

Original Study

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Seismic bearing capacity of shallow strip footing embedded in slope resting on two-layered soil

<https://doi.org/10.2478/sgem-2021-0021>

Received December 15, 2020; accepted June 20, 2021

Abstract: In this paper, the limit equilibrium method with the pseudo-static approach is developed in the evaluation of the influence of slope on the bearing capacity of a shallow foundation. Particle swarm optimisation (PSO) technique is applied to optimise the solution. Minimum bearing capacity coefficients of shallow foundation near slopes are presented in the form of a design table for practical use in geotechnical engineering. It has been shown that the seismic bearing capacity coefficients reduce considerably with an increase in seismic coefficient. Besides, the magnitude of bearing capacity coefficients decreases further with an increase in slope inclination.

Keywords: Pseudo-static, Particle swarm optimisation, Bearing capacity coefficient, Slope inclination, Limit equilibrium

1 Literature Review

Many researchers analysed the bearing capacity based on the static method in which the bearing capacity coefficients were calculated based on static loads on the footings and the weight of the soil in both active and passive conditions. The classical bearing capacity theories started from Rankine (1857), Prandtl (1921), Terzaghi (1943), Meyerhof (1957), Saran et al. (1989) and many others who extensively studied the bearing capacity of shallow footings for static loading case. Terzaghi's bearing capacity theory (1943) was the first general theory for the bearing capacity of soils. Okabe (1924) and Mononobe and Matsuo (1929) were the pioneers in the inclusion of 'seismicity' in the design of structures. IS: 1893-1984 (Part-3) has also adopted the Mononobe and Okabe method for the determination of seismic active and passive earth pressure behind the re-

taining wall. Sarma and lossifelis (1990), Richards et al. (1993), Budhu and Al-Karni (1993) and Kumar and Kumar (2003) considered the seismic forces both on the structures and on the supporting soil mass, which were not considered by Meyerhof (1957). Researchers like Dormieux and Pecker (1995), Paolucci and Pecker (1997), Soubra (1997), Kumar and Rao (2002), Kumar (2003) and Choudhury and Subba Rao (2005) studied the seismic bearing capacity of shallow footings for horizontal ground. Sawada et al. (1994), Sarma (1999) and Askari and Farzaneh (2003) gave the solution for seismic bearing capacity of shallow foundations near the sloping ground. Again, some work for surface footing on the sloping ground was carried out by Zhu (2000), Kumar and Kumar (2003) and Kumar and Rao (2003) using limit equilibrium analysis, method of characteristics, etc. Choudhury and Rao (2006), Castelli and Lentini (2012), Farzaneh and Askari (2013) and Chakraborty and Kumar (2014) determined the seismic bearing capacity of a shallow foundation embedded in sloping ground by using the theorem of limit equilibrium method and limit analysis in conjunction with finite elements and non-linear optimisation technique, respectively. In their analysis, it was found that on increasing slope inclination, the bearing capacity decreased. But the researchers did not analyse the bearing capacity on layered soil. Yamamoto (2010) investigated seismic bearing capacity coefficients of spread and embedded foundations near slope in the analytical method. The pseudo-static approach was used, and the seismic forces consisted of a horizontal load applied to the foundation and inertia of a soil mass. Chakraborty and Kumar (2013) evaluated the bearing capacity factor on the sloping ground by applying lower bound (LB) finite element limit analysis in conjunction with non-linear optimisation. Baazouzi et al. (2016) studied the numerical analysis of the bearing capacity for a strip footing near a cohesionless slope and subjected to a centred load using the finite difference code. Button (1953) was the first to analyse the bearing capacity of strip footing on two layers of clay under static loading conditions. In this analysis, it was postulated that failure surface at the ultimate load is cylindrical, where the centre of the cylindrical curve lies at the edge of the footing.

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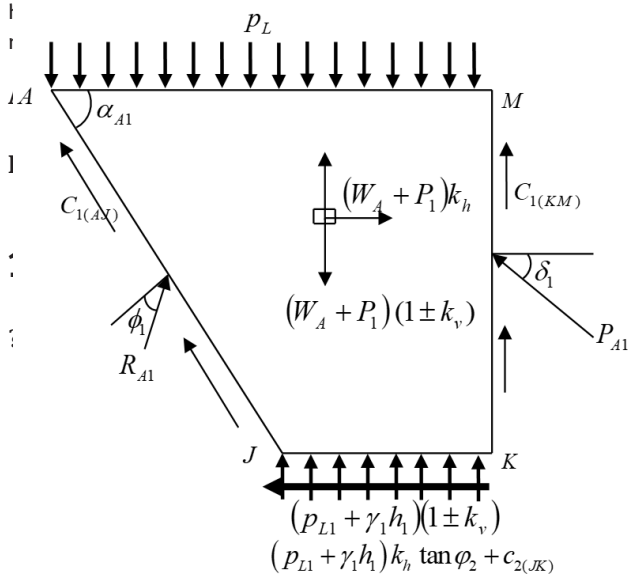


Figure 3: Active wedge in Top layer.

$$P_1 = p_L B_0 \quad (2)$$

Total cohesive force (C_1) on the slip lines AJ and MK is calculated as

$$\begin{aligned} C_{1(MK)} &= c_1 h_1, C_{1(AJ)} = c_1 AJ = c_1 h_1 \sec \alpha_{A1} \\ C_{2(JK)} &= c_2 JK = c_2 (B_0 - h_1 \cot \alpha_{A1}) \end{aligned} \quad (3)$$

The intensity of load at layer thickness h_1 is expressed as (depicted in Fig. 7)

$$p_{L1} = \frac{p_L B_0}{(B_0 + h_1)} \quad (4)$$

Conceding to limit equilibrium conditions, the authors can write

$$\begin{aligned} \sum V &= 0 \\ \Rightarrow C_{1(AJ)} \sin \alpha_{A1} + C_{1(MK)} + R_{A1} \cos (\alpha_{A1} - \phi_1) + \\ &+ P_{A1} \sin \delta_1 + (p_{L1} + \gamma_1 h_1) JK (1 - k_v) - \\ &- (P_1 + W_A) (1 - k_v) = 0 \end{aligned} \quad (5)$$

$$\begin{aligned} \sum H &= 0 \\ \Rightarrow -C_{1(AJ)} \cos \alpha_{A1} + R_{A1} \sin (\alpha_{A1} - \phi_1) - P_{A1} \cos (\delta_1) - \\ &- \{ (p_{L1} + \gamma_1 h_1) JK k_h \tan \phi_2 + c_{2(JK)} JK \} + \\ &+ (P_1 + W_A) k_h = 0 \end{aligned} \quad (6)$$

After solving Equations 5 and 6 and modifying both, the active pressure can be obtained as given below:

$$\begin{aligned} P_{A1} &= p_L B_0 \left\{ \frac{(1 - k_v) \sin (\alpha_{A1} - \phi_1) + k_h \cos (\alpha_{A1} - \phi_1)}{\cos (\alpha_{A1} - \phi_1 - \delta_1)} \right\} \\ &- \left\{ \frac{p_L B_0}{(B_0 + h_1)} (B_0 - h_1 \cot \alpha_{A1}) \right\} \\ &\left\{ \frac{(1 - k_v) \sin (\alpha_{A1} - \phi_1) + k_h \tan \phi_2 \cos (\alpha_{A1} - \phi_1)}{\cos (\alpha_{A1} - \phi_1 - \delta_1)} \right\} + \\ &+ \frac{2B_0 - h_1 \cot \alpha_{A1}}{2} h_1 \gamma_1 \\ &\left\{ \frac{(1 - k_v) \sin (\alpha_{A1} - \phi_1) + k_h \cos (\alpha_{A1} - \phi_1)}{\cos (\alpha_{A1} - \phi_1 - \delta_1)} \right\} - \\ &- \gamma_1 h_1 (B_0 - h_1 \cot \alpha_{A1}) \\ &\left\{ \frac{(1 - k_v) \sin (\alpha_{A1} - \phi_1) + k_h \tan \phi_2 \cos (\alpha_{A1} - \phi_1)}{\cos (\alpha_{A1} - \phi_1 - \delta_1)} \right\} - \\ &- 2c_1 h_1 \frac{\sin (\alpha_{A1} - \phi_1)}{\cos (\alpha_{A1} - \phi_1 - \delta_1)} - c_2 B_0 \frac{\cos (\alpha_{A1} - \phi_1)}{\cos (\alpha_{A1} - \phi_1 - \delta_1)} \\ &- c_1 h_1 \cot \alpha_{A1} \frac{\cos (\alpha_{A1} - \phi_1)}{\cos (\alpha_{A1} - \phi_1 - \delta_1)} + \\ &+ c_2 h_1 \cot \alpha_{A1} \frac{\cos (\alpha_{A1} - \phi_1)}{\cos (\alpha_{A1} - \phi_1 - \delta_1)} \end{aligned} \quad (7)$$

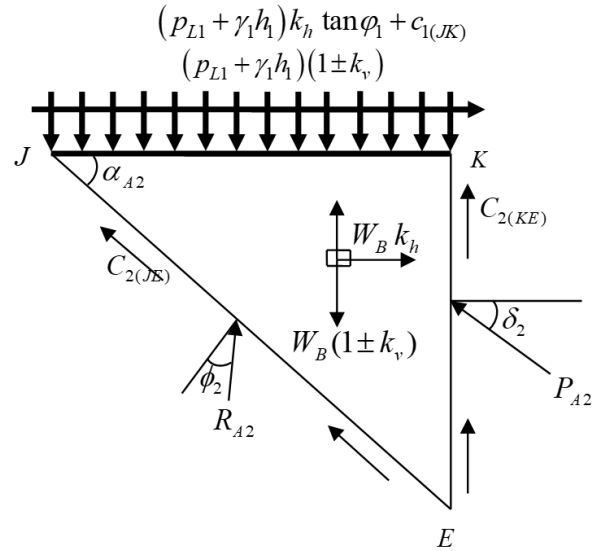


Figure 4: Active wedge in Bottom layer.

2.2 Active pressure at the bottom layer

From Fig. 4, active pressure distributing from the top layer to the wedge JKE.

Using limit equilibrium equations

$$\sum V = 0$$

$$\Rightarrow -C_{1(MK)} - C_{1(GD)} \sin \alpha_{p1} +$$

$$+ R_{p1} \cos (\phi_1 + \alpha_{p1}) - P_{p1} \sin \delta_1 +$$

$$+ (q_2 + \gamma_1 h_1) KG (1 - k_v) -$$

$$- (Q_1 + W_c) (1 - k_v) = 0$$

$$\sum H = 0$$

$$\Rightarrow -C_{1(GD)} \cos \alpha_{p1} + C_{1(GK)} -$$

$$- R_{p1} \sin (\phi_1 + \alpha_{p1}) + P_{p1} \cos \delta_1 +$$

$$+ (q_2 + \gamma_1 h_1) KG k_h \tan \phi_2 +$$

$$+ (Q_1 + W_c) k_h = 0$$

Solving Equations 19 and 20 and modifying both equations, passive resistance can be expressed as:

$$P_{p1} = \left[\frac{h_1 \cot \alpha_{p1} + h_2 \cot \alpha_{p2}}{2} h_1 - \right.$$

$$- \frac{1}{4} \left(h_1 \cot \alpha_{p1} + h_2 \cot \alpha_{p2} - \frac{D_f}{\tan i} + \frac{B_0}{2} \right)^2 \tan \alpha_{p1} \gamma_1$$

$$\left\{ \frac{(1 - k_v) \sin (\phi_1 + \alpha_{p1}) - k_h \cos (\phi_1 + \alpha_{p1})}{\cos (\phi_1 + \alpha_{p1} + \delta_1)} \right\}$$

$$- \gamma_1 h_1 h_2 \cot \alpha_{p2} \left\{ \frac{(1 - k_v) \sin (\phi_1 + \alpha_{p1}) + k_h \tan \phi_2 \cos (\phi_1 + \alpha_{p1})}{\cos (\phi_1 + \alpha_{p1} + \delta_1)} \right\}$$

$$- \gamma_1 \frac{\tan \left(\frac{D_f}{\tan i} - \frac{B_0}{2} \right) (2D_f - B_0 \tan i)}{2D_f + \tan i (2h_1 - B_0)} h_2 \cot \alpha_{p2}$$

$$\left\{ \frac{(1 - k_v) \sin (\phi_1 + \alpha_{p1}) + k_h \tan \phi_2 \cos (\phi_1 + \alpha_{p1})}{\cos (\phi_1 + \alpha_{p1} + \delta_1)} \right\}$$

$$+ \gamma_1 \tan i \left(\frac{D_f}{\tan i} - \frac{B_0}{2} \right)^2 \left\{ \frac{(1 - k_v) \sin (\phi_1 + \alpha_{p1}) - k_h \cos (\phi_1 + \alpha_{p1})}{\cos (\phi_1 + \alpha_{p1} + \delta_1)} \right\} +$$

$$c_1 h_1 \left\{ \frac{\sin (\phi_1 + \alpha_{p1})}{\cos (\phi_1 + \alpha_{p1} + \delta_1)} \right\}$$

$$+ c_1 \sin \alpha_{p1} \left[\frac{h_1}{2} \cos \alpha_{p1} - \frac{h_2}{2} \cot \alpha_{p2} \sec \alpha_{p1} + \frac{1}{2} \frac{D_f}{\tan i} \sec \alpha_{p1} - \frac{B_0}{4} \sec \alpha_{p1} \right]$$

$$\left\{ \frac{\sin (\phi_1 + \alpha_{p1})}{\cos (\phi_1 + \alpha_{p1} + \delta_1)} \right\}$$

$$+ c_1 \cos \alpha_{p1} \left[\frac{h_1}{2} \cos \alpha_{p1} - \frac{h_2}{2} \cot \alpha_{p2} \sec \alpha_{p1} + \frac{1}{2} \frac{D_f}{\tan i} \sec \alpha_{p1} - \frac{B_0}{4} \sec \alpha_{p1} \right]$$

$$\frac{\cos (\phi_1 + \alpha_{p1})}{\cos (\phi_1 + \alpha_{p1} + \delta_1)} - c_1 h_2 \cot \alpha_{p2} \frac{\cos (\phi_1 + \alpha_{p1})}{\cos (\phi_1 + \alpha_{p1} + \delta_1)}$$

