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THE MANAGEMENT OF CATASTROPHE INSURANCE RISK THROUGH THE ANTICIPATION OF THE PRICE OF CAT BOND

Summary: The application of financial instruments from the capital market aims at the management of securitization process of the catastrophe risk. This is important with respect to the results of losses caused by catastrophe incidents. The article develops the structure of CAT bonds and their pricing with the use of model of stochastic process of interest rate. The CAT bonds are designed to finance the results of catastrophe incidents. They are similar to the contingent claim capital but in reality they are the financial market instruments. The elaborated approach is illustrated by the distribution of the bond price in the configuration of trigger level for CAT bond forgiveness and its volatility. The direction of possible research is determined by the consideration of moral hazard and basis (market) risk in the pricing process.

Key words: CAT bonds, contingent claim, moral hazard, basis risk.

1. Introduction

The article refers to the insurance instruments, including moral hazard and basis risk, considered in the literature [Szkutnik 2010]. This is aimed at the management of the insurance risk of a catastrophic type. The basis for the presented considerations are the financial instruments the application of which, in the process of managing the insurance company, aims at the securitization of the catastrophe risk. This means that they are aimed at financing the results of catastrophe incidents. The CAT bonds are financing tools in the case considered in this article. These bonds have a similar character as a contingent claim capital CC, but in real conditions they are instruments of the financial market; they are catastrophe-linked bonds.

The goal of the presented considerations is the formulation of a contingent claim model to price bonds issued directly by the insurer, by the SPV company dependent on the main insurer when there is no default risk for claims initiated by catastrophe incident. In market reality the pricing of such bonds was justified in recent decades by the increasing frequency of catastrophe incidents. The anticipation of results of possible catastrophes with big aggregated losses, expressed indirectly in specifically constructed indices of losses together with considering them in the terms and conditions of issued bonds, is the anticipation of their price. To abbreviate the heart of the problem, we state that proper pricing influences the losses of the insurer when a catastrophe incident is realized. Therefore, the proper pricing of bonds is an element of insurance operations management; for there is a total or partial depreciation of the face value of the CAT bond which influences the finance of the insurance company.

A specific project of CAT bonds has a historical context. In retrospect, one could observe a constant evolution of CAT designs and, consequently, also their pricing. Even though we can price default-risky CAT bonds, it is also possible to consider default-free CAT bonds prices [*Ryzyko…* 2009]. We also assume that there is a moral hazard, which is associated with the event of claims pricing (the event that is inadequate to losses) that leads to losses either of the insurer (the most frequent case) or the bondholders and insurance policies holders. It should be remembered that catastrophe risk is partially insured in a traditional way by entities that are subject to such a risk and this procedure also involves insurance companies, typically reinsurance companies. We also consider basis risk, which is a symptom of the influence of the capital market condition, as well as of market risk.

In the model concerning the dynamics of value of insurance company assets, significant assumptions are adopted for interest rate risk and credit risk that, in a specific way, affect all other forms of risk. This is explained in a more detailed manner in the part related to the specification of assumptions of the assets value model.

It should be taken into account that the presented empirical examples [*Ryzyko…* 2009] may accede to Polish reality concerning the manifestation of catastrophe risk, even though the capital market itself has not worked out proper projects of instruments which can be applied in the securitization of such a risk. Polish literature has shown the attempt to price a catastrophe bond in the aspect of the so-called two-factor function of the investor's usefulness, whose payee would be municipal authorities.

2. Structural aspect of CAT bonds

In Polish literature the above mentioned catastrophe bonds were considered as debt instruments, burdened with the risk of the payee's insolvency. In Polish conditions the project of such instruments was addressed to self-government authorities in order to secure the region against the results of floods, droughts, forest fire etc. The structure of such bonds and also their efficiency consists in the fact that in case of catastrophe, which stimulates beforehand the issue of such instruments and generates financial losses and claims, the issuer is not obliged to pay back the nominal value of the bond and to continue the interest payment until they expire. However, when no catastrophe took place, the bond holder (investor) is the beneficiary of such a favorable arrangement.

Significant characteristics of the American CAT bonds which determine their structure are:

1. Existence of debt-forgiveness trigger (the moment of catastrophe occurrence or the value of the observed technical loss index).This results from the CAT bond issuing conditions, whose generic design allows for the payment of interest and/or the return of principal forgiveness, and the extent of forgiveness can be total, partial or scaled to the size of loss.
2. Debt-forgiveness may be triggered by the insurer's (or reinsurer's) actual losses or on the composite index of insurers' losses during a specific period.
3. A CAT bond hedge enables the insurer (reinsurer) to avoid the credit risk (a way to gain capital).
4. CAT bonds provide a hedge to the insurer by forgiving existing debt (the insurer's); thus, the value of this hedge is independent of the bondholders' assets and the issuer has no risk of nondelivery on the hedge.
5. From the bondholder's (investor's) perspective, the estimation of the value of CAT bond is determined by:
   a) the default risk,
   b) the potential moral hazard occurrence and 'behaviour',
   c) the basis risk of the issuing company.
6. Moral hazard can increase the claim payments at the expense of the investors principal reduction and affect the bond price.
7. The CAT bond's basis risk refers to the gap between the insurer's actual loss and the composite index of losses that prevents the insurer from receiving complete risk hedging.

3. Trends in pricing CAT bond designs with the consideration of moral hazard and basis risk

The effect of moral hazard and basis risk was originally studied not from the perspective of the CAT bonds pricing but from the point of view of the influence of these types of risk upon the hedging process as a form of financing economic enterprises with various provenience, uncertain, yet burdened with significant losses in the case of these enterprises' failure. The potentially new, restricted catastrophe-linked securities, related to the securitization of the effects of catastrophic events Center et al.1997; Laurenzano1998] were also created. Bouzouita and Young [1998] focused on the regulations of the specific actions and applications from the perspective of risk management.

Among others, Cox and Schebach [1992], Cummins and Geman [1995] and Chang and Yu [1996] focused on the pricing of CAT futures and CAT call spreads under the condition of deterministic interest rate and specific property claims services (PCS) loss processes. There was also the pricing of one-year zero-coupon CAT bond [Litzenberg et al.1996] which was further compared to the CAT bond price estimated by hypothetical catastrophe loss distribution. Zajdenweber [1998] followed Litzenberg, Beaglehole and Reynolds [1996], but he changed the catastrophe loss distribution to the stable Levy distribution. Louberge, Kellezi and Gilli [1999] numerically estimated the CAT bond price under the assumptions that the catastrophe loss follows a pure Poisson process, the individual losses have an independently identical lognormal distribution, and the interest rate model is a binomial random process.

All the above listed pricing elaborations failed to incorporate a commonly acceptable stochastic interest rate process and catastrophe loss process as well as the default risk of the CAT bonds.

The article by Jin-Ping Lee and Min-The Yu [2002] develops a contingent claims model to price default-risky catastrophe-linked bonds, where interest rates have a stochastic character. Moreover, it allows for more generic loss processes and practical considerations of moral hazard, basis risk and default risk. There are estimations of both default-free and default-risky CAT bond prices. The results show that both moral hazard and basis risk drive down the bond prices substantially; therefore, these results should not be ignored in pricing the CAT bonds. There are also shown relations between the bond prices and the scale of claims caused by the catastrophe incident, loss volatility, and trigger level, dependent on the loss index, the issuing company's capital position, debt structure and interest rate uncertainty. The elaboration on this subject is also important from the practical point of view. This results from the fact that under accepted assumptions in the offered models of assets value, interest rate, loss model and the priced CAT bond hedge enable the issuer to avoid the credit risk that may arise with traditional reinsurance or catastrophe-linked options, which has already been mentioned.

