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The Impact of Market Liquidity on the Effectiveness of Option Valuation with the Black-Scholes-Merton Model Using the Example of the WIG20 Index

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Abstract: The Black-Scholes-Merton model is one of the most popular option pricing models used in market practice. This model is based on unrealistic assumptions, including the lack of transaction costs. While it is not possible to satisfy all the conditions of the model, it is logical to assume that perfectly liquid markets will meet them better, which will help to reduce the risk of error. The aim of the article was to measure the impact of liquidity to the divergence of the Black-Scholes-Merton model compared to real market closing prices. The result of the research demonstrates a moderate dependence between the volume of the WIG20 index, the volume of option transactions and a negative correlation with the *ILLIQ* illiquidity indicator introduced by Amihud (2002). The research led to the conclusion that there is a positive correlation between the liquidity and the divergence between BSM model and the market prices.

Keywords: options, Warsaw Stock Exchange, WIG20, Black-Scholes-Merton model, pricing models, derivatives, index options.

1. Introduction

Options are risk transfer financial instruments with hedging and speculative functions. Knowledge of option pricing is crucial for professional investors and financial institutions, participating in derivatives markets. One of the most popular methods of options valuation is the Black-Scholes-Merton model. Due to its universality, the model is widely used, and helps with the measuring the options value. Some simplifications have been applied to build the valuation model. These include the ability to trade any number of shares and no transaction costs. While these assumptions are unrealistic, it can be assumed that more liquid financial markets will reflect them better.

The aim of the article was to investigate the impact of liquidity on the effectiveness of the Black-Scholes-Merton model based on the example of the WIG20 index option. This knowledge can be essential to understand the divergence between the BSM and market price and therefore, facilitate reducing the risk of incorrect valuation. From the practical standpoint, this could help with a more accurate estimation of volatility as an input parameter in the model.

The subject of the effectiveness of option pricing models is not a new concept in the literature, however the research so far has concentrated on a comparison of individual models, including BSM to indicate the most effective in the given period and market (Bates, 1995; Rastogi, 2019). This paper focused on the BSM model. Thus, in regard to the market liquidity impact on option pricing, in the literature there are positions that indicate a more attractive valuation of options in less liquid markets (Brenner, 2001) or indicate higher expected rates of return in the case of less liquid options, called illiquidity premia (Christoffersen, Goyenko, Jacobs, & Karoui, 2018). This article examines this impact for the only options traded the on Warsaw Stock Exchange – WIG20 index options.

The author examined the effectiveness of the option pricing model from the perspective of the model risk, understood as a situation when the model used is not able to correctly estimate the market value of the financial instrument. Model risk may manifest itself as the result of errors in estimating model parameters, adopting an incorrect model form or incorrect model application. In the BSM model at the time of valuation, all parameters except volatility are known. Therefore, it could be assumed that the correct estimation of the variability would allow to eliminate the model risk to a minimum. However, it should be noted that even with the same input parameters, transaction prices may differ from each other. To most accurately approximate the volatility used in the study, the volatility published by the Warsaw Stock Exchange was used.

Capital market liquidity is a complex concept. Market liquidity can be understood as the ease of entering transactions. This ease may result from many factors, such as low transaction costs. In this article the basic methods of measuring market liquidity are reviewed, then two selected methods are used as the basis for examining the impact of liquidity on the effectiveness of option pricing models on the Warsaw Stock Exchange.

The article consists of four sections. The first one presents the Black-Scholes-Merton model and its assumptions, while the second one is focused on the selected methods of measuring market liquidity. In the third part, the research methods and results are described. The conclusion of the study are presented in the final section.

2. The Black-Scholes-Merton option valuation model

An option is a contract which gives a one-party right to purchase or sell the underlying asset at a predefined time in future at a specific price, called the strike price. This is a risk-transfer financial instrument, traded across global exchanges and over-the--counter markets. The valuation of the option is an important topic for professional investors and banks, due to various reasons, for example supporting the investment decision-making process and the requirements of financial reporting.

