

## Original Study

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# Assessment of spread foundation settlement using statistical determination of characteristic values of subsoil properties

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**Abstract:** According to Eurocode 7, limit state design codes generally draw more attention to ultimate limit states than to serviceability limit states. This paper presents the problem of settlement assessment of spread foundations on clays when the foundation design is governed by the serviceability limit state requirements. The paper presents the test results of geotechnical parameters for heavily preconsolidated boulder clays (sandy clay saCl and silty sandy clay sasiCl), which prevail on the Warsaw University of Life Sciences – SGGW campus. The test results were used for settlement calculations of spread foundations. Based on the results of field and laboratory tests, the problem of spatial variability assessment and determination of the characteristic value of soil parameters was addressed. Classical statistics and Bayesian analysis were used in the statistical analysis of the test results. Settlements of spread foundations were calculated based on the soil parameters obtained from cone penetration tests (CPTs) and dilatometer tests (DMTs). Special attention was drawn to the selection of the characteristic values of soil parameters. Determination of the characteristic value of the constrained modulus  $M_k$  was performed using two methods: according to the well-known and frequently used formula proposed by Schneider (1997; 1999) and according to the European draft standard prEN 1997-1:2022-09. Settlement calculations of spread foundations were carried out taking into account changes in the stresses and the constrained modulus in the subsoil. The calculated

settlements were verified by field measurements performed during the construction of the object. Comparison of settlements obtained from the characteristic values of the constrained modulus  $M_k$  estimated according to prEN 1997-1:2022-09 with the measured settlements indicates that the calculated values were significantly higher than the measured values. Smaller differences between the measured and calculated settlements were obtained when the characteristic values of the constrained modulus  $M_k$  were determined from Schneider's formula, while smaller differences were obtained when the mean values of the constrained modulus  $M_{mean}$  were used.

**Keywords:** spread foundation, settlement, characteristic values, constrained modulus, cone penetration tests (CPTs), dilatometer tests (DMTs)

## 1 Introduction

According to Eurocode 7, limit state design codes generally draw more attention to ultimate limit states than serviceability limit states. However, in many practical cases, serviceability limit states determine the final dimensions of the foundation (Bond, Harris, 2008; Simpson et al., 2009). In engineering practice, the constrained modulus is often used to estimate vertical displacements in a one-dimensional strain state. The constrained modulus can be determined in oedometer tests on undisturbed samples taken at selected depths. Based on the results of cone penetration test (CPT) and dilatometer test (DMT) using empirical relationships, a profile of the change of constrained modulus with depth can be determined. Empirical relationships shown in the literature should be verified for given soils before application (Młynarek et al., 2023).

The distribution of parameter variability depends on the randomness of the ground properties and measurement uncertainties, and the distribution of measured parameters is not always normal (Lacasse,

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Nadim, 1994). When determining the characteristic value of a geotechnical parameter, the following should be taken into account in particular: the method of separating layers depending on the origin and structure of the subsoil, the heterogeneity of subsoil properties and the dispersion of results, the method of “deriving” values from existing local correlations, local studies and “comparable experiences” the existing knowledge and experience, accuracy of recognition of the active zone, and ability of the structure to redistribute loads (Wysokiński et al., 2011). Wysokiński et al. (2011) reported, based on the results of laboratory tests collected from many publications, the values of the coefficient of variation of the deformation modulus  $V_x = 2\%–42\%$ . According to the draft standard prEN 1991-1:2022 in the second case of calculations, when the value of  $V_x$  is assumed, it is proposed to assume  $V_x = 20\%–70\%$  for the deformation and shear moduli.

For many years, there has been an ongoing discussion in the geotechnical society regarding the use of various statistical procedures to evaluate geotechnical parameters in geotechnical design (Wysokiński et al., 2011; Rabarijoely et al., 2013; 2019a; 2019b; 2021; Olek et al., 2014; Lesny, 2017; Straż, Borowiec, 2021; Godlewski et al., 2023). Special attention was paid to determine the characteristic values of geotechnical parameters (Jaksa et al., 1997; Batog, Hawrysz, 2010; Yoon et al., 2010; Pohl, 2011; Puła, 2014; Puła, Zaskórski, 2015; Löfman, 2016; Lesny 2017; Ching et al., 2020; Zhang et al., 2020). A problem of statistical characterization of the geotechnical properties of subsoil (Nguyen et al., 2023) and the influence of spatial variability of shear strength (Nguyen et al., 2017; 2019; 2021; Sulewska, Lechowicz, 2024) was analyzed.

Foundation settlements within the serviceability limit states were calculated based on the characteristic values of geotechnical parameters (according to PN-B-03020:1981, prEN 1990:2022-09, prEN 1997-1:2022-09). The method of determining the values of geotechnical parameters has changed in subsequent geotechnical European standards (EN 1997-1:2004, prEN 1997-1:2022-09).

According to Polish standard PN-B-03020:1981, the mean value of geotechnical parameter is taken as the characteristic value. Using Schneider’s formula, the characteristic value of the geotechnical parameter, in the case when its smaller value is unfavorable, is calculated as the mean value minus half a standard deviation. The analysis of the formula from prEN 1997-1:2022 shows that the characteristic value of the geotechnical parameter  $X_k$  depends on the mean value  $X_{\text{mean}}$ , the standard deviation  $S_x$ , the coefficient of variation  $V_x$ , and the statistical coefficient  $k_n$ , which depends on the number of results and the type of population.

This paper presents the problem of settlement assessment of spread foundations on clays when the serviceability limit state requirements govern the foundation design. The presented study aims to analyze how the method applied to determine the characteristic values of the constrained modulus for cohesive soils influences the calculated values for foundation settlements. Values of the constrained modulus were determined using CPT and DMT. Characteristic values of the constrained modulus were determined using two methods: according to the well-known and frequently used formula proposed by Schneider (1997; 1999) and Schneider and Fitze (2013) and according to the European draft standard prEN 1997-1:2022-09. Settlement values calculated from the characteristic values of the constrained modulus were compared with the measured values of foundation settlement.

## 2 Methods for determining the characteristic values of the constrained modulus

### 2.1 Determination of the mean values of geotechnical parameters according to the draft standard prEN 1997-1:2022-09

The European draft standard EN 1997-1:2004 recommends that the characteristic value of the geotechnical parameter  $X_k$  should be selected as a conservative estimate of the value that determines the occurrence of a limit state. The characteristic value of the geotechnical parameter  $X_k$  is the most probable value of a given parameter at which the considered limit state will occur. Conservative estimation of the mean value involves selecting the mean value from a limited set of geotechnical parameter values with a confidence level of 95%. The characteristic value of parameter  $X_k$  is determined on the basis of mean values  $X_{\text{mean}}$  derived from laboratory or field tests, alternatively based on derived values using formulas or empirical relationships.

