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COMPARISON OF STATISTICAL VERSUS STOCHASTIC MODELS FOR WORK INDEX DETERMINATION IN QUARTZ-MARBLE MIXTURES

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Abstract: The required work for ore trituration is represented by the Bond Work Index value and is determined by the grindability test for ball mills. This article examines the grinding behavior of ore blends with different mechanical properties in standard ball mills. The goal of this research was to compare statistic and stochastic models of the Work Index value for mixtures of quartz and marble at different proportions of each material. Quartz and marble bearing rocks were selected for this study due to the high difference between the Work Index value of each material, making the variability of the results more evident. Work Index values obtained for each mixture are shown, from which a deterministic model was proposed defined by data regression. The novelty of this research lies in the non-linear model, which was the best fit for the Work Index value of the quartz-marble blends. Our methodology allows us to build more accurate models and can be used for quartz-marble blends and other materials.

Keywords: *Work Index, Bond grindability, mineral mixture, Monte Carlo, stochastic model*

1. INTRODUCTION

In most countries, mining is one of the fundamental pillars for developing other industrial processes. However, mining development needs energy resources that can be

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expensive. In the mining industry, the comminution stage is one of the most energy-intensive, which considerably increases material processing costs (Pedrosa et al. 2019; Aras et al. 2019; Lucay et al. 2019). For this reason, it is necessary to optimize comminution stage energy consumption, thus minimizing operational costs (Annicchiarico 2018). Nothing can prove the importance of grinding experiments for mill sizing better than the fact that 5% of total energy consumption in developed countries goes towards mineral grinding and crushing processes (Mucsi 2008).

Establishing a controlled, efficient system for mineral grinding circuits has been the topic of many studies and the primary cause of concern for operators across many years (Hadizadeh et al. 2017). Implementing a mathematically modeled system for optimizing and simulating mineral grinding processes has been constantly studied in mineral processing for decades (Farzanegan, Mirzaei 2015). However, the pressure exerted on industries due to high-grade resources' exhaustion will make all technological methods for milling become more prominent in the future, allowing for the reprocessing and exploitation of lower-grade resources and obligating industries to support much finer crushing processes (Singh et al. 2019).

Compared to all other mining stages and processes, milling can consume 10 times more energy than employed in the crushing stage and 100 times more than used in the blasting stage. Since ore deposit rocks are heterogeneous, we must understand ore mixtures' behavior in the milling stage in order to improve efficiency, update protocols and regulate the equipment associated to it. However, Mucsi (2008) affirms that for several decades there has been increased demand in the production of a wide variety of fine materials and minerals. Extracted ore grindability is especially relevant here, since the value of this parameter is the main overall energy need indicator for size reduction processes. Understanding and characterizing ore grindability variability is one of the most important parts of the geometallurgical framework, and is essential for optimizing production, since the information about ore blocks' hardness, mineralogy, and metallurgical response guides mining and processing planning and management (Heiskari et al. 2019). Grain size and the association of the minerals contained in the ore body usually vary, directly affecting the processing plant feed. For this reason, geometallurgical models must provide the most optimal operating parameters, the grinding size for any rock unit, and the target liberation degree (Lund et al. 2015).

Citing Farzanegan and Mirzaei (2015) there have been progressive advances in numerical modeling algorithms and applied methodologies oriented towards resolving optimization problems. For Mucsi (2013) in general contexts, in order to process minerals, there are three commonly applicable laws describing the existing relation between grinding energy use and the resulting fineness of the processed material: Bond's law, Rittinger's law and Kick-Kirpicsev's law.

The milling process uses various specific energy consumption models. One is the Bond Work Index (WI), a grindability index that represents the required energy to reduce the size of a specific material (Bond 1961). The Standard Bond method is the most accepted to design ball mills, but it is a long process that consists of at least eight milling cycles, so several researchers have focused on simplifying this method by trying to perform faster WI calculations. (Gharehgheshlagh 2016). The value obtained depends on the nature of the material and the type of comminution equipment used to reduce its size, which can be determined through the grindability test for ball mills proposed by Bond in 1961 (Yan, Eaton 1994). Aras et al. (2019) affirm that the Bond method is widely used in designing grinding circuits, selecting grinding equipment, determining power requirements and performance evaluation. Therefore, it is important to predict the Bond work index using some mechanical tests on practical and easy rocks without needing to use a mill.