As far as moral hazard is concerned, it should be remembered [Lawędziak, Szkutnik 2006a, 2006b] that this is initiated by insurers themselves. This is related to the insurer's cost of loss pricing. Sometimes this cost exceeds the issuer's (SPV company) profits (insurer) that result from the debt value (the issuer) which occurs at the time of catastrophe. This stems from the specific character of CAT bonds. This means that the insurer has an incentive to pay the claims more generously when the loss amount is near the trigger set in the debt-forgiveness provision. Doherty [1997] pointed out that **moral hazard results from less loss and control effort by the insurer issuing CAT bonds**, because these efforts will increase the amount of debt that must be repaid. Bantwal and Kunreuther [1999] also noted the tendency for insurers to write additional policies in a catastrophe-prone area, spending less time and money in their auditing of losses after a disaster. The effect of moral hazard may increase the claim payments at the expense of the bondholders' principal reduction and affect the bond price.

Another important aspect which must be considered in pricing a CAT bond is the basis risk. As is already known [*Zastosowanie…* 2004], the basis risk of CAT bond refers to the gap between the insurer's actual loss and the composite index of losses that prevents the insurer from receiving complete risk hedging. The basis risk may cause insurers to default on their debt in the case of high individual loss but low index of loss, and therefore affects the bond price. However, there exists a balance between the basis risk and moral hazard. If one uses an independently calculated index to define CAT bonds payments, then the insurer's opportunity to cheat the bondholders is reduced or eliminated. This is equal to a lesser scope of moral hazard behaviour or even its elimination. However, the basis risk is created.

4. The concept of stochastic structure of pricing CAT bonds

The structure of pricing CAT bonds [Jin-Ping Lee, Min-The Yu 2002] meets the usually unconsidered assumptions of pricing CAT bond models halfway.

The estimation model of pricing CAT bonds is presented with respect to practical assumptions concerning:

– default risk,
– moral hazard,
– basis risk.

In this model it is necessary to define:

– assets value dynamics A_t,
– interest rate dynamic s$r_{t,}$
– aggregate loss dynamics $C_{i,t}$for the issuing company *i* and, relevantly, $C_{\mathrm{index},t}$ in a composite index of losses (e.g. a PCS index).

Additionally, the models also define relevant processes with respect to risk-neutralized pricing measure.

The next part of the chapter provides the numerical analysis and discusses the results [*Ryzyko*… 2009].

4.1. Asset dynamics model

The typical way to model asset dynamics assumes a lognormal diffusion process for the asset value; for example, as in Merton [1977] and Cummins [1988]. The main disadvantage of this modelling approach is that it fails to take into account the explicit impact of stochastic interest rates on the asset value. This is important for modelling the insurer's asset value, because it is quite common for insurers to hold a large proportion of fixed-income assets in their portfolios. In particular, insurers that issue CAT bonds mainly invest their proceeds from CAT bonds sales in high grade, interest rate-sensitive investments such as commercial papers and treasury securities.

Apparently the determination of the insurer's total asset value as consisting of two risk components: interest rate risk and credit risk, allows [Duan et al. 1995] for the measurement of the effect of the interest rate risk on CAT bond prices.

From a theoretical point of view, "credit risk" term, which is mentioned in the introduction, refers to all risks that are orthogonal to the interest rate risk.

Specifically, **the value of the insurer's total assets** is described by the process expressed by stochastic equation (1), where the instantaneous drift μ_A that is a trend resulting from credit risk effect is most important . The model also takes into account the instantaneous interest rate elasticity of the insurer's assetsφ, the already mentioned instantaneous interest rate r_t at time t, the volatility of credit risk process $\sigma_{A,}$ as well as credit risk $W_{A,,t}$ expressed by the Wiener process:

$$\frac{dA_t}{A_t} = \mu_A dt + \varphi dr_t + \sigma_A dW_{A,t} \qquad (1)$$

4.2. The instantaneous interest risk model

Model (1), under the assumption that the instantaneous interest rate is modelled according to the square-root process of Cox, Ingersoll and Ross [1985], this means – the square root of a given number is the number which when multiplied by itself equals this number, avoids the existence of the negative interest rate that can appear in Vasicek's model [1977] and is described as follows:

$$dr_t = \kappa(m - r_t)dt + v\sqrt{r_t}\, dZ_t$$

where: κ denotes the mean-reverting force measurement;
m is the long-run mean of the interest rate;
v is the volatility parameter for the interest rate;
Z_t is a Wiener process independent of $W_{A,,t}$ and leads to the **asset dynamics model** (2):

$$\frac{dA_t}{A_t} = (\mu_A + \varphi vm - \varphi\, \kappa\, r_t)\, dt + \varphi\, v\, \sqrt{r_t}\, dZ_t + \sigma_A dW_{A,t} \qquad (2)$$

4.3. The risk-neutralized dynamics of the insurer's assets

For the asset dynamics model (2) to be neutralized with respect to risk, it is necessary to use **device of risk neutralization**.

The dynamics for the interest process under the risk-neutralized pricing measure, denoted by Q, can be written as

$$dr_t = \kappa^* \,(m^* - r_t)\, dt + v\, \sqrt{r_t}\, d\, Z_t^*$$

where: κ^*, m^* and Z^* are defined as

$$\kappa^* = \kappa + \lambda_r$$

$$m^* = \frac{\kappa \times m}{\kappa + \lambda_r}$$

$$dZ_t^* = dZ_t + \frac{\lambda_r \times \sqrt{r_t}}{v} \times dt.$$

The term λ_r is the market price of interest rate risk and is a constant under Cox, Ingersoll and Ross [1985]; Z_t* is a Wiener process under Q, the formulation of which can be found in Ritchken [1996].

Thus, the insurer's asset dynamics can be risk neutralized to

$$\frac{dA_t}{A_t} = r_t dt + \varphi\, v\, \sqrt{r_t}\, dZ_t^* + \sigma_A dW_t^* \tag{3}$$

where W* is a Wiener process Q and is independent of Z_t*.

4.4. Aggregate loss dynamics

Following the typical setting for loss dynamics in the actuarial literature [Bowers et al. 1986], the **aggregate loss model** can be expressed as a compound Poisson process, a sum of jumps.

To estimate the impact of basis risk on the CAT bond price, the term $C_{i,t}$ that denotes the aggregate loss for the issuing company i is introduced; the term $C_{index,t}$ represents that for a composite index of losses (e.g. a PCS index). These two processes can be described as follows:

$$C_{i,t} = \sum_{j=1}^{N(t)} X_{i,j}, \tag{4}$$

and

$$C_{index,t} = \sum_{j=1}^{N(t)} X_{index,j}, \tag{5}$$

where the process $\{N(t)\}_{t\geq 0}$ is the loss number process described by Poisson process with intensity λ. Symbols $X_{i,j}$ and $X_{index,j}$ denote the Mount of losses caused by the jth catastrophe during the specific period for the issuing insurance company and the composite index of losses, respectively. It is assumed that terms $X_{i,j}$ and ($X_{index,j}$), for $j = 1, 2, \ldots, N(T)$, are mutually independent and have identical lognormal distribution, and they are also independent of the loss number process, and their logarithmic means and variances are denoted by $\mu_i(\mu_{index})$ and $\sigma_i^2\ (\sigma_{index}^2)$, respectively. In addition, assume that ρ correlation coefficients of the logarithms of $X_{i,j}$ and ($X_{index,j}$), for different $j = 1, 2, \ldots, N\ (T)$ are identical.

4.5. Loss dynamics under the risk-neutralized pricing measure

To carry out the pricing of CAT bonds one needs to know the loss dynamics under the risk neutralized pricing measure Q. When the loss process has sudden jumps, **the market is then called incomplete** and there is no unique pricing measure.

