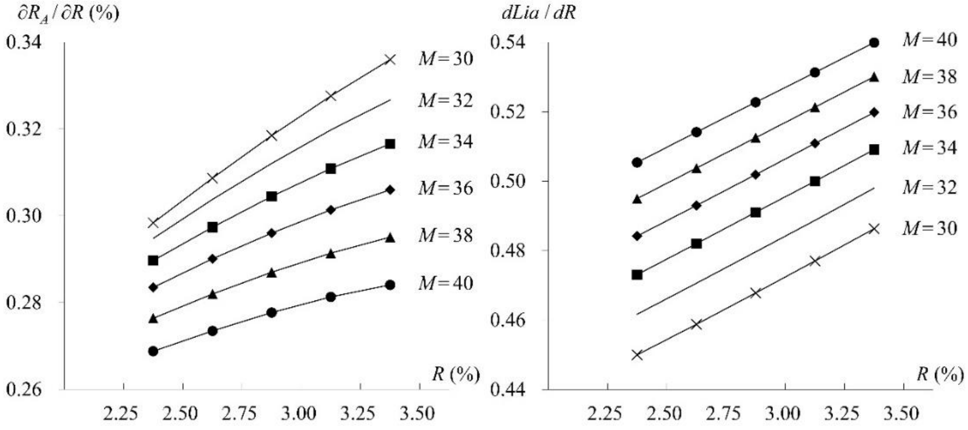
To elaborate on the issue of the guaranteed interest rate in the life insurance policy, the authors computed Equation (5) where $P = R$, illustrated the numerical findings in Table 4, and showed the following proposition.

Proposition 3. An increase in the guaranteed interest rate increases the loan rate and the liabilities of the life insurance company.

When the regulatory authority forces the insurer to increase the required guaranteed interest rate in order to protect the policyholders, the insurer must now provide a return to a larger cost base. One possible way the insurer may attempt to augment its total returns is to reduce its loans. If loan demand faced by the insurer is relatively rate-elastic, a less loan portfolio is possible at an increased loan rate. The rationale is that an increase in the guaranteed interest rate makes loans more costly to grant. In response to this, the insurer has an incentive to reduce the number of loans it grants by charging a higher loan rate.

Next, the authors examined the impact on the liabilities from changes in the required guaranteed interest rate. The positive direct effect captures the increased Lia due to an increase in R, holding the optimal loan rate constant. This is because an increase in the guaranteed rate makes the insurer have a higher cost burden, enhancing the life insurance liabilities of the company, *ceteris paribus*. The negative indirect effect arises because an increase in R decreases Lia through the optimal loan rate adjustment. A decreased amount of loans held by the insurer due to an increase in the guaranteed rate decreases the life insurance liabilities. The negative indirect effect is insufficient to offset the positive direct effect to give an overall positive response of life insurance liabilities to an increase in the guaranteed interest rate. This result is consistent with that of Grosen and Jørgensen (2000), namely that the interest rate guarantees issued with participating life insurance policies have threatened the solvency of the issuing companies.

It is of interest to further analyse the effects of the required guaranteed interest rate on the company's liabilities at various levels of shadow banking entrusted loan activities. The results observed from Figure 3 are stated in the following proposition.



Notes: unless otherwise indicated, $\alpha = 0.90$, $R_S = 4.00\%$, $\sigma = 0.30$, $\beta = 0.60$, $B = 200$, $m = 0.1\%$, and $\delta = 0.85$. The values at various levels of entrusted loan rates are computed based on an appropriate optimal loan rate of 5.90%.

Fig. 3. Responsiveness of life insurance liabilities to the required guaranteed interest rate at various levels of shadow banking entrusted loans

Source: self-created.

Proposition 4. As the shadow banking entrusted loan increases, the positive effect of the guaranteed rate on the loan rate is decreased, and that on the life insurance liabilities is increased.

At a given level of the entrusted loan, the study shows that an increase in the guaranteed interest rate enhances the loan rate and further the life insurance liabilities, as stated earlier. Furthermore, as the amount of the entrusted loans increases, the positive effect of the guaranteed rate on the loan rate is reduced, and the positive effect of the guaranteed rate on the liabilities is reinforced. Entrusted loans as such make the insurer more prone to loan risk-taking when the guaranteed interest rate is increased, thereby increasing the life insurance liabilities. The authors argue that it is important to take into account the role played by the shadow banking entrusted loans when the policyholder is considered to be protected, affecting the stability of the life insurance company.

