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SUBJECTIVE GROWTH MODELS IN LONG-TERM FORECASTING OF DEVELOPMENT TECHNOLOGIES

Summary: The article concerns the application issues of forecasting on the basis of subjective growth models, known from Polish literature as formal second type models. The function's form and the parameters of such models are determined on the grounds of expert opinions, the forecasts are constructed by means of extrapolation of the model. Expert opinions, collected for the need of *foresight* research, allowed for the construction of prognostic subjective growth models. On their basis the 'initial' forecasts for the development of energetic technologies were determined; there was a proper combination of these forecasts as well as an attempt to estimate their uncertainty.

Key words: foresight, experts' forecasts, subjective models, forecast uncertainty.

1. Introduction

The dynamic development of civilization, the economy and technology have created a need for modelling and forecasting of new phenomena, which is confirmed by the ever increasing popularity of *foresight*-type research in Poland and around the world. The main cause of difficulties with it is the lack of an adequate amount of empirical data, allowing for the "classical construction" of a mathematical model of reality. The routine approach in this type of situation is to utilize heuristic forecasting methods based on experts' opinions which can be collected for example by a Delphi survey. The research shows, however, that the accuracy of forecasts formulated directly by experts is rarely satisfying, especially in comparison with forecasts obtained on the basis of a formal forecasting model [Russo, Schoemaker 1989]. These difficulties intensify for example when the construction of an entire trajectory of forecasts, stretching forward for many periods in *foresight* research sometimes up to a few decades, is required for the needs of a long term developmental scenario. An alternative to "traditional" heuristic methods could then be the construction of a so-called formal subjective model (formal second type model), whose parameters are determined on the basis of subjective information gained from experts. Depending on the scope of information possessed, this could be a cause and effect model (see, e.g. [Little 1970; Lilien, Rangaswamy 2004]) or a growth model (see [Gardner 1991; Shim 2000; Dittmann 2004]).

The practical aspects of forecasting on the basis of subjective growth models constitute the basic subject of consideration of this article. The main goal is to show the utility of such models in analyzing data collected during *foresight* research. The proposals concerning the methods of creating forecasts and assessing their degree of uncertainty will be illustrated by a practical example of their utilization in the construction of base growth scenarios for new energy technology required by the *foresight* study "A Zero-Emissions Energy Economy under Conditions of Sustainable Development of Poland until 2050", realized by the Main Mining Institute in Katowice.

2. Subjective growth models

Known from the topical literature, subjective growth models serve to describe the dynamics of sales of new products [Gardner 1991; Shim 2000; Dittmann 2004]. The forecaster accepts an assumption about the form of the functional model based on the expected shape of the product's life curve. To this end, the following functions are utilized:

1) linear:

$$Y_t = \alpha + \beta t \tag{1}$$

2) exponential:

$$Y_t = \alpha(1 + g)^t \tag{2}$$

and, if additionally we assume the market's finite potential:

3) reverse exponential (with horizontal asymptote)

$$Y_t = \alpha - \beta g^t, \qquad g < 1 \tag{3}$$

4) logistic

$$Y_t = \frac{1}{\alpha - \beta g^t} \tag{4}$$

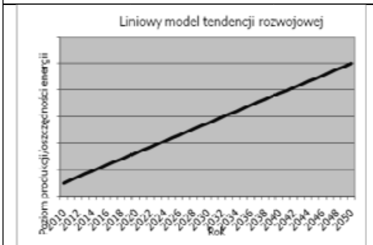
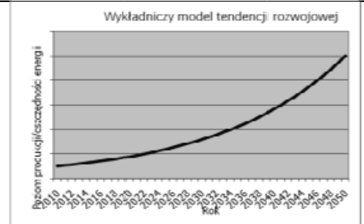
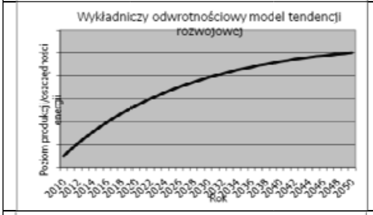
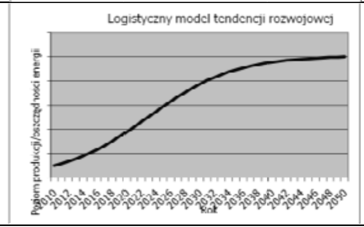
where: t – temporal variable; α, β, g – the model's parameters.