Thus, let us follow Merton [1976] and assume that **economic conditions are only marginally influenced** by localized catastrophes such as earthquakes and hurricanes, and that the loss number process $\{N\ (t)\}$ and the amount of losses $X_{i,j}$

($X_{index,j}$),are directly related to **idiosyncratic 'shocks'** to the capital markets, this means **the factors that influence the capital markets in an inexpedient way.**

These factors − catastrophic shocks represent 'nonsystematic' risk and have a zero risk premium, which they generate.

By assuming that such a jump risk is nonsystematic and diversifiable, attaching a risk premium to the risk is unnecessary. Apparently this assumption is important because one cannot apply a risk-neutral evaluation to situations in which the size of the jump is systematic. This point is minutely discussed by Naik and Lee [1990], Cummins and Geman [1995], Cox and Pedersen [2000].

Therefore, it can be accepted that the aggregate loss processes expressed by equations (4) and (5) retain their original distributional characteristics after changing from the physical probability measure to the risk-neutralized pricing measure.

5. The influence of basis risk and moral hazard on the price and payment of CAT bonds

Apparently once the risk-neutral process of asset dynamics, loss and interest rate are known, it is possible to estimate the CAT bond price by the discounted expectation of its various payoffs in the risk-neutral world. The specification of payoffs of the CAT bond may be carried out under alternative considerations concerning the payoff risk. In this aspect, we can first consider the basic case in which there is no default risk [*Ryzyko*...2009], and then also the case of the default-risky CAT bonds with potential basis risk and moral hazard.

5.1. Default-free CAT bonds

To price the CAT bond it is assumed that this is a hypothetical discount bond whose payoffs (PO_T) at maturity (i.e. time T) are as follows:

$$PO_T = \begin{cases} a \times L & \text{gdy } C_T \leq K \\ r \times p \times a \times L & \text{gdy } C_T > K \end{cases} \tag{6}$$

where: K is the trigger level set in the CAT bond provisions;

- C_T − aggregate loss at maturity;
- $r \times p$ − the portion of the capital needed to be paid to bondholders when the forgiveness trigger has been pulled;
- L – the face amount of the issuing company's total debts which includes the face amount of the CAT bond;
- A − the ratio of the CAT bond's face amount to total outstanding debts.

The price of the CAT bond with the payoffs specified in equation (6)is carried out under the assumption that the state variables θ and η,which determined the term structure of interest rate and the aggregate loss "behave" well enough to be able to apply the risk-neutral approach of Cox and Ross [1976] and Harrison and Pliska [1981]. More specifically, under the risk-neutralized pricing measure Q, the CAT bond price on the issuing date (i.e. time 0) can be expressed by the term $E^{*}_{\theta,\eta}$ which denotes its expected value in a risk-neutral world.

In the next specifications of the model it is assumed that the state variablesθ, which for the purpose of valuing catastrophe risk bonds determine the term structure of interest rates, are independent of the state variables η, which relate to catastrophe risk variables. Under these assumptions, the price of CAT bond dependent on the price factor, denoted by $B_{CIR}(0,T)$, which concerns the default-free bond, which can be found in literature [Cox et al.1985], can be written as follows:

$$P_{CAT}(0) = B_{CIR}(0,T) \times \sum_{j=0}^{\infty} \exp\left(-\lambda \times T\right) \times$$
$$\times \frac{\left(\lambda \times T\right)^{j}}{j!} \times F^{j}(K) + r\rho \times (1 - \sum_{j=0}^{\infty} \exp\left(-\lambda \times T\right) \times \frac{\left(\lambda \times T\right)^{j}}{j!} \times F^{j}(K) \tag{7}$$

where $F^{j}(K) = P\,(X_{i,1} + X_{i,2} + \ldots + X_{i,j} \leq K)$,
F^{j} denotes the jth convolution of F,
$B_{CIR}\,(0, T) = A\,(0, T) \times \exp[-B(0,T)\cdot r]$,
where

$$A(0,\, T) = \left[\frac{2 \times \gamma \times \exp\left(\kappa + \gamma) \times \frac{T}{2} \right)}{(\kappa + \gamma) \times \exp\left(\gamma^{T} - 1\right) + 2 \times \gamma} \right]^{\frac{2 \times \kappa \times m}{v^{2}}},$$

$$B(0,\, T) = \frac{2 \times [\exp(\gamma^{T}) - 1]}{(\kappa + \gamma) \times [\exp(\gamma^{T}) - 1] + 2 \times \gamma},$$

$$\gamma = \sqrt{\kappa^{2} + 2 \times v^{2}}\,.$$

5.2. Approximation of the aggregate loss distribution and the bond price-analytical solution

Under the assumption that the catastrophe loss amount components are independent and identically lognormally distributed, the exact distribution of the aggregate loss at maturity date, denoted as $f(C_T)$, is obviously not known in the exact form. However, an approximate analytical form of this probability distribution can be set up. For this purpose we approximate the exact distribution by a lognormal distribution, denoted as $g(C_T)$, with specified moments. Jarrow and Rudd [1982], Turnbull and Wakeman [1991], Nielson and Sandmann [1996] used the same assumptions in approximating the values of Asian options and the so-called basket options.

The application of this approach requires only the setting of the two first moments of distribution defined by function $g(C_T)$, as equal to the moments of exact (but unknown) distribution of the aggregate loss at maturity $f(C_T)$. Let us write it as follows:

$$\text{first order moment } \mu_g = E[C] = \lambda \cdot T \cdot \exp\left\{\mu_i + \frac{1}{2} \times \sigma_i^2\right\} \tag{8}$$

$$\text{central second order moment} \sigma_g = Var[C] = \lambda \cdot T \cdot \exp\left\{2 \times \mu_i + 2 \times \sigma_i^2\right\} \tag{9}$$

thus, μ_g and σ^2_g denote the mean and variance of the approximating distribution g(C), respectively.

The value of the CAT bond can be written as follows:

$$\begin{aligned} B_{apr}(0) = B_{CIR}(0,T) \times \int_0^K \frac{1}{\sqrt{2 \times \pi \times \sigma_g \times C_T}} \times \exp\left\{-\frac{1}{2} \times (\ln C_T - \mu_g)^2\right\} dC_T + \\ + rp \times \int_K^\infty \frac{1}{\sqrt{2 \times \pi \times \sigma_g \times C_T}} \times \exp\left\{-\frac{1}{2} \times (\ln C_T - \mu_g)^2\right\} dC_T \end{aligned} \tag{10}$$

where $B_{apr}(0)$ is the approximate analytical CAT bond price at time 0.

This formula is similar to the one introduced by Litzenberg, Beaglehole and Reynolds [1996], except that they use a constant interest rate in the model.

The final empirical part of the article presents the comparison of the analytical solution with the estimates based on the numerical method without the approximating assumptions. Now we are going to discuss default-risky CAT bonds.

5.3. Default-risky CAT bonds

The assumption of no default mentioned above is for the derivation of CAT bonds' analytical presentation. In the case of practical considerations of default risk, basis risk, and moral hazard relating to CAT bonds, their payoffs will be specified and then they will be valued using the numerical method, which will be presented in the final section.

In the case when the insurer becomes insolvent and defaults, let us assume that the CAT bondholders have priority for salvage over the other debtholders because the proceeds from CAT bond sales are usually invested in a trust fund and can be liquidated only for the purpose of paying limited claims or returning to the bondholders. If the insurer is solvent, then CAT bondholders can receive the full principal of CAT bonds when the underlying losses are lower than the trigger level; otherwise, they can be repaid only part of the principal.

5.4. Determination of payoffs with no basis risk

In the first considered case we specify the default-risky payoffs when there is no basis risk. This means that the insurer's debt is forgiven when its actual loss is larger than a specific amount of loss.

