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ERROR CORRECTION MODELS AND REAL CONVERGENCE – CASE OF SLOVAKIA[1]

Abstract

On January 1, 2009 the Slovak Republic entered the Economic and Monetary Union. This step is the confirmation that the country has fulfilled nominal criteria for monetary integration. The main aim of the monetary integration is sustainable economic growth and convergence of the real economy to most advanced EU countries.

Long lasting mutual trend of analyzed indicators is searched by cointegration. In a short term there might be some deviations which are studied by error correction models (ECM). The output of the contribution is twofold. Firstly we will analyze the long-lasting equilibrium of the real economy, and secondly we will estimate the duration whithin which the searched variables are returning to the long-lasting equilibrium in the case of the short-term deviations.

1. Introduction

The Convergence Report 2008 submitted by European Commission examined the state of economic convergence in 10 Member States of the European Union under review [6]. The report deals with the nominal convergence criteria and sustainability of economic convergence. The common analytical framework was applied individually to 10 Member States under review. The convergence must be achieved on a lasting basis and not just at a given point in time. For this reason, the country examinations elaborate on the sustainability of convergence. In this respect, economic developments in analyzed countries are reviewed from a backward-looking perspective, covering, in principle, the past 10 years. This helps to better determine

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the extent to which current achievements are the result of genuine structural adjustment, which should lead to the sustainability of economic convergence. The last Convergence Report approved that the Slovak Republic met criteria for the Economic and Monetary Union entry.

From the Slovak perspective it is also important that the entry into monetary union can bring new dynamics to further increase of the living standard of the Slovak citizens. The real convergence can be in practice measured by economic performance, the level of labor productivity, the level of prices and wages. Commonly used indicator to measure the economic convergence is the Gross Domestic Product in the purchasing power parity (GDP_{PPP}).

Besides the nominal and real convergence the economic theory proved also the importance of the structural convergence. The structural convergence is analysing the convergence of the economy according to its sectors, employment, innovations, research, economic reforms, social policy and environmental policy. It is also analysing the harmonization of the economic cycles and the synchronization of economic shocks [10].

Analyses of nominal, real and structural convergence of the country are important not only before the entry to monetary union but also after that important step. The contribution deals with the problematics of the real convergence focusing on the quantitative aspect of the real convergence. Analyses are focused on the past, presence and future of the real convergence of the Slovak Republic as well. The contribution consists of 4 sections. After introduction the second part is devoted to the methodology and data sources. Third part brings the outcome of the cointegration analyses and the last part is conclusion.

2. Methodology and data

When economists refer to "real economic convergence", quite often they refer to **β convergence and σ convergence**. Another method how to measure convergence is **cointegration**.

Barro and Sala-i-Martin [4] and Sala-i-Martin [16] published useful distinction between two types of convergence in growth empirics: β convergence and σ convergence. When the partial correlation between growth in income over time and its initial level is negative, there is β convergence. When the variability of real *per capita* income across a group of economies falls over time, there is σ convergence.

2.1. An introduction to β convergence concept

β convergence is a concept which is estimating the speed of convergence of the individual country to the average of clustered countries. More dynamic growth of less developed countries has an impact on convergence acceleration of less developed countries comparing to developed ones. Based on that the negative relationship is assumed between initial level of GDP_{PPP} and its average rate of growth. All countries are closing the gap between their present income and their balanced growth level. In the literature [10] implicit and conditional β convergence are distinguished. Implicit β convergence is linear regression relationship of panel database, where dependent variable is growth rate of GDP_{PPP} and independent variable is absolute level of GDP_{PPP}:

$$\log(GDP_{PPP_{i,t}}) - \log(GDP_{PPP_{i,t-1}}) = \alpha_i + \beta_i \cdot \log(GDP_{PPP_{i,t}}) + \varepsilon_t, \quad (1)$$

where:

GDP_{PPP} – gross domestic product in the i-th country and t-quarter,
α_i – level constant,
β_i – beta coefficient determining the convergence or divergence,
ε_t – stochastic error,
$i = 1, 2, ..., N$ – country identificator,
$t = 1, 2, ..., T$ – time identificator.

