PRACE NAUKOWE AKADEMII EKONOMICZNEJ WE WROCŁAWIU
Nr 1133 — 2006
Inwestycje finansowe i ubezpieczenia
– tendencje światowe a polski rynek

Eva Rublikova, Magdalena Rutkowska
Economic University Bratislava, University of Łódź

ESTIMATING CYCLICAL COMPONENT IN CPI OF SLOVAKIA AND POLAND

1. Autoregressive Process for Modelling a Cycle

Time series Y_t observed over the period t = 1, 2, ..., T can be usually decomposed additively into two statistically independent components:

$$y_t = \mu_t + \varepsilon_t,$$

$E(\mu_t, \varepsilon_s) = 0$ for all t,s, where (1)

y_t often the logarithm of the observed series Y_t,

μ_t trend ,

ε_t cyclical component.

In the classical approach to trend modelling, μ_t is regarded as being a smooth function designed to capture the long-run, secular or growth component of y_t, but the implicit view of the cyclical component ε_t is that it cannot therefore exhibit any long-term features itself.

Consequently, departures of y_t from μ_t must be temporary only, which is the same as assuming that ε_t is stationary, or briefly:

$$E(\varepsilon_t) = 0 \quad E(\varepsilon_t^2) = \sigma_\varepsilon^2 \prec \infty \text{ for all t and k} \neq 0. \tag{2}$$

Series ε_t will then have by Wold´s decomposition theorem the linear filter representation:

$$\varepsilon_t = u_t + \psi_1 u_{t-1} + \psi_2 u_{t-2} + ... = \sum_{j=0}^{\infty} \psi_j u_{t-j}, \; \psi_0 = 1 . \tag{3}$$

Random components u_t for t = 0, ± 1, ± 2, ± 3, ... are sequences of mean zero, constant variance σ_u^2 or i.i.d. random variables, so that $E(u_t, u_{t-k}) = 0$ for all k ≠ 0, which we refer to as white noise.

Stationarity requires that ψ-weights in the linear filter representation (3) are absolutely sum able, i.e. the ψ-weights are said to converge.

Although (3) may appear complicated, many realistic models for cyclical component result from particular choices of the ψ-weights. For example AR(2) process is capable of modelling the cyclical fluctuations of economic time series. This process can be written as:

$$\varepsilon_t = \phi_1 \varepsilon_{t-1} + \phi_2 \varepsilon_{t-2} + u_t$$

or

$$(1 - \phi_1 B - \phi_2 B^2)\varepsilon_t = (1 - g_1 B)(1 - g_2 B)\varepsilon_t = u_t, \quad (4)$$

where the roots g_1 and g_2 of associated characteristic equation $\phi_2(B) = 0$ are given by

$$g_1, g_2 = (\phi_1 \pm \sqrt{(\phi_1^2 + 4\phi_2)})/2 \quad (5)$$

For stationarity, it is required that the roots are $|g_1| < 1$ and $|g_2| < 1$, and it can be shown that these conditions imply the following set of restrictions put on ϕ_1 and ϕ_2:

$$\phi_1 + \phi_2 < 1 \quad \phi_2 - \phi_1 < 1 \quad -1 < \phi_1 < 1.$$

The roots can both be real or they can be a pair of complex numbers, which would produce an autocorrelations following a damped sine wave and hence an ε_t containing cyclical fluctuations. The roots will be complex if $\phi_1^2 + 4\phi_2 < 0$, although a necessary condition for complex roots is simply that $\phi_2 < 0$. When the roots are complex, they take the form, whereupon the autocorrelations becomes the damped sine wave

$$\rho_k = \frac{(\mathrm{sgn}(\phi_1))^k d^k \sin(2\pi fk + F)}{\sin F}, \quad (6)$$

where $d = \sqrt{-\phi_2}$ is damping factor and f and F are the frequency and phase of wave, this is obtained from

$$f = \frac{\cos^{-1}(|\phi_1| / 2d)}{2\pi} \quad (7)$$

and

$$F = \tan^{-1}(\frac{1 + d^2}{1 - d^2} \tan 2\pi f) \quad (8)$$

respectively. The period of cycle is then defined as 1/f.