Basis risk refers to the risk that the losses that individual insurers incur will not have an anticipated correlation with the underlying loss index of the CAT bond. Basis risk might reduce the hedging effect of CAT bonds and increase the default probability of the issuing company. In the case of no basis risk the default-risky payoffs of CAT bonds may be written as follows:

$$PO_{i,T} = \begin{cases} a \times L, & \text{when } C_{i,T} \le K \text{ and } C_{i,T} \le A_{i,T} - a \times L \\ rp \times a \times L, & \text{when } K < C_{i,T} \le A_{i,T} - rp \times a \times L \\ \max\{A_{i,T} - C_{i,T}, 0\}, & \text{in different cases} \end{cases} \tag{11}$$

where: $PO_{i,T}$ – are the payoffs at maturity for the CAT bond forgiven on the issuing company's own actual losses;

$A_{i,T}$ – are the issuing company's asset value at maturity;

$C_{i,T}$ – is the issuing company's aggregate loss at maturity;

L, a, K and rp are defined as in the cases discussed before.

5.5. Determination of payoffs with basis risk

In this case we compare the payoffs with the basis risk payoffs when the debt is forgiven at the composite index of losses.

In the case of the CAT bond being forgiven on the composite index of losses, the default-risky payoffs can be written as follows:

$$PO_{indeks,T} = \begin{cases} a \times L, & \text{when } C_{indeks,T} \le K \text{ and } C_{i,T} \le A_{i,T} - a \times L \\ rp \times a \times L, & \text{when } C_{indeks,T} > K \text{ and } K < C_{i,T} \le A_{i,T} - rp \times a \times L \\ \max\{A_{i,T} - C_{i,T}, 0\}, & \text{in different cases} \end{cases} \tag{12}$$

where: $C_{indeks,T}$ − is the value of the composite index of maturity;

a, L, $r\rho$, $A_{i,T}$, $C_{i,T}$ and K denote the same values as in equation (13).

$$P_i(0) = \frac{1}{a \times L} \times E_0^*[\exp\{-\bar{r} \times T \times PO_{i,T}\} \tag{13}$$

where: $P_i(0)$ – is the default-risky CAT bond price with no basis risk;

$E_0{}^*$ − denotes expectations taken on the issuing date under pricing measure Q;

$\bar{r}$ – is the average risk-free interest rate between the issuing date and maturity date;

$\frac{1}{a \times L}$ – the coefficient used to normalize the CAT bond prices for a face amount of one dollar,

and

$$P_{indeks}(0) = \frac{1}{a \cdot L} \times E_0^*[\exp\left\{-\bar{r} \times T \times PO_{index,T}\right\} \tag{14}$$

where: $P_{index}(0)$ – is the default-risky CAT bond price with basis risk at time 0;

E_0*, $\bar{r}$, $\frac{1}{a \times L}$ – defined as in formula (12).

5.6. Determination of payoffs with consideration of moral hazard

The third of the possible versions of pricing default-risky bonds concerns the structure of modelling payoffs with the moral hazard.

We may encounter such a situation when the CAT bond is forgiven on the issuing company's own losses because the issuing company has the priority and an incentive to settle claims, which has been already mentioned, more "generously" when the loss incurred approaches the trigger level. This model assumes that the issuing company "relaxes" its settlement policy once the accumulated losses fall into the range close to the trigger. This assumption is justified with the fact the accumulation of losses would cause an increase in expected losses for the catastrophic event. In this case the change of the loss process shall be specified as follows:

$$\mu_i' = \begin{cases} (1+\alpha) \times \mu_i, & \text{when } (1-\beta) \times K \leq C_{i,j} \leq K, \\ \mu_i, & \text{in a different case} \end{cases} \tag{15}$$

where: μ' is a logarithmic mean of the losses incurred by the $(j+1)$ *th* catastrophe when the accumulated loss $C_{i,j}$ falls in the specified range;

$(1-\beta)\ K \leq C_{i,j} \leq K$;

α is a positive constant, reflecting the percentage increase in the mean;

β is a positive constant that specifies the range of moral hazard behaviour or the solution, which does not have to be fully specified and which is introduced *ad hoc* for the purpose of determining the approach towards moral hazard when it is necessary, but without prior assumption of complex conditionings that led to its occurrence.

In the aspect of the last assumption concerning constant βthat modifies the change of loss process it should be marked that literature does not settle this aspect of moral hazard and, therefore, it should be rather regarded as a challenge that inspires other scientists to model moral hazard.

The presented analytical structure of the complex contingent contract is the basis for its complementation and modification. However, presented in the current form, it does not allow for the numerical estimation of the CAT bond price. It has already been signalled that this will be the subject of the exemplification of the application of CAT bond pricing model on the basis of the data simulated by the Monte Carlo method.

6. The exemplification of the model structure of CAT bonds with the application of the Monte Carlo simulation

For the purpose of explaining a some what difficult, from the formal point of view, analytical structure of the CAT bond estimation, we carry out the exemplification of application of model structure for this bond [Jin-Ping Lee, Min-teh Yu2002; *Ryzyko*… 2009]. The CAT bond prices were estimated [Jin-Ping Lee, Min-the Yu2002] by the Monte Carlo simulation. Let us consider the pricing of default-free bonds. We will not take into account the default-risky bonds with moral hazard and basis risk.

The initial step in pricing the CAT bond is establishing the set of parameters and base values. To assess the comparative effects of these parameters on CAT bond prices deviations from the base values are also established. To make it simple, it is assumed that **the total amount of the issuing company's debts, which include CAT bonds, has a face value of $100 and that the maturity of the CAT bond is equal to one year.** The simulations are run on a weekly basis with 20,000 paths. The given parameters and base values are presented in Table 1.

Table 1. Parameters and base values

Types of parameters and base values	Values
Asset parameters	
A insurer's assets	Assets to liabilities ***A/L***: 1.1, 1.2 and 1.3
μ_A drift due to credit risk	
Φ interest rate elasticity of asset	0, –3, –5
σ_A volatility of credit risk	5%
W_A Wiener process for credit shock	

Source: own elaboration on the basis of [Jin-Ping Lee, Min-teh Yu2002].

The initial asset/liability (or capital) position (*A/L*) ratios are set to be 1.1, 1.2, and 1.3, respectively. The average *A/L* ratio for the insurance sector equalled about 1.3 on a book-value basis over the past ten years. The interest rate elasticity of the insurer's assets is set at 0, –3, and –5, respectively to measure how the insurer's interest rate risk affects CAT bond prices. The volatility of the asset return that is caused by the credit risk is set at the level of 5%.

Table 2 includes the interest rate parameters. The initial spot interest rate r and the long-run interest rate m are both set at 5%. The magnitude of mean-reverting force κ is set to be 0.2, while the volatility of the interest rate v is set at 10%. The market prices λ_r of interest rate are set at 0 and –0.01, respectively. All these term structure parameters are included within the range typically used in the previous literature.

Table 2. Interest rate parameters

Type of parameters	Values
Interest rate parameters	
r initial instantaneous interest rate	5%
κ magnitude of mean-reverting force	0.2
m long-run mean of interest rate	5%
v volatility of interest rate	10%
Market prices λ_r of interest rate risk	0, –0.01
ZWiener process of interest rate shock	.

Source: own elaboration on the basis of [Jin-Ping Lee, Min-teh Yu 2002].