Conditional β convergence is a more complex concept than implicit β convergence. The assumption on the invert relationship between initial level of GDP_{PPP} and its growth rate holds, but this method is generating the long-lasting steady state (balanced state) of growth for analyzed countries as well. Variables which are influencing steady state used to be money growth, government expenditures, exchange rate and inflation. These variables are extending the regression model:

$$\log(GDP_{PPP_{i,t}}) - \log(GDP_{PPP_{i,t-1}}) = \alpha_i + \beta_i \cdot \log(GDP_{PPP_{i,t}}) + \gamma_i \cdot (X_{i,t}) + \varepsilon_t, \quad (2)$$

where:

GDP_{PPP} – gross domestic product in the i-th country and t-quarter,
α_i – level constant,
β_i – beta coefficient determining the convergence or divergence,
ε_t – stochastic error,
γ – regression coefficient for steady state of variables,
X – variables which are influencing the steady state,
$i = 1, 2, ..., N$ – country indentificator,
$t = 1, 2, ..., T$ – time identificator.

More information on implicit and conditional β convergence provided the research done by Hrubina and Gavliak [10].

2.2. An introduction to σ convergence concept

When the dispersion of real *per capita* income across a group of economies falls over time, there is σ convergence. σ convergence is the ratio of mean-root-square error and mean value of log variable gross domestic demand in purchasing power parity as it is shown by equation 3. In statistical terminology we call such a ratio a variation coefficient:

$$\frac{\sigma\,[GDP_{PPP_{i,\,l}}]}{E\,[GDP_{PPP_{i,\,l}}]} < \frac{\sigma[\,GDP_{PPP_{i,\,k}}]}{E\,[GDP_{PPP_{i,\,k}}]},\ l = 1,2\ldots t_{1:2004},\quad k = t_{2:2004},\ldots T. \tag{3}$$

If σ convergence holds, and differences between economic growth are steadily vanishing, than the variation coefficient should decline.

2.3. An introduction to Vector Error Correction and Cointegration Theory

The finding that many macro time series may contain a unit root has spurred the development of the theory of non-stationary time series analysis. In 1987 Engle and Granger pointed out that a linear combination of two or more non-stationary series may be stationary. If such a stationary, or $I(0)$, linear combination exists, the non-stationary (with a unit root) time series are said to be cointegrated. The stationary linear combination is called the cointegrating equation and may be interpreted as a long-run equilibrium relationship between the variables.

A vector error correction (VEC) model is a restricted VAR that has cointegration restrictions built into the specification, so that it is designed for use with non-stationary series that are known to be cointegrated. The VEC specification restricts the long-run behaviour of the endogenous variables to converge to their cointegrating relationships while allowing a wide range of short-run dynamics. The cointegration term is known as the error correction term since the deviation from long-run equilibrium is corrected gradually through a series of partial short-run adjustments.

As a simple example, consider a two-variable system with one cointegrating equation and no lagged difference terms. The cointegrating equation is

$$y_{2,t} = \beta\, y_{1,t}$$

and the VEC is

$$\Delta y_{1,t} = \gamma_1 (y_{2,\text{t-1}} - \beta y_{1,\text{t-1}}) + \varepsilon_{1,t},$$
$$\Delta y_{2,t} = \gamma_2 (y_{2,\text{t-1}} - \beta y_{1,\text{t-1}}) + \varepsilon_{2,t}.$$

In this simple model, the only right-hand side variable is the error correction term. In long run equilibrium, this term is zero. However, if y_1 and y_2 deviated from long run equilibrium last period, the error correction term is non-zero and each variable adjusts to partially restore the equilibrium relation. The coefficients and measure the speed of adjustment.

In this model, the two endogenous variables y_1 and y_2 will have non-zero means but the cointegrating equation will have a zero intercept. To keep the example simple, despite the fact that the use of lagged differences is common, we have included no lagged differences on the right-hand side.

If the two endogenous variables y_1 and y_2 have no trend and the cointegrating equations have an intercept, the VEC has the form

$$\Delta y_{1,t}=\gamma_1 (y_{2,t-1} - \mu - \beta y_{1,t-1}) + \varepsilon_{1,t},$$
$$\Delta y_{2,t}=\gamma_2 (y_{2,t-1} - \mu - \beta y_{1,t-1}) + \varepsilon_{2,t}.$$

Another VEC specification assumes that there are linear trends in the series and a constant in the cointegrating equations, so that it has the form

$$\Delta y_{1,t}=\delta_1 + \gamma_1 (y_{2,t-1} - \mu - \beta y_{1,t-1}) + \varepsilon_{1,t},$$
$$\Delta y_{2,t}=\delta_2 + \gamma_2 (y_{2,t-1} - \mu - \beta y_{1,t-1}) + \varepsilon_{2,t}.$$

Similarly, there may be a trend in the cointegrating equation, but no separate trends in the two VEC equations. Lastly, if there is a separate linear trend outside the parentheses in each VEC equation, then there is an implicit quadratic trend in the series.

