Kamil Pankowski

e-mail: biuro@kamilpankowski.pl

ORCID: 0009-0008-4395-8493

Wroclaw University of Economics and Business

The Impact of Fluctuations in the Number of Economic Entities in Sectors on Inter-Sectoral Dependencies: Application of Predictions in the Leontief Input-Output Model

DOI: 10.15611/2025.44.3.16

JEL Classification: C67, E37

Quote as: Pankowski, K. (2025). The Impact of Fluctuations in the Number of Economic Entities in Sectors on Inter-Sectoral Dependencies: Application of Predictions in the Leontief Input-Output Model. In A. Witek-Crabb & J. Radomska (Eds.), *New Trends in Business Management. Strategy, Branding, Teamwork* (pp. 203-220). Publishing House of Wroclaw University of Economics and Business.

Abstract: The objective of this paper is to examine the impact of fluctuations in the number of economic entities across different sectors on inter-sectoral dependencies. The research is based on data from the Central Statistical Office and the application of the Leontief input-output model. The goal of the study is to recognise dependencies between sectors and determine which sectors are most susceptible to changes in other parts of the economy. In the analysis, advanced analytical tools were used, including the input-output model which allows for a precise assessment of inter-sectoral flows. The results of the research indicate notable differences in the resilience of individual sectors to changes, as well as the presence of potential supply gaps that may arise from a reduction in the number of entities in certain sectors. The conclusions drawn from the analyses can serve as a foundation for managerial decision-making and for developing effective risk management strategies.

Keywords: fluctuations, Leontief model, inter-sectoral analysis, economic risk, supply gap, prediction

1. Introduction

Modern economies operate in an extremely dynamic and complex environment, which requires constant analysis and adaptation to changing conditions. The analysis of changes in the number of economic entities across different sectors is therefore essential for assessing the economic health of a country and for effective risk management. This study focuses on examining data from the Central Statistical Office (Główny Urząd Statystyczny – GUS) to gauge these changes and determine their impact on sectoral relations and interactions between sectors.

The main objective of the study is to examine how changes in the number of entities within one sector affect other sectors. The analysis is based on check and quantifying intersectoral dependencies, which allows for determining which sectors are most susceptible to disruptions in other areas of the economy.

However, the time frame (2022-2024) may influence the results, as these years follow the COVID-19 pandemic which caused significant economic disruptions. The post-pandemic recovery period may not fully reflect long-term sectoral dependencies, and further studies should incorporate data from more stable economic periods.

To achieve this goal, advanced analytical tools were used, such as Computable General Equilibrium (CGE) models and the Social Accounting Matrix (SAM) (Shoven & Whalley, 1992), including Wassily Leontief's input-output model (Leontief, 1986). The model was developed by Wassily Leontief, an American economist and Nobel laureate in Economics in 1973 for "the development of the input-output method and for its application to important economic problems" (The Nobel Prize, 1973). Input-output analysis, originally developed by Leontief, has since evolved to address not only linear but also non-linear systems. Sahani and Prasad (2023) explore this complexity in their analysis, providing insights into how this methodology can be adapted for non-linear scientific contexts. Due to its ability to analyse inter--industry flows, the model, after modification, allows for a precise estimation of how fluctuations in one sector, such as a reduction in the number of entities or deregistration from the REGON database, affect other sectors. As a result, it will be possible not only to check sectors with the greatest interdependencies but also those that demonstrate greater resilience to economic shocks.

This analysis also aims to examine potential supply gaps that may arise as a result of a decrease in the number of entities in a given sector (Mankiw, 2016). A supply gap, defined as a mismatch between supply and demand for a particular good or service, can result from the elimination of a key supplier from the market. The analysis of such gaps will allow for the development of appropriate remedial strategies, such as stimulating the growth of new entities within the sector or increasing imports.

Changes in the number of economic entities are a natural phenomenon in a market economy, serving as a cleansing mechanism that allows for restructuring and the emergence of new, more efficient entities. However, major changes in the number of entities can also pose a threat to financial stability, especially when they occur in strategic sectors of the economy. Therefore, it is important that the analysis of these phenomena is conducted systematically and comprehensively.