The European standard EN 1997-1:2022-09 recommends that the mean  $X_{\text{mean}}$  value of the geotechnical parameter is calculated according to formula (1):

$$X_{\text{mean}} = \frac{1}{n} \sum X_i \quad (1)$$

where  $X_i$  is the result of the determination of the parameter concerned and  $n$  is the number of determinations.

The measure of variability of a geotechnical parameter in a given soil layer is the coefficient of variability  $V_x$  calculated according to formula (2):

$$V_x = \frac{S_x}{X_{\text{mean}}} \quad (2)$$

where  $S_x$  is the standard deviation, most often from a sample, in the case of a limited number of test results. The standard deviation for sample  $S_x$  is expressed by formula (3):

$$S_x = \sqrt{\frac{\sum_{i=1}^n (X_i - X_{\text{mean}})^2}{n-1}} \quad (3)$$

## 2.2 Determination of the mean values of geotechnical parameters using Bayesian analysis

Assessment of a parameter in classical mathematical statistics is based on a random sample taken from the population. In an alternative approach, derived from Bayes' theorem (Alén, 1998; Alén, Sällfors, 1999), the assessment may be based not only on a random sample, but also on the so-called *a priori* information. *A priori* knowledge may be either expert knowledge or a result from previous research. Population parameters to be estimated include parameter  $\theta$ , for example, the mean, and the standard deviation from the population; in Bayesian analysis, they are treated as random variables. Bayes' theorem for random variables with a continuous probability distribution can be expressed as formula (4):

$$f(\theta | x) = \frac{f(x | \theta) \cdot f(\theta)}{\int_{\Omega} f(x | \theta) \cdot f(\theta) d\theta} \quad (4)$$

where  $f(\theta | x)$  is the *a posteriori* density function of parameter  $\theta$ , after the sample's result  $x$  has been observed,  $f(\theta)$  is the *a priori* distribution density function of parameter  $\theta$ ,  $f(x | \theta)$  is the credible function, that is, the density function of the conditional observation's result  $x$  with a given value of  $\theta$ , and  $\Omega$  is the set of possible values of parameter  $\theta$ . Therefore, on the basis of Bayes' theorem, the *a priori* density function of parameter  $\theta$  is actualized with the use of information from a sample.

A common case is the estimation of an unknown parameter  $\theta$ , which is the mean in a normal population for which the standard deviation  $\sigma_0$  is known. The *a priori* knowledge about the mean  $\theta$  of this population can be used, which shows that parameter  $\theta$  is a random variable with a normal distribution with parameters  $m_1$  and  $\sigma_1$ , while the mean of the drawn  $n$ -element sample is  $m_2$ . Therefore, the *a posteriori* distribution of the random variable  $\theta$  is also normal, with the mean  $m$  and standard deviation  $\sigma$  calculated as follows (formulae 5 and 6):

$$m = \frac{(1/\sigma_1^2) \cdot m_1 + (n/\sigma_0^2) \cdot m_2}{(1/\sigma_1^2) + (n/\sigma_0^2)} \quad (5)$$

$$\sigma = \frac{1}{(1/\sigma_1^2) + (n/\sigma_0^2)} \quad (6)$$

where  $m$  and  $\sigma$  are the mean and standard deviation, respectively, of the *a posteriori* distribution of the random variable  $\theta$ ,  $m_1$  and  $\sigma_1$  are the mean and standard deviation, respectively, of the *a priori* distribution of the random variable  $\theta$ ,  $m_2$  is the mean of the drawn  $n$ -element sample,  $\sigma_0$  is the known standard deviation, and  $n$  is the number of elements in the sample.

Using Bayesian analysis, the mean values and standard deviation for cone resistance  $q_c$  and constrained modulus  $M$  from CPT, as well as for dilatometer modulus  $E_d$  and constrained modulus  $M$  from DMT were calculated based on formulae (5) and (6). Calculated mean values of constrained modulus  $M$  were used to calculate the settlements shown in Table 4.

## 2.3 Determination of the characteristic values of geotechnical parameters using Schneider's formula

Schneider (1997; 1999) took into account the need for a careful estimation of geotechnical parameters and proposed the often used formula (7) for assessing the characteristic value  $X_k$  of geotechnical parameters (Schneider, 1997; 1999; Schneider, Fitze, 2013; Puła, 2014; Sulewska, Lechowicz, 2024):

$$X_k = X_{\text{mean}} - 0.5 \cdot S_x \quad (7)$$





Figure 1: View of building No. 34 at the campus of the Warsaw University of Life Sciences – SGGW. <https://wzim.sggw.edu.pl/wydzial/>

## 2.4 Determination of characteristic values of geotechnical parameters according to the draft standard prEN 1997-1:2022-09

The draft standard prEN 1997-1:2022-09 introduces the concept of a representative value of ground properties  $X_{\text{rep}}$ . Representative values of ground properties  $X_{\text{rep}}$  are specific geotechnical properties of a given subsoil layer.

Two cases can be considered, depending on the sensitivity of the limit state of the ground to the spatial variability of a given property:

- case A – if the verified limit state of the ground is insensitive to the spatial variability of a given ground property in the volume of soil involved, the representative value of a given parameter  $X_{\text{rep}}$  is its nominal value  $X_{\text{nom}}$ , that is, the mean, according to formula (8):

$$X_{\text{rep}} = X_{\text{nom}} \quad (8)$$

- case B – if a given limit state is sensitive to the spatial variability of the ground, the representative value of parameter  $X_{\text{rep}}$  is its characteristic value  $X_k$ , according to formula (9):

$$X_{\text{rep}} = X_k \quad (9)$$

The characteristic value of the geotechnical parameter  $X_k$  may be calculated according to formula (10):

