RESEARCH PAPERS OF WROCŁAW UNIVERSITY OF ECONOMICS No. 91
Econometrics 28 2010

Włodzimierz Szkutnik
Academy of Economics, Katowice

ANTICIPATION OF THE PRICE OF CAT BOND IN THE MANAGEMENT OF THE PROCESS OF SECURITIZATION OF CATASTROPHE INSURANCE RISK

Abstract: The application of financial instruments from the capital market aims at the management of securitization process of the catastrophe risk. This is important with respect to the results of losses caused by catastrophe incidents. The article develops the structure of CAT bonds and their pricing with the use of model of stochastic process of interest rate. The CAT bonds are designed to finance the results of catastrophe incidents. They are similar to the contingent claim capital but in reality they are the financial market instruments, i.e., the catastrophe bonds. The elaborated approach is illustrated by the distribution of the bond price in the configuration of trigger level for CAT bond forgiveness and its volatility. The direction of possible research is determined by the consideration of moral hazard and basis (market) risk in the pricing process.

Key words: securitization, catastrophe bonds, moral hazard, basis risk.

1. Introduction

The application of financial instruments of the capital market in the management of securitization process of the catastrophe risk is a form of financing the results of catastrophe incidents. The CAT bonds will be the financing tools in the case considered in this article. These bonds have a similar character as contingent claim capital CC, but in real conditions they are instruments of the financial market; they are catastrophe-linked bonds.

The main goal of presented considerations is the formulation of a contingent claim model to price bonds issued directly by the insurer, by SPV company dependent on the main insurer when there is no default risk for claims initiated by catastrophe incident. In the market reality the pricing of such bonds was justified, in the last decades, by the increasing frequency of catastrophe incidents. The anticipation of results of possible catastrophes with big aggregated losses, expressed indirectly in specifically constructed indices of losses together with considering them in the terms and conditions of issued bonds, is the anticipation of their price. To abbreviate the heart of the problem, we state that the proper pricing influences the losses of the insurer when a catastrophe incident is realized. Therefore, the proper pricing of bonds is an element of insurance operations management. CAT bond provisions have debt-forgiveness triggers whose generic design allows for the payment of interest and/or the return of principal forgiveness, and the extent forgiveness can be total, partial, or scaled to the size of loss; there is total or partial depreciation of the face value of the CAT bond, which influences the finance of the insurance company.

A specific project of CAT bonds has a historical context. In retrospect, one could observe a constant evolution of CAT designs and, consequently, also their pricing. Even though we can price default-risky CAT bonds, it is also possible to consider default-free CAT bonds prices. It may also be assumed that there is moral hazard which is associated with the event of claims pricing (the event that is inadequate to losses) that leads to losses either of the insurer (the most frequent case) or the bondholders and insurance policies holders. It should be remembered that catastrophe risk is partially insured in a traditional way by entities that are subject to such a risk and this procedure involves also insurance companies, typically reinsurance companies. In application of financial instruments there occurs also basis risk, which is a symptom of influence of capital market condition as well as of market risk.

In the model concerning the dynamics of value of insurance company assets significant assumptions will be adopted for interest rate risk and credit risk that, in a specific way, affect all other forms of risk. It will be explained in a more detailed manner in the part related to specification of assumptions of the assets value model.

It should be taken into account that the presented empirical examples [Szkutnik 2009] may accede to Polish reality concerning the manifestation of catastrophe risk, even though the capital market itself has not worked out proper projects of instruments which can be applied in the securitization of such a risk. National literature has shown the attempt to price a catastrophe bond in the aspect of the so-called two-factor function of the investor's usefulness, whose payee would be municipal authorities.

In Polish literature the above mentioned catastrophe bonds were considered as debt instruments, burdened with the risk of the payee's insolvency. In Polish conditions the project of such instruments was addressed to self-government authorities in order to secure the region against the results of floods, droughts, forest fire, etc. The structure of such bonds and also their efficiency lies in the fact that in case of catastrophe incident, which stimulates beforehand the issue of such instruments and generates financial losses and claims, the issuer is not obliged to pay back the nominal value of the bond and to continue the interest payment until they expire. However, when no catastrophe incident took place, the bond holder (investor) is the beneficiary of such a favourable arrangement.

Among others, Cox and Schwebach [1992], Cummins and Geman [1995] and Chang and Yu [1996] focused on the pricing of CAT futures and CAT call spreads under the condition of deterministic interest rate and specific property claims services

(PCS) loss processes. There was also the pricing of one-year zero-coupon CAT bond [Litzenberg, Beaglehole, Reynolds 1996] which was further compared to the CAT bond price estimated by hypothetical catastrophe loss distribution. Zajdenweber [1998] followed Litzenberg, Beaglehole and Reynolds, but he changed the catastrophe loss distribution to the stable Lévy distribution. Loubergé, Kellezi and Gilli [1991] numerically estimated the CAT bond price under the assumptions that the catastrophe loss follows a pure Poisson process, the individual losses have independently identical lognormal distribution, and the interest rate model is a binomial random process.