The study further examined the relations among participation levels, life insurance liabilities, and the optimal loan rate. The results of Equation (5), where $P = \delta$ obtained from Table 5, are stated in the following proposition.

Table 5
Responsiveness of life insurance liabilities to participation level

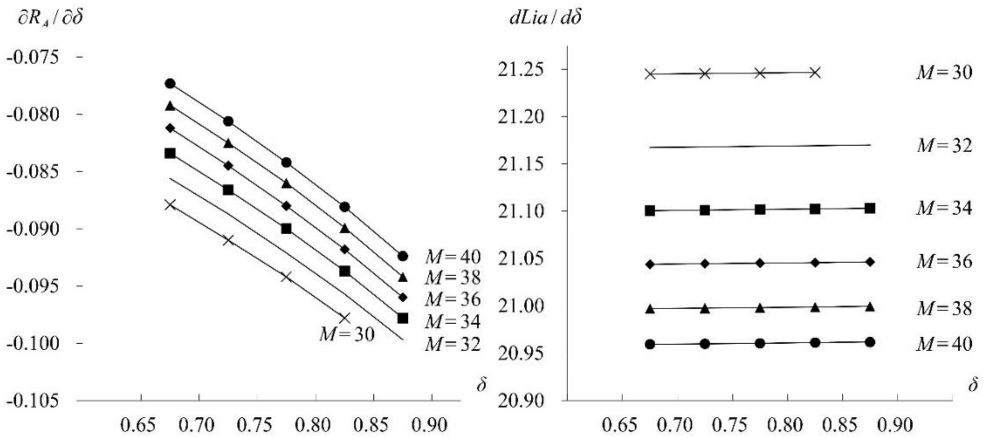
	(R_A (%), A)						
δ	(5.80, 33.9)	(5.90, 33.8)	(6.00, 33.5)	(6.10, 33)	(6.20, 32)	(6.30, 30)	(6.40, 27)
	$\partial R_A / \partial\delta$						
0.65→0.70	-	-0.0812	-0.0043	0.0291	0.0525	0.1256	-
0.70→0.75	-	-0.0845	-0.0045	0.0303	0.0546	0.1301	-
0.75→0.80	-	-0.0880	-0.0047	0.0316	0.0568	0.1350	-
0.80→0.85	-	-0.0918	-0.0049	0.0329	0.0592	0.1403	-
0.85→0.90	-	-0.0960	-0.0051	0.0344	0.0618	0.1459	-
	$dLia / d\delta$: total effect						
0.65→0.70	-	21.0438	21.0213	20.8720	20.4637	18.4702	-
0.70→0.75	-	21.0444	21.0217	20.8669	20.4438	18.3770	-
0.75→0.80	-	21.0450	21.0221	20.8613	20.4223	18.2768	-
0.80→0.85	-	21.0457	21.0226	20.8553	20.3991	18.1688	-
0.85→0.90	-	21.0464	21.0231	20.8487	20.3737	18.0521	-

Notes: unless otherwise indicated, $\alpha = 0.90$, $R_S = 4.00\%$, $\sigma = 0.30$, $\beta = 0.60$, $B = 200$, $m = 0.1\%$, $R = 3.00\%$, and $M = 36$. The shaded areas represent the corresponding values with an approximate optimal loan rate of 5.90%. The positive indirect effect $(\partial Lia / \partial R_A)(\partial R_A / \partial\delta)$ reinforces the positive direct effect $(\partial Lia / \partial\delta)$.

Source: self-created.

Proposition 5. An increase in the participation level decreases the loan rate and increases the life insurance liabilities of the company.