Table 3. Catastrophe loss parameters

Types of parameters	Values
Catastrophe loss parameters	
μ_i mean of the logarithm of the amount of catastrophe losses for the insurer	2
μ_{indeks} mean of the logarithm of the amount of catastrophe losses for the composite loss index	0.2
σ_i standard deviation of the logarithm of the amount of catastrophe losses for the insurer	0.5, 1, 2
σ_{indeks} standard deviation of the logarithm of the amount of catastrophe losses for the composite loss index	0.5, 1, 2
ρ_x correlation coefficient of the logarithms of amounts of catastrophe losses of the insurer and the composite loss index	0.5, 0.8, 1
$N(t)$ Poisson process for the occurrence of catastrophes	0.5, 1, 2
Other parameters	
K trigger levels	100, 110, 120
Rp the ratio of principal needed to be paid if debt forgiveness has been triggered	0.5
a the ratio of the amount of CAT bond to total debts	0.1
α moral hazard intensity	20%
β the ratio set below the trigger that will cause the insurer's moral hazard	20%
L the total amount of insurer's debts	100

Source: own elaboration on the basis of [Jin-Ping Lee, Min-teh Yu 2002].

Table 3 presents the catastrophe loss parameters and other parameters, including the trigger levels for debt-forgiveness and the ratio of principal needed to be paid if debt forgiveness is triggered.

The occurrence intensities of catastrophe losses are set to be 0.5, 1, and 2, respectively, to reflect the frequencies of catastrophic incidents per year. Also assume that the parameter values for catastrophe loss are the same for individual insurers and the composite loss index. We set the logarithmic mean μ_i and μ_{indeks} to be 2, and the logarithmic standard deviations, σ_i and σ_{indeks} to be 0.5, 1 and 2. The values for the index and individual insurers can be differently modified, but they increase the numerical dimension of calculations and do not broaden the analysis of basis risk. The analysis focuses on the coefficient of correlation ρ_x between the individual loss and the loss index rather than on their means and standard deviations. The portion of principal needed to be repaid, $r \times p$, is set at 0.5 when debt forgiveness has been triggered. The ratio of the amount of CAT bonds to the insurer's total debt a, is set at 0.1. Additionally, there are three different trigger levels K set at 100, 110, and 120.

6.1. Numerical pricing of default-risky CAT bonds with the alternative consideration of moral hazard

Table 4 includes the prices of default bonds with the alternative occurrence and non-occurrence of moral hazard at the alternative values of initial capital position (*A/L*), catastrophe intensity, loss variance and the interest rate elasticity of the issuing company's assets. Three (upper, middle, lower) values reported in each cell of Table 4 represent the corresponding estimate under the interest rate elasticity of 0, –3 and –5, respectively. A higher (absolute) value of interest rate elasticity corresponds to higher asset volatility and default risk of the issuer. Thus, one would expect the upper value of each cell (the CAT bond price for $\varphi = 0$) to be higher than the middle value (the CAT bond price for $\varphi = -3$) and lower value (the CAT bond price for $\varphi = -5$).

It is also expected that the higher the initial capital position (*A/L*) of the issuing company is, the lower the default risk and the higher the CAT bond prices are. The rise of the bond price is caused by the occurrence intensity and catastrophe loss volatility. Both estimates were computed using 20000 simulation runs. Bond prices per face amount to one dollar. The upper value, middle value, and lower value in each cell are CAT bond prices computed when the interest rate elasticities of the asset are 0, –3, –5, respectively.

Let us observe that the default risk premium decreases with the *A/L* ratio and increases with occurrence intensity and loss volatility. The default risk premium can go as high as 1.015 basis points for the case of *A/L* = 1.1, $(\lambda, \sigma) = (2, 2)$, and $\varphi = -5$.

To incorporate the effect of moral hazard it is assumed that when the accumulated loss amounts to 80% of the trigger level, the insurer settles the catastrophe claims more generously and therefore increases the expected loss of the catastrophe by 20%,

Table 4. Prices of default –risky CAT bonds with and without moral hazard (λ_r= –0.01)

(λ,σ_i)	A/L=1.1			A/L=1.2			A/L=1.3		
	Triggers K								
	100	110	120	100	110	120	100	110	120
	No moral hazard								
(0.5,1)	0.94891 0.94864 0.94847	0.94919 0.94898 0.94876	0.94924 0.94898 0.97876	0.94926 0.94910 0.94896	0.94968 0.94958 0.94944	0.95002 0.94963 0.94953	0.94940 0.94931 0.94977	0.94993 0.94984 0.95029	0.95021 0.95998 0.95062
(2,1)	0.93121 0.93078 0.92959	0.93292 0.93282 0.93150	0.93335 0.93335 0.93231	0.93360 0.93321 0.93268	0.93688 0.93611 0.93563	0.93845 0.93769 0.93706	0.93479 0.93474 0.93442	0.93827 0.93821 0.93761	0.94046 0.94021 0.93942
(2,2)	0.75871 0.75743 0.75658	0.76281 0.76095 0.76024	0.76323 0.76191 0.76229	0.76500 0.76476 0.76317	0.77104 0.77075 0.76317	0.77470 0.77446 0.77212	0.77044 0.76974 0.76955	0.77696 0.77621 0.77554	0.78314 0.78211 0.78063
	With moral hazard								
	A/L=1.1			*A/L*=1.2			A/L=1.3		
	100	110	120	100	110	120	100	110	120
(0.5,1)	0.94695 0.94667 0.94634	0.94738 0.94705 0.94672	0.94743 0.94709 0.94677	0.94746 0.94726 0.94712	0.94817 0.94778 0.94774	0.94841 0.94792 0.94788	0.94781 0.94767 0.94757	0.94853 0.94838 0.94824	0.94881 0.94867 0.94847
(1,2)	0.84697 0.84587 0.84551	0.84892 0.84759 0.84727	0.84925 0.84831 0.84727	0.85073 0.85054 0.84988	0.85444 0.85363 0.85268	0.85615 0.85525 0.85426	0.85394 0.85384 0.85369	0.85770 0.85750 0.85698	0.85079 0.86050 0.85936
(2,2)	0.72243 0.72064 0.71921	0.72628 0.72416 0.72254	0.72690 0.72521 0.72440	0.72928 0.72877 0.72764	0.73584 0.73463 0.73278	0.74017 0.73853 0.73645	0.73542 0.73456 0.73378	0.74226 0.74117 0.74001	0.74883 0.74702 0.74539

Source: simulated data elaborated on the basis of [Jin-Ping Lee, Min-teh Yu 2002].

which is reflected by the coefficient of moral hazard intensity α = 0.2. The moral hazard, therefore, increases the default risk and lowers the bond price. For example, in the case of (λ, σ) = (2.2), φ = –5, the price decreases about 350 basis points with the moral hazard. The magnitude of the moral hazard effect increases with (λ, σ_i) and absolute value of φ, and decreases with the *A/L* ratio.

Figure 1 presents the price of default-risky CAT bonds with no moral hazard (λ, σ) = (2,2), and φ = 0, at various trigger levels of *A/L* bond payoff. The triggers of 1.1, 1.15, 1.2, 1.3 are marked on the axis of abscissae. We observe that the CAT bond price increases with the increase of the trigger. This increase of bond price is also influenced by the increase of intensity λ, and loss volatility rate σ_i.

Figure 2 presents the price of default-risky CAT bonds with moral hazard and parameters of intensity, volatility and elasticity: (λ, σ)= (2,2), and φ = –5, at various trigger levels of *A/L* bond payoff. The triggers of 1.1, 1.15, 1.2, 1.3 are marked on the axis of abscissae. We also observe that CAT bond price increases with the increase of the trigger. This increase of bond price is, as previously noted, also influenced by the increase of intensity λ, and loss volatility rate σ_i.