All the above listed pricing elaborations failed to incorporate a commonly acceptable stochastic interest rate process and catastrophe loss process as well as the default risk of the CAT bonds.

The article by Jin-Ping Lee and Min-The Yu [2002] develops a contingent claims model to price default-risky catastrophe-linked bonds, where interest rates have a stochastic character. Moreover, it allows for more generic loss processes and practical considerations of moral hazard, basis risk and default risk. There are estimations of both default-free and default-risky CAT bond prices. The results show that both moral hazard and basis risk drive down the bond prices substantially; therefore, these results should not be ignored in pricing the CAT bonds.

The elaboration on this subject is important also from the practical point of view. This results from the fact that under accepted assumptions in the offered models of assets value, interest rate, loss model, the priced CAT bond hedge enable the issuer to avoid the credit risk that may arise with traditional reinsurance or catastrophe-linked options, which has already been mentioned.

As far as moral hazard is concerned, it should be remembered that (which is also emphasized in literature) it is initiated by the insurers themselves. This is related to the insurer's cost of loss pricing. Sometimes this cost exceeds the issuer's (SPV company) profits (insurer) that result from the debt value (the issuer) which occurs at the time of catastrophe incident. The effect of moral hazard may increase the claim payments at the expense of the bondholders' principal reduction and affect the bond price.

Another important aspect, which must be considered in pricing a CAT bond, is the basis risk. The basis risk may cause insurers to default on their debt in the case of high individual loss but low index of loss, and therefore affects the bond price. However, there exists a balance between the basis risk and moral hazard. If one uses an independently calculated index to define the CAT bonds payments, then the insurer's opportunity to cheat the bondholders is reduced or eliminated. This is equal to a lesser scope of moral hazard behaviour or even its elimination. However, the basis risk is created, which results from the increase share of capital market instruments in catastrophe risk hedging.

2. The concept of stochastic structure of pricing CAT bonds

The structure of pricing CAT bonds [Jin-Ping Lee, Min-The Yu 2002] meets the usually unconsidered assumptions of pricing CAT bond models halfway. The estimation model of pricing CAT bonds is presented with respect to practical assumptions concerning:

– default risk,
– moral hazard,
– basis risk.

In this model it is necessary to define:

– assets value dynamics A_t,
– interest rate dynamics r_t,
– aggregate loss dynamics $C_{i,t}$ for the issuing company i and, relevantly, $C_{index,t}$ in a composite index of losses (e.g., a PCS index).

Additionally, the models define also relevant processes with respect to:

– risk-neutralized pricing measure.

The last part of the paper provides the numerical analysis and discusses the results [Szkutnik 2009].

2.1. Asset dynamics model

The typical way to model asset dynamics assumes a lognormal diffusion process for the asset value; for example, as in [Merton 1977; Cummins 1988]. The main disadvantage of this modelling approach is that it fails to take into account the explicit impact of stochastic interest rates on the asset value. This is important for modelling the insurer's asset value, because it is quite common for insurers to hold a large proportion of fixed-income assets in their portfolios. In particular, insurers that issue CAT bonds mainly invest their proceeds from CAT bonds sales in high grade, interest rate-sensitive investments such as commercial papers and treasury securities.

It turns out that determination of the insurer's total asset value as consisting of two risk components:

– interest rate risk,
– credit risk,

allows for the measure of the effect of the interest rate risk on CAT bond prices [Duan, Moreau, Sealey 1995].

From a theoretical point of view, term "credit risk", which was mentioned in the introductory part, refers to all risks that are orthogonal to the interest rate risk. Specifically, the value of the insurer's total assets is described by the process expressed by stochastic equation (1), where the most important is the instantaneous drift μ_A that is a trend resulting from credit risk effect. The model also takes into account the instantaneous interest rate elasticity of the insurer's assets ϕ, the already mentioned instantaneous interest rate r_t at time t, the volatility of credit risk process σ_A, as well as credit risk $W_{A,t}$ expressed by the Wiener process:

$$\frac{dA_t}{A_t} = \mu_A \, dt + \phi \, dr_t + \sigma_A \, dW_{A,t}. \tag{1}$$

2.2. The instantaneous interest risk model

Model (1), under the assumption that the instantaneous interest rate is modelled according to the square-root process of Cox, Ingersoll and Ross [1985], it means – the square root of a given number is the number which when multiplied by itself equals this number, which avoids the existence of negative interest rate that can appear in Vasicek's model [1977] and is described as follows:

$$dr_t = \delta(m - r_t) \, dt + v\sqrt{r_t} \, dZ_t,$$

where: δ – the mean-reverting force measurement, m is the long-run mean of the interest rate, v – the volatility parameter for the interest rate, Z_t – a Wiener process independent of $W_{A,t}$ and leading to the **asset dynamics model** (2):

$$\frac{dA_t}{A_t} = (\mu_A + \phi\delta m - \phi\delta r_t) \, dt + \phi \, v \sqrt{r_t} \, dZ_t + \sigma_A \, dW_{A,t}. \tag{2}$$

2.3. The risk-neutralized dynamics of the insurer's assets

For the asset dynamics model (2) to be neutralized with respect to risk, it is necessary to use **device of risk neutralization**.