"Prices for options and corporate obligations", a study published by F. Black and M. Scholes in cooperation with R. Merton in 1973, was a significant achievement in the option valuation field. It introduced a widely used option valuation model named the Black-Scholes model. One year later, R. Merton published his "Theory of rational option pricing" which improved the model by adding the dividend element. The improved model, including the dividend value, was named the Black-Scholes--Merton model and is based on the following assumptions:

1. The short-term interest rate is fixed and constant.
2. The price of the underlying asset follows a random walk, with a variance rate proportional to the square of the share price. Therefore, the distribution of the rates of return of the underlying asset has a log-normal nature.
3. The dividend yield is continuous.
4. The option is European so it can be exercised only at the expiry date.
5. There are no transaction costs.
6. There is an elevated share rate, paying only the short-term interest rate.
7. There are no transaction limits for short sale.
8. There is no arbitrage.

It should be noted that the above assumptions are difficult to meet in the real market. For example, WIG 20 rate of returns for the analysed period did not meet the criteria of a log-normal nature for the Shapiro-Wilk or Lilliefors tests.

However, the application of the above assumptions allows for the construction of a model in which the option price will depend on the option price, time to expiry and other parameters, which will be constant over time. Then, assuming there is no possibility of arbitrage, one can create a position consisting of a long position on the underlying and a short position in the option, the value of which will not depend on the price of the underlying, but only on fixed parameters and price.

According to the assumptions of the model, at time zero, at price S_0, strike price X, time to expiry T, price of European call (c) or put (p) option, can be calculated as:

$$c = S_0 e^{-qT} N(d_1) - X e^{-rT} N(d_2),$$

$$p = X e^{-rT} N(-d_2) - S_0 e^{-qT} N(-d_1),$$

where

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r - q + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$$

$$d_2 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r - q - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$$

$N(x)$ is the cumulative probability distribution function for a variable that is normally distributed with a mean of zero and a standard deviation of 1.0 (Hull, 2000, p. 250).

The above formulas were used for the calculation of theoretical option values in this paper. As input parameters for the valuation, option indicators data published by the Warsaw Stock Exchange were used. Each option series had own set of the indicators, and this fact was considered for all the calculations. The input parameters can be defined as:

1. The S_0 price is the closing price of the WIG20 index for a given hour on a given trading day,

2. The risk-free rate is determined for each option expiry date in accordance with the following steps:

a) WIBOR capitalisation_continuous = ln (1 + WIBOR capitalisation annual × (t / 365)) / (t / 365),

b) WIBID capitalisation_continuous = ln (1 + WIBID annual capitalisation × (t / 365)) / (t / 365),

where: t – date for which a given interest rate is determined (e.g. for WIBOR 1 week = 7 days, for WIBOR 1 month = 30 days).

c) Then WIMEAN rates should be calculated. WIMEAN are the average of the WIBOR and WIBID interest rates for the terms – 1 week, 2 weeks, 1 month, 3 months, 6 months, and 9 months, calculating the interest rates for each expiry date by the linear interpolation of the available WIMEAN rates.

3. Implied volatility calculated for each option series is based on the arithmetic mean of the best buy and sell offers in the continuous trading range in the period from 1 hour and 10 minutes before the end of continuous trading, to 5 minutes before the end of continuous trading.

4. The continuous dividend rate, with the capitalisation of the portfolio of a given index for a given closing day *KAP_Index,* the amount of dividend on KD shares and the number of shares of a given company in the portfolio of a given index Package (Pakiet), were determined using the following formula:

$$DY_{Index} = \frac{\sum KD \times Pakiet}{KAP\_Index}.$$

The detailed methodology for calculating the interest rate, implied volatility and dividend rate, is presented in "Methodology for calculating Greek coefficients for options on WIG20" (https://www.gpw.pl/pub/GPW/files/metodologia_wledzniki_opcje.pdf).