The parameters are determined on the basis of an expert's or group of experts judgments, which concern:

- in the case of linear and exponential functions- the value of two random variables: the product's sales levels in the first period its presence on the market (Y_1) and the product's sales levels in some later period, after stabilization (Y_n).
- in the case of reverse exponential and logistic functions- the value of three random variables: the product's sales levels in the first period of its presence on the market (Y_1), the product's sales levels in some later period (Y_n), as well as the level of the market's saturation (Y_∞).

Formulas allowing for the determination of the parametersα, β, g along with charts of the appropriate functions (1) – (2) are presented in Table 1. The forecast y_T^* for the arbitrary period $T > 1$ is determined by extrapolating the constructed model.

Table 1. Formulas to determine parameters of selected subjective growth models

Graph of the Function	Determination of the model's parameters		
	α	β	g
	$\alpha = y_1 - \beta$	$\beta = \frac{y_n - y_1}{n-1}$	
	$\alpha = \frac{y_1}{1+g}$		$g = \sqrt[n-1]{\frac{y_n}{y_1}} - 1$
	$\alpha = y_\infty$	$\beta = \frac{\alpha - y_1}{g}$	$g = \sqrt[n-1]{-\frac{y_n - \alpha}{\alpha - y_1}}$
	$\alpha = \frac{1}{y_\infty}$	$\beta = \frac{\alpha - \frac{1}{y_1}}{g}$	$g = \sqrt[n-1]{-\frac{\frac{1}{y_n} - \alpha}{\alpha - \frac{1}{y_1}}}$

Source: own elaboration.

3. A proposal for the assessment of uncertainty of forecasts

After completing the appropriate algebraic calculations stemming from the formulas placed in Table 1, each of the models (1) – (4) can be recorded as a function with the following form[1] (see: [Poradowska 2006]):

$$Y_t = f(t, Y_1, Y_n, Y_\infty), \tag{5}$$

[1] Models (5), (6) are represented in the versions corresponding to the reverse exponential and logistic models. In the case of linear or exponential models, the amounts linked to the variable Y_∞ do not appear.

whose argumentation is: the temporal variable *t* as well as the variables Y_1, Y_n, Y_∞, whose determination of the expected values y_1, y_n, y_∞ served to establish the parameters of the model.

For a y_T^* forecast, achieved by the extrapolation of the model according to the rule of forecasting according to expected value:

$$y_T^* = f(T, y_1, y_n, y_\infty), \tag{6}$$

we can determine a measure of acceptability which is analogous to the well-known *ex ante forecasting error* – an estimate of the standard deviation of forecasting error. This can be called the **standard forecast uncertainty** and labeled $u(y_T)$ (see: [Poradowska 2006]). Its value can be determined utilizing the law of transposition of standard deviation, in accordance with which we have (compare: [*Guide*... 1995])[2]:

- when Y_1, Y_n, Y_∞ are independent arbitrary variables:

$$u(y_T) = \left\{ \left[\frac{\partial f}{\partial y_1} u(y_1) \right]^2 + \left[\frac{\partial f}{\partial y_n} u(y_n) \right]^2 + \left[\frac{\partial f}{\partial y_\infty} u(y_\infty) \right]^2 \right\}^{\frac{1}{2}} \tag{7}$$

- when Y_1, Y_n, Y_∞ are mutually correlated[3]:

$$\begin{aligned} u(y_T) = \Bigg\{ & \left[\frac{\partial f}{\partial y_1} u(y_1) \right]^2 + \left[\frac{\partial f}{\partial y_n} u(y_n) \right]^2 + \left[\frac{\partial f}{\partial y_\infty} u(y_\infty) \right]^2 + \\ & + 2\Bigg[\frac{\partial f}{\partial y_1}\frac{\partial f}{\partial y_n} u(y_1)u(y_n)r(y_1, y_n) + \frac{\partial f}{\partial y_1}\frac{\partial f}{\partial y_\infty} u(y_1)u(y_\infty)r(y_1, y_\infty) + \\ & + \frac{\partial f}{\partial y_n}\frac{\partial f}{\partial y_\infty} u(y_n)u(y_\infty)r(y_n, y_\infty) \Bigg]\Bigg\}^{\frac{1}{2}} \end{aligned} \tag{8}$$