Linear interpolation of WI values for mineral blends can be misleading. For example, the WI of mineral blends does not correspond to the weighted average WI of all components (Yan, Eaton 1994). Also, the blend WI can be greater than the WI of the hardest material present (Hosten, Avsar 1998). The WI values obtained when studying the behavior of different cement clinker and slag blends were always below the individual components' weighted average (Öner 2000). The WI value was shown to decrease considerably when magnetite percentage was increased to 50% in the breakage of iron oxides (Shad et al. 2018). The WI values were shown to be lower for individual samples than for binary and tertiary blends. However, Tavares and Kallemback (2013) argue that the weighted average is true based on the material blend in the mill load after the final milling test cycle.

The goal of this research was to compare statistic and stochastic models of the Work Index value for mixtures of quartz and marble at different proportions of each material. We selected these types of rocks because their individual WI is very different, which allowed us to carry out a more extensive analysis.

2. MATERIALS AND METHODS

The study consisted in the determination of WI values for quartz and marble blends by using a Bond standard grindability test for ball mills in order to generate a statistical-stochastic model which would allow us to predict the value for any given mixture ratio based on the proportion of the components. Later, computational simulations were performed in order to generate a stochastic-statistic model that produces more precise values by using the Monte Carlo method.

2.1. MATERIALS AND EXPERIMENTAL PROCEDURE

Quartz and marble samples were prepared in three crushing stages at laboratory scale. For this purpose, a jaw crusher, a roller crusher and a cone crusher were respectively employed under an open circuit configuration and separating the material 100% under

#6 Tyler Mesh (3.327 mm). It is important to consider that determining energy consumption required for ore grinding in a Bond ball mill during testing requires samples of standardized size, since different sizes can cause distortions in obtained index values (Magdalinovic et al. 2012).

In order to determine WI values for each mixture ratio, the standard Bond grindability test for ball mills was used, repeating the process five times to take into account the potential variability of each measurement using the average values.

Eventually, Work Index values were obtained according to Eq. (1):

$$W_1 = \frac{44.5}{P_{100^{0.23}} * Gpb^{0.82} * \left(\frac{10}{\sqrt{P_{80}}} - \frac{10}{\sqrt{F_{80}}} \right)}, \tag{1}$$

where:

W_1 – Bond Work Index [kWh/t],

P_{100} – test sieve mesh size [μm],

Gbp – mill grindability index [gr/rev],

F_{80} – sieve mesh size passing 80% of the feed before grinding [μm],

P_{80} – sieve mesh size passing 80% of the product after grinding [μm].

2.2. GRINDABILITY TEST FOR BALL MILLS

Bond states that the WI value applicable to fine materials is obtained in a standard 12″ long ball mill spinning at 70 RPM with rounded corners and a smooth surface (except for the 4″ × 8″ loading door). The mills must also contain the balanced steel ball load indicated in Table 1(Bond 1961).

Table 1. Ball loading for standard grindability test

Ball diameter [inch]	Approximate number of balls	Ball weight [g]	Surface [inch^2]
1.45	43	8.803	285
1.17	67	7.206	289
1.00	10	672	32
0.75	71	2.011	126
0.61	94	1.433	110
Total	285	20.125	842

The mill was fed using the previously crushed material under controlled conditions (100% under #6 Tyler Mesh). An apparent volume of 700 cc was achieved, which was measured using a graduated test tube. The mass of the material was measured and sieved as part of the granulometric analysis protocol before being grinded in the ball mill under dry conditions, simulating a 250% circulating load. To close the circuit, a set of meshes between #28 and #325 (Tyler) was used.

The test was started by grinding the material with 100 revolutions; the mill was emptied along with the ball loading, and the material (700 cc) was sieved using the selected cutoff mesh of the circuit (using coarser protective meshes when required).