The dynamics for the interest process under the risk-neutralized pricing measure, denoted by Q, can be written as

$$dr_t = \delta^* (m^* - r_t) \, dt + v \sqrt{r_t} \, d Z_t^*,$$

where δ^*, m^* and Z^* are defined as

$$\delta^* = \delta + \lambda_r,$$

$$m^* = \frac{\kappa \cdot m}{\kappa + \lambda_r},$$

$$dZ_t^* = dZ_t + \frac{\lambda_r \cdot \sqrt{r_t}}{v} \cdot dt.$$

The term λ_r is the market price of interest rate risk and is a constant under Cox, Ingersoll and Ross [1985]; Z_t^* is a Wiener process under Q, the formulation of which can be found in [Ritchken 1996]. Thus, the insurer's asset dynamics can be risk neutralized to

$$\frac{dA_t}{A_t} = r_t\, dt + \phi\, v\, \sqrt{r_t}\; dZ_t{}^* + \sigma_A\, dW_t{}^*, \tag{3}$$

where W^* is a Wiener process Q and is independent of $Z_t{}^*$.

2.4. Aggregate loss dynamics

Following the typical setting for loss dynamics in the actuarial literature [Bowers et al. 1986], the aggregate loss model can be expressed as a compound Poisson process, a sum of jumps.

To estimate the impact of basis risk on the CAT bond price the term $C_{i,t}$ that denotes the aggregate loss for the issuing company i is introduced; the term $C_{\text{index},t}$ represents that for a composite index of losses (e.g., a PCS index). These two processes can be described as follows:

$$C_{i,t} = \sum_{j=1}^{N(t)} X_{i,j}, \tag{4}$$

and

$$C_{\text{index},t} = \sum_{j=1}^{N(t)} X_{index,j}. \tag{5}$$

Where the process $\{N(t)\}_{t\geq 0}$ is the loss number process described by Poisson process with intensity λ. Symbols $X_{i,j}$ and $X_{index,j}$ denote the Mount of losses caused by the j-th catastrophe during the specific period for the issuing insurance company and the composite index of losses, respectively. It is assumed that terms $X_{i,j}$ and ($X_{\text{index},j}$), for $j = 1, 2, \ldots, N(T)$, are mutually independent and have identical lognormal distribution, and they are also independent of the loss number process, and their logarithmic means and variances are denoted by μ_i (μ_{index}) and σ_i^2 (σ_{index}^2), respectively. In addition, assume that ρ correlation coefficients of the logarithms of $X_{i,j}$ and ($X_{\text{index},j}$), for different $j = 1, 2, ..., N(T)$ are identical.

2.5. Loss dynamics under the risk-neutralized pricing measure

To carry out the pricing of CAT bonds one needs to know the loss dynamics under the risk neutralized pricing measure Q. When the loss process has sudden jumps, the market is called then incomplete and there is no unique pricing measure.

Thus, follow Merton [1977] and assume that the economic conditions are only marginally influenced by localized catastrophes such as earthquakes and hurricanes, and that the loss number process $\{N(t)\}$ and the amount of losses $X_{i,j}$ ($X_{\text{index},j}$) are directly related to idiosyncratic "shocks" to the capital markets, it means that the factors influence the capital markets in an inexpedient way.

These factors catastrophic shocks represent “non-systematic” risk and have a zero risk premium, which they generate.

By assuming that such a jump risk is nonsystematic and diversifiable, attaching a risk premium to the risk is unnecessary. It turns out that this assumption is important because one cannot apply a risk-neutral evaluation to situations in which the seize of the jump is systematic. This point is minutely discussed by Naik and Lee [1990], Cummins and Geman [1995], Cox and Pedersen [2000].

Therefore, it can be accepted that the aggregate loss processes expressed by equations (4) and (5) retain their original distributional characteristics after changing from the physical probability measure to the risk-neutralized pricing measure.

3. Default-free CAT bonds

It turns out that once the risk-neutral process of asset dynamics, loss and interest rate are known, it is possible to estimate the CAT bond price by the discounted expectation of its various payoffs in the risk-neutral world. The specification of payoffs of the CAT bond may be carried out under alternative considerations concerning the payoff risk. In this aspect, we can consider first the basic case, in which there is no default risk, and then also the case of the default-risky CAT bonds with potential basis risk and moral hazard.