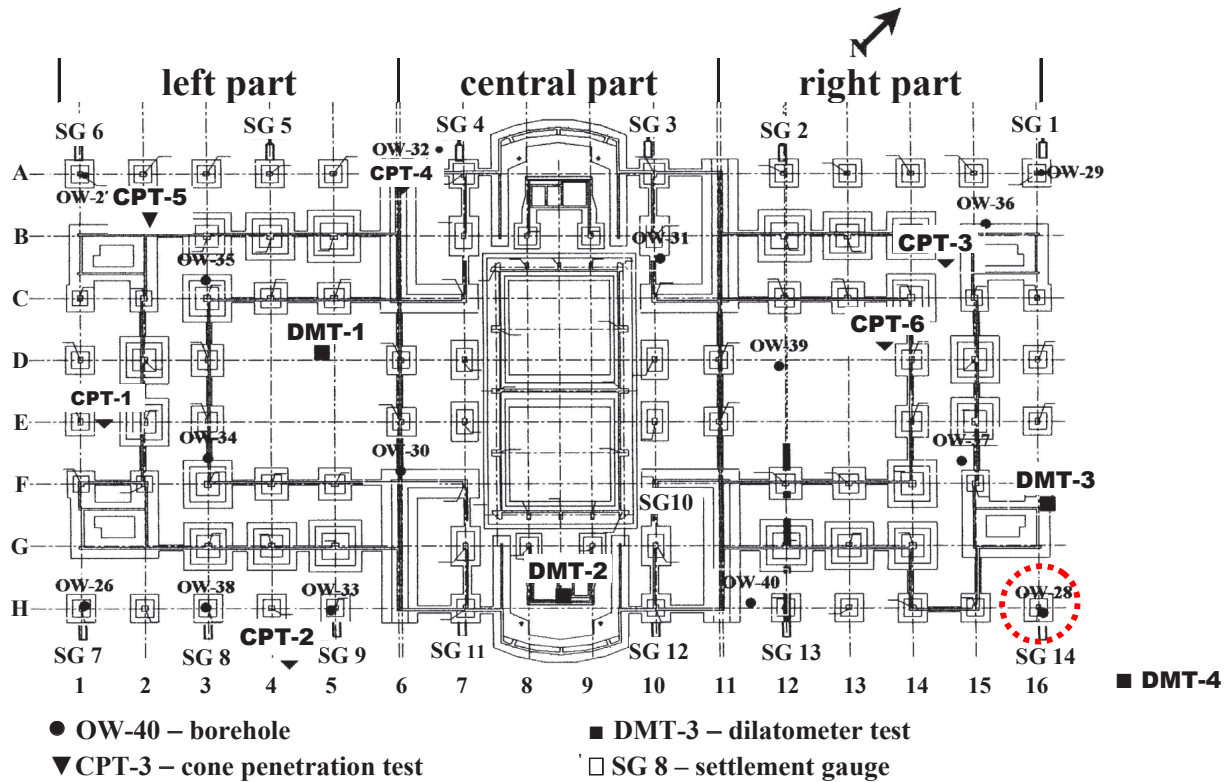
$$X_k = X_{\text{mean}}[1 \mp k_n V_x] = X_{\text{mean}} \left[ 1 \mp \frac{k_n S_x}{X_{\text{mean}}} \right] \quad (10)$$

where  $k_n$  is the factor depending on the number  $n$ ,  $\mp$  means that  $k_n V_x$  should be subtracted when the lower value of  $X_k$  is required or added when its upper value is required.

## 3 Description of the test site

The test site is located at the campus of the Warsaw University of Life Sciences – SGGW, where new buildings were constructed. Assessment of the foundation settlement of building No. 34, with dimensions 120 m × 57 m and height of 25 m, with a basement (B) and five floors (5F), is considered in this paper (Figure 1). Since the settlement gauges were installed before the construction of the third floor, the building was divided into two stages: B–2F and 3F–5F.

Due to expected differential settlements, the building was constructed as three dilated parts (Figure 2). In each side part, two halls with two floor heights were located. The building was constructed as a monolithic reinforced



**Figure 2:** View of the pad footings of building No. 34 – location of test points: CPT – cone penetration test, DMT – dilatometer test, SG – settlement gauge.

concrete structure with columns and slabs in the side parts and as a reinforced concrete structure based on walls and columns in the central part of the building. Building structural loads varying between 5 and 12 MN were transmitted by the columns with a 7.8 m spacing to the ground by pad footings. Square pad footings were designed with dimensions from 3.4 m × 3.4 m up to 6.5 m × 6.5 m. Pad footing H-16 with dimensions 3.4 m × 3.4 m (Figure 2) with a benchmark SG14 is considered in this paper. The unit load from all stages of building construction under the pad footing H-16 was 441 kPa.

During the construction, settlement gauges (SG1–SG14) were installed on selected columns of the three dilated parts of building No. 34 (Figure 2). Settlement measurements began after completion of the second floor (2F); therefore, they were concerned with the second stage of building construction (floors 3F–5F). They proved that the displacements of the three dilated parts were different. Settlement gauges installed in the middle part and in the left part under loading caused by the second stage of building construction (3F–5F) were 1–2 mm and in the right part were 6–7 mm. In the paper, the results of settlement calculation for the selected footing H-16 in the right part caused by the second stage of building

construction (3F–5F) are presented. To compare the measured and calculated settlements of the pad footing H-16, settlement calculations were carried out with a unit load from the second stage of building construction (3F–5F) of 227 kPa.

## 4 Field and laboratory tests

### 4.1 Ground characteristics

In general, with the exception of the 2–3 m thick surface layer, the tested subsoil consists of Quaternary moraine deposits underlain by fine sands. The moraine deposits consist of two layers: layer III comprising brown boulder clay of the Wartanian glaciation and layer IV comprising gray boulder clay of the Odranian glaciation. The large variation in the values of geotechnical parameters in the tested cohesive subsoil results from the heterogeneity of the soil caused by sedimentation conditions and diagenetic processes that the soil has been subject to in its geological history. The analyzed cohesive soils of post-glacial origin (Pleistocene), deposited as moraine sediments of the Riss

**Table 1:** Index properties of the analyzed boulder clays and thickness of layers and sublayers.

Properties\thickness	Boulder clay (brown) III layer	Boulder clay (gray) IV layer	IVa (m)	IVb (m)	IVc (m)
Water content $w_n$ (%)	10.0–11.0	10.5–13.5			
Unit density $\rho$ (t·m <sup>-3</sup> )	2.1–2.2	2.1–2.2			
Liquid limit $w_L$ (%)	21.0–24.9	22.0–26.0			
Plasticity index $I_p$ (%)	11.0–13.0	12.0–14.0			
Consistency index $I_c$ (-)	0.93–1.0	0.89–0.96			
Content of fraction (%):					
Sand (0.063–2 mm) Sa	62–63	56–59			
Silt (0.002–0.063 mm) Si	25–28	28–29			
Clay ( $\leq 0.002$ mm) Cl	10–12	13–15			
The thickness of layers and sublayers below the foundation level (m) (depth of foundation level = 4.4 m below ground level)	CPTs 0.8–1.0 DMTs	5.2	1.2 1.0	1.6 1.8	2.4 2.4

CPTs = cone penetration tests, DMTs = dilatometer tests

glaciation, originating from two stages, that is, cooling within one glaciation, that is, formed in two-time stages and differing visually in color (brown and gray boulder clays) and large spatial variability of geotechnical properties characteristic for boulder clays.