3. Capital market liquidity measurement

The comprehensive concept of capital market liquidity is difficult to define. Liquidity can be understood as the ease of trading in given markets or financial instruments. The liquidity of a given financial instrument is influenced by many factors, including transaction costs, demand pressure, bid-ask spread, difficulty of locating counterparty and trading volume (Amihud, 2002). The group of the most popular liquidity measures includes (Porcenaluk, 2013; Wojtasiak, 2003):

1. Bid-ask spread – the difference in prices from the best sell and buy offers, lower values may indicate greater liquidity.
2. Number of transactions – shows the activity of investors, higher values may indicate greater liquidity of a given financial instrument.
3. Trading volume – the number of financial instruments that have changed owners, higher values may indicate greater liquidity.
4. Turnover value – the turnover volume multiplied by the prices in concluded transactions, higher values may indicate greater liquidity.
5. Amount of free-float financial instruments – the number of financial instruments that are not held by long-term investors, higher values may indicate greater liquidity.
6. Ratio of the volume of purchase offers to sale offers – a value close to 1 is the potential balance between the liquidity of the demand and supply sides.

From the investor's perspective, these measures should be compensated by the expected rate of return. This fact should be considered in the financial instrument valuation. Standard asset valuation theory assumes the existence of perfectly liquid markets where any security can be traded at no cost at all times (Amihud, 2002, p. 6). The assumptions of the Black-Scholes-Merton model are also confirmed as it moves towards an ideal market. It is therefore logical to assume that an increase in market illiquidity will result in a discrepancy between the model's valuation and the market price. The more illiquid the market, the greater the divergence might be observed if the statement is true.

To measure the market liquidity influence on the Black-Scholes-Merton model result, an application of appropriate market liquidity indicator is required. The measurement of the liquidity is a complex task, due to the difficulty in capturing of all the liquidity characteristics in one tool or indicator. For instance, research on the bid-ask spreads requires the availability of high-frequency data for the respective period. Due to the unavailability of such data, researchers must use substitutes, for example daily rate return or daily volume return. It should be noted that measuring the market liquidity will always be affected by error risk due to the following (Amihud, 2002, p. 37):

1. A single indicator cannot capture all liquidity dimensions.
2. The result acquired on empirically obtained data may be distorted by a single event occurring in the analysed period.
3. Using low-frequency data increases error risk.

Market liquidity measures can be divided into groups based on transaction costs or their estimation (Corwin & Schultz, 2012; Roll, 1984) and investor activity indicators based on volume, turnover value, and rate of return (Amihud, 2002). One of the most popular liquidity measures is bid-ask spread. However, this measure cannot be applied in the Polish market due to the unavailability of transaction data. Even one-minute intraday data will be insufficient to accurately determine the amount of the spread.

The *ILLIQ* market illiquidity measure, introduced by Amihud, is one of the most popular indicators used to measure liquidity and can be applied to various time frames. Due to this fact and the universality of the method, this indicator was selected for further research.

Amihud defines stock illiquidity as the average ratio of the daily absolute return to the trading volume on that day, $|R_{iyd}|\ j = VOLD_{iyd}$: R_{iyd} is the return on stock i on day d of year y and $VOLD_{iyd}$ is the respective daily volume in dollars. This ratio gives the absolute (percentage) price change per dollar of daily trading volume, or the daily price impact of the order flow. The *ILLIQ* value can be calculated using the following formula:

$$ILLIQ_{iy} = 1/D_{iy} \sum_{t=1}^{Diy} \left|R_{iyd}\right| / VOLD_{ivyd},$$

where D_iy – number of days for which data are available for the financial instrument i in year y.

The result should be interpreted as follows: the higher the index value, the more illiquid the market. In the context of this research, lower values will be interpreted as moments of greater market liquidity, for which less divergence of option valuation should be expected.

4. Impact of liquidity on the divergence between the Black-Scholes-Merton model and the transaction prices

As demonstrated in the previous section, the accuracy of the data allows to reduce the information noise and measurement error. It follows that high frequency data at a single transaction level would allow for a more accurate measurement. However, due to the unavailability of such data for the Polish market and the difficulties related to the analysis of such a volume of data for a wider range of dates, a substitute should be employed. The Stooq.pl service provides historical stock market data in the following intervals: 5 minutes, hourly, and daily, accessible on the database website (https://stooq.pl/db/h/). The daily data were rejected due to the high risk of making a mistake in estimating the value of options using the Black-Sholes-Merton model. This risk comes from the fact that when analysing the options' daily closing prices, information from which point during the session the price comes from is unavailable. Hence, in the case of low liquidity on a given series of options, the difference in the theoretical price resulted from the Black-Scholes-Merton model and the market closing price may come from the difference between the closing price of the WIG20 index and the index price at the time of execution of the option transaction. Therefore, hourly data were used to reduce this risk. The 5-minute data contained too short a range of data to be considered reliable in the context of the study.