The article presents only the case of default-free CAT bonds. To price the CAT bond it is assumed that this is a hypothetical discount bond whose payoffs (PO_T) at maturity (i.e., time T) are as follows:

$$PO_T = \begin{cases} a \cdot L & \text{when } C_T \le K \\ r \cdot p \cdot \ a \cdot L & \text{when } C_T > K \end{cases}, \tag{6}$$

where: K – the trigger level set in the CAT bond provisions, C_T – aggregate loss at maturity, $r{\cdot}p$ – the portion of the capital needed to be paid to bondholders when the forgiveness trigger has been pulled, L – the face amount of the issuing company’s total debts; which includes the face amount of the CAT bond, a – the ratio of the CAT bond’s face amount to total outstanding debts.

The price of CAT bond with the payoffs specified in equation (6)is carried out under the assumption that the state variables θ and η,which determined the term structure of interest rate and the aggregate loss, “behave” good enough to be able to apply the risk-neutral approach of Cox and Ross [1976] and Harrison and Pliska [1981]. More specifically, under the risk-neutralized pricing measure Q, the CAT Bond price on the issuing date (i.e., time 0) can be expressed by the term $E^*_{\theta,\eta}$ which denotes its expected value in a risk-neutral world.

In further specifications of the model it is assumed that the state variables θ, which for the purpose of valuing catastrophe risk bonds determine the term structure of interest rates, are independent of the state variables η which are related to catastrophe risk variables. Under these assumptions, the price of CAT bond dependent on the price factor, denoted by $B_{CIR}(0,T)$, which concerns the default-free bond, which can be found in literature [Cox, Ingersoll, Ross 1985], can be written as follows:

$$P_{CAT}(0) = B_{CIR}(0,T) \cdot$$
$$\cdot \sum_{j=0}^{\infty} \exp(-\lambda \cdot T) \cdot \frac{(\lambda \cdot T)^j}{j!} \cdot F^j(K) + r\rho \cdot (1 - \sum_{j=0}^{\infty} \exp(-\lambda \cdot T) \cdot \frac{(\lambda \cdot T)^j}{j!} \cdot F^j(K), \tag{7}$$

where:

$F^j(K) = P(X_{i,1} + X_{i,2} + ... + X_{i,j} \leq K)$, F^j denotes the j-th convolution of F,

$B_{CIR}(0, T) = A(0, T) \cdot \exp[-B(0,T) \cdot r]$,

where

$$A(0, T) = \left[\frac{2 \cdot \gamma \cdot \exp\left(\kappa + \gamma) \cdot \frac{T}{2} \right)}{(\kappa + \gamma) \cdot \exp\left(\gamma^T - 1\right) + 2 \cdot \gamma} \right]^{\frac{2 \cdot \kappa \cdot m}{v^2}},$$

$$B(0, T) = \frac{2 \cdot [\exp(\gamma^T) - 1]}{(\kappa + \gamma) \cdot [\exp(\gamma^T) - 1] + 2 \cdot \gamma},$$

$$\gamma = \sqrt{\kappa^2 + 2 \cdot v^2}.$$

4. Approximation of the aggregate loss distribution and the bond price-analytical solution

Under the assumption that the catastrophe loss amount components are independent and identically lognormally distributed the exact distribution of the aggregate loss at maturity date, denoted as $f(C_T)$, is obviously not known in the exact form. However, an approximate analytical form of this probability distribution can be set up. For this purpose, we approximate the exact distribution by a lognormal distribution, denoted as $g(C_T)$, with specified moments. Jarrow and Rudd [1982], Turnbull and Wakeman [1991], Nielson and Sandmann [1996] used the same assumptions in approximating the values of Asian options and the so-called basket options.

The application of this approach only requires the setting of two first moments of distribution defined by function $g(C_T)$, as equal to the moments of exact(but unknown) distribution of the aggregate loss at maturity $f(C_T)$. Let write it as follows:

first order moment $\mu_g = E[C] = \lambda T \exp\left\{ \mu_i + \frac{1}{2} \cdot \sigma_i^2 \right\}$, (8) central second order moment σ_g = Var [C]= λT $\exp\{2\cdot\mu_i + 2\cdot\sigma_i^2\}$, (9)

thus, μ_g and $\sigma^2{}_g$ denote the mean and variance of the approximating distribution g(C), respectively.