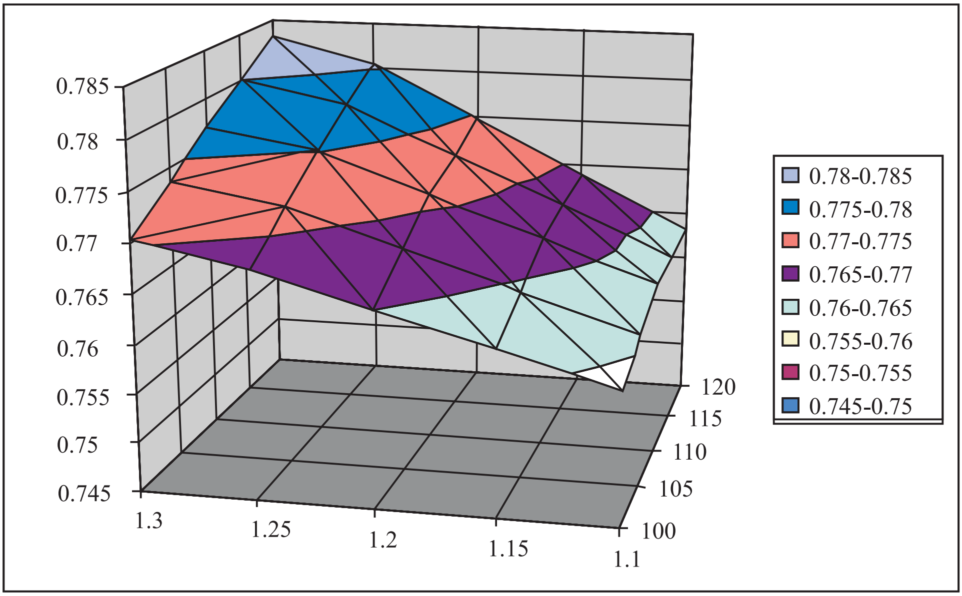

Figure 1. Prices of default-risky CAT bonds without moral hazard (λ, σ)= (2,2), and φ = 0

Source: own elaboration.

The above figure reflects the case when loss volatility is constant (σ_i = 2) and asset/liability ratio *A/L*= 1.1. The illustration from Figure 3 suggests that in extreme cases, when loss intensity is low (level of λ = 0.5), irrespective of the trigger of bond payoff, the risky bond premium with moral hazard reflects the payoff in connection

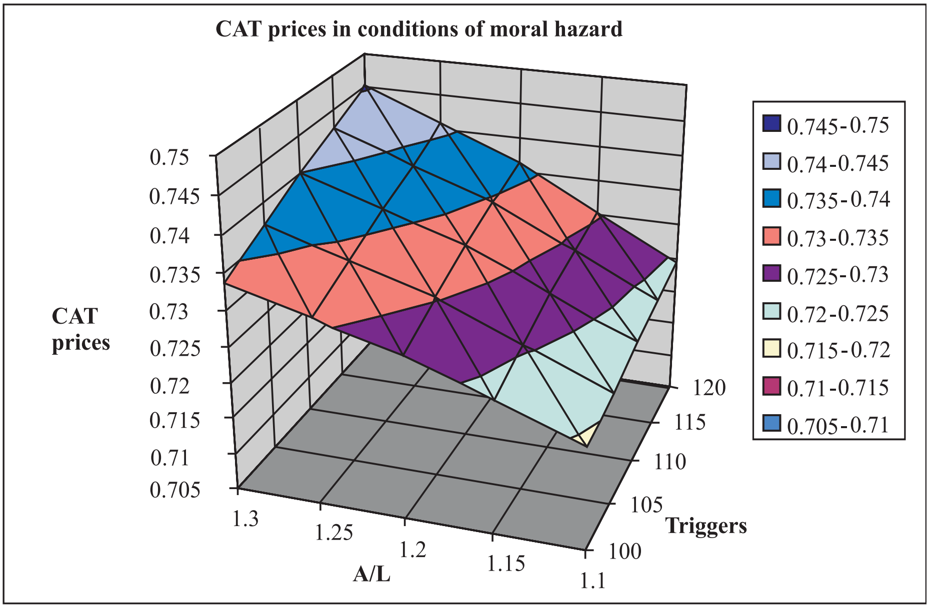

Figure 2. Prices of default-risky CAT bonds with moral hazard (λ, σ)= (2,2), and φ = –3

Source: own elaboration.

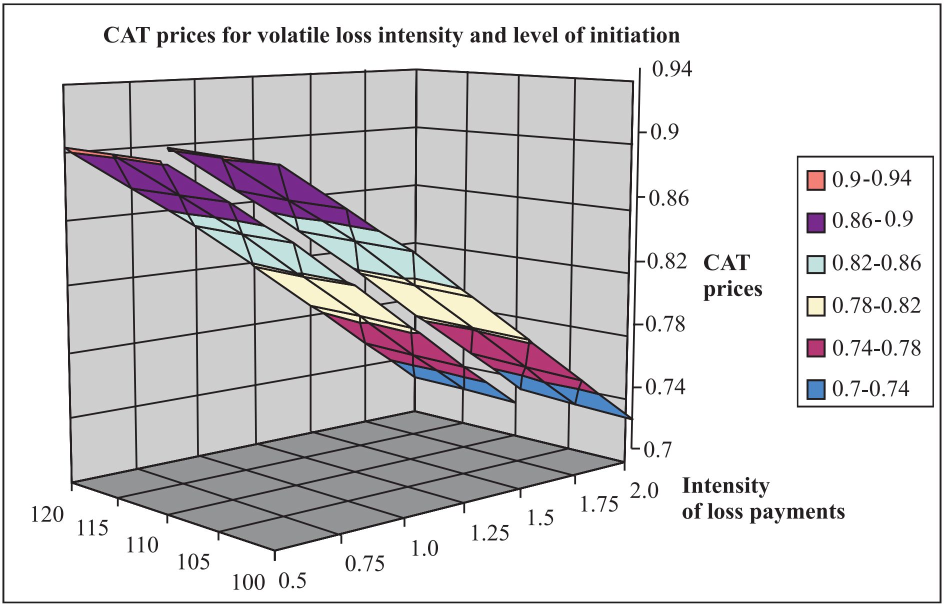

Figure 3. Prices of default-risky CAT bonds with A/L=1.1 with moral hazard and φ = –5 and constant volatility σ_i

Source: own elaboration.

with the incurred losses. The increase of loss intensity λ to level 2 significantly decreases the bond payoff. The CAT bond prices illustrated with and without moral hazard under alternative values of parameters (λ, σ_i) in Figures 1 and 2, as well as in Figure 3 that presents the CAT bond prices at variable trigger levels and variable loss intensity and with moral hazard, show the clear dependence of these prices on basis risk measured by *A/L* ratio and on the loss intensity λ (Figure 4). These figures show, at the same time, a slight influence of the trigger level at a constant basis risk *A/L* on the CAT bond price. The significant price differences indicate that the moral hazard is an important factor and should be taken into consideration when pricing the CAT bonds. Bantwal and Kunreuther [1999] also pointed out that moral hazard may explain the CAT bond premium puzzle.

6.2. Numerical pricing of default-risky CAT bonds with the alternative consideration of moral hazard

Table 5 presents the impact of basis risk on CAT bond prices. Because the difference in CAT pricing caused by the elasticity of interest rate is low, we consider only the case when $\Phi = -3$, and we focus on the discussion of the influence of basis risk. The table includes full results only for trigger K= 100 and 110 and A/L ratio = 1.1. For A/L = 1.2 and 1.3 the values of CAT bond are given only for ($\lambda, \sigma_i, \sigma_{index}$) = (2, 2, 2), thus only for the highest loss intensity and the highest loss volatility and loss index.