The index properties of boulder clays and the thicknesses of layers and sublayers are presented in Table 1. The tested soils can be classified as preconsolidated low-plasticity sandy clay saCl or silty sandy clay asiCl clays (according to EN ISO 14688–2:2018). The building was founded below the surface of brown boulder clay. Because of the difference in thickness of boulder clays below the foundation level and the properties of gray boulder clay, differential settlements were expected. Compression curves obtained from incremental loading oedometer tests conducted up to 30 MPa (Lechowicz et al., 2017) indicate that the preconsolidation stress  $\sigma'_p = 800$ –1000 kPa and is much higher than the effective stresses induced by loading of building construction. In building settlement calculations, the secondary constrained modulus values were only used.

## 4.2 Results of CPT and DMT

To determine the geotechnical parameters of the subsoil under building No. 34, four CPTs and four DMTs were carried out. The profiles of the cone resistance  $q_c$ , sleeve friction  $f_s$ , and friction ratio  $R_f$  from CPT are shown in Figure

3. The profiles of the material index  $I_p$ , horizontal stress index  $K_p$ , and dilatometer modulus  $E_d$  from DMT are shown in Figure 4. CPT and DMT test results indicate that the gray boulder clay is softer than the brown boulder clay.

Variability of the thickness of foundation sublayers was determined based on the profiles of cumulative values of cone resistance  $\Sigma q_c$  from CPTs and cumulative values of dilatometer modulus  $\Sigma E_d$  from DMTs (Figure 5). In the case of the gray boulder clay (layer IV), three sublayers (IVa, IVb, and IVc) were distinguished as shown in Figure 5.

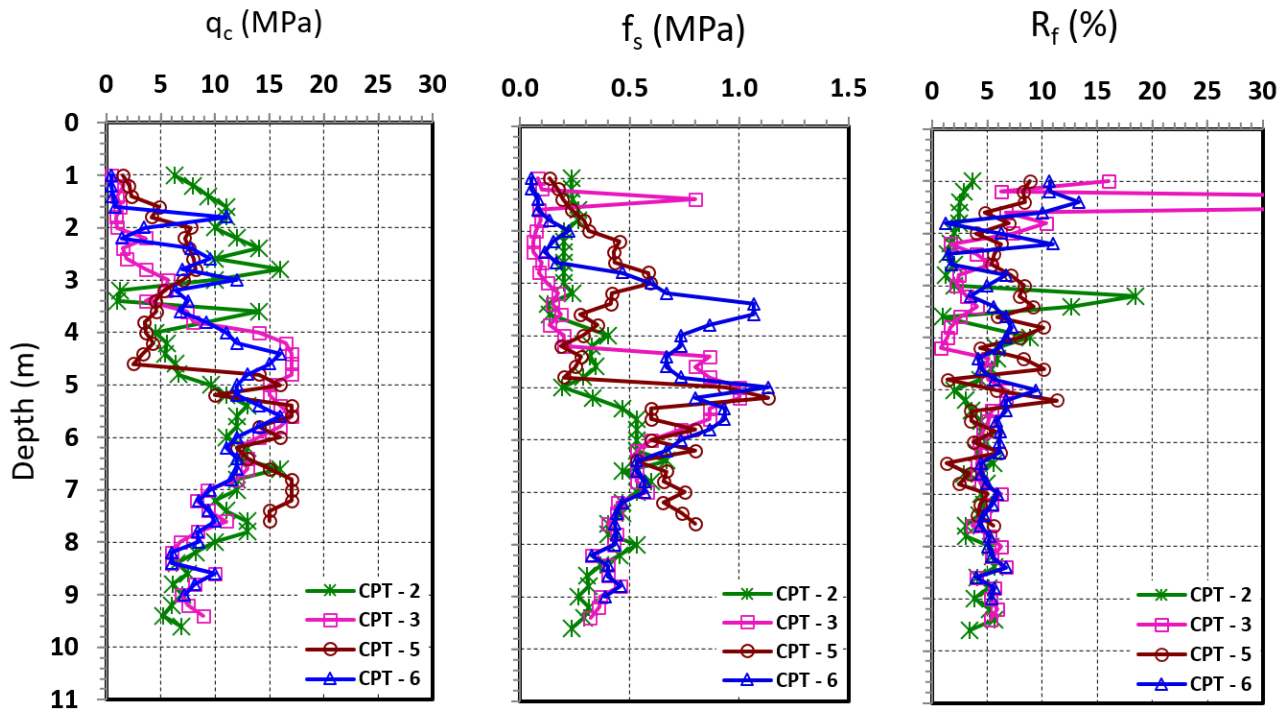
## 4.3 Evaluation of the constrained modulus $M$ based on CPT and DMT results

To evaluate the value of the constrained modulus  $M$  from CPT, the empirical correlations proposed by Senneset et al. (1989) were used as follows:

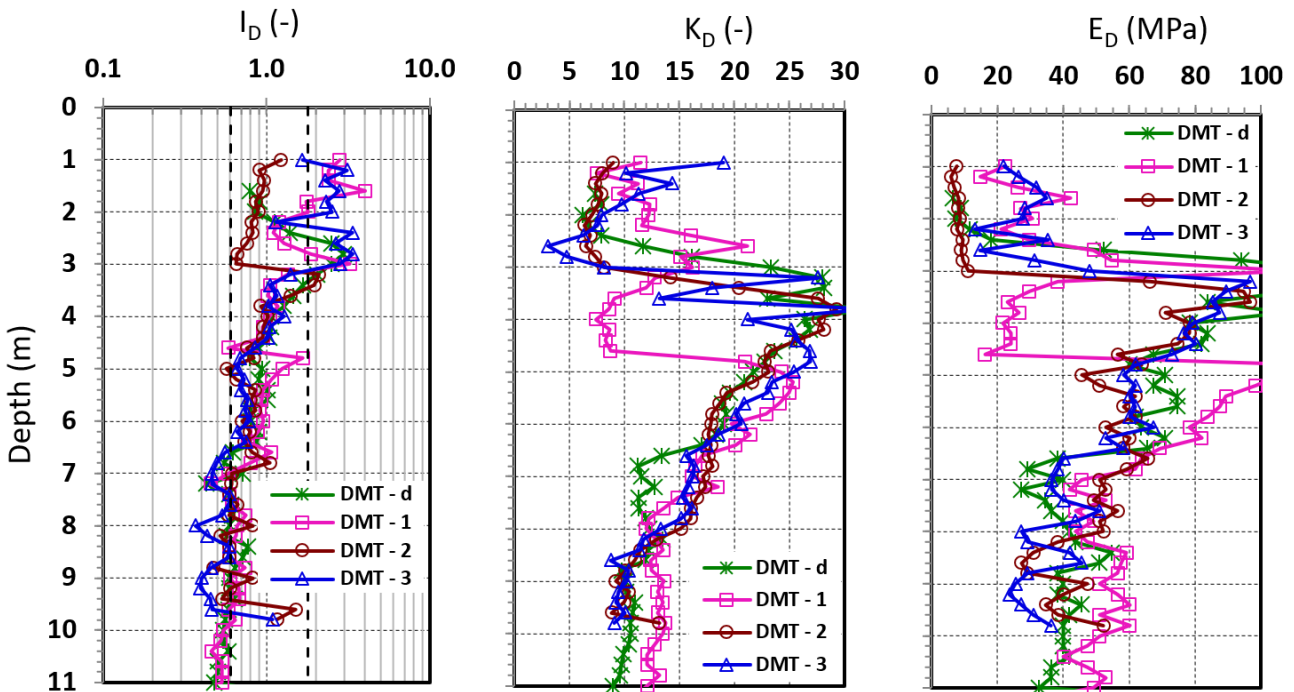
$$M = \alpha q_n \quad (11)$$

where  $\alpha$  is the empirical coefficient,  $q_n = (q_c - \sigma_{v0})$  is the net cone resistance,  $\sigma_{v0} = \Sigma(\gamma \cdot h)$  is the total overburden stress,  $\gamma$  is the unit weight, and  $h$  is the layer thickness. Values of the constrained modulus  $M$  from CPT were calculated based on the empirical correlation (11) with the empirical coefficient  $\alpha = 8.25$  proposed by Kulhawy and

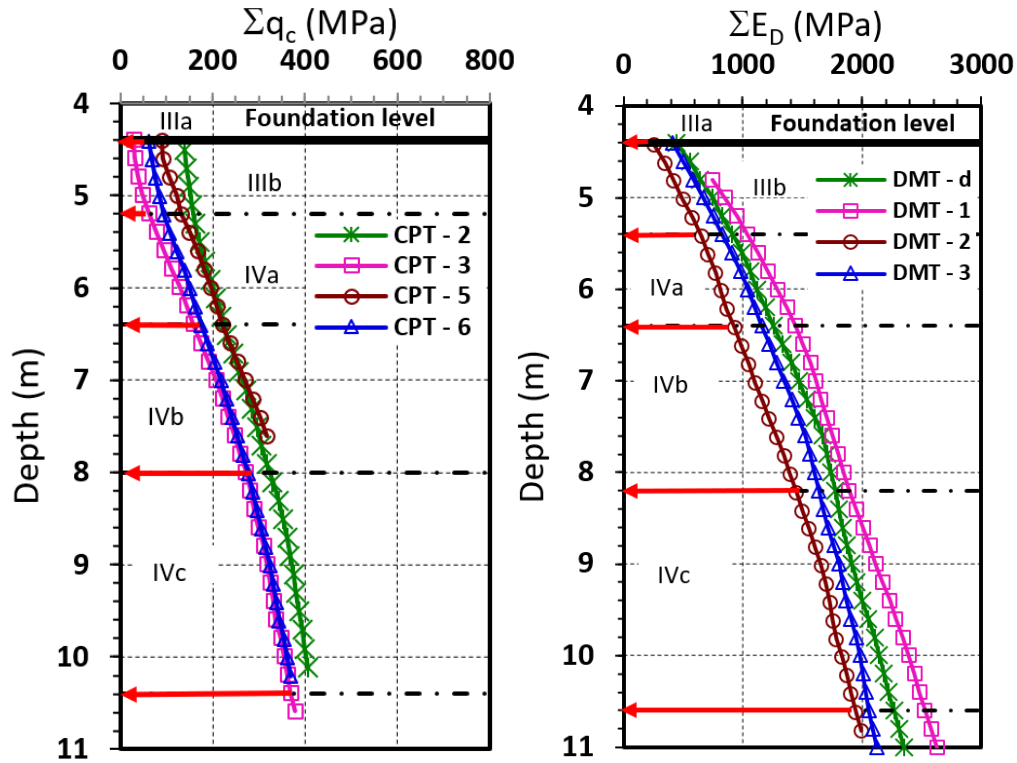




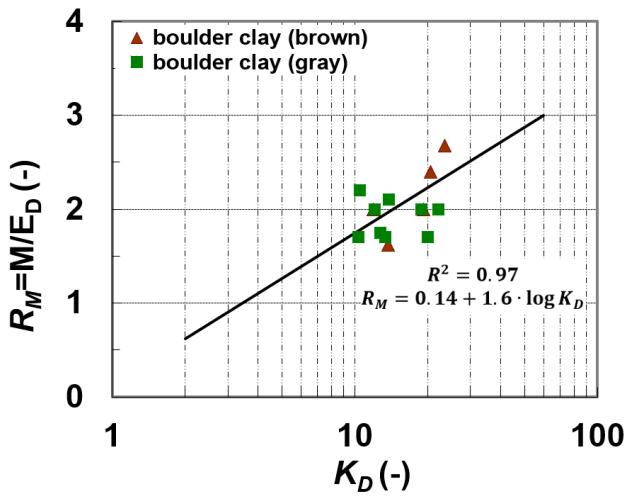
**Figure 3:** Profiles of cone resistance  $q_c$ , sleeve friction  $f_s$ , and friction ratio  $R_f$  from CPT.  
CPTs = cone penetration tests



**Figure 4:** Profiles of material index  $I_D$ , horizontal stress index  $K_D$ , and dilatometer modulus  $E_D$  from DMT.  
DMTs = dilatometer tests



**Figure 5:** Profiles of cumulative values  $\Sigma q_c$  from CPTs and cumulative values  $\Sigma E_D$  from DMTs below the foundation level. CPTs = cone penetration tests, DMTs = dilatometer tests



**Figure 6:** Relationship between factor  $R_M$  and horizontal stress index  $K_D$ .  $R^2$  = determination coefficient.