where: $u(y_1), u(y_n), u(y_\infty)$ are the determinations of the standard deviation of the variables Y_1, Y_n, Y_∞, $\frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial y_n}, \frac{\partial f}{\partial y_\infty}$ – partial derivatives (the ratio of sensitivity) of function *f* according to the appropriate variables Y_1, Y_n, Y_∞ calculated at point $t = T, Y_1 = y_1, Y_n = y_n, Y_\infty = y_\infty$, $r(y_1, y_n), r(y_1, y_\infty), r(y_n, y_\infty)$ – determination of the ratio of linear correlation between pairs of variables, accordingly Y_1 and Y_n, Y_1 and Y_∞, Y_n and Y_∞.

The appropriate partial derivatives appearing in examples (7) and (8) are presented in Table 2.

[2] Translated into Polish: *Wyrażanie niepewności pomiaru. Przewodnik*, Główny Urząd Miar, Warszawa 1999.

[3] If only one pair of variables is mutually correlated, the ratio of correlation of the remaining two pairs of variables should be substituted in example (8) with zeros.

Table 2. Partial derivatives of function (5) according to initial variables Y_1, Y_n, Y_∞

Type of Function \ Partial Derivative	$\frac{\partial f}{\partial y_1}$	$\frac{\partial f}{\partial y_n}$	$\frac{\partial f}{\partial y_\infty}$
linear	$\frac{n-T}{n-1}$	$\frac{T-1}{n-1}$	
exponential	$\frac{n-T}{n-1}\left(\frac{\hat{y}_n}{\hat{y}_1}\right)^{\frac{T-1}{n-1}}$	$\frac{T-1}{n-1}\left(\frac{\hat{y}_1}{\hat{y}_n}\right)^{\frac{n-T}{n-1}}$	
reverse exponential	$\frac{n-T}{n-1}\left(\frac{\hat{y}_\infty-\hat{y}_n}{\hat{y}_\infty-\hat{y}_1}\right)^{\frac{T-1}{n-1}}$	$\frac{T-1}{n-1}\left(\frac{\hat{y}_\infty-\hat{y}_1}{\hat{y}_\infty-\hat{y}_n}\right)^{\frac{n-T}{n-1}}$	$1-\frac{\partial f}{\partial y_1}-\frac{\partial f}{\partial y_n}$
logistic $\varepsilon_1=\frac{n-T}{n-1}\left(\frac{\hat{y}_n^{-1}-\hat{y}_\infty^{-1}}{\hat{y}_1^{-1}-\hat{y}_\infty^{-1}}\right)^{\frac{T-1}{n-1}}$ $\varepsilon_n=\frac{T-1}{n-1}\left(\frac{\hat{y}_1^{-1}-\hat{y}_\infty^{-1}}{\hat{y}_n^{-1}-\hat{y}_\infty^{-1}}\right)^{\frac{n-T}{n-1}}$	$\left(\frac{y_T^*}{\hat{y}_1}\right)^2\times\varepsilon_1$	$\left(\frac{y_T^*}{\hat{y}_n}\right)^2\times\varepsilon_n$	$\left(\frac{y_T^*}{\hat{y}_\infty}\right)^2\times(1-\varepsilon_1-\varepsilon_n)$

Source: own elaboration.

The assessment of standard deviations $u(y_1), u(y_n), u(y_\infty)$ of the variables Y_1, Y_n, Y_∞ can be determined in two ways:
- on the basis of the distribution of subjective probability of variables Y_1, Y_n, Y_∞ (when utilizing the judgments of individual experts),
- as the simple or weighted arithmetical averages of several possible values of these variables (when utilizing the judgments of a group of experts).

The problem of determining the possible correlation between the pairs of variables Y_1, Y_n, Y_∞ was discussed in [Poradowska 2009].