Table 5. Risky prices of CAT bonds vs. basis risk ($\lambda_r = -0{,}01$, $\Phi = -3$)

Triggers	K = 100			K = 110		
($\lambda, \sigma_i, \sigma_{index}$)	$\rho_x = 0.5$	$\rho_x = 0.8$	$\rho_x = 1$	$\rho_x = 0.5$	$\rho_x = 0.5$	$\rho_x = 1$
	***A/L*=1.1**					
(0.5, 0.5, 1)	0.95122	0.95122	0.95122	0.95122	0.95122	0.95122
(0.5, 1, 1)	0.94833	0.94854	0.94885	0.94853	0.94867	0.94906
(0.5, 2, 2)	0.89340	0.89808	0.97876	0.89448	0.89859	0.90835
(1, 0.5, 0.5)	0.95123	0.95123	0.95123	0.95123	0.95123	0.95123
(1, 1, 1)	0.94327	0.94383	0.94552	0.94413	0,94467	0,94607
(1, 2, 2)	0.83091	0.84144	0.85737	0.83322	0.84387	0.86005
(2, 0.5, 0.5)	0.95127	0.95127	0.95127	0.95127	0.95127	0.95127
(2, 1, 1)	0.92610	0.92876	0.93332	0.92861	0.93075	0.93523
(2, 2, 2)	0.71399	0.73115	0.76071	0.71764	0.73464	0.76521
($\lambda, \sigma_i, \sigma_{index}$)	***A/L*=1.2**					
(2, 2, 2)	**0.72460**	**0.74141**	**0.76736**	0.72943	0.74604	0.77496
($\lambda, \sigma_i, \sigma_{index}$)	A/L=1.3					
(2, 2, 2)	0.73357	0.74992	0.77300	0.73896	0.75536	0.78101

Source: simulated data elaborated on the basis of [Jin-Ping Lee, Min-teh Yu 2002].

It should be noted that with the low coefficient of correlation between an individual loss and loss index ρ_{index}, the issuing company has a high basis risk. When $\rho_x = 1$, no basis risk exists and the Bond prices are supposed to be the same as their corresponding values in Table 4. In the case of $\rho_x = 0.8$ or 0.5, basis risk exists and we observe that it drives the CAT bond prices down and that its magnitude increases with the loss frequency and loss volatility. For example, in the case when $(\lambda, \sigma_i, \sigma_{indeks}) = (2, 2, 2)$, $K = 110$ and $A/L = 1.1$, the CAT bond price drops 305 basis points when ρ_x decreases from 1 to 0.8 and falls another 170 basis points as 0.8 goes to 0.5. The price differences caused by the basis risk are absolutely significant in this setting. With A/L ratio = 1.2 the decrease of CAT bond prices equals 290 and 166 basis points, respectively, while, in the most favorable case, when $A/L = 1.3$ the decreases of CAT bond values equal 256 and 164 basis points, respectively.

We should also observe that the effect of basis risk decreases with the A/L ratios and increases with trigger levels, loss intensity and loss volatility.

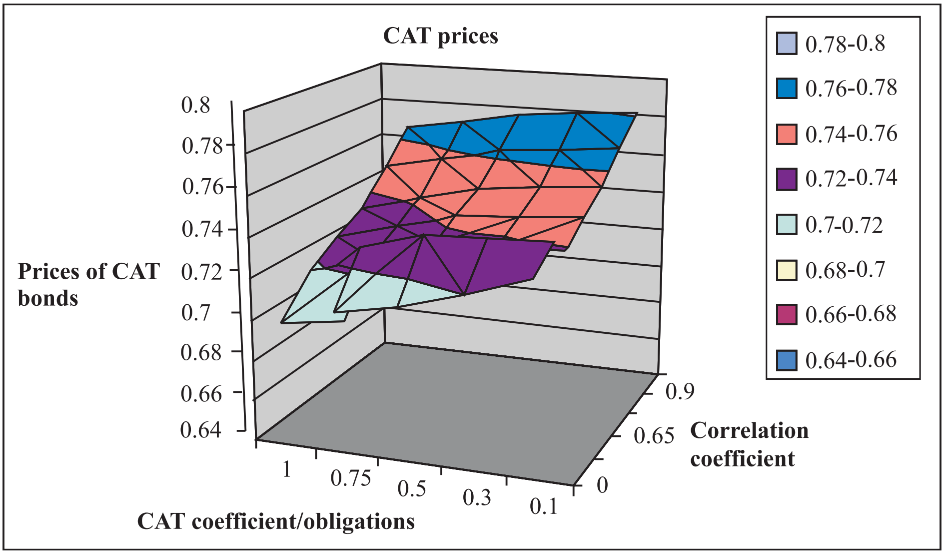

Figure 4. Default-risky CAT bond prices at A/L=1.1, $\varphi = -3$, trigger level $K = 110$ and $\lambda_r = -0{,}01$

Source: own elaboration.

Figure 4 illustrates this situation. One can notice how the CAT bond prices relate to basis risk (loss correlation, ρ_x) and the debt structure ratios (the amount of CAT bonds issued to the amount of total debt, a) for the case of $(\lambda, \sigma_i, \sigma_{index}) = (2, 2, 2)$, $K = 110$. The major conclusion is that the CAT bond price increases with the loss correlation at an increasing rate. Thus, the loss correlation decreases the basis risk premium at an increasing rate. It also implies that insurers with low loss correlation are subject to a substantial discount in their CAT bond prices.

Figure 4 also indicates also that the CAT bond price decreases with the debt structure ratio *a* at an increasing rate, especially at low A/L ratios, shown in Figure 4, indicating that CAT bond debt increases the default premium at an increasing rate. Therefore, insurers with more CAT bonds in debt have to pay a higher premium for their bonds.

7. Conclusion

The model of pricing the CAT bond considered in the article takes into account stochastic interest rates and more generic catastrophe loss processes. It is also possible to ”measure” the impacts of default-risk, moral hazard, and basis risk that are associated with CAT bonds. In the case of no default risk it is stated that the CAT bond prices computed numerically are very close to the ones computed by the approximating solution, except when the loss volatility is high. Then, the approximated prices reach higher values.

In the sequence of four cases considered in the article, the prices of CAT bonds with default risk are estimated. There is also the analysis of premium in the conditions of default risk that is changing along with the amount of claims caused by a given catastrophic event, variability of loss, flexibility of interest rate of the insurer assets, the coefficient of initial capital and the structure of liabilities. The premium is also estimated with regard to the influence of moral hazard upon prices (value) of CAT bonds. In this context, it is stated that moral hazard significantly lowers the value of bonds. The intensity of influence of moral hazard increases together with the intensity of a catastrophic event, variability of loss and also the interest rate risk of the insurer assets and it decreases along with the level of launching bond payment and the initial value of the insurer capital.

In addition to the considerations related to the behaviour of CAT bond prices, basis (systematic) risk is also considered. Basis risk significantly decreases the prices of CAT bonds and it contributes to the decrease of the rate of these prices. The influence of basis risk increases along with the trigger level, intensity of a catastrophic event, variability of loss and it decreases along with the initial value of the capital.

The model considered in this article may be viewed as a general way of assessing the default-risk. The exemplary applications of this model are presented here. Structural restrictions in this model link the bond price to basic characteristics of assets, liabilities, and interest rates. This allows to value bonds with unique features through the use of numerical analysis. It is important to note that this model can be easily extended to analyze other default-risky liabilities, not only these concerning CAT bonds, but also insurance-linked securities.

Literature

Bantwal V.J., Kunreuther, *A CAT bond Premium Puzzle?*, Working Paper, Financial Institutions Center, The Warton School, University of Pennsylvania 1999.

Bowers N.L., Gerber H.U., Hickman J.C., Jones D.A., Nesbitt C.J., *Actuarial Mathematics*, The Society of Actuaries, Itasca 1986.