(21)

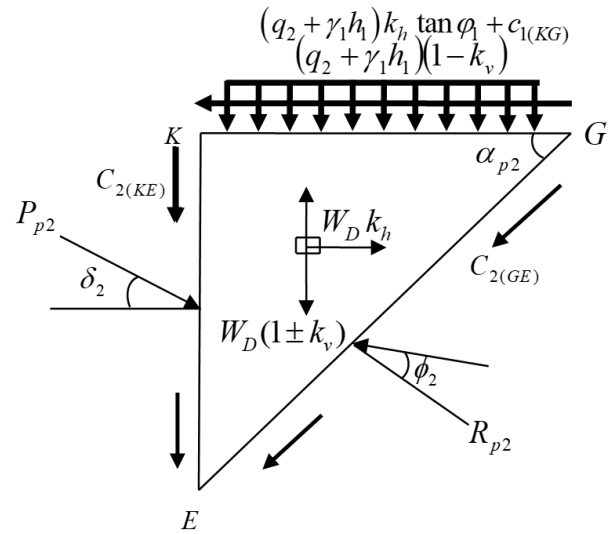


Figure 6: Passive wedge in Bottom layer.

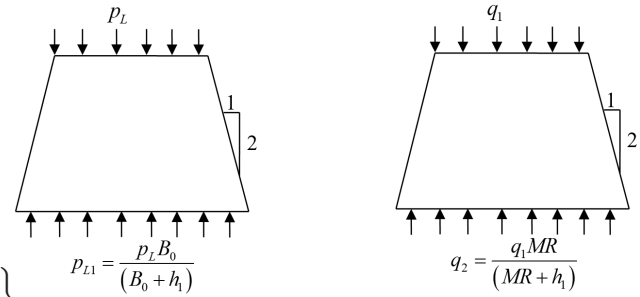


Figure 7: Load spread mechanism.

$$W_D = \frac{1}{2} h_2^2 \cot \alpha_{p2} \gamma_2 \quad (22)$$

According to 2 : 1 load distribution method, intensity of surcharge load at depth h_1 can be written as

$$q_2 = \frac{q_1 MR}{(MR + h_1)}$$

$$= \frac{\frac{\gamma_1}{2} \tan i \left(\frac{D_f}{\tan i} - \frac{B_0}{2} \right)^2}{\left(\frac{D_f}{\tan i} - \frac{B_0}{2} \right) + h_1} \quad (23)$$

Base shear between two passive wedges can be coded as

$$(q_2 + \gamma_1 h_1) k_h \tan \phi_2 + c_1 \quad \text{and} \quad (q_2 + \gamma_1 h_1) k_h \tan \phi_1 + c_1 \quad (24)$$

Cohesive forces in slip lines KE, KG and GE are given as

2.4 Passive resistance at the bottom layer

Total weight of the passive wedge KGE, as shown in Fig. 6, is calculated as

Case 1: Considering the effective area of active zones

Case 2: Considering the effective area of passive zones

$$\bar{\gamma} = \frac{\left(2 - \frac{h_1}{B_0} \cot \alpha_{A1}\right) \frac{h_1 \gamma_1}{2} + \left(1 - \frac{h_1}{B_0} \cot \alpha_{A1}\right) \frac{h_2 \gamma_2}{2}}{\left(2 - \frac{h_1}{B_0} \cot \alpha_{A1}\right) \frac{h_1}{2} + \left(1 - \frac{h_1}{B_0} \cot \alpha_{A1}\right) \frac{h_2}{2}} \quad (33)$$

$$\bar{\gamma} = \frac{\left\{ \left(\frac{h_1}{B_0} \cot \alpha_{p1} + 2 \frac{h_2}{B_0} \cot \alpha_{p2} \right) \frac{h_1}{2B_0} - \frac{1}{4} \left(\frac{h_1}{B_0} \cot \alpha_{p1} + \frac{h_2}{B_0} \cot \alpha_{p2} - \frac{D_f}{B_0 \tan i} + \frac{1}{2} \right)^2 \tan \alpha_1 \right\} \gamma_1 + \frac{1}{2} \left(\frac{h_2}{B_0} \right)^2 \cot \alpha_{p2} \gamma_2}{\left\{ \left(\frac{h_1}{B_0} \cot \alpha_{p1} + 2 \frac{h_2}{B_0} \cot \alpha_{p2} \right) \frac{h_1}{2B_0} - \frac{1}{4} \left(\frac{h_1}{B_0} \cot \alpha_{p1} + \frac{h_2}{B_0} \cot \alpha_{p2} - \frac{D_f}{B_0 \tan i} + \frac{1}{2} \right)^2 \tan \alpha_1 \right\} + \frac{1}{2} \left(\frac{h_2}{B_0} \right)^2 \cot \alpha_{p2}} \quad (34)$$

The bearing capacity factor ($N_{\gamma''}$) is a function of several parameters including cohesion, surcharge and unit weight. It can be expressed as:

$$N_{\gamma''} = \left(\frac{a_1}{e_1} + \frac{b_1}{e_1} + \frac{2\bar{c}}{\bar{\gamma}B_0} \frac{d_1}{e_1} \right) \quad (35)$$

The details of the equations a_1 , b_1 , e_1 and d_1 are given in **Appendix II**.

Where $\bar{c}$ is averaged cohesion in each layer in the slip line is shown by

$$\bar{c} = \frac{c_1 h_1 + c_2 h_2}{h_1 + h_2} \quad (36)$$

a_1 , b_1 , e_1 and d_1 are dimensionless equations.

In seismic condition, ($N_{\gamma''}$) can be expressed as ($N_{\gamma E}$) and in static condition, ($N_{\gamma''}$) can be expressed as ($N_{\gamma S}$), whereas ($N_{\gamma''}$) is the unity factor for the simultaneous resistance of unit weight, surcharge and cohesion.

3 Results and Discussion

The bearing capacity factor ($N_{\gamma''}$) has been computed by PSO algorithm, with the optimum ($N_{\gamma''}$) optimized w.r.t. variables α_{A1} , α_{A2} , α_{p1} and α_{p2} . The minimum value is taken as an optimised value. The design charts of the obtained bearing capacity coefficients ($N_{\gamma''}$) for different values of slope angle (i) are shown in Figs 8 and 9 ($\phi = 30^\circ, 40^\circ, \frac{D_f}{B_0} = 1$). It has been observed that ($N_{\gamma''}$) decreases when the slope angle (i) increases. The results obtained from MATLAB are summarised in design Tables 1 and 2. Using these design tables, the bearing capacity coefficients of the strip footing near slope are easily obtained with sufficient accuracy from the engineering point of view. Undrained bearing capacity is expressed as N_{cs} in the design tables.

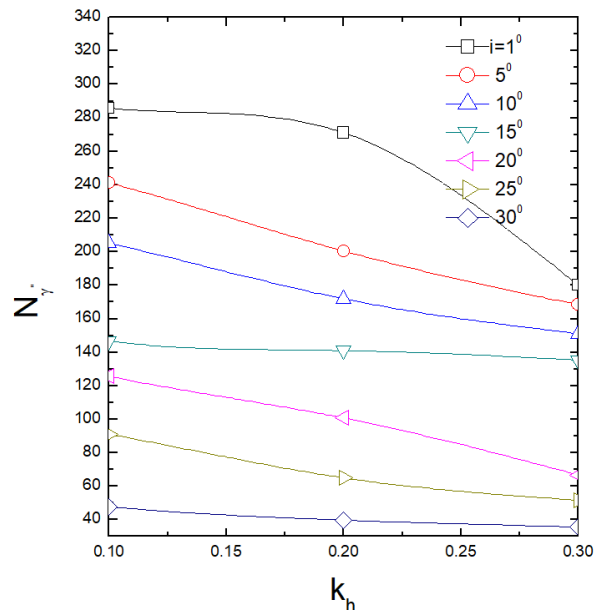


Figure 8: Design chart of Bearing capacity coefficient at $\phi_2 = 30^\circ$, $\delta_2 = \phi_2/2$, $i = 15^\circ$, $\delta_1/\delta_2 = 0.8$, $k_v = k_h/2$, $\gamma_1/\gamma_2 = 0.8$, $D_f/B_0 = 0.5$, $h_1/B_0 = 0.25$, $2c_2/B_0\gamma_2 = 0.2$, $c_1/c_2 = 0.8$.

3.1 Computation of N_{cs}

Merifield et al. (1999) investigated the bearing capacity of a strip footing resting on a two-layer clay deposit with a horizontal ground surface and proposed a modified bearing capacity factor N_c^* , which can be expressed as:

$$N_c^* = \frac{q_u}{c_1} \quad (37)$$

where q_u = ultimate bearing capacity and c_1 = undrained shear strength of the top layer.

Similarly, a dimensionless undrained bearing capacity factor N_{cs} is defined in the present study, which is a function of the parameters $c_u/\gamma B_0$, D_f/B_0 , c_1/c_2 , i , k_h and it can be described by the following equation:

$$N_{cs} = q_{ult}/c_1 = f(c_u/\gamma B_0, D_f/B_0, c_1/c_2, i, k_h) \quad (38)$$

Table 1: Undrained seismic bearing capacity N_{cs} for strip footing placed adjacent to two layered slope with $i=15^\circ$.

k_h	D_f/B_0	c_1/c_2	$c_1/\gamma B_0$				k_h	D_f/B_0	c_1/c_2	$c_1/\gamma B_0$			
			1	2	4	6				1	2	4	6
0	0.25	0.25	5.52	5.54	5.56	5.56	0.50	0.50	0.25	4.54	4.60	4.62	4.64
		0.5	5.52	5.54	5.56	5.56			0.5	4.52	4.60	4.61	4.63
		0.75	5.42	5.37	5.43	5.46			0.75	4.52	4.60	4.60	4.62
		1	4.74	4.40	4.41	4.42			1	4.36	4.42	4.44	4.47
		1.5	3.05	3.17	3.22	3.22			1.5	3.32	3.45	3.52	3.54
		2	2.4	2.52	2.58	2.6			2	1.55	2.82	2.90	2.92
		3	-	1.80	1.85	1.88			3	-	2.12	2.22	2.24
		4	-	0.6	1.51	1.53			4	-	0.91	1.84	1.90
		5	-	-	1.29	1.31			5	-	-	1.61	1.64
		0.25	4.50	4.46	4.25	4.49			0.25	4.40	4.49	4.52	4.53
	0.75	0.5	4.5	4.48	4.53	4.59	0	1	0.5	4.39	4.49	4.52	4.53
		0.75	4.4	4.4	4.54	4.58			0.75	4.38	4.49	4.52	4.53
		1	4.35	4.2	4.45	4.49			1	4.38	4.44	4.47	4.48
		1.5	4.05	3.84	3.93	3.96			1.5	3.88	4.2	4.3	4.33
		2	3.45	3.24	3.36	3.4			2	1.87	3.62	3.76	3.82
		3	-	2.44	2.62	2.68			3	-	2.7	3.07	3.13
		4	-	1.12	2.20	2.28			4	-	1.38	2.64	2.71
		5	-	-	1.92	2.0			5	-	-	2.27	2.42
		0.25	4.42	4.46	4.50	4.52			0.25	4.44	4.50	4.52	4.53
		0.5	4.41	4.43	4.48	4.51			0.5	4.41	4.49	4.52	4.53
	1.25	0.75	4.40	4.42	4.47	4.50			0.75	4.41	4.49	4.52	4.54
		1	4.39	4.41	4.45	4.48			1	4.38	4.44	4.47	4.48
		1.5	4.01	3.59	3.74	3.85		1.5	1.5	4.17	4.49	4.52	4.44
		2	3.67	3.2	3.42	3.45			2	2.34	4.18	4.31	4.37
		3	-	2.64	2.92	2.97			3	-	3.1	3.54	3.74
		4	-	1.67	2.59	2.67			4	-	1.92	3.3	3.43
		5	-	-	2.36	2.48			5	-	-	2.87	3.12
		0.25	4.84	4.79	4.8	4.81			0.25	4.12	4.2	4.23	4.24
		0.5	4.82	4.78	4.8	4.81			0.5	4.13	4.2	4.24	4.24
		0.75	4.64	4.79	4.8	4.81			0.75	4.13	4.2	4.24	4.24
		1	4.12	4.15	4.2	4.21			1	4.06	4.15	4.19	4.21
	0.25	1.5	-	3.03	3.09	3.11		0.5	1.5	-	3.37	3.46	3.5
		2	-	2.42	2.49	2.51			2	-	2.76	2.87	2.91
		3	-	-	1.82	1.87			3	-	-	2.21	2.25
		4	-	-	1.47	1.51			4	-	-	1.85	1.89
		5	-	-	-	1.28			5	-	-	-	1.65
		0.25	4.11	4.14	4.18	4.2			0.25	4.06	4.15	4.18	4.19
		0.5	4.08	4.13	4.18	4.19			0.5	4.05	4.14	4.19	4.19
		0.75	4.06	4.14	4.18	4.19			0.75	4.04	4.13	4.18	4.19
		1	4.04	4.15	4.2	4.21			1	4.02	4.11	4.14	4.17
	0.75	1.5	-	3.74	3.85	3.89	0.1	1	1.5	-	4.07	4.18	4.2
		2	-	3.13	3.31	3.36			2	-	3.48	3.71	3.79
		3	-	-	2.61	2.68			3	-	-	3.01	3.1
		4	-	-	2.14	2.28			4	-	-	2.34	2.67
		5	-	-	-	2.02			5	-	-	-	2.38
		0.25	4.08	4.14	4.18	4.19			0.25	4.05	4.14	4.18	4.19
		0.5	4.06	4.14	4.18	4.2			0.5	4.04	4.14	4.18	4.19
		0.75	4.05	4.13	4.18	4.19			0.75	4.05	4.14	4.18	4.19
		1	4.04	4.15	4.2	4.21			1	4.06	4.14	4.19	4.2
	1.25	1.5	-	4.14	4.19	4.2		1.5	1.5	-	4.14	4.19	4.2
		2	-	3.84	4.07	4.16			2	-	3.95	4.18	4.2
		3	-	-	3.37	3.48			3	-	-	3.67	3.82
		4	-	-	2.54	3.05			4	-	-	2.72	3.38
		5	-	-	-	2.68			5	-	-	-	2.98