The value of the CAT bond can be written as follows:

$$B_{apr}(0) = B_{CIR}(0,T)\cdot\int_0^K \frac{1}{\sqrt{2\cdot\pi}\cdot\sigma_g\cdot C_T}\cdot\exp\left\{-\frac{1}{2}\cdot(\ln C_T-\mu_g)^2\right\}dC_T + \\ +rp\cdot\int_K^\infty \frac{1}{\sqrt{2\cdot\pi}\cdot\sigma_g\cdot C_T}\cdot\exp\left\{-\frac{1}{2}\cdot(\ln C_T-\mu_g)^2\right\}dC_T, \quad (10)$$

where $B_{apr}(0)$ is the approximate analytical CAT Bond price at time 0. This formula is similar to the one introduced by Litzenberg, Beaglehole and Reynolds [1996], except that they use a constant interest rate in the model.

The final empirical part of the article presents the comparison of the analytical solution with the estimates based on the numerical method without the approximating assumptions.

The presented analytical structure of the complex contingent contract is the basis for its complementation and modification. However, presented in the current form, it does not allow for numerical estimation of the CAT bond price. It has already been signalled that this will be the subject of the exemplification of the application of CAT bond pricing model on the basis of the data simulated by the Monte Carlo method.

5. The exemplification of the model structure of default-free CAT bonds

For the purpose of explaining a bit difficult, from the formal point of view, analytical structure of the CAT bond estimation, we will carry out the exemplification of application of model structure for this bond [Jin-Ping Lee, Min-The Yu 2002; Szkutnik 2009]. The CAT bond prices were estimated [Jin-Ping Lee, Min-The Yu 2002)] by the Monte Carlo simulation. Consider the pricing of default-free bonds. We will not take into account the default-risky bonds with moral hazard and basis risk.

The initial step in pricing the CAT bond is the established set of parameters and base values. To assess the comparative effects of these parameters on CAT bond prices deviations from the base values are also established. For simplicity, it is assumed that **the total amount of the issuing company's debts, which include CAT bonds, has a face value of $100 and that the maturity of the CAT bond is equal to one year**. The simulations are run on a weekly basis with 20,000 paths. The given parameters and base values are presented in Table 1.

Table 1. Parameters and base values

Types of parameters and base values	Values
Asset parameters	
A – insurer's assets	Assets to Liabilities *A*/*L*: 1.1, 1.2 and 1.3
μ_A – *drift due to credit risk*	
Φ – interest rate elasticity of asset	0, –3, –5
σ_A – volatility of credit risk	5%
W_A – Wiener process for credit shock	

Source: own elaboration on the basis of [Jin-Ping Lee, Min-The Yu 2002].

The initial asset/liability (or capital) position (*A*/*L*) ratios are set to be 1.1, 1.2, and 1.3, respectively. The average *A*/*L* ratio for the insurance sector equalled about 1.3 on a book-value basis over the past ten years. The interest rate elasticity of the insurer's assets is set at 0, –3, and –5, respectively to measure how the insurer's interest rate risk affects CAT bond prices. The volatility of the asset return that is caused by the credit risk is set at the level of 5%.

Table 2 includes the interest rate parameters. The initial spot interest rate r and the long-run interest rate m are both set at 5%. The magnitude of mean-reverting force δ is set to be 0.2, while the volatility of the interest rate v is set at 10%. The market prices λ_r of interest rate are set at 0 and –0.01, respectively. All these term structure parameters are included within the range typically used in the previous literature.

Table 2. Interest rate parameters

Type of parameters	Values
Interest rate parameters	
r – initial instantaneous interest rate	5%
$\boldsymbol{\delta}$ **– magnitude of mean-reverting force**	0.2
m – long-run mean of interest rate	5%
v – volatility of interest rate	10%
Market prices λ_γ of interest rate risk	0, –0.01
Z – Wiener process of interest rate shock	

Source: own elaboration on the basis of [Jin-Ping Lee, Min-The Yu 2002].

Table 3 presents the catastrophe loss parameters and other parameters, including the trigger levels for debt-forgiveness and the ratio of principal needed to be paid if debt forgiveness is triggered.

The occurrence intensities of catastrophe losses are set to be 0.5, 1, and 2, respectively, to reflect the frequencies of catastrophic incidents per year. Also assume that the parameter values for catastrophe loss are the same for individual insurers and the composite loss index. We set the logarithmic mean μ_i and μ_{indeks} to be 2, and the logarithmic standard deviations, σ_i and σ_{indeks} to be 0.5, 1, and 2. The values for the index and individual insurers can be differently modified, but it increases the numerical dimension of calculations and it does not broaden the analysis of basis risk. The analysis will focus on the coefficient of correlation ρ_x between the individual loss and the loss index rather than on their means and standard deviations. The portion of principal needed to be repaid, $r{\cdot}p$, is set at 0.5 when debt forgiveness has been triggered. The ratio of the amount of CAT bonds to the insurer's total debt a, is set at 0.1. Additionally, there are three different trigger levels K set at 100, 110, and 120.