The undersize material was weighted, separating this portion and adding non-segregated fresh load material to the oversize particles in order to reconstruct the original load that was fed to the mill in each cycle, completing the 700 cc of material. All of this material was returned to the mill along with the ball loading, being grinded by the determined number of revolutions required to generate a 250% circulating load, repeating this process when the required state of equilibrium was reached.

The number of required revolutions was obtained based on the results of the previous cycle (grams of fine material produced per number of rotations).

The grinding cycles continued until reaching equilibrium state, where growth direction or grindability index decay is commonly reversed (grams/revolution) when compared to the last three cycles.

Once the equilibrium state was reached, the particle size distribution of the undersize product was thoroughly analyzed to determine P_{80} value. Then, the grindability index was calculated for the ball mill by averaging the last three values of net grams of undersize material by revolution produced by the mill.

Normally, a deterministic model is obtained using average values. For the proposed investigation, the final model contemplates the use of probability distributions that describe a broader spectrum of possible values for the WI.

2.3. MONTE CARLO SIMULATION METHOD

A Monte Carlo simulation is a mathematical method that relies on repeated randomness to solve problems that might be deterministic in principle. This methodology is a powerful tool that can be applied in a wide range of problems. It is particularly valuable when facing cases with limited available information, due to technical difficulties or high experimentation costs (Kroese et al., 2014).

In this study, WI values for each analyzed blend ratio were simulated with the Monte Carlo method using computational tools. First, the PDFs that best fitted actual value distributions for each ratio were determined, then the number of required simulations was determined using heuristic convergence experimentation.

The average value of all simulations for each blend ratio was selected to replace the values that defined the curve of the deterministic model and a new regression analysis was performed. This allowed the construction of a more robust predictive model for WI determination in marble and quartz mixtures.

3. RESULTS AND DISCUSSION

Aiming to build the empirical model, values were obtained for quartz-marble mixtures using different proportions of each individual component as described above.

3.1. BOND WORK INDEX FOR EXPERIMENTAL MINERAL MIXTURES

The following table shows WI values obtained via applying the standard grindability test for ball mills for each ratio.

Table 2. Obtained WI values for mixtures

Mixture Q : M [%]		Bond Work Index [kWh/t]					Averages [kWh/t]
0	100	5.53	5.4	5.32	5.22	5.75	5.44
20	80	7.26	7.53	7.62	7.22	8.06	7.54
40	60	10.91	9.92	10.2	9.84	10.4	10.25
50	50	11.57	11.6	11.7	11.7	10.8	11.48
60	40	12.64	12.8	12.6	12.9	12.9	12.76
80	20	13.62	13.6	13.8	14.5	14.1	13.92
100	0	14.02	13.7	14.1	13.7	13.5	13.80

Considering the standard WI values of each material found in scientific literature Fred Bond as a benchmark (Quartz WI = 13.46 kWh/t and Marble WI = 5.38 kWh/t), when comparing WI values according to the literature versus the empirically obtained averages, differences of 0.34 kWh/t are observed for quartz and 0.06 kWh/t for marble. The following table contains descriptive value statistics:

Table 3. Descriptive Statistics for determined work index values

Mixture Q : M [%]	Average [kWh/t]	Standard Deviation	Coefficient of variation [%]
0 : 100	5.44	0.205	3.77
20 : 80	7.54	0.338	4.49
40 : 60	10.25	0.435	4.24
50 : 50	11.48	0.401	3.49
60 : 40	12.76	0.135	1.06
80 : 20	13.92	0.371	2.66
100 : 0	13.80	0.254	1.84

By adjusting a set of experimentally measured values to characteristic statistical distributions, the variability of the process itself can be included in WI calculation, which is achieved using stochastic methods such as the Monte Carlo method.

It should be noted that the ratios with higher variability correspond to those containing high marble proportions, followed by the 40–60% quartz-marble ratio.

3.2. DETERMINISTIC MODEL FOR WORK INDEX VALUE PREDICTION IN MINERAL MIXTURES

A regression analysis was performed with the obtained Work Index values of each mixture ratio.