The interpretation of Proposition 5 follows a similar argument as in the case of a change in R. Basically, an increase in the participation level encourages the company to shift investments to its loan portfolio from the default-free liquid assets. In an imperfect policy loan market, the insurer must reduce the size of its loan rate in order to increase the number of loans. Next, partially differentiating Equation (4) with respect to δ, one obtains a constant value of $DOC(\alpha V, Z)$, a short barrier call option. The positive direct effect is understood because an increase in the participation level increases the policyholder protection at the cost of the insurer, enhancing the life insurance liabilities of the insurer, *ceteris paribus*. The indirect effect is captured by the impact on Lia from increases in δ at various levels of the optimal R_A. As mentioned previously, an increase in δ decreases R_A, resulting in increasing loans held by the company. Furthermore, the liabilities of the insurer are increased because increasing the loans related to liquidity reveals the life insurance company to be more prone to risk-taking. The indirect effect, in this case, reinforces the direct effect. This implies that an increase in the participation level of the life insurance contracts to protect policyholders increases the liabilities for the insurer.



Notes: unless otherwise indicated, $\alpha = 0.90$, $R_S = 4.00\%$, $\sigma = 0.30$, $\beta = 0.60$, $B = 200$, $m = 0.1\%$, and $R = 3.00\%$. The responsiveness of company liabilities to participation level is valued with a corresponding optimal set $(R_A\ (\%), A) = (5.90, 33.8)$ where $M = 32 \sim 40$, and $(R_A\ (\%), A) =$ (5.90, 33.8) when $\delta = 0.65 \sim 0.85$, and (5.80, 33.9) when $\delta = 0.90$ where $M = 30$

Fig. 4. Responsiveness of life insurance liabilities to participation level at various levels of shadow banking entrusted loan

Source: self-created.

It is of interest to further explain the relationship between life insurance liabilities and participation level through the optimal loan rate adjustment for various levels of the shadow banking entrusted loan. The results of Figure 4 are stated in the following proposition.

Proposition 6. As the entrusted loan increases, the negative effect of the participation level on the loan rate is decreased, and the positive effect of the participation level on the company's liabilities is decreased.

The results observed in Figure 4 indicate that the negative effect of δ on R_A (and thus on the insurer's profit) is more significant when the insurer gets less involved in the entrusted loan activities than when the company gets more involved. When the policyholders are protected by increasing the participation level of the life insurance contracts, the profits of the company are decreased significantly when the company gets less involved in the entrusted loan activities. Similarly, the positive effect of δ on Lia is more significant when the insurer gets less involved in the entrusted loan activities than when the company gets more. Under the increased protection to the policyholders by the participation policy, the insurer may attempt to augment its entrusted loan activities in order not to significantly suffer from the decreased profits and the increased life insurance liabilities. Therefore, a higher likelihood of bankruptcy of life insurance companies during financial turmoil may be offset by such conducting shadow banking.

CONCLUSION

In the presented article, the authors extended the premature default risk model of Grosen and Jørgensen (2002), and investigated the question of how to value an equity-linked life insurance contract when considering balance-sheet insurer loans and shadow banking entrusted loans. This model allows the inclusion of a more realistic loan market and insurance shadow bank conditions along with the more appropriate behavioural mode of loan rate-setting. In the numerical analysis part based on the theoretical model, the authors performed several comparative static analyses to see how the shadow banking entrusted loans affect the loan rate and life insurance liabilities, in particular at various levels of knockout barrier, and how the required guaranteed interest rate and the participation level affect the loan rate and life insurance liabilities at various levels of entrusted loans. Several results were derived that should be of interest to investors, analysts, supervising agencies, and policymakers. For example, the life insurance company's profit is positively related to the entrusted loans, to the required guaranteed interest rate, but negatively to the participation. In addition, the liability of the insurer is positively related to the entrusted loans, to the guaranteed rate, and to the participation level. In conclusion, it was shown that shadow banking entrusted loans assist with life insurance policyholder protection.

Within the framework presented in this paper, the authors analysed the issues of policyholder protection together with considering shadow banking in the premature default risk environment, which in general is enough to accommodate further extensions. For example, the use of reinsurance or regulatory capital requirements can be introduced and their effects on the profit-sharing policy and default risk can be discussed. In this paper, the authors focused on the market mechanisms of insurer spread behaviour and policyholder protection. Nevertheless, the results generally apply to a financial environment in distress, explicitly relevant to reinsurance under capital regulations in response to a financial crisis.

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