Table 2: Undrained seismic bearing capacity N_{cs} for strip footing placed adjacent to two layered slope with $i=30^\circ$.

k_h	D_f/B_0	c_1/c_2	$c_1/\gamma B_0$				k_h	D_f/B_0	c_1/c_2	$c_1/\gamma B_0$			
			1	2	4	6				1	2	4	6
0	0.25	0.25	5.12	5.16	5.18	5.19	0.5	0.5	0.25	4.06	4.14	4.21	4.23
		0.5	5.12	5.16	5.18	5.19			0.5	4.05	4.13	4.19	4.22
		0.75	4.82	4.91	4.96	4.97			0.75	4.03	4.11	4.17	4.20
		1	3.8	3.96	4.03	4.05			1	3.8	3.96	4.06	4.08
		1.5	2.23	2.81	2.9	2.93			1.5	2.32	3.01	3.10	3.12
		2	—	2.18	2.3	2.33			2	—	2.45	2.61	2.65
		3	—	1.25	1.68	1.72			3	—	1.44	2	2.06
		4	—	—	1.33	1.38			4	—	—	1.65	1.72
		5	—	—	1	1.17			5	—	—	1.24	1.5
		0.25	3.82	3.97	4.06	4.09			0.25	3.80	3.92	4.04	4.06
		0.5	3.83	3.97	4.06	4.09			0.5	3.80	3.92	4.04	4.06
		0.75	3.82	3.97	4.07	4.09			0.75	3.80	3.91	4.04	4.06
		1	3.8	3.96	4.06	4.09			1	3.82	3.96	4.03	4.05
		1.5	2.51	3.25	3.37	3.41			1.5	2.64	3.44	3.59	3.64
		2	—	2.73	2.9	2.95			2	—	2.96	3.17	3.23
	0.75	3	—	1.67	2.31	2.38		1	3	—	1.86	2.56	2.67
		4	—	—	1.92	2.05			4	—	—	2.17	2.34
		5	—	—	1.5	1.8			5	—	—	1.74	2.07
		0.25	3.83	3.96	4.03	4.06			0.25	3.8	3.96	4.03	4.05
		0.5	3.83	3.96	4.03	4.05			0.5	3.79	3.96	4.02	4.04
		0.75	3.83	3.96	4.03	4.05			0.75	3.8	3.96	4.03	4.05
		1	3.83	3.96	4.03	4.05			1	3.8	3.96	4.03	4.05
		1.5	2.74	3.65	3.84	3.9		1.5	1.5	2.8	3.84	4.01	4.04
		2	—	3.2	3.45	3.53			2	—	3.41	3.71	3.8
		3	—	2.1	2.9	3.02			3	—	2.33	3.18	3.32
		4	—	—	2.43	2.64			4	—	—	2.69	2.94
		5	—	—	2.02	2.35			5	—	—	2.28	2.64
		0.25	4.25	4.31	4.34	4.34			0.25	3.57	3.68	3.74	3.76
		0.5	4.25	4.31	4.34	4.35			0.5	3.57	3.69	3.74	3.76
		0.75	4.25	4.31	4.33	4.33			0.75	3.57	3.69	3.74	3.76
		1	3.44	3.6	3.67	3.7			1	3.44	3.61	3.68	3.71
	1.25	1.5	—	2.6	2.7	2.73		0.5	1.5	—	2.82	2.94	2.97
		2	—	2.03	2.15	2.19			2	—	2.29	2.46	2.5
		3	—	—	1.57	1.61			3	—	—	1.89	1.95
		4	—	—	1.23	1.3			4	—	—	1.48	1.64
		5	—	—	—	1.1			5	—	—	—	1.4
		0.25	3.45	3.61	3.68	3.7			0.25	3.44	3.61	3.68	3.7
		0.5	3.45	3.61	3.68	3.7			0.5	3.44	3.61	3.68	3.7
		0.75	3.45	3.61	3.68	3.7			0.75	3.45	3.61	3.68	3.71
		1	3.45	3.61	3.68	3.7			1	3.45	3.61	3.68	3.7
	0.25	1.5	—	3.04	3.18	3.22		0.1	1.5	—	3.23	3.44	3.5
		2	—	2.55	2.75	2.81			2	—	2.78	3.04	3.12
		3	—	—	2.2	2.28			3	—	—	2.5	2.62
		4	—	—	1.74	1.96			4	—	—	2.03	2.27
		5	—	—	—	1.74			5	—	—	—	1.97
		0.25	3.45	3.61	3.68	3.67			0.25	3.44	3.61	3.68	3.7
		0.5	3.44	3.61	3.68	3.67			0.5	3.44	3.61	3.68	3.7
		0.75	3.44	3.61	3.68	3.67			0.75	3.44	3.61	3.68	3.7
		1	3.45	3.61	3.68	3.7			1	3.45	3.62	3.68	3.7
	0.75	1.5	—	3.47	3.68	3.71		1.5	1.5	—	3.61	3.68	3.7
		2	—	2.95	3.3	3.38			2	—	3.13	3.59	3.68
		3	—	—	2.76	2.91			3	—	—	3.06	3.24
		4	—	—	2.18	2.54			4	—	—	2.33	2.86
		5	—	—	—	2.22			5	—	—	—	2.53

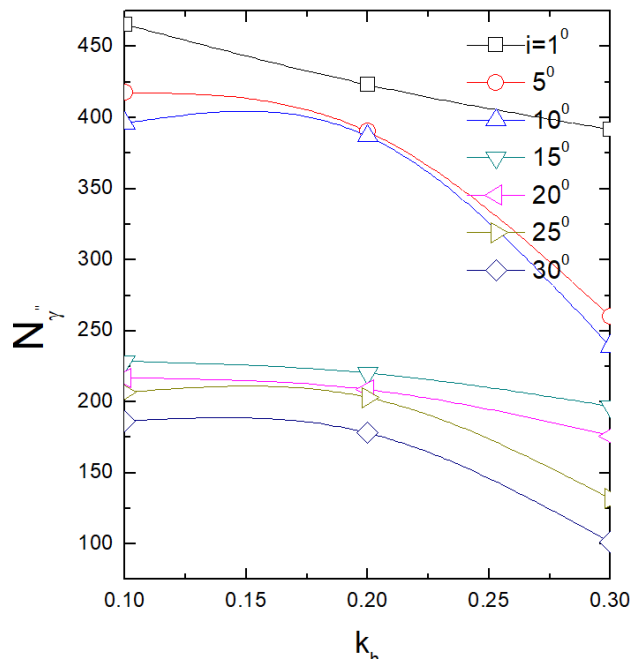


Figure 9: Design chart of Bearing capacity coefficient at $\phi_2 = 40^\circ$, $\delta_2 = \phi_2/2$, $i = 15^\circ$, $\delta_1/\delta_2 = 0.8$, $k_v = k_h/2$, $\gamma_1/\gamma_2 = 0.8$, $D_f/B_0 = 1$, $h_1/B_0 = 0.25$, $2c_2/B_0\gamma_2 = 0.2$, $c_1/c_2 = 0.8$

Ranges of various parameters are given as follows:

$$\frac{\phi_1}{\phi_2} = 0.6, 0.8, 1 \quad \frac{\gamma_1}{\gamma_2} = 0.6, 0.8, 1 \quad \frac{h_1}{B_0} = 0.1, 0.25, 0.5$$

$$i = 10^\circ, 15^\circ, 20^\circ, 25^\circ \quad k_v = 0, \quad k_h/2, \quad k_h$$

$\frac{\delta_1}{\delta_2}$ is the ratio of wall friction angles between the top and bottom layers.

Since the heuristic algorithms give us low ramification and high execution and these methods are relatively new, they can be applied in the geotechnical problem. Out of these methods, a brief discussion on PSO is given here as it is used in the analysis.

3.2 Particle swarm optimisation

Kennedy and Eberhart (1995) developed PSO as a simulation of birds swarm. A swarm is a group of individuals with defined rules for individual behaviours and communication. The ability of each individual to deal with the previous experiences of the swarm is called swarm intelligence. This capability guides the swarm towards its optimum goal. PSO is a population-based search technique where a population of particles start their journey in a space concerning the current best position (Hossain and El-Shafie 2014; Hajihassani et al. 2017). Reynolds (1987) described three simple rules for the behaviours of individuals inside a swarm, which were used as one of the ba-

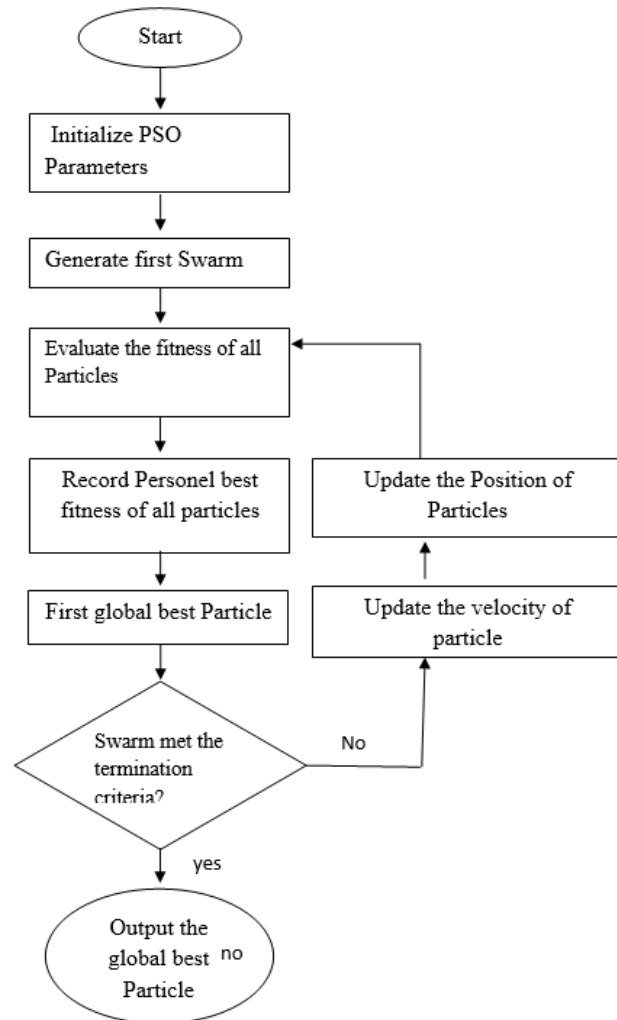


Figure 10: Flowchart of PSO algorithm.

sic concepts of PSO by Kennedy and Eberhart (1995). Although these simple rules model the behaviour of individuals, their combination produces a complicated behaviour for the swarm.