Higher order AR models will exhibit cyclical fluctuations as long as they admit a pair of complex roots, i.e., if AR (p) can be factorised as

$$\Phi_p(B)\varepsilon_t = (1 - d\exp(2\pi fi)B)(1 - d\exp(-2\pi fi)B)\prod_{j=3}^{p}(1 - g_j B)\varepsilon_t = u_t . \quad (9)$$

A second class of models is obtained simply by truncating the infinite lag order in the Wold decomposition at a finite lag q, thus defining MA (q) process as:

$$\varepsilon_t = u_t + \theta_1 u_{t-1} + \ldots + \theta_q u_{t-q} = \Theta_q(B)u_t . \quad (10)$$

It is easily shown that such process will have autocorrelation function that cut-off after lag q. We may also define mixed models: the ARMA (p, q) process takes the form

$$\Phi_p(B)\varepsilon_t = \Theta_q(B)u_t . \quad (11)$$

2. Estimating the Cyclical Component

The typical procedure for estimating the cyclical component is simply to calculate it as the residual left after fitting a trend model, i.e. as:

$$\hat{\varepsilon}_t = y_t - \hat{\mu}_t . \quad (12)$$

Having obtained this estimated series, autoregressive models or mixed ARMA models may be fitted to it.

Figure 1 shows development of CPI (previous year = 100) in Slovakia and in Poland respectively during the period from January 1993 till August 2005.

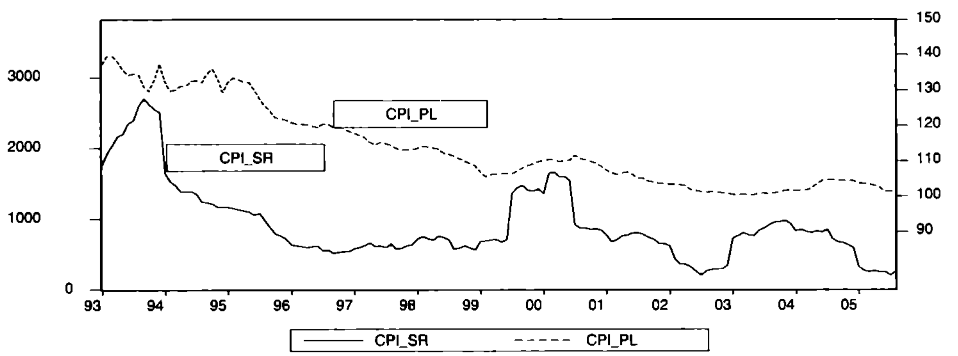

Fig. 1. CPI in Slovakia and Poland from January 1993 – August 2005
source: www.statistics.sk and NBP Bulletins (1994-2005).

From fig. 1 it is possible to see that Slovak CPI has greater variability than Poland CPI. Cycle is more evident in case of Slovak data than in Poland's case.

It is highly recommend to begin our analysis with fitting appropriate trend function to Slovak's and Poland's logarithm of CPI time series respectively and then the residual analysis will be done in order to look for the appropriate model of the cycle.

Modelling log (CPI_SR)

Linear trend in Slovak logarithm of CPI was estimated as $\log CPI\_SR_t = 7,238 - 0,0075t$ with statistically significant coefficients but with autocorrelated residuals – as it is possible to see on the button of the fig. 2. There are two different cycle lengths.

The analysis of the autocorrelation and partial autocorrelation function of residuals has following properties: ACF of residuals decay exponentially; PACF has two coefficients statistically significant at 5% level of significance i.e. $r_{11} = 0,947$ and $r_{22} = -0,244$ in absolute value are greater than $0,162 = 2/\sqrt{152}$. It means that the residual series could be generated by AR(2) process.

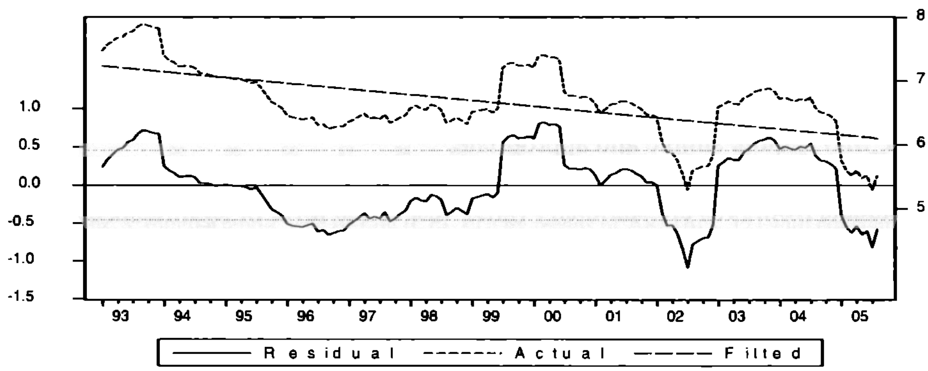

Fig. 2. Linear trend for log CPI_SR, January 1993-August 2005
source: own calculations.