Testing for Cointegration. Given a group of non-stationary series, we determined whether the series are cointegrated. If they are, we would like to identify the cointegrating (long-run equilibrium) relationships. VAR-based cointegration test was implemented using the methodology developed by Johansen [12]. Johansen's method is to test the restrictions imposed by cointegration on the unrestricted VAR involving the series.

Johansen's Cointegration Test. Consider a VAR of order p:

$$y_t = A_1 y_{t-1} + ... + A_p y_{t-p} + Bx_t + \varepsilon_t,$$

where y_t is a k-vector of non-stationary $I(1)$ variables, x_t is a d vector of deterministic variables, and ε_i is a vector of innovations. We can rewrite the VAR as:

$$\Delta y_t = \Pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + Bx_t + \varepsilon_t,$$

where

$$\Pi = \sum_{i=1}^{p} A_i - I, \quad \Gamma_i = -\sum_{j=i+1}^{p} A_j.$$

Granger's representation theorem asserts that if the coefficient matrix Π has reduced rank $r < k$, then there exist $k \times r$ matrices α and β each with rank r such that

$\Pi=\alpha\beta$ and $\beta' y_t$ is stationary $I(0)$, r is the number of cointegrating relations (the cointegrating rank) and each column of β is the cointegrating vector. The elements of α are known as the adjustment parameters in the vector error correction model. Johansen's method is to estimate the Π matrix in an unrestricted form, then test whether we can reject the restrictions implied by the reduced rank of Π.

The Cointegrating Relations (Vector). Each column of the β matrix gives an estimate of a cointegrating vector. The cointegrating vector is not identified unless we impose some arbitrary normalization. The normalization was adopted so that the r cointegrating relations were solved for the first r variables in the y_t vector as a function of the remaining $k - r$ variables.

Deterministic Trend Assumptions. Our series may have non-zero means and deterministic trends as well as stochastic trends. Similarly, the cointegrating equations may have intercepts and deterministic trends. The asymptotic distribution of the LR test statistic for the reduced rank test does not have the usual χ^2 distribution and depends on the assumptions made with respect to deterministic trends. We provided tests for the following five possibilities considered by Johansen (see [12], pp. 80-84 for details):

1. Series y have no deterministic trends and the cointegrating equations do not have intercepts:

$$H_2(r): \prod y_{t-1} + Bx_t = \alpha\beta' y_{t-1} .$$

2. Series y have no deterministic trends and the cointegrating equations have intercepts:

$$H_1^*(r): \Pi y_{t-1}\ Bx_t = \alpha(\beta' y_{t-1} + \rho_0) .$$

3. Series y have linear trends but the cointegrating equations have only intercepts:

$$H_1(r): \Pi y_{t-1}\ Bx_t = \alpha(\beta' y_{t-1} + \rho_0) + \alpha_\perp \gamma_0 .$$

4. Both series y and the cointegrating equations have linear trends:

$$H_1(r): \Pi y_{t-1}\ Bx_t = \alpha(\beta' y_{t-1} + \rho_0 + \rho_1 t) + \alpha_\perp \gamma_0 .$$

5. Series y have quadratic trends and the cointegrating equations have linear trends:

$$H(r): \Pi y_{t-1}\ Bx_t = \alpha(\beta' y_{t-1} + \rho_0 + \rho_1 t) + \alpha_\perp (\gamma_0 + \gamma_1 t) ,$$

where $\alpha_\perp$ is the (non-unique) $k \times (k - r)$ matrix such that $\alpha'\alpha_\perp = 0$ and $rank(|\alpha|\alpha_\perp) = k$.

For cointegration analyses quarterly input data from Eurostat datasource were choosen. We analyzed detrended data from the first quarter of 2000 till the the first quarter of 2008 by EViews 4 software [7].

3. Empirical results of cointegration analyses

Cointegration analyses between growth rate of Slovak Gross Domestic Product in Purchasing Power Parity (GDP_{PPP} SR) and growth rate of the Euroarea Gross Domestic Product in Purchasing Power Parity (GDP_{PPP} EA), based on Johansen cointegration test, did not prove the existence of one cointegration relationship (proved by *max-eigenvalue* statistics on the 5% probability level). The reason might be in divergence of price development within the Euroarea. Despite the initial plans to harmonize the price development across the Euroarea by the single monetary policy, the divergence is persistent. It is affected not only by external shocks (prices of oil, foods, etc.) but also by inert behaviour of economic agents.