1. Individuals avoid collision with others
2. Individuals go towards the goal of the swarm
3. Individuals go to the centre of the swarm

The process of decision-making related to individuals is another basic concept of PSO. Each individual of the swarm makes decisionbased on the following two factors:

1. Own experiences of the individual that is its bestresults so far
2. The experiences of other individuals in the swarm that is the best results in the whole swarm

Figure 10 illustrates the standard flowchart of PSO. At the starting step of the original PSO, a certain num-

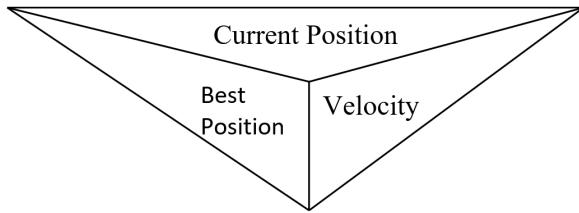


Figure 11: Schematic structure of a particle in PSO (Kalatehjari 2013).

ber of individuals, called particles, are distributed in the search space by using a random pattern (Kennedy and Eberhart 1995; Cheng et al. 2007; Aote et al. 2013). Each particle is representative of a feasible solution. Figure 11 shows the schematic structure of a particle in PSO involving three divided parts as its current position, best position and velocity. The current position, best position and velocity of particles record the current coordinates, best coordinates and velocity vectors of a particle in D-dimensional space, respectively, where D starts from 1 (Kalatehjari 2013). Consequently, for a particle in D-dimensional space, a 3D-dimensional particle is desirable. PSO aims to meet the termination criteria which are defined as the criteria for terminating the iterative search process. To select an appropriate termination criterion, it should be noted that the termination condition does not cause a premature converge and it should protect against oversampling of the fitness (Engelbrecht 2007). The following termination criteria are frequently used in PSO:

1. Termination when the maximum number of iterations is exceeded
2. Termination when a satisfactory solution is found based on the condition of each problem
3. Termination when no improvement is achieved over a certain number of iterations

These criteria are applied to ensure that PSO can converge on a feasible solution. Although PSO has some limitation which is explained by Gbenga et al. (2016) and Aote et al. (2013), PSO tries to make the objective function as a minimum or maximum depending on the problem to be solved. To lead the swarm towards this aim, the fitness value of each particle is determined by evaluating its current position by the objective function. After evaluation of fitness of all particles, Equation 39 (velocity equation) is used to calculate the velocity of particles based on their best position and the position of the best particle in the swarm. Using Equation 40, particle positions can be updated according to their current positions and velocities. This iterative process continues until reaching the

termination criteria. Equations 39 and 40 are as follows (Kennedy and Eberhart 1995):

$$v_{n(i)} = v_{n(i-1)} + u(0, \vartheta_1) (b_{p_{n(i)}} - X_{n(i)}) + u(0, \vartheta_2) (b_{g_{n(i)}} - X_{n(i)}) \quad (39)$$

$$X_{n(i+1)} = X_{n(i)} + v_{n(i)} \quad (40)$$

where v is the velocity of an n th particle in the past iteration and $v_{n(i-1)}$ is the velocity of the n th particle in the current iteration. The vectors of random numbers of an n th particle are presented by $u(0, \vartheta_1)$ and $u(0, \vartheta_2)$, $b_{p_{n(i)}}$ is the best position of the n th particle so far, $b_{g_{n(i)}}$ is the position of the best particle of the swarm so far, and $X_{n(i-1)}$ and $X_{n(i)}$ are the positions of the n th particle in the current and next iterations, respectively. Input parameters are taken for optimisation as follows:

$$\begin{aligned} &\text{Input } h_1/B_0, \phi_2, \phi_1/\phi_2, \delta_2, \delta_1/\delta_2, 2\bar{c}/\bar{\gamma}B_0, \\ &k_h, k_v, \xi, \alpha_{A1} = 20^\circ - 80^\circ, \alpha_{A2} = 30^\circ - 80^\circ \\ &\alpha_{p1} = 30^\circ - 80^\circ, \alpha_{p2} = 30^\circ - 60^\circ \end{aligned}$$

3.3 Parametric study

3.3.1 Weak soil layer over strong soil layer

Aparametric study was done for the variation of pseudo-static seismic bearing capacity coefficients with different soil parameters as shown in Figs 12–18.

3.3.2 Variations of seismic bearing capacity coefficient for different values of slope angle (i) using PSO algorithm

Figure 12 shows the variation of seismic bearing capacity coefficient ($N_{\gamma E}$) at $\phi_2 = 30^\circ$, $\delta_2 = \frac{\phi_2}{2}$, $\frac{\phi_1}{\phi_2} = 0.8$, $\frac{\delta_1}{\delta_2} = 0.8$, $k_v = \frac{k_h}{2}$, $\frac{\gamma_1}{\gamma_2} = 0.8$, $\frac{D_f}{B_0} = 0.5$, $\frac{h_1}{B_0} = 0.5$, $\frac{2c_2}{\gamma_2 B_0} = 0.2$ and $\frac{c_1}{c_2} = 0.8$ with k_h . From the plot, it is seen that $N_{\gamma E}$ decreases with the increase of angle of inclination (i). As the slope angle is increased, the area of the slope is decreased; therefore, the failure zone is decreased, resulting in much smaller bearing capacity.

3.3.3 Variations of seismic bearing capacity coefficient for different values of $\frac{\phi_1}{\phi_2}$ using PSO algorithm

Figure 13 depicts the variations of seismic bearing capacity coefficient ($N_{\gamma E}$) at $\phi_2 = 30^\circ$, $\delta_2 = \frac{\phi_2}{2}$, $i = 15^\circ$, $\frac{\delta_1}{\delta_2} = 0.8$,

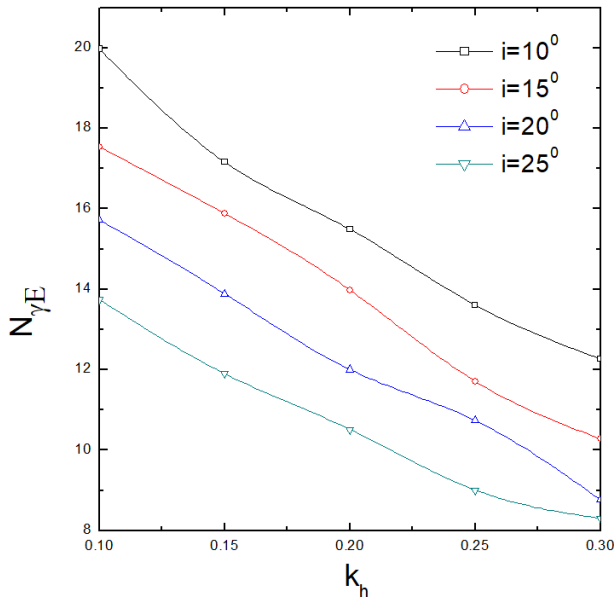


Figure 12: Variation of $N_{\gamma E}$ with k_h for $\phi_2 = 30^\circ$, $\delta_2 = \phi_2/2$, $\phi_1/\phi_2 = 0.8$, $\delta_1/\delta_2 = 0.8$, $k_v = k_h/2$, $\gamma_1/\gamma_2 = 0.8$, $D_f/B_0 = 0.5$, $h_1/B_0 = 0.25$, $2c_2/B_0\gamma_2 = 0.2$, $c_1/c_2 = 0.8$.

$k_v = \frac{k_h}{2}$, $\frac{\gamma_1}{\gamma_2} = 0.8$, $\frac{D_f}{B_0} = 0.5$, $\frac{D_f}{B_0} = 0.5$, $\frac{2c_2}{\gamma_2 B_0} = 0.2$ and $\frac{c_1}{c_2} = 0.80$ with k_h . From the plot, it is seen that $N_{\gamma E}$ increases with an increase in the value of $\frac{\phi_1}{\phi_2}$. An increase in $\frac{\phi_1}{\phi_2}$ ratio increases the strength of the soil (or internal resistance of the soil) against the shearing resistance, which results in increasing the bearing capacity. Here, ϕ_1 value is increased while keeping the ϕ_2 value constant.

3.3.4 Variations of seismic bearing capacity coefficient for different values of $\frac{\gamma_1}{\gamma_2}$ using PSO algorithm

Figure 14 shows the variations of seismic bearing capacity coefficient ($N_{\gamma E}$) at $\phi_2 = 30^\circ$, $\delta_2 = \frac{\phi_2}{2}$, $i = 15^\circ$, $\frac{\delta_1}{\delta_2} = 0.8$, $k_v = \frac{k_h}{2}$, $\frac{\phi_1}{\phi_2} = 0.8$, $\frac{h_1}{B_0} = 0.25$, $\frac{D_f}{B_0} = 0.5$, $\frac{2c_2}{\gamma_2 B_0} = 0$ and $\frac{c_1}{c_2} = 0$ with k_h . From the plot, it is seen that coefficient $N_{\gamma E}$ increases with an increase in the value of $\frac{\gamma_1}{\gamma_2}$. Here, the ratio $\frac{\gamma_1}{\gamma_2}$ is increased while keeping γ_2 as a constant.

3.3.5 Variations of seismic bearing capacity coefficient for different values of $\frac{h_1}{B_0}$ using PSO algorithm

Figure 15 shows the variations of seismic bearing capacity coefficient ($N_{\gamma E}$) at $\phi_2 = 30^\circ$, $\delta_2 = \frac{\phi_2}{2}$, $i = 15^\circ$, $\frac{\delta_1}{\delta_2} = 0.8$, $k_v = \frac{k_h}{2}$, $\frac{\phi_1}{\phi_2} = 0.8$, $\frac{\gamma_1}{\gamma_2} = 0.80$, $\frac{D_f}{B_0} = 0.5$, $\frac{2c_2}{\gamma_2 B_0} = 0.2$ and $\frac{c_1}{c_2} = 0.8$ with k_h . From the plot, it is seen that $N_{\gamma E}$ decreases with an increase in the value of $\frac{h_1}{B_0}$. h_1 is the depth

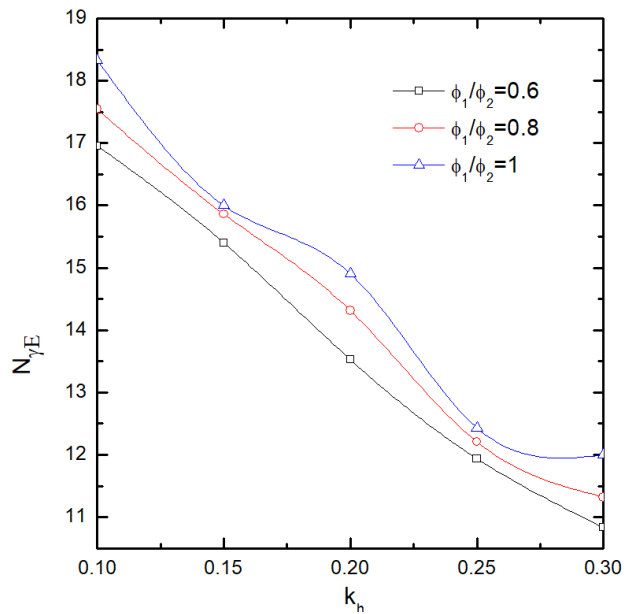


Figure 13: Variation of $N_{\gamma E}$ with k_h for $\phi_2 = 30^\circ$, $\delta_2 = \phi_2/2$, $i = 15^\circ$, $\delta_1/\delta_2 = 0.8$, $k_v = k_h/2$, $\gamma_1/\gamma_2 = 0.8$, $D_f/B_0 = 0.5$, $h_1/B_0 = 0.25$, $2c_2/B_0\gamma_2 = 0.2$, $c_1/c_2 = 0.8$.

of the top layer and it is considered in the analysis that it is weaker than the bottom layer. So, a weaker layer will provide less resistance, and hence increase in the thickness of this layer decreases the value of bearing capacity coefficients.

3.3.6 Variations of seismic bearing capacity coefficient for different values of $\frac{\delta_1}{\delta_2}$ using PSO algorithm

Figure 16 shows the variation of seismic bearing capacity coefficient ($N_{\gamma E}$) at $\phi_2 = 30^\circ$, $\delta_2 = \frac{\phi_2}{2}$, $i = 15^\circ$, $\frac{h_1}{B_0} = 0.25$, $k_v = \frac{k_h}{2}$, $\frac{\phi_1}{\phi_2} = 0.8$, $\frac{\gamma_1}{\gamma_2} = 0.80$, $\frac{D_f}{B_0} = 0.5$, $\frac{2c_2}{\gamma_2 B_0} = 0.2$ and $\frac{c_1}{c_2} = 0.8$ with k_h . From the plot, it is seen that coefficient $N_{\gamma E}$ increases with an increase in the value of $\frac{\delta_1}{\delta_2}$.