Squared residuals also decay exponentially; PACF has $r_{11} = 0,836$ and $r_{55} = -0,213$ statistically significant at 5 % level of significance. It means that ARCH effect is present what could also be proved by the Lagrange Multiplier test where $LM = 152R^2 = 152 \cdot 0,70 = 106,4$ with *P*-value 0,000.

Linear trend model estimation of log CPI_SR together with AR(2) process is as follows:

$$\underset{}{LCPI\_SR_t} = \underset{(0,02)}{7,235} - \underset{(0,0003)}{0,0075t} + \underset{(0,081)}{1,1649\hat{\varepsilon}_{t-1}} - \underset{(0,082)}{0,2204\hat{\varepsilon}_{t-2}}. \quad (13)$$

$R^2 = 0,942$; s.e. = 0,13; *D-W*= 2,01.

Figure 3 shows how well the model AR(2) with the constant and linear trend fit the data of log (CPI_SR).

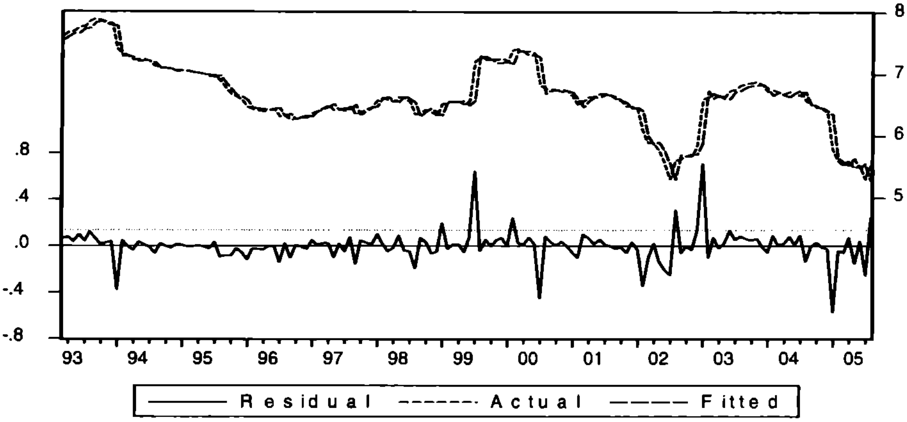

Fig. 3. AR (2) process with constant and trend for Log CPI_SR, January 1993-August 2005
source: own calculations.

Residuals of the model (13) are not autocorrelated; there is no ARCH effect because *LM* = 0,03 with *P*-value 0,8554. We can conclude that the model for log(CPI_SR) is quite simple and appropriate.

To confirm the results about the cycle's length the Fourier analysis was done. The residuals from (13) are stationary (ADF test at 1% level of significance), so the analysis can be carried out. The most significant length of cycle is 2,313 months, next is 18,5 months and the third most significant length of cycle is 3,217 months. This fully certifies the above computations.

Modelling log (CPI_PL)

Quadratic trend for logarithm of polish CPI was estimated as: $\log CPI\_SR_t = 4{,}949 - 0{,}0045t + 0{,}000016t^2$ with statistically significant coefficients but with autocorrelated residuals, what is possible to see on the button part of the fig. 4.