The simplest and most intuitive adjustment model for WI interpolation is described via a simple linear regression.

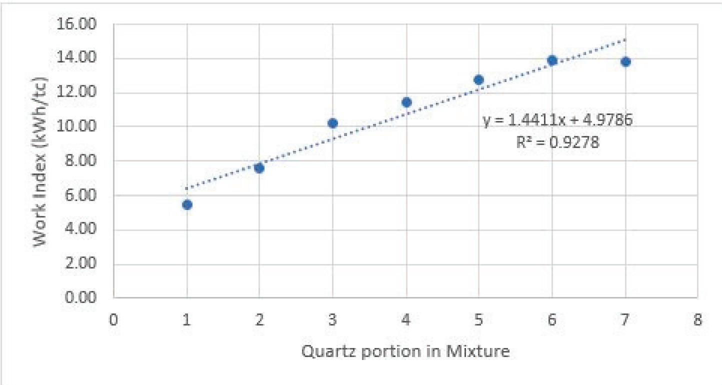

Fig. 1. Deterministic lineal model graph

As the simplest interpretation, this model is the most used industrywide to interpolate WI with an adjustment reflected in an R^2 of around 0.938.

The analysis showed that the values are best fitted by a third degree polynomial function, thus, a deterministic model for obtaining work index values for quartz-marble mixture ratios was obtained. The model is presented in Fig. 2 and corresponds to the following algebraic expression:

$$W_{1(\mathrm{Mix})} = -14.931x^3 + 14.728x^2 + 8.550x + 5.417, \tag{2}$$

where:

x – quartz proportion in the mixture [%],

$W_{1(\mathrm{Mix})}$ – Bond Work Index for the given mixture ratio.

The portion of quartz was considered as the dependent variable for this regression model. If the dependent variable were to be changed for the portion of marble present in the mixture, the curve would simply be the axial reflection of the one shown in Figs. 1 and 2. Using this approach, the polynomic bias term is equivalent to the work index value of the less resistant material present in the mixture. Given the characteristics of a polynomial regression, it is safe to assume this behavior is expected to be present in other mixtures with different grindability properties.

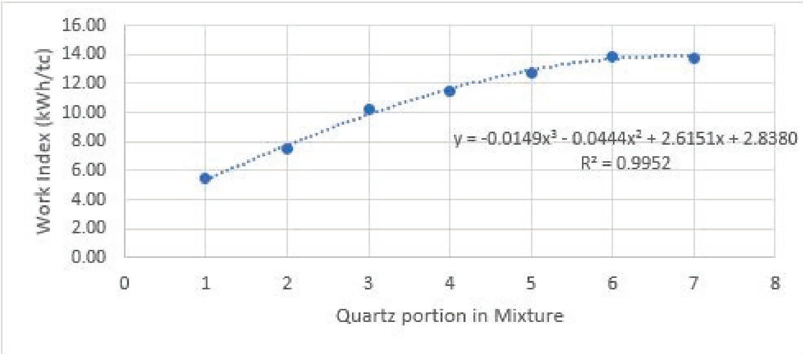

Fig. 2. Deterministic third degree polynomial model graph

3.3. WORK INDEX VALUES SIMULATION

The variability of the determined values obtained for each mixture ratio generally followed a Pareto distribution with the exception of the 50/50 ratio, which followed a Gumbel distribution (Table 4).

Using numerical experimentation, it was determined that the values converged around 100 iterations of the random process for each one of the ratios. The following table shows the descriptive statistics for the simulated values using the Monte Carlo method.

Table 4. Descriptive Statistics for simulated work index values for each mixture ratio

Mixture (Q : M)	Average (kWh/t)	Std. Dev.	Coef. Var. (%)
0 : 100	5.445	0.230	4.220
20 : 80	7.536	0.320	4.250
40 : 60	10.251	0.413	4.030
50 : 50	11.509	0.250	2.170
60 : 40	12.768	0.184	1.440
80 : 20	13.919	0.327	2.350
100 : 0	13.816	0.375	2.710

Considering those results, the average values were considered to be representative of the whole blend ratio, given that the standard deviations did not show a significant variation related to the WI values.