3.3.7 Variations of seismic bearing capacity coefficient for different values of k_v using PSO algorithm

Figure 17 shows the variation of $N_{\gamma E}$ at $\phi_2 = 30^\circ$, $\delta_2 = \frac{\phi_2}{2}$, $i = 15^\circ$, $\frac{h_1}{B_0} = 0.25$, $\frac{c_1}{c_2} = 0.80$, $\frac{\phi_1}{\phi_2} = 0.8$, $\frac{\gamma_1}{\gamma_2} = 0.80$, $\frac{D_f}{B_0} = 0.5$, $\frac{2c_2}{\gamma_2 B_0} = 0.2$ and $\frac{\delta_1}{\delta_2} = 0.8$ with k_h . From the plot, it is seen that $N_{\gamma E}$ decreases with an increase in k_v . It is obvious because increase in the value of k_v increases the disturbance of base soil and this decreases the value of $N_{\gamma E}$.

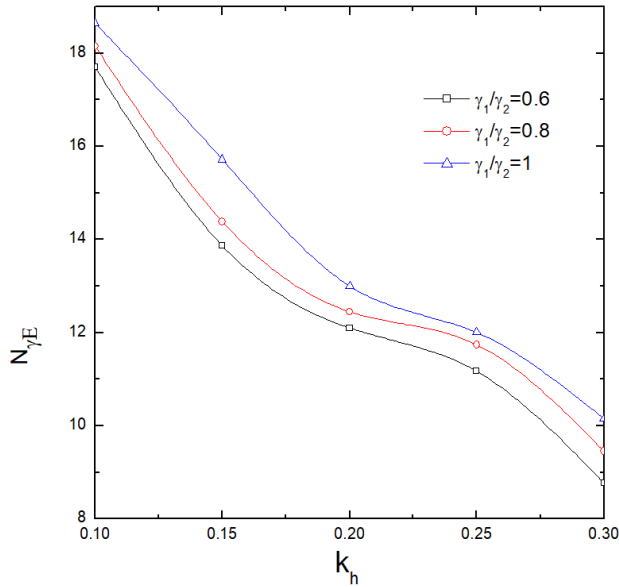


Figure 14: Variation of $N_{\gamma E}$ with k_h for $\phi_2 = 30^\circ$, $\delta_2 = \phi_2/2$, $i = 15^\circ$, $\delta_1/\delta_2 = 0.8$, $k_v = k_h/2$, $\phi_1/\phi_2 = 0.8$, $D_f/B_0 = 0.5$, $h_1/B_0 = 0.25$, $2c_2/B_0\gamma_2 = 0.2$, $c_1/c_2 = 0.8$.

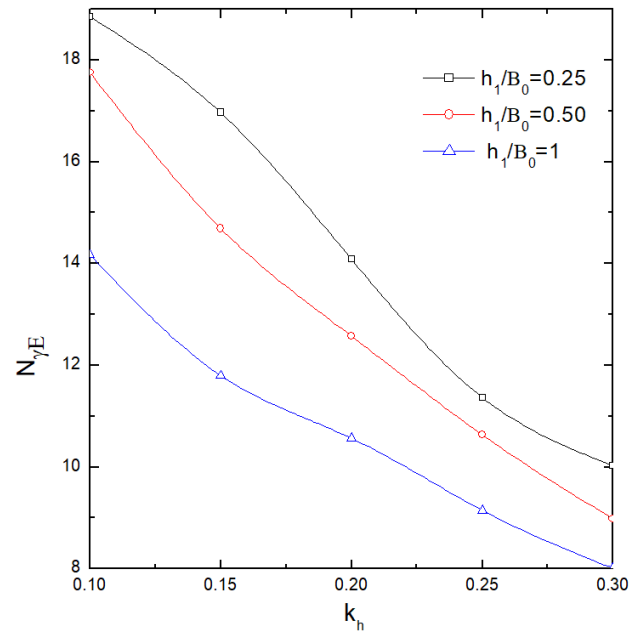


Figure 15: Variation of $N_{\gamma E}$ with k_h for $\phi_2 = 30^\circ$, $\delta_2 = \phi_2/2$, $i = 15^\circ$, $\delta_1/\delta_2 = 0.8$, $k_v = k_h/2$, $\phi_1/\phi_2 = 0.8$, $D_f/B_0 = 0.5$, $\gamma_1/\gamma_2 = 0.80$, $2c_2/B_0\gamma_2 = 0.2$, $c_1/c_2 = 0.8$.

3.3.8 Variations of seismic bearing capacity coefficient for different values of $\frac{c_1}{c_2}$ using PSO algorithm

Figure 18 shows the variation of $N_{\gamma E}$ at $\phi_2 = 30^\circ$, $\delta_2 = \frac{\phi_2}{2}$, $i = 15^\circ$, $\frac{h_1}{B_0} = 0.25$, $k_v = \frac{k_h}{2}$, $\frac{\phi_1}{\phi_2} = 0.8$, $\frac{\gamma_1}{\gamma_2} = 0.80$, $\frac{D_f}{B_0} = 0.5$, $\frac{2c_2}{\gamma_2 B_0} = 0.2$ and $\frac{\delta_1}{\delta_2} = 0.8$ with k_h . From the plot, it is seen that the coefficient $N_{\gamma E}$ increases with an increase in the value of $\frac{c_1}{c_2}$. By increasing the $\frac{c_1}{c_2}$ ratio, intermolecular attraction among the soil particles increases, which results in an increase in the bearing capacity.

3.3.9 Strong soil layer over weak soil layer

3.3.10 Variations of seismic bearing capacity coefficient for different values of using $\frac{\phi_1}{\phi_2}$ PSO algorithm

Figure 19 depicts the variations of seismic bearing capacity coefficient ($N_{\gamma E}$) at $\phi_2 = 20^\circ$, $\delta_2 = \frac{\phi_2}{2}$, $\frac{h_1}{B_0} = 0.25$, $\frac{\delta_1}{\delta_2} = 1.1$, $k_v = \frac{k_h}{2}$, $\frac{\gamma_1}{\gamma_2} = 1.1$, $\frac{D_f}{B_0} = 0.5$, $\frac{2c_2}{\gamma_2 B_0} = 0$ and $\frac{c_1}{c_2} = 0$ with k_h . From the figure, it is seen that the coefficient $N_{\gamma E}$ increases with an increase in the value of $\frac{\phi_1}{\phi_2}$. As the upper layer is considered a strong layer for this parameter, increasing the internal soil friction bearing capacity will be increased. Here, the ϕ_1 value is increased, keeping the ϕ_2 value constant.

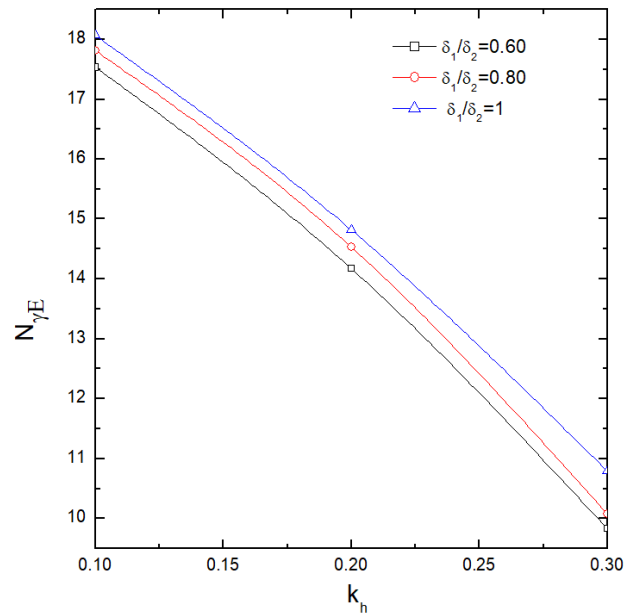


Figure 16: Variation of $N_{\gamma E}$ with k_h for $\phi_2 = 30^\circ$, $\delta_2 = \phi_2/2$, $i = 15^\circ$, $c_2/c_2 = 0.8$, $k_v = k_h/2$, $\phi_1/\phi_2 = 0.8$, $D_f/B_0 = 0.5$, $h_1/B_0 = 0.25$, $\gamma_1/\gamma_2 = 0.8$.

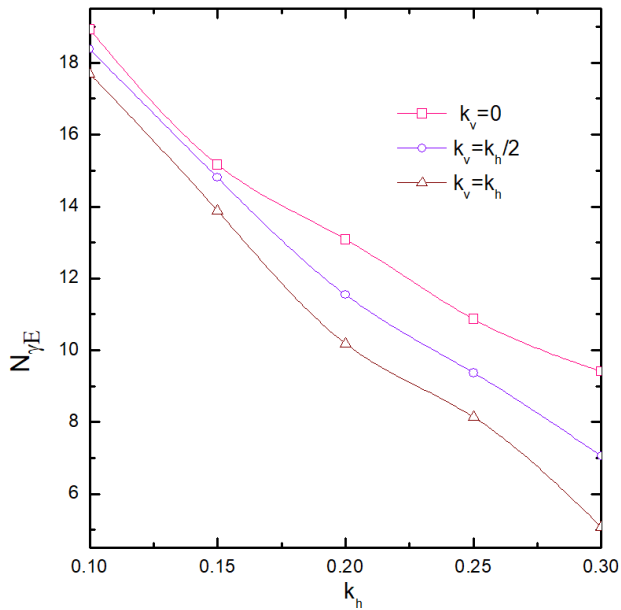


Figure 17: Variation of $N_{\gamma E}$ with k_h for $\phi_2 = 30^\circ$, $\delta_2 = \phi_2/2$, $D_f/B_0 = 0.5$, $i = 20^\circ$, $h_1/B_0 = 0.25$, $\gamma_1/\gamma_2 = 0.8$, $\delta_1/\delta_2 = 0.8$, $\phi_1/\phi_2 = 0.8$, $2c_2/B_0\gamma_2 = 0$, $c_1/c_2 = 0$.

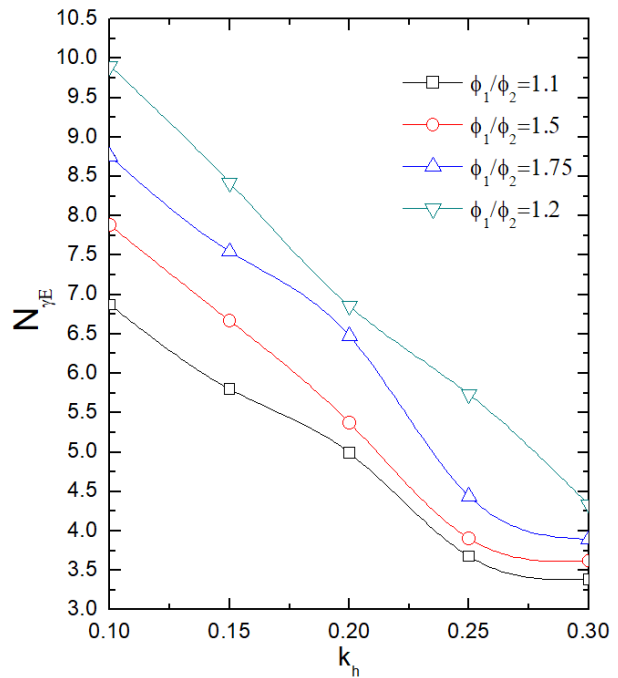


Figure 19: Variation of $N_{\gamma E}$ with k_h for $\phi_2 = 20^\circ$, $\delta_2 = \phi_2/2$, $i = 20^\circ$, $\delta_1/\delta_2 = 1.1$, $k_v = k_h/2$, $\gamma_1/\gamma_2 = 1.1$, $D_f/B_0 = 0.50$, $2c_2/B_0\gamma_2 = 0$, $c_1/c_2 = 0$.

3.3.11 Variations of seismic bearing capacity coefficient for different values of using $\frac{\gamma_1}{\gamma_2}$ PSO algorithm

Figure 20 shows the variations of seismic bearing capacity coefficient ($N_{\gamma E}$) at $\phi_2 = 30^\circ$, $\delta_2 = \frac{\phi_2}{2}$, $\frac{h_1}{B_0} = 0.25$, $\frac{\delta_1}{\delta_2} = 1.1$, $k_v = \frac{k_h}{2}$, $\frac{\phi_1}{\phi_2} = 1.1$, $\frac{D_f}{B_0} = 0.5$, $\frac{2c_2}{\gamma_2 B_0} = 0$ and $\frac{c_1}{c_2} = 0$ with k_h . From the figure, it is seen that the coefficient $N_{\gamma E}$ increases with an increase in the value of $\frac{\gamma_1}{\gamma_2}$. The ratio $\frac{\gamma_1}{\gamma_2}$ is increased keeping γ_2 as a constant. Here, γ_1 is the unit weight of the strong soil layer and γ_2 is the unit weight of the weak soil layer.