Bouzouita R., Young A.J., *Catastrophe insurance option*, "Journal of Insurance Regulation" 1998, No. 7(2), p. 170-178.

Canter M.S., Cole J.B., Sandor R.I., *Insurance derivatives: a new asset class for the capital markets and a new hedging tool for the insurance industry*, "Journal of Applied Corporate Finance" 1997, Vol. 10, p. 69-83.

Cox S.H., Schwebach R.G., *Insurance futures and hedging insurance price risk*, "Journal of Risk and Insurance" 1992, No. 59, p. 628-644.

Cox J., Ingersoll J., Ross S., *The term structure of interest rates*, "Econometrica" 1985, No. 53, p. 385--407.

Cox J., Ross S., *Valuation of options for alternative stochastic processes*, "Journal of Financial Economics" 1976, No. 3, p. 145-166.

Cox S.H., Pedersen H.W., *Catastrophe risk bonds*, "North American Actuarial Journal" 2000, No. 4(4), p. 56-82.

Cummins J.D., *Risk-based premiums for insurance guarantee funds*, "Journal of Finance" 1988, No. 43, p. 593-607.

Cummins J.D., Geman H., *Pricing catastrophe futures and call spreads: an arbitrage approach*, "Journal of Fixed Income" 1995, No. 4, p. 46-57.

Chang C.J., Yu M.T., *Pricing catastrophe insurance futures call spreads: a randomized operational time approach*, "Journal of Risk and Insurance" 1996, No. 63(4), p. 599-617.

Doherty N., *Financial innovation in the management of catastrophe risk*, "Journal of Applied Corporate Finance" 1997, Vol. 10(3).

Duan J.C., Moreau A., Sealey C.W., *Deposit insurance and bank interest rate risk: pricing and regulatory implication*, "Journal of Banking and Finance" 1995, No. 19, p. 1091-1108.

Harrison J.M., Pliska S.R., *Martingales and stochastic integrals in the theory of continuous trading,* "Stochastic Processes and their Applications" 1981, No. 11, p. 215-260.

Jarrow R., Rudd A., *Approximate option valuation for arbitrary stochastic processes*, "Journal of Financial Economics" 1982, No. 10, p. 347-369.

Jin-Ping Lee, Min-Teh Yu, *Pricing default-risky CAT bonds with moral hazard and basis risk*, "Journal of Risk and Insurance" 2002, Vol. 69, No 1.

Lawędziak B. Szkutnik W., *Aspekt sekurytyzacji i hazardu moralnego w prognozowaniu działalności lokacyjnej firmy ubezpieczeniowej*, [in:] *Prognozowanie w zarządzaniu firmą,* Prace Naukowe Akademii Ekonomicznej nr 1112, Wyd. AE, Wrocław 2006a.

Lawędziak B. Szkutnik W., *Ryzyko realizacji wysokich roszczeń w aspekcie modelowej równowagi hazardu moralnego i ryzyka bazowego*, *Metody matematyczne, ekonometryczne i informatyczne w finansach i ubezpieczeniach, część 1*, Wyd. AE, Katowice 2006b.

Laurenzano V.I., *Securitization of insurance risk: a perspective for regulator*, "Journal of Insurance Regulation", No. 17(2), p. 179-185.

Litzenberger R.H., Beaglehole D.R., Reynolds C.E., *Assessing catastrophe reinsurance-linked securities as a new asset class*, "Journal of Portfolio Management" 1996, No. 23(3), p. 76-86.

Loubergé H., Kellezi E., Gilli M., *Using catastrophe-linked securities to diversify insurance risk: a financial analysis of CAT bonds*, "Journal of Insurance Issues" 1999, No. 22(2), p. 125-146.

Merton R., *Option pricing when underlying stock returns are discontinuous*, "Journal of Financial Economcs" 1976, No. 3, p. 125-144.

MertonR., *An analytic derivation of the cost of deposit insurance and loan guarantee*, "Journal of Banking and Finance" 1977, No. 1, p. 3-11.

Naik V, Lee M., *General equilibrium pricing of options on the market portfolio with discontinuous returns*, "Review of Financial Studies" 1990, No. 3(4), p. 493-521.

Nielson J., Sandmann K., *The pricing of Asian options under stochastic interest rate*, "Applied Mathematical Finance" 1996, No. 3, p. 209-236.

Ritchken P., *Derivative Markets:Theory, Strategy, and Applications*, Harper Collins, New York 1996.

Ryzyko zobowiązań i działań w inwestycjach gospodarczych oraz ubezpieczeniowych. Aspekt modelowania i oceny sekurytyzacji ryzyka, red. W. Szkutnik, Prace Naukowe Akademii Ekonomicznej, Wyd. AE, Katowice 2009.

Szkutnik W., *Anticipation of the price of CAT and in the management of the process of securitization of catastrophe insurance*, [in:] *Econometrics 28, Forecasting,* ed. P. Dittmann, Research Papers of Wrocław University of Economics No. 91. Publishing House of Wrocław University of Economics, Wrocław 2010, p. 151-165

Turnbull S., Wakeman L., *A quick algorithm for pricing European average option*, "Journal of Financial and Quantitative Analysis" 1991, No. 26,p. 377-389.

Vasicek O.A.,*An equilibrium characterization of term structure*, "Journal of Financial Economics" 1977, No. 5, p. 177-188.

Zajdenweber D., *The valuation of catastrophe-reinsurance-linked securities,* Paper presented at American risk and insurance association meeting, 1998.

Zastosowanie metod ekonometryczno-statystycznych w zarządzaniu finansami w zakładach ubezpieczeń, red. W. Ronka-Chmielowiec, Wyd. AE, Wrocław 2004.

ANTYCYPACJA CENY OBLIGACJI CAT W ZARZĄDZANIU KATASTROFALNYM RYZYKIEM UBEZPIECZENIOWYM

Streszczenie: Zastosowanie instrumentów finansowych rynku kapitałowego ma na celu zarządzanie procesem sekurytyzacji ryzyka katastrofalnego. Jest to istotne ze względu na skutki, jakie przynoszą straty wywołane zjawiskami katastrofalnymi. W artykule omówionazostała struktura obligacji CAT i ich wycena w oparciu o model stochastycznego procesu stopy procentowej. Obligacje CAT przeznaczone są do finansowania skutków zdarzeń katastrofalnych. Mają one charakter zbliżony do kapitału warunkowego CC (*contingentclaim*), ale w realnych warunkach są one instrumentami rynku finansowego – obligacjami katastrofalnymi. Przedstawione podejście zostało zilustrowane rozkładem cen obligacji w układzie poziomu uruchomienia umorzenia obligacji CAT i jej zmienności. Rozpatrzony został kierunek badań wyznaczony przez uwzględnienie w wycenie hazardu moralnego i ryzyka bazowego (rynkowego). W sekwencji przypadków rozpatrzonych w artykule szacowane są ceny obligacji CAT o ryzyku odmowy wypłaty oraz analizowana jest składka w warunkach ryzyka odmowy wypłaty zmieniająca się wraz z wielkością roszczeń wywołanych danym zdarzeniem katastrofalnym, zmiennością straty, elastycznością stopy procentowej aktywów ubezpieczyciela, współczynnikiem początkowego kapitału i strukturą zobowiązań. Składka szacowana jest także ze względu na wpływ moralnego hazardu na ceny (wartość) obligacji CAT. W tym kontekście stwierdzono, że moralny hazard w znacznym stopniu obniża wartość obligacji. Intensywność wpływu moralnego hazardu wzrasta wraz z intensywnością zdarzenia katastroficznego, zmiennością straty i ryzykiem stopy procentowej aktywów ubezpieczyciela, a maleje wraz z poziomem uruchomienia wypłat z obligacji i początkową wartością kapitału ubezpieczyciela.