3.4. STATISTIC STOCHASTIC MODEL FOR WORK INDEX VALUE PREDICTION IN MINERAL MIXTURES

The same methodology that was applied to the values obtained via experimentation was applied to the simulated values, thus obtaining a polynomic regression curve that fits the simulated values generated via the Monte Carlo method in order to generate a more precise model.

Given the advantages the Monte Carlo method provides when generating pseudo-random numbers that take potential measurement variability into account (through the grindability tests), the procedure for obtaining the new model consisted in using the Monte Carlo method in a set of observations adjusted to a PDF and then applying a regression function to obtain its polynomic coefficients. Figure 3 shows a diagram of the process.

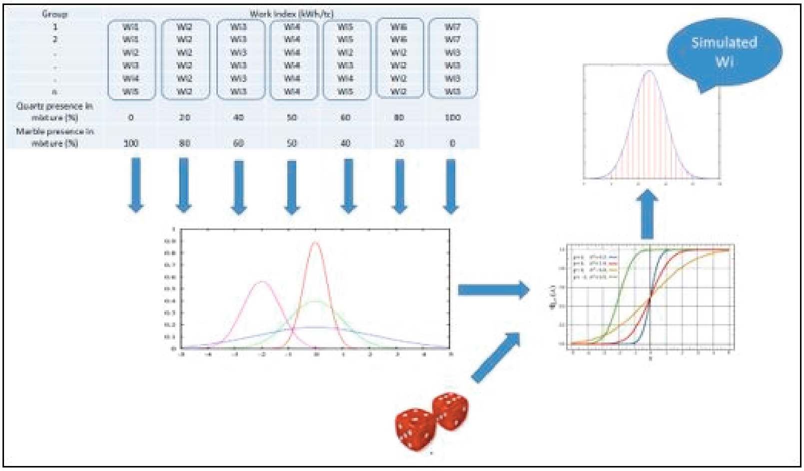

Fig. 3. Diagram of the Monte Carlo method application for WI values

Following this methodology, Table 5 shows the PDF that best fitted the simulated data.

Table 5. Probability density functions for simulated WI values for each mixture ratio

Mixture (Q : M)	Prob. distribution	Simulated WI (kWh/t)
0 : 100	Pareto	5.445
20 : 80	Pareto	7.536
40 : 60	Pareto	10.251
50 : 50	Gumbel	11.509
60 : 40	Pareto	12.768
80 : 20	Pareto	13.919
100 : 0	Pareto	13.816

The statistic-stochastic model was defined as shown in Fig. 4, whose curve was obtained by plotting the simulated values shown in Table 5, generating the following polynomic function:

$$W_{1(\mathrm{Mix})} = -14.924x^3 + 14.710x^2 + 8.571x + 5.418, \tag{3}$$

where:

$W_{1(\mathrm{Mix})}$ – Bond Work Index for the given mixture ratio,
x – quartz proportion in the mixture [%].

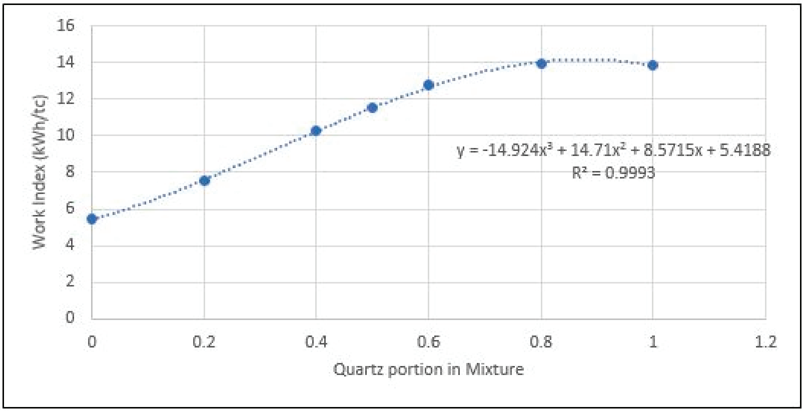

Fig. 4. Graph of the statistic-stochastic model
as a function of the relative presence of quartz [%]

The models shown in Figs. 1 and 3 show that the Work Index values in mineral mixtures are not simply a linear weighted average of its components' feed masses and the curves should not suffer violent peaks when proportions are changed while the mass is kept constant. Therefore, a polynomic regression is considered to be reasonable for this kind of experiment.