Table 3. Catastrophe loss parameters

Types of parameters	Values
Catastrophe loss parameters	
μ_i – mean of the logarithm of the amount of catastrophe losses for the insurer	2
μ_{indeks} – mean of the logarithm of the amount of catastrophe losses for the composite loss index	0.2
σ_i – standard deviation of the logarithm of the amount of catastrophe losses for the insurer	0.5, 1, 2
σ_{indeks} – standard deviation of the logarithm of the amount of catastrophe losses for the composite loss index	0.5, 1, 2
ρ_x – correlation coefficient of the logarithms of amounts of catastrophe losses of the insurer and the composite loss index	0.5, 0.8, 1
$N(t)$ – Poisson process for the occurrence of catastrophes	
Other parameters	
K – trigger levels	100, 110, 120
Rp – the ratio of principal needed to be paid if debt forgiveness has been triggered	**0.5**
a – the ratio of the amount of CAT bond to total debts	0.1
α – moral hazard intensity	20%
β – the ratio set below the trigger that will cause the insurer's moral hazard	20%
L – the total amount of insurer's debts	100

Source: own elaboration on the basis of [Jin-Ping Lee, Min-The Yu 2002].

6. The numerical and approximating pricing of default-free CAT bonds

The default-free CAT bond prices were determined by the approximating solution method (Table 4) and numerical method (Table 5). Tables 4 and 5 present the results for the bond prices under alternative sets of occurrence intensities λ and volatility of loss σ_i. Observe that the values of the approximating solution and the values from the numerical method, based on the exact formula, are very close and mostly they fall within the range of 10 basis points. The differences do not change much if the market price λ_r of the interest rate risk changes from 0 to −0.01. All the estimations are calculated with the use of 20,000 simulation paths. The bond prices are set to the face value of one dollar.

Table 4. Alternative pricing of the default-free CAT bonds: Approximating solution with no moral hazard and basis risk

	Triggers		
(λ, σ_i)	100	110	120
(0.5,0.5)	0.95112	0.95117	0.95120
(0.5,1)	0.94981	0.95009	0.95031
(0.5,2)	0.92933	0.93128	0.93293
(1,0.5)	0.95095	0.95196	0.95113
(1,1)	0.94750	0.94829	0.94887
(1,2)	0.90559	0.90933	0.91254
(2,0.5)	0.95038	0.95071	0.95091
(2,1)	0.94015	0.94259	0.94441
(2,2)	*0.85939*	*0.86603*	*0.87183*

Source: own elaboration on the basis of [Jin-Ping Lee, Min-The Yu 2002].

Table 5. Alternative pricing of the default-free CAT bonds: Numerical estimates with no moral hazard and basis risk

	$\lambda_r = 0$			$\lambda_r = -0.01$		
(λ, σ_i)	*100*	*110*	*120*	100	110	120
(0.5,0.5)	0.95126	0.95126	0.95126	0.95119	0.95119	0.95119
(0.5,1)	0.94988	0.95017	0.95040	0.94977	0.95029	0.95062
(0.5,2)	0.92805	0.92952	0.93152	0.92675	0.92903	0.93103
(1,0.5)	0.95126	0.95126	0.95126	0.95119	0.95119	0.95119
(1,1)	0.94689	0.94822	0.94869	0.94825	0.97877	0.94977
(1,2)	0.9.270	0.90660	0.90955	0.90273	0.90682	0.91058
(2,0.5)	0.95121	0.95126	0.95126	0.95110	0.95115	0.95119
(2,1)	0.94070	0.94346	0.94546	0.93016	0.94263	0.94492
(2,2)	*0.85156*	*0.85846*	*0.86446*	0.85065	0.85717	0.86378

Source: own elaboration on the basis of [Jin-Ping Lee, Min-The Yu 2002].

In addition, the approximate CAT bond prices are higher than those estimated by the Monte Carlo simulation for a high value of σ_i. This situation takes place because the approximate lognormal distribution underestimates the tail probability of losses and this underestimation will be more when the loss standard deviation ($\sigma_{i.}$) is high. Let us also note that the CAT bond price increases with trigger levels and that this increase rises with occurrence intensity and loss volatility. For instance, in the case where the occurrence intensity (λ) equals 2 and the loss standard deviation (σ_i) equals 2, the CAT bond prices will increase by 65-69 basis points when the trigger level increases from 100 to 110, while the CAT bond prices will increase by 60-66 basis points when the trigger level increases from 110 to 120.