Mayne (1990). The empirical coefficient  $\alpha$  proposed by Kulhawy and Mayne concerns static penetration tests with measurement of pore water pressure (CPTUs). Experience shows that in the case of preconsolidated cohesive soils, the pore water pressure measured during penetration is close to zero, so cone resistance  $q_c$  from CPT can be

assumed in formula (11) instead of total cone resistance  $qt$  (Młynarek et al., 2023).

Based on the comprehensive analysis of the results of DMTs and oedometer tests of the heavily preconsolidated boulder clays prevailing in the Warsaw region, the values of constrained modulus  $M$  obtained from oedometer tests were compared with the values of dilatometer modulus  $E_D$  to obtain factor  $R_M = M/E_D$ . The empirical correlation to evaluate the constrained modulus  $M$  from DMTs was as follows (12):

$$M = R_M E_D \quad (12)$$

where  $R_M$  is the empirical factor related to the horizontal stress index  $K_D$ .

The empirical correlation between factor  $R_M$  and horizontal stress index  $K_D$  proposed by Lechowicz et al. (2017) shown in Figure 6 is (13):

$$R_M = 0.14 + 1.6 \log K_D \quad (13)$$



**Table 2:** Results of statistical analysis of cone resistance  $q_c$  and constrained modulus  $M$  from CPT.

Test type/measure	Layers	Number of determinations $n$ (-)	Mean value $X_{\text{mean}}$ (kPa)	Min. value $X_{\text{min}}$ (kPa)	Max. value $X_{\text{max}}$ (kPa)	Standard deviation $S_x$ (kPa)	Coefficient of variation $V_x$ (-)
CPT: $q_c$	III + IV together	104	11,098	2,500	17,000	3,666	0.333
CPT: $M$			90,498	19,866	139,524	30,329	0.335
CPT: $q_c$	III	15	8,493	2,500	16,000	3,773	0.444
CPT: $M$			69,384	19,866	131,175	31,093	0.448
CPT: $q_c$	IV	89	11,537	5,200	17,000	3,459	0.300
CPT: $M$			94,056	41,349	139,524	28,707	0.305
CPT: $q_c$	IVa	23	13,857	6,600	17,000	2,751	0.199
CPT: $M$			113,710	53,658	139,524	22,303	0.197
CPT: $q_c$	IVb	31	13,661	10,000	17,000	2,100	0.154
CPT: $M$			111,646	81,312	139,326	17,343	0.155
CPT: $q_c$	IVc	35	8,217	5,200	13,000	1,872	0.228
CPT: $M$			66,428	41,349	105,996	15,509	0.233

CPT = cone penetration test

#### 4.4 Settlement calculations

Settlement calculations of spread foundations were carried out taking into account changes in stresses with depth and different values of the constrained modulus for the distinguished layers. Due to that, the preconsolidation stress  $\sigma'_p$  is much higher than the stresses induced by loading, therefore the settlements of spread foundations were calculated as for preconsolidated soils (Lechowicz et al., 2017). For a given layer, settlement  $s$  was calculated using the following formula (14):

$$S = \frac{\Delta\sigma_z \cdot h}{M} \quad (14)$$

where  $\Delta\sigma_z$  is the increase in vertical stress in a given layer caused by building loading,  $h$  is the thickness of a given layer, and  $M$  is the constrained modulus for a given layer.

To determine the distribution of vertical stress beneath uniformly loaded footing, the Fadum nomogram published by Poulos and Davis (1974) was used. In the case of entire cohesive subsoil (layers III + IV, together), the total settlement was calculated assuming a shared value of constrained modulus. For a cohesive subsoil divided into two layers (III and IV), the total settlement was calculated assuming separate values of constrained modulus for each layer. In the case of separation of three

sublayers in layer IV, the different values of constrained modulus for each layer (III, IVa, IVb, and IVc) were used.

## 5 Statistical analysis of CPT and DMT results

### 5.1 Measured $q_c$ and $E_p$ , and evaluated constrained modulus $M$

Profiles of cone resistance  $q_c$  from CPT and constrained modulus  $M$  were subjected to statistical analysis. Statistical analysis was performed for the entire cohesive subsoil, for the subsoil divided into two layers, that is, brown boulder clay (layer III) and gray boulder clay (layer IV), as well as for a layer of gray boulder clay subdivided into three sublayers (IVa, IVb, and IVc). For each dataset, the mean, minimum, and maximum values with standard deviation were calculated according to formula (3) and the coefficient of variation was calculated according to formula (2). Results of statistical analysis of the cone resistance  $q_c$  and the constrained modulus  $M$  are shown in Table 2.

Statistical analysis of the results from CPT carried out for the entire cohesive subsoil indicates large variability of the constrained modulus  $M$  with the coefficient of variation  $V_x = 0.335$ . Large variability of the constrained

**Table 3:** Results of statistical analysis of the dilatometer modulus  $E_d$  and the constrained modulus  $M$  from DMTs.

Test type/measure	Layers	Number of determinations $n$ (-)	Mean value $X_{\text{mean}}$ (kPa)	Min. value $X_{\text{min}}$ (kPa)	Max. value $X_{\text{max}}$ (kPa)	Standard deviation $S_x$ (kPa)	Coefficient of variation $V_x$ (-)
DMT: $E_d$	III + IV together	137	55,527	16,214	116,592	19,621	0.353
DMT: $M$			117,863	26,676	273,266	53,331	0.452
DMT: $E_d$	III	20	85,336	16,214	116,592	20,287	0.238
DMT: $M$			201,715	26,676	273,266	51,754	0.257
DMT: $E_d$	IV	117	50,432	23,683	87,444	14,233	0.282
DMT: $M$			103,529	40,265	205,619	38,277	0.370
DMT: $E_d$	IVa	20	68,106	45,544	87,444	11,218	0.165
DMT: $M$			157,888	105,708	205,619	26,800	0.170
DMT: $E_d$	IVb	36	54,552	29,039	74,582	11,225	0.206
DMT: $M$			115,362	52,880	164,621	27,986	0.243
DMT: $E_d$	IVc	61	42,205	23,683	60,118	9,634	0.228
DMT: $M$			78,724	40,265	117,792	21,111	0.268

DMTs = dilatometer tests

modulus  $M$  was particularly obtained for layer III with the coefficient of variation  $V_x = 0.448$ . The separation of three sublayers within layer IV resulted in the reduction of the coefficient of variation from  $V_x = 0.305$  to  $V_x = 0.155$ – $0.233$ .