Historical quotations of all (335) option series for the period from 7 June 2021 to 1 April 2022, presented in hourly intervals, were used to determine the value of the options with the Black-Scholes-Merton model. For each trading hour and option series, the difference between the theoretical value and the closing price, interpreted as the market value, was determined. Then, for each trading day the average daily difference was calculated. The charts below show the dependence of daily divergences of transaction prices obtained by the Black-Scholes-Merton model depending on volatility, daily WIG20 index volume, daily option volume and *ILLIQ* index.

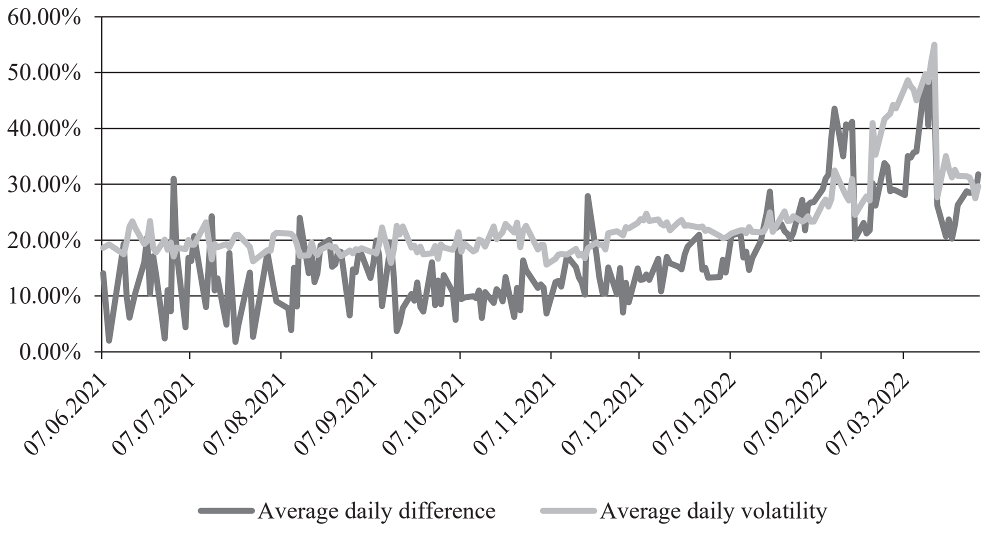

Fig. 1. Average daily difference between the market and the Black-Scholes-Merton model valuation and daily volatility

Source: own elaboration based on Stooq.pl data (https://stooq.pl/db/h/).

