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FIRST-PRICE SEALED-BID AUCTION WITH ADDITIONAL PRICE NEGOTIATIONS: THEORETICAL ANALYSES AND RESULTS OF EXPERIMENTS

1. Introduction

Auctions, which play a significant role in Western countries' economies,[1] are also becoming more and more popular in CEE countries. They are a simple, efficient and transparent procedure of making market transactions. An auction is a very broad term which can mean any design in which goods are allocated to economic agents. In standard auctions in which few buyers compete for one unit (or many units) of a particular good an auction mechanism allocates the good(s) to the bidder(s) offering the highest price(s). In a procurement auction the whole procedure is reversed. Few sellers compete and one of them will provide a specified good (or specified goods) and the auction mechanism allocates the good(s) to the bidder(s) offering the lowest price(s).

There exist many auction designs and typically the law does not enforce an application of a particular one. The economic agents can run either the dynamic auctions (with ascending or descending prices) or a sealed-bid one, in which no interaction between participants is possible. There are dozens of specific solutions that can be applied, especially in case of multi-unit auctions, where additional decisions have to be made concerning for example the activity rules or winner determination issue. Different auction designs can lead to different outcomes, i.e.

[1] For example a couple of years ago just the public procurement auctions were estimated to constitute about 16% of GDP in EU and about 20% of GDP in the USA. Auctions are widely used to sell such commodities as treasury bills, electricity, foreign exchange, spectrum licenses, natural gas, emission permits and many others.

different winners, prices, seller's revenues, and so on. The huge part of auction theory is thus devoted to the problem of auction efficiency [Kuśmierczyk 2009].

In this paper a special form of auction is analyzed, which seems to comprise the elements of two most popular auction designs: first-price sealed-bid auction and an ascending-bid auction. This form is quite often used in practice and the paper's goal is to analyze its results both theoretically and experimentally.

2. Theoretical analysis

In Polish auction's practice the following auction design can be quite often observed. The auctioneer starts an auction (either a standard one, or a procurement auction) as a first-price sealed-bid auction. This means that each of the participants of the auction is supposed to make a sealed offer. The offers from all the participants are disclosed on a predetermined day. In a standard first-price sealed-bid auction that would be the end: the winner would be the bidder who offered the highest price and he would pay a price that he offered.[2] But in a procedure analyzed herein the disclosure of the offers is not the end, as the auctioneer now starts additional price negotiations with the two best bidders. In what follows the auctioneer allows the second best bidder to improve his offer, so as to outbid his competitor. Of course the competitor would typically also have a chance to react if the second best bidder decided to raise his bid.

In practice there are plenty additional rules that the auctioneer might apply, that would influence the auction proceedings. Just to name few:

1. The auctioneer does not always have to start the negotiations' phase, basing his decisions on the bidders' offers. So if he is satisfied with the price offered in a first-price sealed-bid phase he might give up the second phase. That would of course introduce an additional element of uncertainty for the bidders.

2. The auctioneer might not inform a second-best bidder about his competitor's offer and just reveal that his offer was not the highest one. In this situation the bidder would be given a chance of increasing his offer without knowing by how much he needs to increase it to outbid his competitor.

3. The auctioneer might not inform the bidders if their offer was the highest one and just reveal that the offers were "close." In this case each of the bidders would be given a chance of increasing his price without knowing if this was actually needed.

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[2] From now on all the analyses will be held for the case of the standard auction, i.e. an auction in which a group of buyers compete for a single object and the one who offers the highest price is the winner. In the procurement auctions the whole procedure is reversed, as the winner is the bidder who offered the lowest price. The case of procurement auctions will not be analyzed separately, which is a standard procedure in auction theory textbooks [Krishna 2002]. The reason is that procurement auctions do not need a separate analyses and theory as all the conclusions from standard auction just have to be reversed.

For the analysis in this paper it will be assumed that the auctioneer always starts the second phase and that he runs it as a standard English auction, i.e. the bidders increase their bids in turns. The starting price for the second phase is the maximal price from the first phase, and it is increased by the minimal increments. Moreover it is assumed here that the object auctioned has a private value for each of the bidders[3].

Bidder *i*'s strategy can be described as: $\left(b_1^i, b_2^i\right)$, where b_1^i is the price offered in the 1st phase (sealed-bid auction) and b_2^i is the maximal price to which he would agree to bid the price up in the 2nd phase (English auction) if he proceeds to it. For example let us say that there were 4 bidders in the auction and their strategies were the following:

- Bidder #1: (5400, 6200),
- Bidder #2: (3000, 7000),
- Bidder #3: (4800, 6500),
- Bidder #4: (4300, 5000).