4. Subjective growth models in forecasting the development of new technologies – a case study

The problems of forecasting on the basis of subjective growth models presented above proved useful in the construction of initial[4] forecasts regarding the development of new technology, prepared for the needs of the *foresight* study: "A Zero-Emissions Energy Economy under Conditions of Sustainable Development of Poland until 2050".

[4] The forecasts constructed were corrected in later stages of research based on additional information gained from experts.

For 72 specified energy technologies in this study experts determined:

- the size of production/energy conservation in the year 2010,
- the minimum (*a*), maximum (*b*) and most probable (*w*) size of production/energy conservation in 2020-2050,
- the expected shape of the life cycle curve of technology in the period 2010-2050 (the selection was made among linear, exponential, reverse exponential and logistic functions).

The procedure of determining forecasts for the upcoming years 2010-2050 as well as the assessment of their uncertainty can be divided into the following stages:

4.1. Establishing the values of y_1, y_n, y_∞, indispensable to the determination of parameters *α, β, g*

The size of production/energy conservation, given by experts for 2010, was considered to be the deterministic variable Y_1. The possible potential of the market during the forecasted period was recognized as the maximum value determined by experts for 2050. It was moreover assumed that the size of production/energy conservation in 2020 and 2050 were independent arbitrary variables Y_{11} and Y_{41} with a triangular distribution, where the limits were three values ascribed by experts: the minimum (*a*), maximum (*b*) and most probable (*w*). Thus:

y_1 – value given by experts for 2010,

y_{11} – expected value of the triangular distribution[5], determined by three values given by experts for 2020,

y_{41} – expected value of the triangular distribution, determined by three values given by experts for 2050,

y_∞ – maximum value determined by experts for 2050.

4.2. Assessment of standard deviation $u(y_1), u(y_n), u(y_\infty)$

The deterministic set size for 2010 was accepted as $u(y_1) = 0$. The values $u(y_{11})$ as well as $u(y_{41})$ were determined as the standard deviations for the appropriate triangular distributions[6]. There lacked expert opinions on the basis of which the uncertainty of the maximum sizes for 2050 could be ascertained, thus an arbitrary uniform distribution for the variable Y_∞ was accepted, with a range of $[0{,}75y_\infty; 1{,}25y_\infty]$[7], from which:

$$u(y_\infty) = \frac{y_\infty}{2\sqrt{12}} \tag{9}$$

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[5] The expected value of the triangular distribution is set as: $m = \frac{a+w+b}{3}$.

[6] The equation $s = \sqrt{\frac{(b-w)^2 + (b-a)(w-a)}{18}}$ was utilized.

[7] This was a consequence of the assumption that the maximum expert error committed in determining the size was approximately 25%.

4.3. Construction of combined forecasts for the period 2010 − 2050

For each of the considered technologies, two initial forecast models were constructed:

(1) on the basis of the values of $y1$, $y11$ (as well as potentially y_∞),

(2) on the basis of the values of $y1$, $y41$ (as well as potentially y_∞).

The form of the functions in both models was uniform, in accordance with the technological development curve indicated by the experts for the period 2010--2050.

Based on models (1) and (2) the appropriate forecasts $y(1)_T^*$ and $y(2)_T^*$ were determined for the time periods T = 1,…,41 as well as their standard uncertainties $u_1(y_T)$ and $u_2(y_T)$. The problems described in sections 1. and 2. of this article were utilized for this purpose.

Subsequently, for each T=1,…,41 a combined prognosis was determined, as:

$$y_T^* = w(1)y(1)_T^* + w(2)y(2)_T^*, \tag{10}$$

where:

$$w(1) = \frac{\frac{1}{u_1(y_T)}}{\frac{1}{u_1(y_T)} + \frac{1}{u_2(y_T)}}, \quad w(2) = \frac{\frac{1}{u_2(y_T)}}{\frac{1}{u_1(y_T)} + \frac{1}{u_2(y_T)}} \tag{11}$$