Furthermore, an interesting effect was observed in mixtures of the 80 : 20 (Q : M) ratio, where the values were very close to that of the hardest individual component (quartz) while in some observations, the values were even higher. In this sense, Aras et al. (2019) found that in predicting WI value there are four rock properties with the most influence on obtaining high precision levels. These are Schmidt hardness (RL), resistance to uniaxial compression (σ_c), point load (IS) and density (ρ). These authors predicted WI value via artificial neuron networks concluding that marble samples were stronger and more resistant than travertine samples, and recommend that for WI prediction, values are trustworthy as a function of the correlation coefficients obtained via the artificial neuron network method.

The model was validated using the experimental values as a reference, obtaining a difference of 1.3%, which is low considering the randomness that characterizes the milling process. Other models have shown a higher error; for example, Gharehgheshlagh (2016) estimated WI using several materials in thirteen samples, with an average error of 2.6%, although the results obtained were in accordance with the standard Bond method. However, for Chakrabarti (2000) the critical condition of obtaining the exact WI has decreased notably in the last four decades due to advances in operation, instrumentation, design and grinding circuit control.

4. DISCUSSION

Based on the adjustments obtained by the 3 models, in Table 6 we can observe that the linear model presents a lower correlation between the three alternatives, while the grade 3 polynomial obtained for both deterministic and simulated results via Monte Carlo has a very similar correlation.

Table 6. R^2 adjustment level summary by model

Model type	R^2
Linear model	0.928
Grade 3 polynomial model	0.995
Grade 3 polynomial model simulated with Monte Carlo	0.999

From Table 6, we can observe that, while complicating the model allows progressively better adjustments to measured data, the difference between adjustments for a grade-3 polynomial and a model simulated from variable experimental results presents differences attributable to variability recorded on empirical WI measurements. Given the low magnitudes of these differences, they are considered insignificant in this particular case.

It is interesting to indicate that the stochastic model requires greater experimental effort for its formulation, given the repetitions necessary to characterize the statistical distributions which describe the behavior of the variable, but these results also allow for better responses to variability observed in experiments. While in this case the measured variability is small and insignificant, it indicates that a stochastic-type model could be a better solution when observed variability is greater.

On an industrial scale, it can be said that the three models analyzed can correctly evaluate WI values with indications that the simulated stochastic model becomes more robust and representative in direct relation with the degree of WI variability measured.

5. CONCLUSIONS

Table 7 presents the principal differences between conventional and stochastic WI obtention methods.

Table 7. Principal differences between conventional and stochastic methods

Conventional WI obtention method	Stochastic WI obtention method
Variability not considered in calculation	Variability considered in calculation
Based on experimental lab analysis	Based on probability distributions adjusted to experimental results
Limited interpolations	Admits interpolation with various forms
Needs less experimentation	Needs more data to characterize probability distributions
No computer resources required	Computer resources needed

The proposed study allows the comparison of models. WI values obtained from comminution tests with quartz-marble blends were then simulated with the Monte Carlo method. It was found that the values were best fitted to a third degree polynomial function, thus discovering a non-linear model for WI determination.

We can observe that adding more marble at the beginning of the process produces less accurate results in the grinding test. The selection of quartz and marble samples produced good differentiation of the obtained WI values as expected, thus allowing controlled experimentation. This was key for the posterior regression process that showed non-linear behavior of the different blend ratios. One limitation of this study was the execution of the milling process on a laboratory scale; however, the values obtained serve as a reference for pilot tests that can be executed on an industrial scale.

Although the three models compared could satisfactorily fit results on an industrial scale, it can be said that the three models can correctly realize WI values with indications that the simulated stochastic model becomes more robust and representative as a function of the variability observed in WI measurements.

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