3.3.12 Variations of seismic bearing capacity coefficient for different values of $\frac{c_1}{c_2}$ using PSO algorithm

Figure 21 shows the variations of ($N_{\gamma E}$) at $\phi_2 = 30^\circ$, $\delta_2 = \frac{\phi_2}{2}$, $\frac{h_1}{B_0} = 0.25$, $\frac{\delta_1}{\delta_2} = 1.1$, $k_v = \frac{k_h}{2}$, $\frac{\gamma_1}{\gamma_2} = 1.1$, $\frac{D_f}{B_0} = 0.5$, $\frac{2c_2}{\gamma_2 B_0} = 0.2$, $\frac{\phi_1}{\phi_2} = 0$ with k_h . From the plot, it is seen that the coefficient $N_{\gamma E}$ increases with an increase in the values of $\frac{c_1}{c_2}$. Here, c_2 is the cohesive force on the weak soil layer and c_1 is the cohesive force on the strong soil layer. Hence, the ratio $\frac{c_1}{c_2}$ is increased while keeping c_2 constant. So, the value $N_{\gamma E}$ will increase.

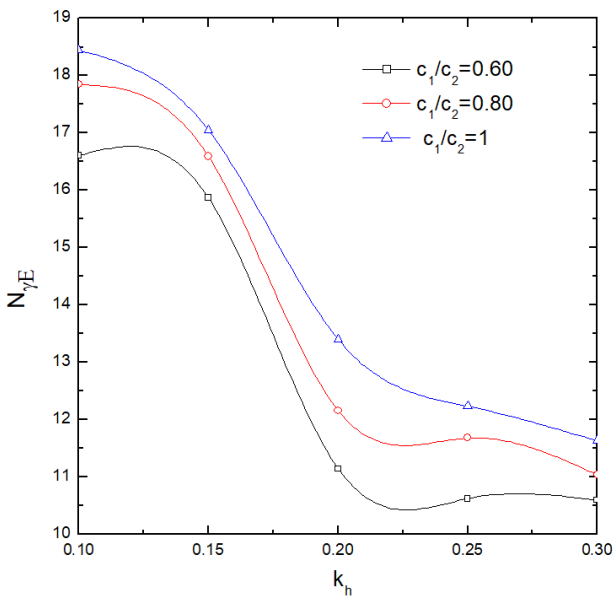


Figure 18: Variation of $N_{\gamma E}$ with k_h for $\phi_2 = 30^\circ$, $\delta_2 = \phi_2/2$, $i = 15^\circ$, $\delta_1/\delta_2 = 0.8$, $k_v = k_h/2$, $\phi_1/\phi_2 = 0.8$, $D_f/B_0 = 0.5$, $h_1/B_0 = 0.25$, $\gamma_1/\gamma_2 = 0.8$.

Table 3: Comparison of variation of seismic bearing capacity (q_{uE}) KN/m^2 with γ (KN/m^3) for the case of $\phi = 30^\circ$, $c = 9.8 \text{ KN/m}^2$, $i = 20^\circ$, $B_0 = 10 \text{ m}$.

k_h	Sawada et al. (1994)		Askari and Farzaneh (2003)		Yamamoto (2010)		Present Analysis	
	$\gamma=9.8$	$\gamma=19.8$	$\gamma=9.8$	$\gamma=19.8$	$\gamma=9.8$	$\gamma=19.8$	$\gamma=9.8$	$\gamma=19.8$
0.1	1798	3321	1066	1856	1013	1795	1614.14	3139.92
0.2	1770	3269	829	1307	755	1283	654	1167

Table 4: Comparison of the variation in N_{cs} with different $c_u/\gamma B_0$, D_f/B_0 , c_1/c_2 for $i=30^\circ$.

$c_u/\gamma B_0$	N_{cs} (Present study)				N_{cs} (Wu et al. 2020)			
	$D_f/B_0=0.5$	$D_f/B_0=1.5$	$D_f/B_0=1.5$	$D_f/B_0=0.5$	$D_f/B_0=0.5$	$D_f/B_0=1.5$	$D_f/B_0=1.5$	$D_f/B_0=0.5$
	$c_1/c_2=0.5$	$c_1/c_2=0.5$	$c_1/c_2=1.5$	$c_1/c_2=1.5$	$c_1/c_2=0.5$	$c_1/c_2=0.5$	$c_1/c_2=1.5$	$c_1/c_2=1.5$
1	4.06	3.78	2.78	2.34	4.05	3.79	2.8	2.34
2	4.15	3.92	3.05	2.95	4.16	3.96	3.01	3.01
4	4.20	4.02	3.11	3.04	4.19	4.02	3.12	3.12
6	4.22	4.04	3.22	3.10	4.2	4.04	3.15	3.15
8	4.24	4.06	3.23	3.14	4.21	4.05	3.17	3.17
10	4.25	4.08	3.24	3.16	4.22	4.6	3.17	3.17

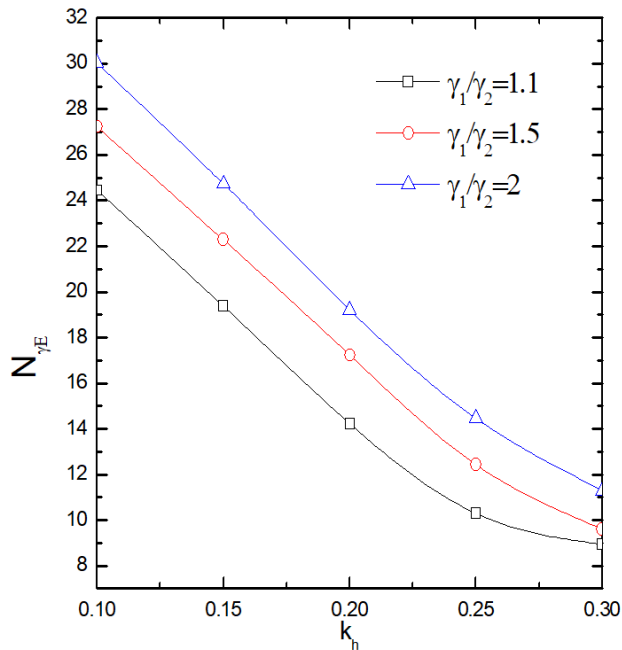


Figure 20: Variation of $N_{\gamma E}$ with k_h for $\phi_2 = 30^\circ$, $i = 20^\circ$, $\delta_2 = \phi_2/2$, $h_1/B_0 = 0.25$, $\delta_1/\delta_2 = 1.1$, $k_v = k_h/2$, $\phi_1/\phi_2 = 1.1$, $D/B_0 = 0.50$, $2c_2/B_0\gamma_2 = 0$, $c_1/c_2 = 0$.

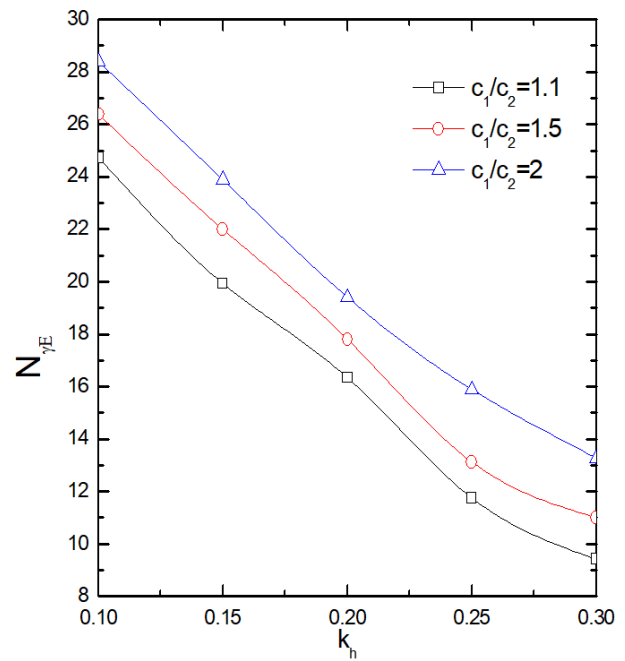


Figure 21: Variation of $N_{\gamma E}$ with k_h for $\phi_2 = 30^\circ$, $i = 20^\circ$, $\delta_2 = \phi_2/2$, $h_1/B_0 = 0.25$, $\delta_1/\delta_2 = 1.1$, $k_v = k_h/2$, $\phi_1/\phi_2 = 1.1$, $D/B_0 = 0.50$, $\gamma_1/\gamma_2 = 1.1$.

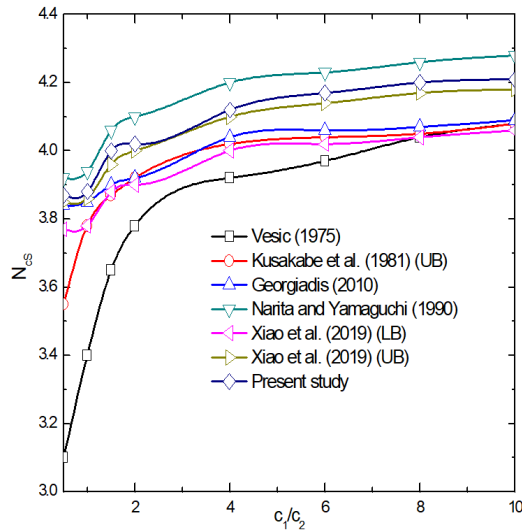


Figure 22: Comparison of variation in static bearing capacity coefficients (N_{cs}) with c_1/c_2 for $i = 30^\circ$.

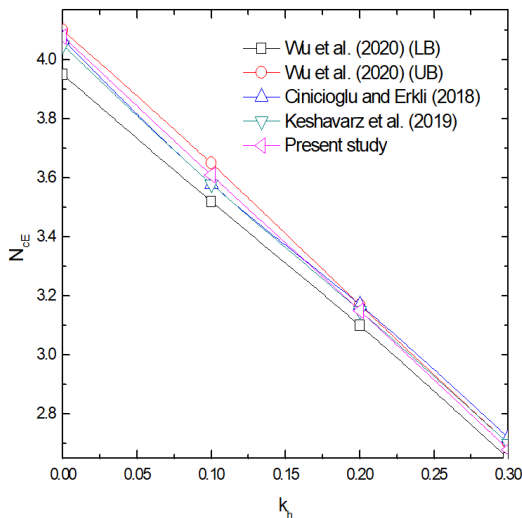


Figure 23: Comparison of seismic bearing capacity coefficients (N_{ce}) with k_h for $D_f/B_0 = 0$, $i = 30^\circ$, $c_1/c_2 = 1$.

4 Comparisons

The bearing capacity coefficient ($N_{\gamma''}$) has been computed by using a computer programming software 'MATLAB' code. PSO algorithm is applied, which can calculate the ultimate bearing capacity, q_{ult} , for various combinations of soil properties in each layer. Table 3 shows a comparison of the ultimate bearing capacity for $D_f/B_0 = 0.25$ between Askari and Farzaneh (2003), Sawada et al. (1994), Yamamoto (2010) and the present analysis, demonstrating the potentiality of the present analysis. It is found that the solutions from Sawada et al. (1994) have high values with the increase of γ , compared with those from Askari and Farzaneh (2003), Yamamoto (2010) and the present

analysis. This fact depends on the assumed failure mechanism. For the case of $\gamma = 9.8, 19.8$, the solutions from the present analysis tend to have lower values than those from Askari and Farzaneh (2003). The proposed solutions of the footing at two-layered slope with different $c_u/\gamma B_0$, D_f/B_0 and c_1/c_2 are compared with the results of Wu et al. (2020), as shown in Table 4. These results are compared for bearing capacity coefficient (N_{cs}). The undrained seismic bearing capacity coefficients can be written as: $N_{cs} = q_{ult}/c_1 = f(c_u/\gamma B_0, D_f/B_0, c_1/c_2, i, k_h)$. q_{ult} is the ultimate bearing capacity of the strip footing. The cases of footings lying on two-layered level ground with the seismic action are compared with the results of Jahani et al. (2019), and the comparison is presented in Table 5. It is observed from Table 5 that the results of the present study agree well with those of previous studies. Consequently, Tables 6–8 display comparisons of the variation in N_{cs} with Xiao et al. (2019) for different slope angles $i = 15^\circ, 30^\circ$ and 45° . The undrained static bearing capacity factors for cohesion (N_{cs}) obtained in the present study for a footing of uniform slope were compared with the UB and LB solutions of Xiao et al. (2019), semi-empirical results reported by Vesic (1975), the upper bound (UB) solutions obtained from Kusakabe et al. (1981), the limit-equilibrium methods of Narita and Yamaguchi (1990) and the finite element results provided by Georgiadis (2010). Comparison of these results is shown in Fig. 22 for the case of slope angle (i) = 30° . In Fig. 22, it is observed that the values of N_{cs} obtained from the UB solutions of Kusakabe et al. (1981) and Georgiadis (2010) lie between the UB and LB solutions of Xiao et al. (2019). The semi-empirical solutions of Vesic (1975) give the lowest values, whereas the values reported by Narita and Yamaguchi (1990) are found to be the highest dimensionless bearing capacity coefficients. From this figure, it is also observed that the values obtained in the present study are closer to the UB solutions obtained by Xiao et al. (2019) and Narita and Yamaguchi (1990). To verify the reliability of the present model, the LB and UB results were compared with the present limit equilibrium method. The present model of $D_f/B_0 = 0$, $i = 30^\circ$ and $c_1/c_2 = 1$ with different seismic coefficients k_h were compared with the previous LB and UB results (Wu et al. 2020), FEM results (Cinicioglu and Erkli 2018) and FELA results (Keshavarz et al. 2019). It can be seen in Fig. 23 that the present seismic bearing capacity coefficients (N_{ce}) closely match with the Upper bound (UB) limit analysis values. Based on the comparisons mentioned above, the proposed model can be proved with relatively minor error.