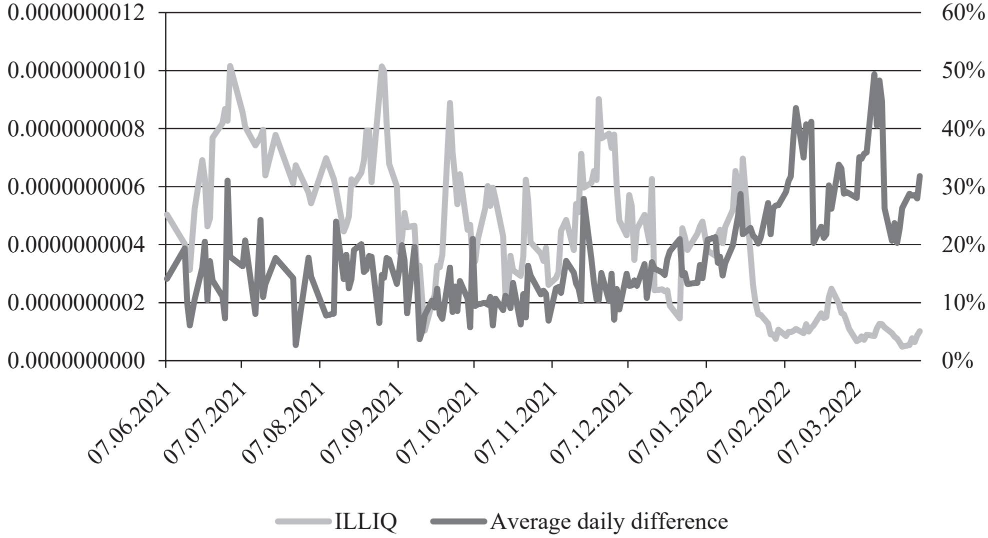

Fig. 2. Average daily difference between the market and the Black-Scholes-Merton model valuation and *ILLIQ*

Source: own elaboration based on Stooq.pl data (https://stooq.pl/db/h/).

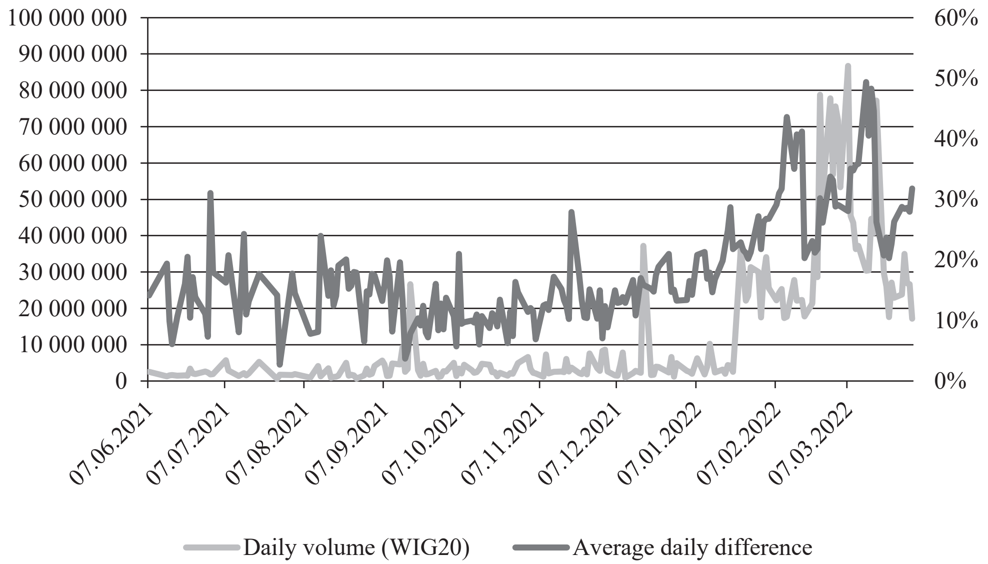

Fig. 3. Average daily difference between the market and the Black-Scholes-Merton model valuation and daily volume of underlying index

Source: own elaboration based on Stooq.pl data (https://stooq.pl/db/h/).