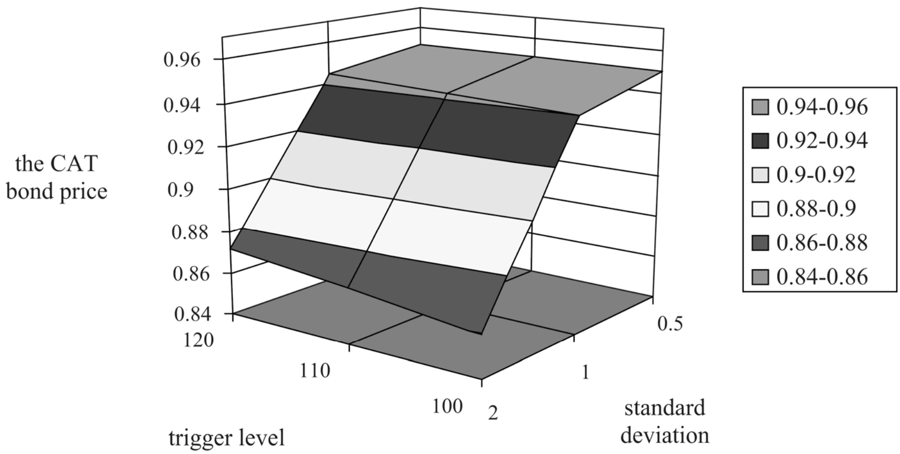

Figure 1. Default-free CAT bond prices- payoffs approximated by the Monte Carlo method

Source: own elaboration.

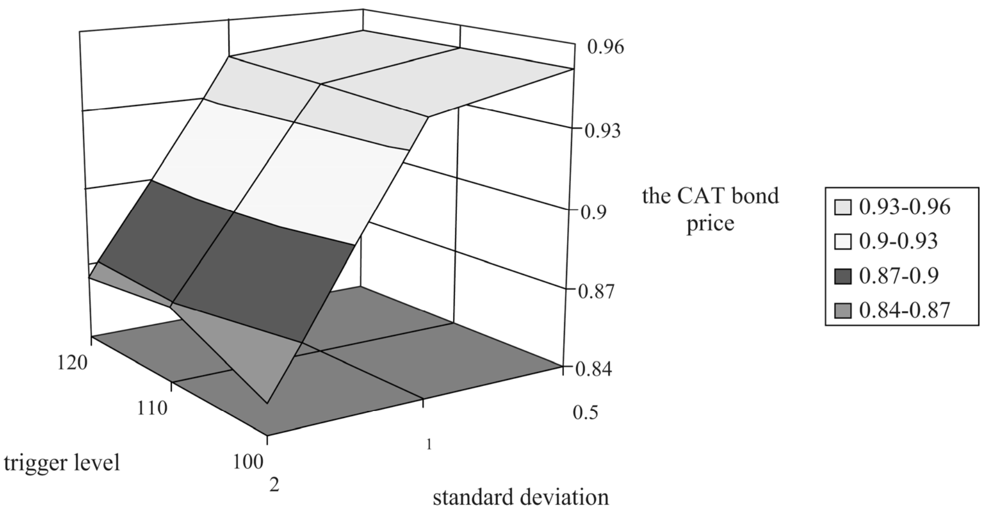

Figure 2. Default-free CAT bond prices – numerical estimate

Source: own elaboration.

7. Conclusion

The model of pricing the CAT bond considered in the article takes into account stochastic interest rates and more generic catastrophe loss processes. It is also possible to "measure" the impacts of default-risk, moral hazard, and basis risk that are associated with CAT bonds. In case of no default risk it is stated that the CAT bond prices computed numerically are very close to the ones computed by the approximating solution, except that when the loss volatility is high. Then, the approximated prices reach higher values.

The model considered in this article may be viewed as a general way of assessing the default-risk. The aspect of anticipation of the bond prices in the catastrophe loss management defines the direction of operation in the insurance sector, which makes it at least partially independent of reinsurance, which is inseparably related to moral hazard. One should remember that the conditions of reaching the balance between moral hazard and basis risk, closely associated with market risk, are known, which introduces the reverse interaction that does not allow for the total liberation from moral hazard. The exemplary applications of this model are presented here. Structural restrictions in this model link the bond price to basic characteristics of assets, liabilities, and interest rates. This allows one to value bonds with unique features through the use of numerical analysis. It is important to note that this model can be easily extended to analyze other default-risky liabilities, not only these concerning CAT bonds, but also insurance-linked securities.

References

Bowers N.L., Gerber H.U., Hickman J.C., Jones D.A., Nesbitt C.J., *Actuarial Mathematics*, The Society of Actuaries, Itasca, Ill., 1986.

Canter M.S., Cole J.B., Sandor R.I., *Insurance derivatives: A new asset class for the capital markets and a new hedging tool for the insurance industry*, Journal of Applied Corporate Finance 1997, Vol. 10, pp. 69-83.

Chang C.J., Yu M.T., *Pricing catastrophe insurance futures call spreads: A randomized operational time approach*, Journal of Risk and Insurance 1996, Vol. 63, No 4, pp. 599-617.