Cointegration analyses between growth rate of Slovak Gross Domestic Product (GDP_{SR}) and growth rate of the Euroarea Gross Domestic Product (GDP_{EA}), based on Johansen cointegration test, showed an existence of one cointegration relationship (proved by *max-eigenvalue* statistics on the 5% probability level). The results are summarized in Table 1.

Pursuing cointegration analysis we rejected model with deterministic trend in the data with respect to expected cyclical development of GDP growth. With regard to statistical insignificance we rejected hypothesis on implementing trend and absolute term into cointegration equation.

The cointegration equation, which is characterizing the long-lasting equilibrium, has the following specification:

$$\begin{matrix} g_{SK} = 3,87 g_{EA} \\ s_b \quad (0,18) \end{matrix} , \tag{4}$$

where:

– *y*-on-*y* growth of the Gross Domestic Product in Slovakia (in %),

– *y*-on-*y* growth of the Gross Domestic Product in Euroarea (in %).

It means that in the analyzed period the balanced economic growth of Slovakia was 3.87 times higher than the balanced economic growth of Euroarea. Regarding the Error Correction Coefficients estimated in the Error Correction Model (they indicate the duration whithin which the searched variables are returning to the long-lasting equilibrium in the case of the short-term deviations.), they indicate that the Slovakia is reacting faster on diversion from the balance. If the Slovak economic growth would be above its balanced level, it should fall down by 16.5% in one quarter. If the Euroarea economic growth were below its balanced level, it should increase by 10% in one quarter.

Table 1. The cointegration test between growth of Slovak Republic and Euroarea

Sample (adjusted): 2001:4 2008:1				
Included observations: 26 after adjusting endpoints				
Trend assumption: No deterministic trend				
Series: RASTHDP_SR RASTHDP_EA12				
Lags interval (in first differences): 1 to 2				
Unrestricted Cointegration Rank Test				
Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	5 Percent Critical Value	1 Percent Critical Value
None **	0.611282	24.83419	12.53	16.31
At most 1	0.010208	0.266783	3.84	6.51
*(**) denotes rejection of the hypothesis at the 5%(1%) level Max-eigen value test indicates 1 cointegrating equation(s) at both 5% and 1% levels				
Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	5 Percent Critical Value	1 Percent Critical Value
None **	0.611282	24.56741	11.44	15.69
At most 1	0.010208	0.266783	3.84	6.51
*(**) denotes rejection of the hypothesis at the 5%(1%) level Max-eigen value test indicates 1 cointegrating equation(s) at both 5% and 1% levels				
Unrestricted Cointegrating Coefficients (normalized by b'*S11*b=I):				
RASTHDP_SR	RASTHDP_EA12			
-0.862681 0.186866	3.340352 0.166020			
Unrestricted Adjustment Coefficients (alpha):				
D(RASTHDP_SR)	0.190893	0.026277		
D(RASTHDP_EA12)	-0.115875	0.027397		
1 Cointegrating Equation(s):		Log likelihood	-2.840635	
Normalized cointegrating coefficients (std.err. in parentheses)				
RASTHDP_SR	RASTHDP_EA 12			
1.000000	-3.872059 (0.17537)			
Adjustment coefficients (std.err. in parentheses)				
D(RASTHDP_SR)	-0.164680			
	(0.05673)			
D(RASTHDP_EA12)	0.099963			
	(0.05393)			

Source: [7], own research.

The results of cointegration analyses proved the real convergence of the Slovak Repulic to Euroarea.

To intensify the economic convergence in the future, the Slovak Republic should also make use of its membership in Euroarea. The implementation of the responsible economic, fiscal and labour market policies, introduction of a new structure of the economy based on the new technologies and by building up the knowledge society the Slovak Republik should speed up its convergence process.

4. Conclusion

The Slovak Republic entry into the Economic and Monetary Union might be a new incentive to the dynamic economic development. The results of cointegration analyses proved the real convergence of the Slovak Repulic to Euroarea. To speed up this process the implementation of the responsible economic, fiscal and labor market policies, the introduction of a new structure of the economy based on the new technologies and the real changes in building the knowledge society in the Slovak Republik are requested.

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MODELE KOREKCJI BŁĘDU I RZECZYWISTA KONWERGENCJA – PRZYPADEK SŁOWACJI

Streszczenie

1 stycznia 2009 r. Słowacja weszła do Europejskiej Unii Monetarnej. Słowacja spełniła kryteria integracji monetarnej. Celami tej integracji są zrównoważony rozwój i konwergencja. W pracy analizowany jest trwały punkt równowagi realnej gospodarki oraz szacowana jest długość trwania w przypadku krótkotrwałych zakłóceń.