Table 5: Comparison of the variation in N_{cs} with different k_h and c_1/c_2 for $i=0^\circ$ and D_f/B_0 .

k_h	N_{cs} (Present study)			N_{cs} (Jahani et al. 2019)		
	$c_1/c_2=0.25$	$c_1/c_2=0.50$	$c_1/c_2=0.75$	$c_1/c_2=0.25$	$c_1/c_2=0.50$	$c_1/c_2=0.75$
0	5.62	3.94	3.06	5.67	4	3.1
0.1	5.20	3.48	2.75	5.25	3.58	2.73
0.2	4.04	3.04	2.37	4.44	3.12	2.4
0.3	3.38	2.65	2.12	3.47	2.7	2.15

Table 6: Comparison of the variation in N_{cs} for $i=15^\circ$.

h_1/B_0	D_f/B_0	c_1/c_2	Xiao et al. (2019)				Present study			
			$c_1/\gamma B_0$				$c_1/\gamma B_0$			
			1	2	4	6	1	2	4	6
0.5	0.25	0.25	5.71	5.73	5.74	5.74	5.68	5.70	5.71	5.72
		0.5	5.71	5.73	5.74	5.74	5.68	5.70	5.71	5.72
		0.75	5.53	5.57	5.58	5.59	5.49	5.55	5.56	5.57
		1	4.51	4.58	4.61	4.63	4.49	4.53	4.58	4.60
	0.50	0.25	4.70	4.75	4.77	4.77	4.68	4.71	4.74	4.74
		0.5	4.70	4.75	4.77	4.77	4.68	4.72	4.75	4.75
		0.75	4.70	4.74	4.77	4.77	4.68	4.71	4.75	4.75
		1	4.51	4.57	4.61	4.62	4.45	4.51	4.59	4.59
1	0.25	0.25	5.71	5.73	5.73	5.75	5.68	5.70	5.71	5.72
		0.5	5.71	5.73	5.73	5.74	5.68	5.70	5.70	5.71
		0.75	5.52	5.57	5.58	5.59	5.50	5.55	5.56	5.57
		1	4.51	4.58	4.61	4.63	4.48	4.55	4.59	4.61
	0.50	0.25	4.70	4.75	4.77	4.78	4.68	4.72	4.74	4.75
		0.5	4.71	4.75	4.77	4.78	4.67	4.74	4.75	4.76
		0.75	4.70	4.75	4.77	4.78	4.66	4.74	4.74	4.75
		1	4.51	4.58	4.62	4.62	4.49	4.55	4.60	4.61
2	0.25	0.25	5.71	5.73	5.74	5.73	5.69	5.72	5.72	5.73
		0.5	5.71	5.73	5.74	5.74	5.69	5.71	5.72	5.73
		0.75	5.52	5.57	5.58	5.59	5.50	5.55	5.56	5.57
		1	4.51	4.58	4.62	4.63	5.49	5.53	5.54	5.55
	0.50	0.25	4.70	4.74	4.77	4.79	4.69	4.72	4.75	4.77
		0.5	4.70	4.76	4.77	4.77	4.68	4.70	4.73	4.75
		0.75	4.70	4.74	4.76	4.78	4.67	4.68	4.70	4.72
		1	4.51	4.58	4.61	4.62	4.47	4.54	4.59	4.60
3	0.25	0.25	5.70	5.73	5.74	5.74	5.69	5.71	5.72	5.72
		0.5	5.70	5.73	5.74	5.74	5.69	5.71	5.72	5.72
		0.75	5.52	5.56	5.58	5.58	5.50	5.55	5.56	5.57
		1	4.51	4.58	4.61	4.62	4.48	4.55	4.58	4.60
	0.50	0.25	4.71	4.75	4.77	4.78	4.69	4.72	4.74	4.77
		0.5	4.70	4.75	4.77	4.78	4.68	4.72	4.74	4.77
		0.75	4.70	4.75	4.77	4.77	4.67	4.71	4.73	4.73
		1	4.51	4.58	4.61	4.63	4.49	4.55	4.59	4.60

Table 7: Comparison of the variation in N_{cs} for $i=30^\circ$.

h_1/B_0	D_f/B_0	c_1/c_2	Xiao et al. (2019)				Present study			
			$c_1/\gamma B_0$				$c_1/\gamma B_0$			
			1	2	4	6	1	2	4	6
0.5	0.25	0.25	5.15	5.20	5.23	5.23	5.11	5.16	5.21	5.21
		0.5	5.15	5.20	5.22	5.23	5.11	5.16	5.21	5.20
		0.75	4.86	4.95	5.00	5.01	4.81	4.88	4.91	4.96
		1	3.88	4.01	4.07	4.09	3.82	4.01	4.03	4.05
	0.50	0.25	4.09	4.18	4.23	4.24	4.05	4.16	4.20	4.21
		0.5	4.09	4.18	4.23	4.24	4.02	4.14	4.18	4.20
		0.75	4.09	4.18	4.23	4.24	4.01	4.12	4.16	4.18
		1	3.88	4.00	4.07	4.09	3.84	3.95	4.02	4.05
1	0.25	0.25	5.15	5.20	5.22	5.23	5.10	5.15	5.20	5.21
		0.5	5.15	5.20	5.22	5.23	5.09	5.13	5.19	5.20
		0.75	4.85	4.95	5.00	5.01	4.80	4.85	4.90	4.93
		1	3.83	3.99	4.06	4.08	3.80	3.92	3.99	4.01
	0.50	0.25	4.10	4.19	4.24	4.25	4.07	4.11	4.20	4.22
		0.5	4.10	4.19	4.23	4.25	4.05	4.10	4.19	4.20
		0.75	4.10	4.19	4.23	4.25	4.04	4.10	4.18	4.19
		1	3.83	3.99	4.06	4.08	3.80	3.95	4.04	4.06
2	0.25	0.25	5.15	5.20	5.22	5.23	5.11	5.17	5.20	5.21
		0.5	5.15	5.20	5.22	5.23	5.10	5.17	5.20	5.21
		0.75	4.85	4.95	5.00	5.01	4.82	4.90	4.96	4.99
		1	3.83	3.99	4.06	4.08	3.81	3.95	4.01	4.04
	0.50	0.25	4.09	4.19	4.23	4.25	4.07	4.15	4.20	4.22
		0.5	4.09	4.19	4.23	4.25	4.05	4.14	4.20	4.21
		0.75	4.09	4.19	4.23	4.25	4.05	4.14	4.20	4.21
		1	3.82	3.98	4.05	4.07	3.80	3.94	4.01	4.04
3	0.25	0.25	5.16	5.20	5.22	5.23	5.11	5.18	5.20	5.21
		0.5	5.16	5.20	5.22	5.23	5.11	5.17	5.20	5.21
		0.75	4.85	4.95	5.00	5.01	4.81	4.92	4.99	5.00
		1	3.83	3.99	4.06	4.08	3.80	3.96	4.03	4.06
	0.50	0.25	4.10	4.19	4.23	4.25	4.07	4.15	4.20	4.23
		0.5	4.09	4.19	4.23	4.24	4.06	4.14	4.20	4.22
		0.75	4.10	4.19	4.23	4.24	4.05	4.12	4.18	4.20
		1	3.83	3.99	4.06	4.08	3.80	3.93	4.03	4.06

Table 8: Comparison of the variation in N_{cs} for $i=45^\circ$.

h_1/B_0	D_f/B_0	c_1/c_2	Xiao et al. (2019)				Present study			
			$c_1/\gamma B_0$				$c_1/\gamma B_0$			
			1	2	4	6	1	2	4	6
0.5	0.25	0.25	4.57	4.63	4.66	4.67	4.54	4.60	4.62	4.64
		0.5	4.57	4.64	4.66	4.68	4.53	4.60	4.61	4.63
		0.75	4.23	4.34	4.40	4.41	4.21	4.31	4.37	4.40
		1	3.44	3.57	3.63	3.65	3.41	3.53	3.55	3.59
	0.50	0.25	3.49	3.62	3.67	3.70	3.45	3.58	3.62	3.67
		0.5	3.49	3.61	3.67	3.70	3.44	3.59	3.60	3.67
		0.75	3.49	3.61	3.68	3.70	3.43	3.56	3.59	3.66
		1	3.44	3.57	3.63	3.65	3.42	3.53	3.56	3.64
1	0.25	0.25	4.57	4.63	4.66	4.67	4.55	4.60	4.63	4.65
		0.5	4.57	4.63	4.66	4.68	4.54	4.58	4.62	4.64
		0.75	4.13	4.28	4.35	4.37	4.11	4.25	4.33	4.35
		1	3.16	3.41	3.51	3.54	3.11	3.38	3.50	3.52
	0.50	0.25	3.49	3.61	3.68	3.69	3.45	3.59	3.64	3.67
		0.5	3.49	3.61	3.68	3.69	3.44	3.58	3.63	3.67
		0.75	3.49	3.61	3.68	3.69	3.43	3.57	3.63	3.67
		1	3.16	3.41	3.51	3.54	3.12	3.38	3.50	3.52
2	0.25	0.25	4.58	4.64	4.66	4.68	4.54	4.60	4.62	4.66
		0.5	4.58	4.63	4.67	4.68	4.53	4.60	4.61	4.64
		0.75	4.11	4.28	4.35	4.38	4.10	4.25	4.32	4.32
		1	3.15	3.41	3.50	3.54	3.11	3.40	3.48	3.50
	0.50	0.25	3.49	3.61	3.67	3.69	3.44	3.58	3.62	3.64
		0.5	3.49	3.61	3.68	3.69	3.42	3.56	3.61	3.63
		0.75	3.49	3.61	3.67	3.69	3.41	3.54	3.58	3.61
		1	3.15	3.41	3.51	3.54	3.10	3.36	3.52	3.52
3	0.25	0.25	4.58	4.64	4.67	4.67	4.55	4.62	4.64	4.65
		0.5	4.58	4.63	4.67	4.68	4.54	4.60	4.62	4.64
		0.75	4.11	4.28	4.35	4.37	4.08	4.24	4.31	4.35
		1	3.15	3.40	3.51	3.54	3.12	3.36	3.41	3.48
	0.50	0.25	3.49	3.61	3.67	3.69	3.44	3.60	3.62	3.65
		0.5	3.48	3.61	3.67	3.69	3.43	3.60	3.61	3.63
		0.75	3.49	3.61	3.67	3.67	3.42	3.58	3.60	3.63
		1	3.15	3.40	3.51	3.54	3.13	3.38	3.49	3.51

5 Conclusions

This paper has investigated the seismic bearing capacity of shallow foundations near slope using pseudo-static limit equilibrium analysis. Linear failure mechanism has been proposed to obtain pseudo-static bearing capacity coefficients of embedded strip footing near slope using limit equilibrium analysis. The PSO technique has been applied to obtain minimum bearing capacity coefficients.

Based on the present investigation, the following conclusions can be drawn:

1. $N_{\gamma E}$ values from the present analysis agreed well with other analyses reported by Askari and Farzaneh (2003), Sawada et al. (1994) and Yamamoto (2010). The present analysis shows a tendency to decrease $N_{\gamma E}$ values by increasing the horizontal seismic coefficient (k_h) and vertical seismic coefficient (k_v).
2. The seismic bearing capacity coefficient N_{cs} was found relatively minor error value with previous researchers.
3. The minimum pseudo-static bearing capacity coefficients are presented in the form of design table for practical use in geotechnical engineering. It has been observed that the magnitude of bearing capacity coefficients decreases with an increase in slope inclination.
4. The seismic bearing capacity decreases with greater undrained shear strength ratio (c_1/c_2) and the effect becomes more significant when the thickness of the top layer decreases.
5. The value of N_{cs} increases with increasing $c_1/\gamma B_0$, whereas this effect recedes with an increase of $c_1/\gamma B_0$. Also, greater value of $c_1/\gamma B$ would decrease the effect of bottom layer.

Data availability statement: The authors confirm that some data and code generated during this study are proprietary or confidential and may only be provided with restrictions (e. g., PSO code).