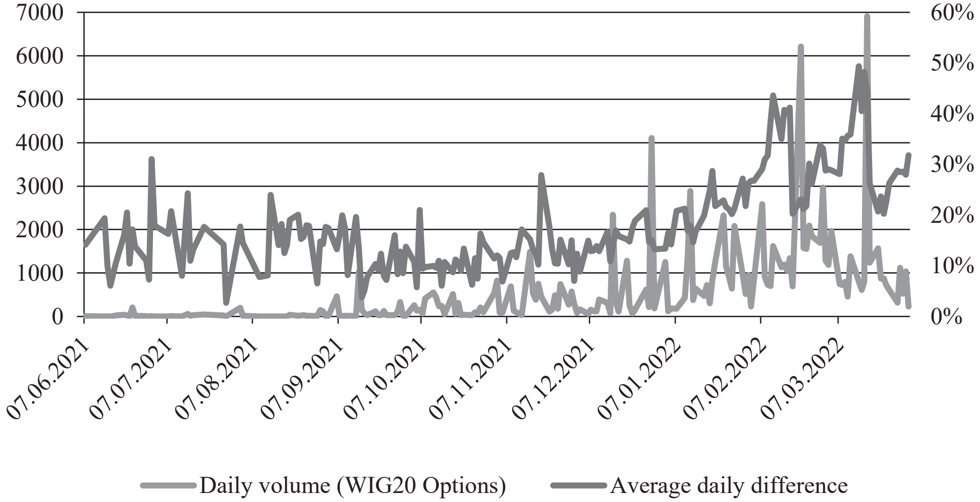

Fig. 4. Average daily difference between the market and the Black-Scholes-Merton model valuation and daily volume of WIG20 Options

Source: own elaboration based on Stooq.pl data (https://stooq.pl/db/h/).

The Pearson linear correlation coefficient was determined for the daily WIG20 volume, WIG20 options and the *ILLIQ* indicator, see the results below:
- for the WIG20 volume: 0.6443,
- for the volume of WIG20 options: 0.4536,
- for *ILLIQ*: -0.5273.

In the case of volume, a positive, moderate correlation can be noticed, and the relationship is more important for the volume on the base index than on the option transactions themselves.

For the *ILLIQ* index, a negative moderate correlation can be observed. Hence, with an increase in market illiquidity measured by this indicator, discrepancies between the valuations obtained by the application of the Black-Scholes-Merton model decreases.

The above correlations were not statistically significant.

5. Conclusion

The Black-Scholes-Merton model of option pricing is based on assumptions which are difficult to meet in the real market. An increase in liquidity should bring the model to the real market. Therefore, one can logically assume that an increase in liquidity increases the model efficiency – understood as the ability to obtain results close to the market prices. However, the results of the research indicate the opposite relationship.

During the period under examination, the divergence between the average daily difference between the BSM model results and transactional prices increased as the volume on either the underlying index or options increased. The use of the selected indicators measuring the market liquidity presented in this paper, does not allow for a more precise estimation of the input parameters. This is because the correlation is moderate and not statistically significant for any of these indicators. Additionally, results of the research show the opposite relationship than the assumed logic would suggest.

The reason of such a phenomenon could be another market dependency, namely that between market volatility and transaction volume. In periods of increased volatility, the transaction volume is usually much higher. The expectations for the future volatility may be more varied, which turns into a more difficult estimation of volatility in the option valuation model.

From the practical standpoint, volume and market liquidity indicators data could be supportive for the estimation of volatility in the BSM model, but not enough for their improvement. The only undisputable conclusion is that in conditions of increased market volatility, the BSM model will be characterised by the higher risk of incorrect result.

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Wpływ płynności na efektywność wyceny opcji modelem Blacka-Scholesa-Mertona na przykładzie indeksu WIG20

Streszczenie: Model Blacka-Scholesa-Mertona jest jednym z najczęściej wykorzystywanych w praktyce modeli wyceny opcji. Model ten oparty jest na pewnych trudnych do spełnienia założeniach, które jednak byłyby bardziej rzeczywiste na idealnie płynnym rynku. Celem badania jest określenie zależności pomiędzy płynnością rynku a rozbieżnościami między rynkowymi cenami a wynikami uzyskanymi na podstawie wyceny modelem Blacka-Scholesa-Mertona. Na podstawie badań dokonanych na godzinowych interwałach dla wszystkich serii opcji notowanych od 7 czerwca 2021 do 1 kwietnia 2022 roku została wykazana umiarkowana zależność pomiędzy wolumenem indeksu WIG20 a wolumenem transakcji opcyjnych oraz ujemna zależność ze wskaźnikiem niepłynności rynku *ILLIQ* wprowadzonym przez Amihuda (2002). Uzyskane wyniki badań wskazują na istnienie pozytywnej korelacji między płynnością rynku a rozbieżnościami pomiędzy wynikami uzyskanymi za pomocą modelu Blacka-Scholesa-Mertona i rynkowymi cenami zamknięcia.

Słowa kluczowe: opcje, GPW, WIG20, model Blacka-Scholesa-Mertona, modele wyceny, instrumenty pochodne.