Cox J., Ingersoll J., Ross S., *The term structure of interest rates*, Econometrica 1985, Vol. 53, pp. 385-407.

Cox J., Ross S., *Valuation of options for alternative stochastic processes*, Journal of Financial Economics 1976, Vol. 3, pp. 145-166.

Cox S.H., Pedersen H.W., *Catastrophe risk bonds*, North American Actuarial Journal 2000, Vol. 4, No 4, pp. 56-82.

Cox S.H., Schwebach R.G., *Insurance futures and hedging insurance price risk*, Journal of Risk and Insurance 1992, Vol. 59, pp. 628-644.

Cummins J.D., *Risk-based premiums for insurance guarantee funds*, Journal of Finance 1988, Vol. 43, pp. 593-607.

Cummins J.D., Geman H., *Pricing catastrophe futures and call spreads: An arbitrage approach*, Journal of Fixed Income 1995, Vol. 4, pp. 46-57.

Duan J.C., Moreau A., Sealey C.W., *Deposit insurance and bank interest rate risk: Pricing and regulatory implication*, Journal of Banking and Finance 1995, Vol. 19, pp. 1091-1108.

Harrison J.M., Pliska S.R., *Martingales and stochastic integrals in the theory of continuous trading*, Stochastic Processes and Their Applications 1981, Vol. 11, pp. 215-260.

Jarrow R., Rudd A., *Approximate option valuation for arbitrary stochastic processes*, Journal of Financial Economics 1982, Vol. 10, pp. 347-369.

Jin-Ping Lee, Min-The Yu, *Pricing default-risky CAT bonds with moral hazard and basis risk*, The Journal of Risk and Insurance 2002, Vol. 69, No 1, pp. 25-44.

Litzenberg R.H., Beaglehole D.R., Reynolds C.E., *Assessing catastrophe reinsurance-linked securities as a new asset class*, Journal of Portfolio Management 1996, Vol. 23, No 3, pp. 76-86.

Loubergé H., Kellezi E., Gilli M., *Using catastrophe-linked securities to diversify insurance risk: A financial analysis of CAT bonds,* Journal of Insurance Issues 1999, Vol. 22, No 2, pp. 125-146.

Merton R., *An analytic derivation of the cost of deposit insurance and loan guarantee*, Journal of Banking and Finance 1977, Vol. 1, pp. 3-11.

Naik V., Lee M., *General equilibrium pricing of options on the market portfolio with discontinuous returns*, Review of Financial Studies 1990, Vol. 3, No 4, pp. 493-521.

Nielson J., Sandmann K., *The pricing of Asian options under stochastic interest rate*, Applied Mathematical Finance 1996, Vol. 3, pp. 209-236.

Ritchken P., *Derivative markets: theory, strategy, and applications*, Harper Collins, New York 1996.

Szkutnik W., *Aspekt ryzyka wyceny, ryzyka rezerw, ryzyka rynkowego w uwarunkowaniach niedotrzymania warunków płatności obligacji, ryzyka kredytowego i ryzyka stopy procentowej*, Studia Ekonomiczne, Zeszyty Naukowe, Wydawnictwo Akademii Ekonomicznej, Katowice 2009.

Turnbull S., Wakeman L., *A quick algorithm for pricing European average option,* Journal of Financial and Quantitative Analysis 1991, Vol. 26, pp. 377-389.

Vasicek O.A., *An equilibrium characterization of term structure*, Journal of Financial Economics 1977, Vol. 5, pp. 177-188.

Zajdenweber D. (1998), *The Valuation of Catastrophe-reinsurance-linked Securities*, paper presented at American Risk and Insurance Association Meeting.

ANTYCYPACJA CENY OBLIGACJI CAT W ZARZĄDZANIU PROCESEM SEKURYTYZACJI KATASTROFALNEGO RYZYKA UBEZPIECZENIOWEGO

Streszczenie: zastosowanie instrumentów finansowych rynku kapitałowego ma na celu zarządzanie procesem sekurytyzacji ryzyka katastrofalnego. Jest to istotne ze względu na skutki, jakie przynoszą straty wywołane zjawiskami katastrofalnymi. W artykule omawia się strukturę obligacji CAT i ich wycenę w oparciu o model stochastycznego procesu stopy procentowej. Obligacje CAT przeznaczone są do finansowania skutków zdarzeń katastrofalnych. Mają one charakter zbliżony do kapitału warunkowego CC (*contingent claim*), ale w realnych warunkach są instrumentami rynku finansowego – obligacjami katastrofalnymi. Opracowane podejście jest zilustrowane rozkładem cen obligacji w układzie poziomu uruchomienia umorzenia obligacji CAT i jej zmienności. Kierunek możliwych badań wyznaczony jest przez uwzględnienie w wycenie hazardu moralnego i ryzyka bazowego (rynkowego).