4.4. Assessment of the degree of uncertainty of the forecasts

For the combined forecasts y_T^* (T = 1,…,41) standard uncertainties $u(y_T)$ were determined, on the basis of the law of propagation of standard deviation, assuming a maximum positive correlation between the forecasts included in the combinations. Using formula (8) the following formula was obtained:

$$u(y_T) = |w(1) \times u_1(y_T)| + |w(2) \times u_2(y_T)| \tag{12}$$

The value of (12) can be interpreted as the standard deviation of forecast error, fixed for time T. As an additional measure of uncertainty, a range with the following scope was accepted:

$$[y_T^* - u(y_T);\ y_T^* + u(y_T)] \tag{13}$$

5. Selected forecasts of technological development

Below, the results pertaining to one of the topical areas of the study are presented – forecasts of the level of energy production from renewable sources of energy. Figure 1 shows the values determined for specific technologies by experts, the preliminary constructions of forecast models (based on estimated size of production for 2010,

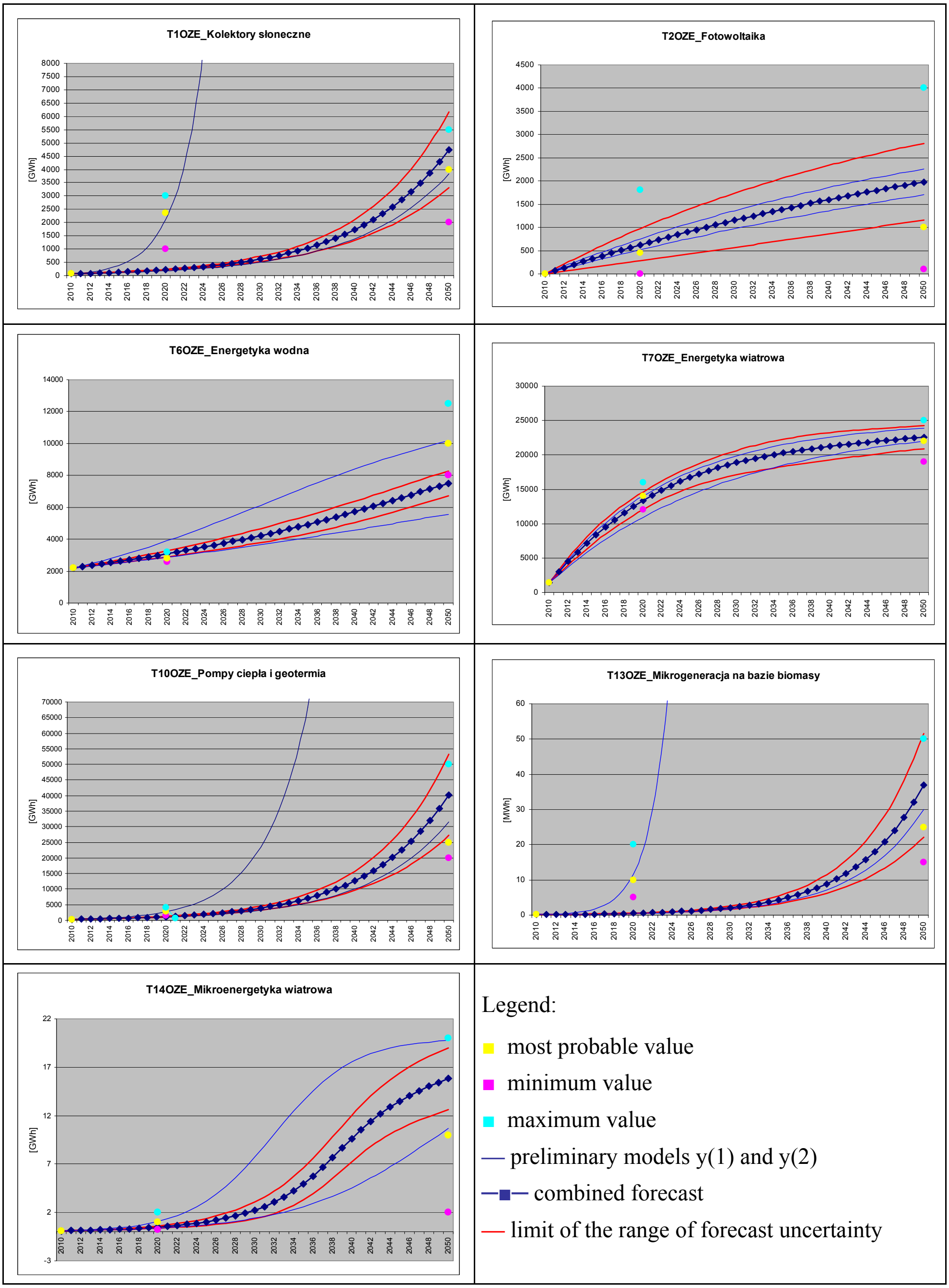

Figure 1. Forecast for the development of renewable energy sources technology along with the degree of uncertainty

Source: own elaboration.

2020, and 2010, 2050), the final forecast for 2010-2050 (obtained as a combination of preliminary forecasts), as well as the range of uncertainty of the forecast $[y_T^* - u(y_T); y_T^* + u(y_T)]$.

A visual appraisal of the graphs presented in Figure 1 allows for the conclusion that the lowest degree of uncertainty is exhibited by the forecast of the development of large-scale wind power technology; the uncertainty range is relatively narrow, and also the forecasts obtained (preliminary as well as combined) are the least removed from the experts' estimates.

It is also significant that in this case the experts' intuition concerning the shape of the technological development curve is convergent with the values determined by them for the chosen years. The cause for such qualitative results here may be the extensive knowledge of the phenomenon of wind power production compared with other technologies, which are just now being introduced into the Polish market.