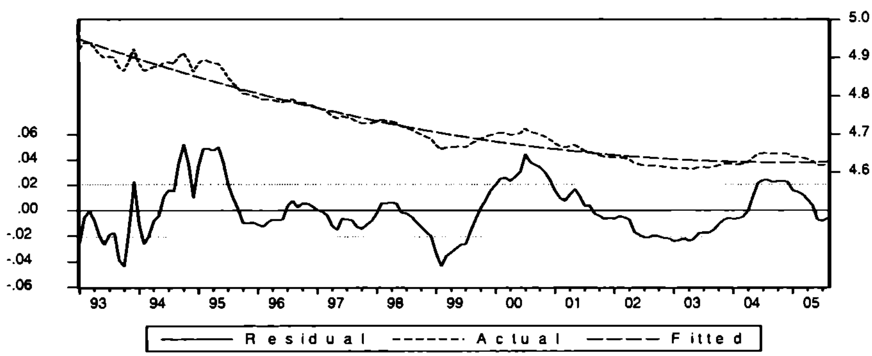

Fig. 4. Quadratic trend for log CPI _PL, January 1993- August 2005
source: own calculations.

Again there are two cycles of different length, but at the beginning of the time series it is possible to see the significant heteroscedasticity of residuals obtained from quadratic trend model.

Analysis of autocorrelation and partial autocorrelation function of residuals from the quadratic trend showed that model ARMA (3, 12) with constant could be suitable model for fitting data log (CPI_PL). Estimation of this model is given by equation (14):

$$\underset{(0,07)}{(1-1,245B}+\underset{(0,10)}{0,610B^2}-\underset{(0,07)}{0,351B^3)}\hat{\varepsilon}_t=(1+\underset{(0,04)}{0,121B^6}+\underset{(0,04)}{0,772B^{12}})u_t \quad (14)$$

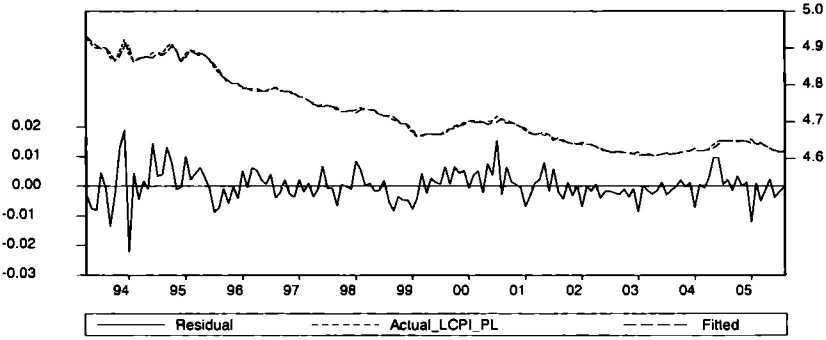

Fig. 5. Fitted Trend-Cycle of Log (CPI_PL) by means of ARMA(3, 12) with constant source: own calculations.

Although residuals of the model (14) are not autocorrelated, ARCH effect measured by Lagrange Multiplier *LM* = 15,6 with *P*-value = 0,0000 is still significant at 5% level of significance. It is the reason why we try to model this heteroscedasticity of residuals of the model (14) by means of ARCH model.

The estimation of the model of quadratic trend together with ARMA(1,1,(12,30)) and GARCH(1,1) for log (CPI_PL) given by E-Views 3 is attached on tab. 1.

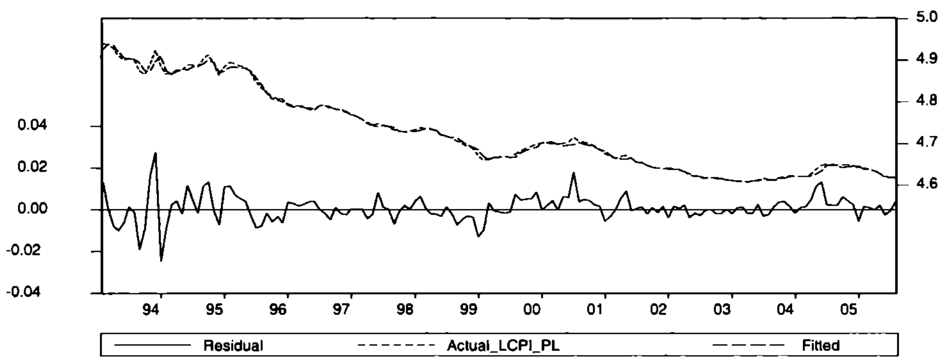

Fig. 6. Fitted log CPI_PL by means of ARMA(1,1 (12,30)) with GARCH(1,1) source: own calculations.

Table 1. Output of estimated model for log CPI_PL, January 1993-August 2005.