Profiles of the dilatometer modulus  $E_d$  from DMTs and the constrained modulus  $M$  were subjected to statistical analysis. For each dataset, the mean, minimum, and maximum values with the standard deviation and the coefficient of variation were calculated. Results of statistical analysis of the dilatometer modulus  $E_d$  and the constrained modulus  $M$  are shown in Table 3.

Statistical analysis of the results from DMTs carried out for the entire cohesive subsoil indicates a large variability of the constrained modulus  $M$  with the coefficient of variation  $V_x = 0.452$ . Compared to the CPT results, less variability of the constrained modulus  $M$  was obtained for layer III with the coefficient of variation  $V_x = 0.257$ . The separation of three sublayers in layer IV reduced the coefficient of variation from  $V_x = 0.370$  to  $V_x = 0.170$ – $0.268$ .

## 5.2 Characteristic values of the constrained modulus $M_k$ from CPTs and DMTs

Calculations of the characteristic values of the constrained modulus  $M_k$  from CPTs and DMTs were carried out using Schneider's formula (7) and according to standard prEN 1997-1:2022-09.

The following assumptions were made for the calculation of the characteristic values of the constrained modulus  $M$  according to standard prEN 1997-1:2022-09:

- The serviceability limit state is sensitive to the spatial variability of the subsoil, that is, settlement calculations constituting case B.
- The calculation method was adopted as for case 3: when the value of  $V_x$  is unknown, then the value of coefficient  $k_n$  according to Table A.1 (prEN 1997-1:2022-09) should be calculated according to the following formula (15):

$$k_n = t_{95,n-1} \sqrt{1 + \frac{1}{n}} \quad (15)$$

where  $t_{95,n-1}$  is Student's  $t$ -distribution, estimated for a 95% confidence level and  $n - 1$  degrees of freedom, and  $n$  is the number of measurements.

- The following calculations were performed in individual datasets for the separated soil layers:
- variable  $M$  was assumed to have a normal distribution,
- $X_{\text{mean}}$  was calculated according to formula (1),
- standard deviations  $S_x$  for the sample were calculated according to formula (3),
- the coefficient of variation  $V_x$  was calculated according to formula (2), and

**Table 4:** Characteristic values of the constrained modulus  $M$  from CPTs and DMTs and the calculated and measured settlements of pad footing H-16.

Test type	Layers	Settlement calculated using											$s_m$ (mm)
		$M_{mean}$		Bayesian statistics		Schneider's formula		prEN 1997-1:2022-09					
		$M_{mean}$ (kPa)	$\Sigma s_{cal}$ (mm)	$M_{mean}$ (kPa)	$\Sigma s_{cal}$ (mm)	$M_k$ (kPa)	$\Sigma s_{cal}$ (mm)	$V_x$ (-)	$k_n$	$(1 - k_n V_x)$	$M_k$ (kPa)	$\Sigma s_{cal}$ (mm)	
CPT	III + IV together	90,498	7.8	90,498	7.8	75,261	9.4	0.335	1.67	0.441	39,910	17.7	7.0
	III	69,384		50,020		53,292		0.448	1.82	0.185	12,840	-	-
	IV	94,056	8.3	100,900	9.1	79,621	10.2	0.305	1.68	0.488	45,899	27.4	7.0
	III	69,384		50,020		53,292		0.448	1.82	0.185	12,840	-	-
	IVa	113,043		125,079		101,420		0.197	1.75	0.655	74,480	-	-
	IVb	111,221		115,218		102,460		0.155	1.73	0.732	81,410	-	-
	IVc	66,428	8.2	66,688	9.1	58,560	9.6	0.233	1.71	0.602	39,990	24.4	7.0
DMT	III + IV together	117,863	6.0	117,863	6.0	91,100	7.8	0.452	1.67	0.245	28,876	24.5	7.0
	III	201,715		195,768		175,165		0.257	1.77	0.545	109,935	-	-
	IV	103,529	5.8	104,441	5.8	84,309	7.1	0.370	1.67	0.382	39,548	14.4	7.0
	III	201,715		195,768		175,165		0.257	1.77	0.545	109,935	-	-
	IVa	157,887		154,300		144,139		0.170	1.77	0.699	110,364	-	-
	IVb	115,361		118,200		101,117		0.243	1.71	0.584	67,371	-	-
	IVc	78,724	5.3	81,241	5.3	68,080	6.1	0.268	1.68	0.550	43,298	9.1	7.0

CPT = cone penetration test, DMT = dilatometer test

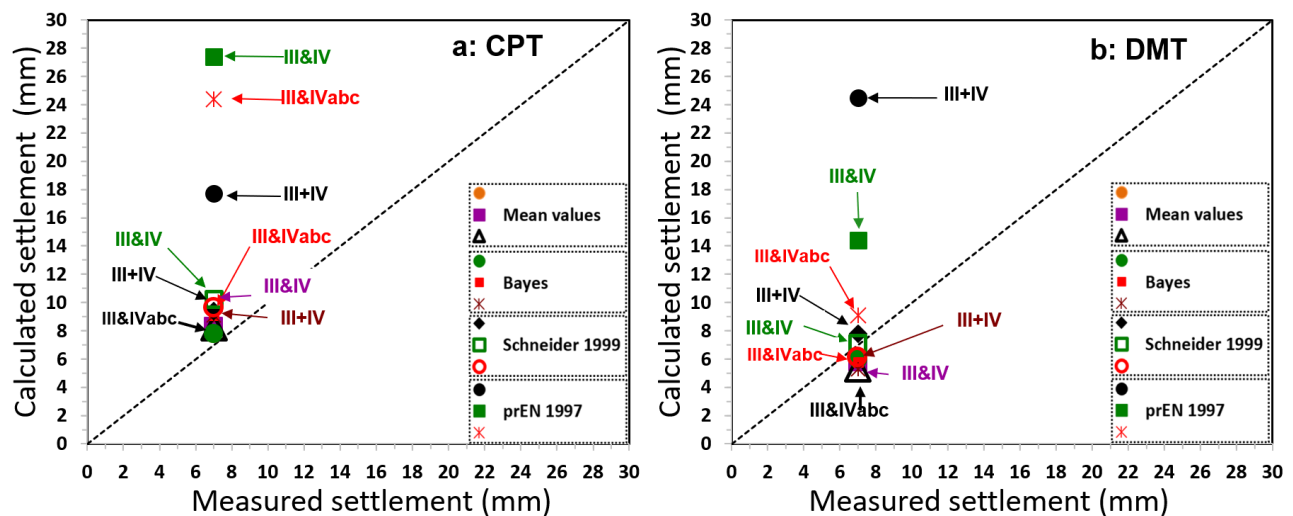
- the values of the coefficient  $k_n$  were read from Table A.7 (prEN 1997-1:2022-09) for the appropriate numbers  $n$ .
- It was decided that the representative value of constrained modulus  $M$  is its smaller characteristic value  $X_k$  calculated according to formula (10) because in the case of settlement calculations, a smaller value of the deformation modulus is unfavorable.