Another example of a technology which is already functioning with success on the Polish market is hydroelectricity. In this case, however, despite a low scope of the range of uncertainty compared with other technologies, a negative determinant of the quality of the forecast is the divergence of expert estimates with the trajectory of the technological development curve determined by them. Here, experts selected a logistic function, whereas their estimates (especially for 2050) indicate exponential growth of the size of energy production. Inconsistency in this realm of expert intuition appears in most of the results obtained, also concerning the remaining topical areas of the entire *foresight* study.

Table 3. Results of the study for 2050: experts' estimates, combined forecast obtained, standard forecast uncertainty, and its relative size (in relation to the forecast)

Renewable Energy Technology		Solar Collectors Panels/Troughs	Photovoltaics	Hydroelectricity Conventional and Pumped-Storage	Large-scale wind power	Heat pumps and Geothermal Energy	Micro generation on the basis of Biomass	Small-scale wind power
Form of function		Exp.	Rev. exp.	logist.	Rev. Exp.	Exp.	Exp.	logist.
Experts' Estimates	*a*	2000	100	8000	19000	20000	15	2
	w	4000	1000	10000	22000	25000	25	10
	b	5500	4000	12500	25000	50000	50	20
Forecast		4744	1980	7504	22592	40243	37	16
Uncertainty of forecast		1434	829	774	1681	13056	15	3
Relative Uncertainty		30%	42%	10%	7%	32%	40%	20%

Source: own elaboration.

In comparison, detailed results for 2050, i.e. the last year of the forecast period, the period fraught with the most uncertainty, are presented in Table 3. In accordance with the conclusions stemming from a visual appraisal of the graphs in Figure 1, the lowest degree of uncertainty is exhibited by forecasts for hydroelectricity and wind power − 10% and 7% of the forecast value respectively. In the case of hydroelectricity, however, a negative fact is that the forecast obtained goes beyond the range marked out by the minimum and maximum experts' estimates, which is caused by conflicting assessments of the size of production and the shape of the technological development curve.

For the remaining technologies the forecasts determined on the basis of subjective growth models remain within the limits of the ranges determined by experts, although their degree of uncertainty is relatively high – in the case of photovoltaic technology (a technology currently in the initial phases of introduction) and micro generation on the basis of biomass (a technology not yet functioning on the Polish market) the standard forecast uncertainties for 2050 amount to as much as 42% and 40% of the value of the forecast respectively.