Dependent Variable: LCPI_PL
Method: ML – ARCH
Sample(adjusted): 1993:02 2005:08
Included observations: 151 after adjusting endpoints
Convergence achieved after 9 iterations
Backcast: 1990:08 1993:01

	Coefficient	Std. Error	z-Statistic	Prob.
C	4.962740	0.027622	179.6670	0.0000
T	-0.004986	0.000659	-7.565552	0.0000
T*T	1.80E-05	3.58E-06	5.029306	0.0000
AR(1)	0.901252	0.044335	20.32827	0.0000
MA(1)	0.372221	0.099921	3.725163	0.0002
MA(12)	-0.269483	0.078754	-3.421851	0.0006
MA(30)	-0.216960	0.068459	-3.169223	0.0015
	Variance Equation			
C	4.37E-06	2.68E-06	1.629416	0.1032
ARCH(1)	0.528122	0.222881	2.369520	0.0178
GARCH(1)	0.515165	0.103470	4.978861	0.0000
R-squared	0.995356	Mean dependent var		4.727060
Adjusted R-squared	0.995060	S.D. dependent var		0.099032
S.E. of regression	0.006961	Akaike info criterion		-7.425808
Sum squared resid	0.006832	Schwarz criterion		-7.225989
Log likelihood	570.6485	F-statistic		3357.824
Durbin-Watson stat	1.929739	Prob(F-statistic)		0.000000
Inverted AR Roots	.90			
Inverted MA Roots	.95	.91 -.19i	.91+.19i	.85 -.40i
	.85+.40i	.77+.55i	.77 -.55i	.61+.70i
	.61 -.70i	.47 -.84i	.47+.84i	.29 -.89i
	.29+.89i	.07 -.95i	.07+.95i	-.10+.95i
	-.10 -.95i	-.31 -.89i	-.31+.89i	-.50+.83i
	-.50 -.83i	-.63+.70i	-.63 -.70i	-.80 -.54i
	-.80+.54i	-.88 -.41i	-.88+.41i	-.93 -.18i
	-.93+.18i	-.99		

source: own calculations.

Residuals of this model are not autocorrelated (*D-W*=1,929), R^2 = 0,995. There is no ARCH effect while *LM* = 1,34 with *P*- value = 0,2867. MAPE = 0,08 %. We can see that all coefficients are statistically significant. Strange in this model is, that there is one seasonal parameter although the series does not allocated statistically significant seasonality. Other interesting feature of this model is, that cycle's length is about thirty months. Figure 6 shows how well the model fit the dataAgain, in order to confirm the results about the cycle's length, the Fourier analysis was done. The residuals from the trend with a ARMA model logarithm of polish CPI indicates that the most significant length of cycle is about 37,5 months, next is 50 months and the third most significant length of cycle is 30 months. This proves the above conclusion that the length of cycle is about 30 months.

3. Conclusions

As is mentioned in [2] deterministic trend models could produce different cyclical components. It was showed that the length of cycle for Poland and Slovakia CPI differ. In Slovakia it is very short and it equals about two months contrary to Poland where it is about 3 years. The similar results were obtained both by means of ARMA models and the Fourier analysis.

References

[1] Davidson R., MacKinnon J.G., *Econometric Theory and Methods,* Oxford University Press, New York 2004.

[2] Harvey A.C., *Forecasting, Structural Time Series Models and the Kalman Filter,* Cambridge University Press, 1989.

[3] Mills T.C., *Modelling Trends and Cycles in Economic Time Series*, Palgrave Mackmillan, New York 2003.

ESTYMACJA SKŁADNIKA CYKLICZNEGO W SŁOWACKICH I POLSKICH SZEREGACH CPI

Streszczenie

Artykuł prezentuje statystyczną analizę cykliczności w miesięcznych szeregach wskaźnika cen dóbr i usług konsumpcyjnych w Polsce i na Słowacji w okresie 1993-2005.

Przedstawiono klasyczne podejście do wyodrębnienia składnika cyklicznego z reszt równania trendu lub stochastycznego stacjonarnego procesu. Trend liniowy wraz z modelem AR(2) został użyty do modelowania trendostacjonarnego procesu dla Słowacji. Z kolei w przypadku polskiego CPI użyto trendu kwadratowego wraz z ARIMA$(p, d, q)(P, D, Q)_s$ i GARCH(p, q). Wyniki zostały również potwierdzone za pomocą analizy Fouriera.