Characteristic values of the constrained modulus  $M$  were determined from CPT and DMT (both separately and jointly) for the entire cohesive subsoil, for subsoil divided into two layers of brown boulder clay and gray boulder clay, as well as for a layer of gray boulder clay subdivided into three sublayers. Table 4 shows the characteristic values of the constrained modulus  $M$ . Analysis of the calculation results indicates that the lowest characteristic values of the constrained modulus  $M$  were obtained using the draft standard prEN 1997-1:2022-09.

## 6 Settlement analysis

The mean values of the constrained modulus  $M_{mean}$  and the characteristic values of the constrained modulus  $M_k$  obtained from CPTs and DMTs were used in settlement calculations of pad footing H-16, considering changes in stresses and the constrained modulus. The mean values of the constrained modulus  $M_{mean}$  were obtained from classic mathematical statistics and Bayesian analysis. The characteristic values of the constrained modulus  $M_k$  were evaluated using Schneider's formula and the draft European standard prEN 1997-1:2022-09. For a given layer, settlement  $s$  was calculated using formula (14). The calculated and measured settlements of pad footing H-16 are shown in Table 4 and Figure 7.

A comparison of settlements indicates that the calculated settlements based on the characteristic values of the constrained modulus  $M_k$  using statistical methods according to prEN 1997-1:2022-09 obtained from CPT were much higher (17.7–27.4 mm) than the measured values (7.0 mm). Large variability of the constrained modulus of layer



**Figure 7:** Comparison of measured and calculated settlements based on the constrained modulus determined from: a – CPT tests; b – DMT tests.

CPTs = cone penetration tests, DMTs = dilatometer tests

III resulted in low characteristic values of the constrained modulus  $M_k$ . The settlement calculated for the entire cohesive subsoil (17.7 mm) based on the shared value of constrained modulus was smaller than the settlement calculated for the cohesive subsoil divided into two layers III and IV (27.4 mm) based on the separate values of constrained modulus for each layer. Low characteristic values of the constrained modulus  $M_k$  of layer III caused that the calculated settlement (24.4 mm) of the cohesive subsoil with separation of three sublayers in layer IV was also higher than the measured value.

Smaller differences between the measured settlements (7.0 mm) and the settlements calculated based on characteristic values of the constrained modulus  $M_k$  using statistical methods according to prEN 1997-1:2022-09 were obtained from DMTs (9.1–24.5 mm) compared to CPTs. This is due to the fact that the constrained modulus from DMTs was determined based on the empirical correlation elaborated for heavily preconsolidated boulder clays prevailing in the Warsaw region. Smaller variability of the constrained modulus of layer III resulted in higher characteristic values of the constrained modulus  $M_k$ . The settlement calculated for the cohesive subsoil divided into two layers III and IV (14.4 mm) based on the separate values of constrained modulus for each layer was smaller than the settlement calculated for the entire cohesive subsoil (24.5 mm) based on the shared value of constrained modulus. A much smaller difference between the measured settlement (7.0 mm) and the calculated settlement (9.1 mm) was obtained when layer IV of the gray boulder clay was subdivided into three sublayers (IVa,

IVb, and IVc). The reduction in the coefficient of variation  $V_x$  values caused this. In this case, the difference between the measured settlements and the settlements calculated based on characteristic values of the constrained modulus  $M_k$  obtained from DMTs using Schneider's formula was much smaller (7.0 and 6.1 mm, respectively).

It should be pointed out that settlements obtained from CPT and DMT calculated based on mean values of the constrained modulus determined from Bayesian analysis or classically according to formula (1), as well as according to Schneider's formula, were much closer to the measured settlement values.

## 7 Conclusions

Based on the presented example of calculating the settlement of cohesive soil layers in subsoil under a spread foundation, the following conclusions can be drawn:

- Calculation of foundation settlements based on CPT and DMT still raises many discussions and doubts; in particular, they concern the interpretation of test results and the method of determining the characteristic values of the soil constrained modulus in the subsoil.
- Depending on the method of analyzing the results, different values of constrained modulus are obtained, which cause large differences in the calculated settlement values.



- Characteristic values of the constrained modulus calculated on the basis of the statistical method according to the draft standard prEN 1997-1:2022-09 are the smallest among the values calculated using other methods, and therefore cause overestimation of the calculated settlement values compared to the measured settlements. The dispersion of the obtained test results, expressed by the coefficient of variation, is of significant influence on the underestimation of the characteristic value of the constrained modulus.
- The variability of the constrained modulus obtained in individual soils was large: for CPT – 44.8% in layer III and 30.5% in layer IV; for DMT – 25.7% in layer III and 37.0% in layer IV.
- The subdivision of layer IV into three sublayers (IVa, IVb, IVc) resulted in the reduction of the coefficient of variation of the constrained modulus: for CPT – 15.5%–23.3%, for DMT – 17.0%–26.8%, which reduced the values of the calculated settlements.
- In general, it should be noted that the method of determining the characteristic modulus according to the draft standard prEN 1997-1:2022-09 results in calculating the settlements higher than the measured settlements.
- Settlements calculated on the basis of mean values of the constrained modulus determined by the Bayesian analysis or classically according to formula (1), as well as according to Schneider's formula, are much closer to the measured settlement values.

## Author Contributions:

S.R. prepared the research program, performed the cone penetration test and dilatometer test, analyzed the test results, and prepared the manuscript. Z.L. prepared the research program, analyzed the test results, and prepared the manuscript. M.J.S. performed the statistical analysis and prepared the manuscript.

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## Conflicts of Interest

The authors declare no conflict of interest.

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