This means that the highest price offered in the sealed-bid phase is 5400 offered by Bidder #1. Bidder #3 would also be invited to the second phase, as his price in the sealed-bid phase, 4800, is the second highest. In the second phase the bidders would compete by subsequently making higher and higher bids. So, Bidder #3 would start by increasing the starting price 5400 (the highest price from the 1st phase) to 5401; Bidder #1 would react by increasing the price to 5402, and so on, they would continue doing it until the maximal price that one of the bidders was willing to pay was reached. In this case Bidder #1 would be the first to quit, as the maximal price he was willing to pay was 6200. The winner would be Bidder #3 who would pay a price 6200.[4] Observe that if the whole auction was run as an English auction, then actually a winner would be Bidder #2 as he was willing to pay as much as 7000. But as he did not pass to the second phase, he did not have a chance to reveal his willingness to pay that much.

To put it more formally. Let us denote by I the index of the bidder who made the highest bid in the first phase, and by II the index of the bidder who made the second-highest bid in the first phase. Then the final price is:

$$p = \begin{cases} b_I^1 & \text{if } b_{II}^2 < b_I^1 \\ b_{II}^2 & \text{if } b_I^1 < b_{II}^2 < b_I^2 \\ b_I^2 & \text{if } b_I^2 < b_{II}^2 \end{cases}. \tag{1}$$

[3] A private-value auction is the one in which each bidder's evaluation of the object (or estimation of costs in case of procurement auction) is independent. In common-value auctions the value of the object is the same to every auction's participant, whereas in interdependent-value auctions valuations comprise both the objective (common) and subjective (private) elements.

[4] The final price could be 6200 or 6201 depending on who was the first to offer a price 6200. For convenience it will be always assumed that it is the lower one.

In the first case the second phase does not start at all as the maximal price the second-best bidder was willing to pay in the English phase was lower than the price offered by the best bidder in the sealed-bid phase. The winning price is the price offered in the first phase. In the second and third case there is an English phase and the price is lower of values b_I^2, b_{II}^2.

Now we will proceed to determine the optimal strategy of the auction participant. Let us denote by v_i the private value of the auctioned good for the *i*-th participant. Bearing in mind that the auction is a private-value one, it is easy to figure out that:

$$b_i^2 = v_i, \tag{2}$$

i.e. a bidder in the second phase should be willing to bid the price up as long as it does not exceed the value of good for him, and quit when it does.

The determination of the optimal value of bid in the first phase is less obvious as it depends on the bidder's expectations, on behaviour of other bidders or the attitude towards the risk. There is a situation though when the calculation of this optimal strategy is relatively easy, i.e. when the assumptions of the Revenue Equivalence Principle are met

The Revenue Equivalence Principle (REP) is the most important theorem in auction theory, which states that the expected value of payments of the auction participants, and hence an expected value of price in every auction is independent of the actual auction design if certain assumptions are met. Among those assumptions there are: a publicly known common distribution from which each bidder's valuation is independently drawn and risk-neutrality of bidders.[5]

Assuming that REP assumptions are met, the calculation of the optimal value of b_i^1 is easy but leads to a bit unexpected results, that an optimal strategy is to bid 0 in the first phase.

Theorem. Let us say that each risk-neutral bidder's valuation v_i is independently drawn from a common distribution, and the auction consists of two phases: first, there is a sealed-bid phase, and later two bidders with the highest offers compete in the ascending-bid auction with the highest offer from the first phase treated as the initial bid of the bidder who made it. Then, the optimal bidding strategy of bidder i is $(0, v_i)$, meaning that he should offer 0 in the first phase and bid maximally till the level of v_i in the second phase.

Proof. The proof is very easy and is based on the Revenue Equivalence Principle. It has been already demonstrated that the optimal strategy in the second phase was to bid up to v_i. Let us say that each bidder in the sealed-bid phase offered a price being some fraction of his private value:

[5] See proof and discussion on importance of those assumptions in [Krishna 2002], or [Klemperer 2004].

$$b_i^1 = \alpha \cdot v_i \text{, with } \alpha \in [0,1]. \tag{3}$$

If all the participants used the same strategy then the highest prices in the first phase would be the ones coming from the bidders with the highest valuations; using the same notations as previously those bids would be

$$p = \begin{cases} b_I^1 = \alpha \cdot v_I \\ b_{II}^1 = \alpha \cdot v_{II} \end{cases} \tag{4}$$

Substituting it to (1), using (4), and the fact that optimal strategy in the English auction is to bid up one's value it turns out that the final price will be:

$$p = \begin{cases} \alpha \cdot v_I & \text{if } v_{II} < \alpha \cdot v_I \\ v_{II} & \text{if } \alpha \cdot v_I < v_{II} < v_I \\ v_I & \text{if } v_I < v_{II} \end{cases}. \tag{5}$$