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A Appendix I

$$\begin{aligned}
 P_{A1} = & \frac{p_L B_0}{B_0 + h_1} (B_0 - h_1 \cot \alpha_{A1}) \\
 & \left\{ \frac{(1 - k_v) \sin(\alpha_{A2} - \phi_2) + k_h \tan \phi_1 \cos(\alpha_{A2} - \phi_2)}{\cos(\alpha_{A2} - \phi_2 - \delta_2)} \right\} \\
 & + \frac{1}{2} (B_0 - h_1 \cot \alpha_{A1}) h_2 \gamma_2 \\
 & \left\{ \frac{(1 - k_v) \sin(\alpha_{A2} - \phi_2) + k_h \cos(\alpha_{A2} - \phi_2)}{\cos(\alpha_{A2} - \phi_2 - \delta_2)} \right\} \\
 & + \gamma_1 h_1 (B_0 - h_1 \cot \alpha_{A1})
 \end{aligned}$$

$$\left\{ \frac{(1 - k_v) \sin(\alpha_{A2} - \phi_2) + k_h \tan \phi_1 \cos(\alpha_{A2} - \phi_2)}{\cos(\alpha_{A2} - \phi_2 - \delta_2)} \right\} - 2c_2 h_2 \frac{\sin(\alpha_{A2} - \phi_2)}{\cos(\alpha_{A2} - \phi_2 - \delta_2)} - c_2 h_2 \cot \alpha_{A2} \frac{\cos(\alpha_{A2} - \phi_2)}{\cos(\alpha_{A2} - \phi_2 - \delta_2)} - c_1 \left[h_1 \cot \alpha_{A1} \frac{\cos(\alpha_{A2} - \phi_2)}{\cos(\alpha_{A2} - \phi_2 - \delta_2)} - B_0 \frac{\cos(\alpha_{A2} - \phi_2)}{\cos(\alpha_{A2} - \phi_2 - \delta_2)} \right]$$

$$P_{p2} = \frac{1}{2} h_2^2 \cot \alpha_{p2} \gamma_2$$

$$\left\{ \frac{(1 - k_v) \sin(\phi_2 + \alpha_{p2}) - k_h \cos(\phi_2 + \alpha_{p2})}{\cos(\phi_2 + \alpha_{p2} + \delta_2)} \right\} + \gamma_1 h_1 h_2 \cot \alpha_{p2} \left\{ \frac{(1 - k_v) \sin(\phi_2 + \alpha_{p2}) + k_h \tan \phi_1 \cos(\phi_2 + \alpha_{p2})}{\cos(\phi_2 + \alpha_{p2} + \delta_2)} \right\} + \gamma_1 D_f \frac{(h_1 \cot \alpha_{p1} + h_2 \cot \alpha_{p2})}{(h_1 + h_1 \cot \alpha_{p1} + h_2 \cot \alpha_{p2})} h_2 \cot \alpha_{p2} \left\{ \frac{(1 - k_v) \sin(\phi_2 + \alpha_{p2}) + k_h \tan \phi_1 \cos(\phi_2 + \alpha_{p2})}{\cos(\phi_2 + \alpha_{p2} + \delta_2)} \right\} + 2c_2 h_2 \left\{ \frac{\sin(\phi_2 + \alpha_{p2})}{\cos(\phi_2 + \alpha_{p2} + \delta_2)} \right\} +$$

$$c_1 h_2 \cot \alpha_{p2} \left\{ \frac{\cos(\phi_2 + \alpha_{p2})}{\cos(\phi_2 + \alpha_{p2} + \delta_2)} \right\} + c_2 h_2 \cot \alpha_{p2} \left\{ \frac{\cos(\phi_2 + \alpha_{p2})}{\cos(\phi_2 + \alpha_{p2} + \delta_2)} \right\}$$

B Appendix II

$$\alpha_1 = \frac{\gamma_1}{\gamma} \left\{ \frac{\left(\frac{h_1}{B_0} \cot \alpha_{p1} + 2 \frac{h_2}{B_0} \cot \alpha_{p2} \right) \frac{h_1}{B_0} - \frac{1}{4} \left(h_1 \cot \alpha_{p1} + h_2 \cot \alpha_{p2} - \frac{D_f}{\tan i} + \frac{B_0}{2} \right)^2 \tan \alpha_{p1}}{\left\{ \frac{(1 - k_v) \sin(\phi_1 + \alpha_{p1}) - k_h \cos(\phi_1 + \alpha_{p1})}{\cos(\phi_1 + \alpha_{p1} + \delta_1)} \right\} \left\{ \frac{(1 - k_v) \sin(\phi_1 + \alpha_{p1}) + k_h \tan \phi_2 \cos(\phi_1 + \alpha_{p1})}{\cos(\phi_1 + \alpha_{p1} + \delta_1)} \right\}} + \left(\frac{h_2}{B_0} \right)^2 \cot \alpha_{p2} \frac{\gamma_2}{\gamma} \right\}$$

$$\left\{ \frac{(1 - k_v) \sin(\phi_2 + \alpha_{p2}) - k_h \cos(\phi_2 + \alpha_{p2})}{\cos(\phi_2 + \alpha_{p2} + \delta_2)} \right\} + 2 \frac{\gamma_1}{\gamma} \frac{h_1}{B_0} \frac{h_2}{B_0} \cot \alpha_{p2} \left\{ \frac{(1 - k_v) \sin(\phi_2 + \alpha_{p2}) + k_h \tan \phi_2 \cos(\phi_2 + \alpha_{p2})}{\cos(\phi_2 + \alpha_{p2} + \delta_2)} \right\} - \left(2 - \frac{h_1}{B_0} \cot \alpha_{A1} \right) \frac{h_1}{B_0} \frac{\gamma_1}{\gamma} \left\{ \frac{(1 - k_v) \sin(\alpha_{A1} - \phi_1) + k_h \cos(\alpha_{A1} - \phi_1)}{\cos(\alpha_{A1} - \phi_1 - \delta_1)} \right\} + 2 \frac{\gamma_1}{\gamma} \frac{h_1}{B_0} \left(1 - \frac{h_1}{B_0} \cot \alpha_{A1} \right) \left\{ \frac{(1 - k_v) \sin(\alpha_{A1} - \phi_1) + k_h \tan \phi_2 \cos(\alpha_{A1} - \phi_1)}{\cos(\alpha_{A1} - \phi_1 - \delta_1)} \right\} - \left(1 - \frac{h_1}{B_0} \cot \alpha_{A1} \right) \frac{h_2}{B_0} \frac{\gamma_2}{\gamma} \left\{ \frac{(1 - k_v) \sin(\alpha_{A2} - \phi_2) + k_h \cos(\alpha_{A2} - \phi_2)}{\cos(\alpha_{A2} - \phi_2 - \delta_2)} \right\} - 2 \frac{\gamma_1}{\gamma} \frac{h_1}{B_0} \left(1 - \frac{h_1}{B_0} \cot \alpha_{A1} \right) \left\{ \frac{(1 - k_v) \sin(\alpha_{A2} - \phi_2) + k_h \tan \phi_1 \cos(\alpha_{A2} - \phi_2)}{\cos(\alpha_{A2} - \phi_2 - \delta_2)} \right\}$$

$$b_1 = 2 \frac{\gamma_1}{\gamma} \tan i \left(\frac{D_f}{B_0 \tan i} - \frac{1}{2} \right)^2$$

$$\left\{ \frac{(1 - k_v) \sin(\phi_1 + \alpha_{p1}) - k_h \cos(\phi_1 + \alpha_{p1})}{\cos(\phi_1 + \alpha_{p1} + \delta_1)} \right\} - 2 \frac{\gamma_1}{\gamma} \frac{h_2}{B_0} \cot \alpha_{p2} \frac{\tan \left(\frac{D_f}{B_0} - \frac{1}{2} \right) 2 \left(\frac{D_f}{B_0} - \tan i \right)}{\left\{ 2 \frac{D_f}{B_0} + \tan \left(2 \frac{h_1}{B_0} - 1 \right) \right\}} \left\{ \frac{(1 - k_v) \sin(\phi_1 + \alpha_{p1}) + k_h \tan \phi_2 \cos(\phi_1 + \alpha_{p1})}{\cos(\phi_1 + \alpha_{p1} + \delta_1)} \right\} + 2 \frac{\gamma_1}{\gamma} \frac{h_2}{B_0} \cot \alpha_{p2} \frac{\tan \left(\frac{D_f}{B_0} - \frac{1}{2} \right) 2 \left(\frac{D_f}{B_0} - \tan i \right)}{\left\{ 2 \frac{D_f}{B_0} + \tan \left(2 \frac{h_1}{B_0} - 1 \right) \right\}} \left\{ \frac{(1 - k_v) \sin(\phi_2 + \alpha_{p2}) + k_h \tan \phi_1 \cos(\phi_2 + \alpha_{p2})}{\cos(\phi_2 + \alpha_{p2} + \delta_2)} \right\}$$

$$d_1 = 2 \frac{c_2}{\bar{c}} \frac{h_2}{B_0} \frac{\sin(\phi_2 + \alpha_{p2})}{\cos(\phi_2 + \alpha_{p2} + \delta_2)} + \frac{c_1}{\bar{c}} \frac{h_2}{B_0} \cot \alpha_{p2} \frac{\cos(\phi_2 + \alpha_{p2})}{\cos(\phi_2 + \alpha_{p2} + \delta_2)} + \frac{c_2}{\bar{c}} \frac{h_2}{B_0} \cot \alpha_{p2} \frac{\cos(\phi_2 + \alpha_{p2})}{\cos(\phi_2 + \alpha_{p2} + \delta_2)} + \frac{c_1}{\bar{c}} \frac{h_1}{B_0} \frac{\sin(\phi_1 + \alpha_{p1})}{\cos(\phi_1 + \alpha_{p1} + \delta_1)} + \frac{c_1}{\bar{c}} \sin \alpha_{p1} \left\{ \frac{h_1}{2B_0} \cos \alpha_{p1} - \frac{h_2}{2B_0} \cot \alpha_{p2} \sec \alpha_{p1} + \frac{1}{2} \frac{D_f}{B_0 \tan i} \sec \alpha_{p1} - \frac{1}{4} \sec \alpha_{p1} \right\}$$

$$\begin{aligned}
& \frac{\sin(\phi_1 + \alpha_{p1})}{\cos(\phi_1 + \alpha_{p1} + \delta_1)} + \frac{c_1}{\bar{c}} \cot \alpha_{p1} \\
& \left\{ \frac{h_1}{2B_0} \cos e c \alpha_{p1} - \frac{h_2}{2B_0} \cot \alpha_{p2} \sec \alpha_{p1} + \frac{1}{2} \frac{D_f}{B_0 \tan i} \sec \alpha_{p1} - \frac{1}{4} \sec \alpha_{p1} \right\} \\
& \frac{\cos(\phi_1 + \alpha_{p1})}{\cos(\phi_1 + \alpha_{p1} + \delta_1)} - \frac{c_1}{\bar{c}} \frac{h_2}{B_0} \cot \alpha_{p2} \frac{\cos(\phi_1 + \alpha_{p1})}{\cos(\phi_1 + \alpha_{p1} + \delta_1)} \\
& + 2 \frac{c_1}{\bar{c}} \frac{h_1}{B_0} \frac{\sin(\alpha_{A1} - \phi_1)}{\cos(\alpha_{A1} - \phi_1 - \delta_1)} + \\
& \frac{c_1}{\bar{c}} \frac{h_1}{B_0} \cot \alpha_{A1} \frac{\cos(\alpha_{A1} - \phi_1)}{\cos(\alpha_{A1} - \phi_1 - \delta_1)} \\
& + \frac{c_2}{\bar{c}} \frac{\cos(\alpha_{A1} - \phi_1)}{\cos(\alpha_{A1} - \phi_1 - \delta_1)} + \\
& \frac{c_2}{\bar{c}} \frac{h_1}{B_0} \cot \alpha_{A1} \frac{\cos(\alpha_{A1} - \phi_1)}{\cos(\alpha_{A1} - \phi_1 - \delta_1)} \\
& + 2 \frac{c_2}{\bar{c}} \frac{h_2}{B_0} \frac{\cos(\alpha_{A2} - \phi_2)}{\cos(\alpha_{A2} - \phi_2 - \delta_2)} \\
& + \frac{c_1}{\bar{c}} \frac{\cos(\alpha_{A2} - \phi_2)}{\cos(\alpha_{A2} - \phi_2 - \delta_2)} - \\
& \frac{c_1}{\bar{c}} \frac{h_1}{B_0} \cot \alpha_{A1} \frac{\cos(\alpha_{A2} - \phi_2)}{\cos(\alpha_{A2} - \phi_2 - \delta_2)}
\end{aligned}$$

$$\begin{aligned}
e_1 = & \left\{ \frac{(1 - k_v) \sin(\alpha_{A1} - \phi_1) + k_h \cos(\alpha_{A1} - \phi_1)}{\cos(\alpha_{A1} - \phi_1 - \delta_1)} \right\} \\
& - \left\{ \frac{\left(1 - \frac{h_1}{B_0} \cot \alpha_{A1}\right)}{1 + \frac{h_1}{B_0}} \right\} \\
& \left\{ \frac{(1 - k_v) \sin(\alpha_{A1} - \phi_1) + k_h \tan \phi_2 \cos(\alpha_{A1} - \phi_1)}{\cos(\alpha_{A1} - \phi_1 - \delta_1)} \right\} \\
& + \left\{ \frac{\left(1 - \frac{h_1}{B_0} \cot \alpha_{A1}\right)}{1 + \frac{h_1}{B_0}} \right\} \\
& \left\{ \frac{(1 - k_v) \sin(\alpha_{A2} - \phi_2) + k_h \tan \phi_1 \cos(\alpha_{A2} - \phi_2)}{\cos(\alpha_{A2} - \phi_2 - \delta_2)} \right\}
\end{aligned}$$