6. Conclusion

Subjective forecasting models can be a valuable tool in constructing long-term growth forecasts for new technologies in cases where there is incomplete information, stemming solely from piecemeal experts' opinions about the potential path of development of the forecasted phenomenon through time. In the case of growth models, it is sufficient to possess experts' opinions concerning the level of the phenomenon in two time periods: the initial period and a selected subsequent period (and possibly the level of market saturation) as well as an assumption about the shape of the phenomenon's developmental curve. A prerequisite to effective forecasting is in such a situation "good quality" expert data, requiring the expert's in-depth knowledge in the given field along with the reliability of the forecaster throughout the process of acquiring opinions.

The experiences arising from the preparation of the results of the study "A Zero-Emissions Energy Economy under Conditions of Sustainable Development of Poland until 2050" allow the conclusion that acquiring too much information from experts – in this case information concerning the size of production in three remote time periods as well as the expected shape of the technology's life cycle − can lead to inconsistency in indirect research results, which is not without an influence on the uncertainty of obtained forecasts. It appears that in this situation it would have been more effective to have had the forecaster determine on his own the form of the growth curve's equation based on known experts' estimates, their own intuition and possibly additional consultations with experts.

To assess the degree of uncertainty of forecasts obtained on the basis of subjective growth models the so-called standard forecast uncertainty can be used – standard

deviation of forecasting error, calculated on the basis of a generalized law of propagation of standard deviation. In the situation discussed in the article the value of this measure was dependent on:
– the situation of the three values determined by experts in relation to each other,
– the distance of the time of forecast to the periods examined by experts,
– the form taken on by the growth curves.

Taking into account the great degree of subjectivism in the manner in which standard forecast uncertainty is estimated, it is recommended to take care when interpreting this measure. In contrast to the analogously defined ex ante forecasting error, standard uncertainty should not be treated as an expected forecasting error, however, it can be useful when comparing forecasts obtained from various subjective models, and also in evaluating the "quality" of experts' opinions.

Literature

Dittmann P., *Prognozowanie w przedsiębiorstwie. Metody i ich zastosowanie*, Oficyna Ekonomiczna, Kraków 2004.
Gardner E. Jr., *Forecast with No Data*, "Lotus" 1991, Vol. 7, No. 6, p. 42-45.
Guide to the Expression of Uncertainty in Measurement, ISO, Geneva 1995.
Lilien G.L., Rangaswamy A., *Marketing Engineering*, Trafford Publishing, St. Victoria 2004.
Little J.D., *Models and managers: The concept of decision calculus*, "Management Science" 1970, Vol. 16, No. 8.
Poradowska K., *Ocena dopuszczalności prognoz gospodarczych*, PhD thesis, Wyd. AE Wrocław 2006.
Poradowska K., *Prawo propagacji niepewności w ocenie dopuszczalności prognoz*, Prace Naukowe Uniwersytetu Ekonomicznego we Wrocławiu nr 38, Ekonometria 24, 2009, p. 100-111.
Russo E.J., Schoemaker P.J., *Decision Traps: The Ten Barriers to Brilliant Decision-Making and How to Overcome Them*, Knopf Doubleday Publishing Group, New York 1989.
Shim J. K., *Strategic Business Forecasting*, St. Lucie Press, New York 2000.

SUBIEKTYWNE MODELE TENDENCJI ROZWOJOWEJ W PROGNOZOWANIU ROZWOJU NOWYCH TECHNOLOGII

Streszczenie: Artykuł dotyczy aplikacyjnych zagadnień prognozowania na podstawie subiektywnych modeli tendencji rozwojowej, znanych z literatury polskiej jako modele formalne II rodzaju. Postać funkcyjna i parametry takich modeli są określane na podstawie opinii ekspertów, prognozy konstruuje się poprzez ekstrapolację modelu, natomiast do oszacowania stopnia niepewności prognozy można wykorzystać uogólnienie prawa propagacji odchyleń standardowych, bazujące na teorii prawdopodobieństwa subiektywnego. Opinie ekspertów, zebrane na potrzeby tego badania *foresight*, umożliwiły budowę prognostycznych, subiektywnych modeli tendencji rozwojowej. Na ich podstawie wyznaczono „wstępne" prognozy rozwoju technologii energetycznych, dokonano odpowiedniej kombinacji tych prognoz oraz podjęto próbę oceny ich niepewności.