But by definition $v_I > v_{II}$ and so the auction will be won by the bidder with the highest valuation, who would pay the price equal to

$$p = \begin{cases} \alpha \cdot v_I & \text{if } v_{II} < \alpha \cdot v_I \\ v_{II} & \text{if } \alpha \cdot v_I < v_{II} \end{cases} \tag{6}$$

and so

$$p = \max\{v_{II}; \alpha \cdot v_I\}. \tag{7}$$

It is easy to interpret the result given by equation (7). If all auction participants act rationally and in a first phase offer a price which is a fraction of their valuation,[6] then the bidder with the highest valuation will win the auction. The price that he pays is either equal to his bid in the first phase $(\alpha \cdot v_I)$ if the second bidder does not bid the price up, or equals the maximum price to which the second participant would continue bidding (v_{II}),

But the Revenue Equivalence Principle claims that this price has to be equal to the price paid by the participants in the other auctions, for example an English auction. In an English auction the price paid by the winning bidder would be v_{II}, as this is the price at which the second-best bidder would quit the auction. Thus the following relationship must hold:

$$\max\{v_{II}; \alpha \cdot v_I\} = v_{II}, \tag{8}$$

from which it results that $\alpha = 0$, which concludes the Proof.

[6] It does not have to be a fixed fraction, but it has to be a non-decreasing function of the valuation, which is quite rational: the optimal bid of the bidder with the higher valuation has to be at least as high as the optimal bid of the participant with the lower valuation.

At first it looks a bit absurd to claim that one should offer 0 in the first phase, but the logic is actually quite simple. Assuming that every bidder's optimal strategy must be a non-decreasing function of his/her valuation then a bidder cannot gain anything by deviating from her optimal strategy of 0: in the second phase (s)he will meet the bidder with the highest valuation, and he will outbid her anyway.[7]

This result is of course true only if bidders behave according to REP's assumptions, i.e. they are for example risk-neutral. But what if the bidders are actually either risk-averse, or not perfectly rational, or there are some asymmetries of information? The results of experiments and formal proofs[8] show that in such situations different auction designs do not lead to the same value of expected profits. In these situations a first-price sealed-bid auction typically leads to higher prices (and seller's revenues) than the English auction. The reason why risk-averse bidders in a first-price sealed-bid auction offer prices higher than expected is that they fear losing the auction, and agree for a lower expected value of profits to reduce the risk of being outbid by the others. A sealed-bid auction with negotiations seems to be a bit less risky thanks to the fact that the sealed-bid phase is followed by the English auction: even if I turn out second in the first phase, I still have a chance to outbid my competitor in the second phase. Because of that we should expect the prices offered in the first round to be lower than those in the first-price sealed-bid auction. But what about the final price, bearing in mind that there is still a second phase that will drive the prices up? Will it turn out to be lower or higher than in the first-price sealed-bid auction? Obviously auctioneers using this procedure expect it to make their revenues higher, but if the rational participants take this into consideration, and make bids in the first phase substantially lower than those that they would make in the first-price sealed-bid auction then the final result is not obvious. This result can be tested experimentally.

3. Results of experiments

In order to see what prices are reached in the sealed-bid auction with negotiations and compare them to the results of standard first-price sealed-bid auction the experiments with students from the Wrocław University of Economics

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[7] It might be actually easier to think of the value α as not being exactly equal 0, but as a value very close to 0, let us say 0.0001. If everyone offers in the first phase a price equal to $0.0001 \cdot v$ and I diverge from this by offering a value substantially higher, then it increases my chances of advancing to the second round. But, assuming all the other bidders kept to the strategy $0.0001 \cdot v$, in the next round I would meet the bidder with the highest valuation and I would lose anyway. Nothing can be gained, but something could be lost, because if I was the bidder with the highest valuation, then by offering a price higher than needed in the first round I might actually increase the price finally paid. This shows that offering a price equal (or very close to) 0 is actually a Nash equilibrium in this game.

[8] See [Krishna 2002; Klemperer 2004].

were carried out. This paper presents only the results of some initial studies. The more detailed analyses of the results of these experiments will be carried out in a separate paper.

The experiments were a part of the "Auctions" course and students were motivated by the fact that the profits from experiments were converted to the points which influenced the final grade from the course. The students had already had some knowledge on auction theory, but the case of first-price sealed-bid auction with negotiations was not analyzed during the classes. In total 64 students participated in the experiment which consisted of 12 rounds. In each round a student competed with 3 other students for an object, the value of which was private for each student and was randomly chosen. Before each auction was started students were obliged to participate in a quiz which tested their understanding of the mechanism's rules.

In the first 4 rounds students participated in the standard first-price sealed-bid auction. Each round was arranged in a similar way. A student, knowing his private value, which was independently drawn from a uniform distribution on $\langle 0, 1000 \rangle$ made a price offer. After collecting all the bids and displaying the results a new round was started.

In the next 8 rounds students participated in the first-price sealed-bid auction with negotiations. The private values were again independently drawn from a uniform distribution on $\langle 0, 1000 \rangle$. Actually to allow for the comparison of data the same sets of data that had been used in rounds 1-4 were later used in rounds 5-8 and then again in 9-12. So that the students did not realize that the same data were used, they were permutated within a group and changed by a small number.[9] In each round a student was asked to offer two prices: a bid in the first phase, and maximal price to which they would bid the price up if needed in the next phase. This approach, i.e. an English auction with proxy bidding was used instead of standard English auction to speed up the experiments. The results of the quiz showed that students did not have problems with understanding this rule.[10]

Taking into consideration the theoretical analyses from the previous parts the experiments were supposed to help test the following hypotheses:

H1: The price offered in the sealed-bid phase of the first-price sealed-bid auction with negotiations is bigger than 0.

[9] So actually every student had completely different private values in each round. The data were only the same within the group, but a student had no chance to realize that, as in no rounds he had a chance to look at other students' private values; the only information publicly disclosed was the information about the bids.

[10] Decisions of four students suggested that they could have problems with understanding the rules, because in some rounds their bid in the second phase was lower than in the first phase, which of course is impossible and makes no sense. All the decisions of those students were excluded from the analyses of decisions in rounds 5-12.

H2: The price offered in the sealed-bid phase of the first-price sealed-bid auction with negotiations is lower than the price offered in the standard first-price sealed-bid auction.

H3: The final price reached in the first-price sealed-bid auction with additional negotiations is higher than the price reached in the first-price sealed-bid auction.

To allow for the comparison of data all the bids and prices were calculated as the fraction of private values.[11] Table 1 shows the average values.

Table 1. Comparison of bids and prices in the experiment

1st price sealed-bid		1st price sealed-bid with negotiations		
bid	final price	b_{I}	b_{II}	final price
0.898	0.897	0.858	0.967	0.870

Source: based on own calculations.

As the results of experiments demonstrate the bids made by students in the first phase are much higher than predicted by the theory. According to REP a participant of the first-price sealed-bid auction with a total of four participants should offer a price equal to 0.750 of his private value; the fact that these values were on the average equal to 0.898 is a proof of the risk aversion. A price offered in the first phase of the first-price sealed-bid auction with negotiations (on the average 0.858) is so far from 0 that no statistical tests are needed to ***prove H1 right***.

As for H2 the answer is not that straightforward. In the auction with negotiations the students did indeed offer prices lower than in the auction without negotiations (0.858 compared to 0.898), but is this difference statistically significant? To verify H2 a parametric test of equality of two average values was run; a test statistics U, calculated as in [Domański 1990, p. 113] has a normal distribution (as both samples are absolutely large). The calculated value of test statistics U equals 5.8, which ***proves H2 right*** (the tested hypothesis of equality of average values have to be rejected).

But ***H3 has to be rejected***, which is obvious as the average value of prices in the auction with negotiations is actually lower, than in case of standard first-price sealed-bid auction (0.870 compared to 0.897). As a matter of fact there are statistical data proving the opposite conclusion, i.e. that ***final prices in the auction with negotiations are lower*** (using the same test with the null hypothesis of the equality of average values, we get U = 1.91, which yields the probability p = 0.028).[12]

[11] In case of bids the average is based on all students' data and in case of prices the average is based on winners' data.

[12] For this test the actual values of prices and not the bid/value ratios were taken. The higher value of bid/value ratio is not a proof of the higher final price, as theoretically a bidder with a lower valuation could win with a price close to his valuation (and so a bid/value ratio is high, even though the final price is low).

4. Conclusions

To maximize their revenues auctioneers quite often run additional price negotiations after collecting all the bids in the first-price sealed-bid auction. Assuming that bidders are aware of this opportunity , they should lower the prices they offer in the first phase of the auction. The application of the Revenue Equivalence Principle shows that rational bidders should in the sealed-bid phase offer prices close to 0. The results of the experiments demonstrated that the actual bids were very far from that, as the average offer in the sealed-bid phase was 85.8% of the bidders' valuations. Nevertheless the experiments showed that bidders did take into consideration the fact that there was additional English phase, as their bids were substantially lower than in the standard first-price sealed-bid auction.

What is most interesting, the results of the experiments suggest that running additional phase of price negotiations is actually counterproductive as the final price reached by the auctioneer is lower than in the standard first-price sealed-bid auction. However, this result should be treated with caution. The experiment analyzed here was preliminary and did not for example involve a common-value element of an auction in case of which additional price negotiations would more likely drive the prices up.

Further results of experiments will be a subject of a separate paper.

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