Peter Drucker, one of the most influential business theorists and often regarded as the father of modern management, emphasised the importance of proactive leadership and strategic thinking. A quote frequently attributed to him – "the best way to predict the future is to create it" – although of uncertain origin, reflects the idea that in an economic context, a proactive approach to risk management and strategic planning is essential for long-term success (Drucker, 1993). Predictive models, which can be developed and tested using advanced econometric tools, such as statistical Machine Learning (ML) methods, and Artificial Intelligence (AI), form the foundation of this approach. They allow not only for better forecasting of future trends but also for more effective resource allocation and support for sectors most at risk.

The economist Joseph Schumpeter introduced the concept of "creative destruction", (Schumpeter, 1942) emphasizing that technological innovations can lead to the downfall of old economic structures while simultaneously making room for new, more innovative companies. Changes in the number of entities in economic sectors, in this context, are not merely a negative phenomenon but also an essential element of the economic cycle that drives progress and development. An innovative approach to analysing macroeconomic trends can therefore yield important benefits, allowing for the observation not only of risks but also of potential areas for growth.

Key elements of this analysis are intersectoral cooperation, diversification of activities, and educating entrepreneurs in risk management. As observed by Alfred Marshall in *Principles of Economics*, industrial success often depends on firms' ability to adjust its processes to local and changing conditions. This capacity for adaptation and variation contributes to economic resilience and long-term efficiency. Effective risk management requires not only appropriate analytical tools but also the involvement of all stakeholders in the process of continuous improvement and innovation (Marshall, 1920, p. 268).

People tend to recognize the need for change only in times of crisis. As Taleb (2007) emphasizes, "Black Swans being unpredictable, we need to adjust to their existence rather than naively try to predict them." Therefore, the development and application of predictive models should not aim to eliminate uncertainty but to support better preparedness and adaptability to unexpected events. Their responsible and long-term implementation – both by the state, which supports the economy in times of instability, and by decision-makers at managerial and ownership levels – can enhance economic resilience and improve the capacity to anticipate structural shifts within the economy.

The study will present a detailed analysis of data from the Central Statistical Office (GUS) aimed at checking the main trends and determinants of changes in the number of entities in the Polish economy. The research methodology, analysis results, and recommendations for risk management and support for sectors most vulnerable to disruptions will also be discussed.

The study is particularly relevant for policymakers, restructuring advisors, and firms in high-risk sectors. By identifying intersectoral dependencies, it supports targeted policy interventions and restructuring strategies aimed at mitigating economic disruptions and enhancing sectoral resilience.

2. Analysis of Quarterly Changes in the Number of Entities in Sectors

In the economic analysis of sectors, it is important to recognize those that show sensitivity to macroeconomic changes as well as the risks specific to their activities. In Poland, the classification of economic sectors is based on the Polish Classification of Activities (PKD 2007) which is consistent with the European NACE Rev.2 classification. The PKD serves a statistical function, used for collecting and analysing data on economic entities.

The PKD consists of five levels of detail, ranging from the general section to division, group, class, and subclass, each differing in the level of specificity in describing economic activities. At the highest level, 21 sectors are distinguished, marked by letters of the alphabet (A-U). This classification is used in the process of registering new economic entities as well as in cases of changing or expanding activities. For the purposes of economic analysis, eight main sectors were identified: industry, construction, trade, transport, accommodation and food services, information and communication, services, and other sectors. Table 1 shows the number of active enterprises in each sector by quarter. This categorisation allows for more focused research, which can reveal the impact of these sectors on the overall economic condition. As a result, it is possible to more effectively monitor areas that are potentially at risk or have the greatest development potential. In detailed analyses, it is possible to refer to specific PKD codes, depending on trade between them and the quantitative and value-based transaction data. This level of detail allows for more effective observation of inter-sectoral dependencies, which serves as the basis for further research using, for example, the Leontief model. Table 2 presents the share of each sector in the total number of entities in the economy.

The 21 sectors, which in the list of sections of the Polish Classification of Activities are marked by successive letters of the alphabet. Within this division, the following categories were distinguished:

A – agriculture, forestry, hunting, and fishing; 01-03;
B – mining, open-pit extraction; 05-09;
C – production, industrial processing; 10-33;
D – energy supply, production and supply of electricity, gas, steam, hot water, and air for air-conditioning systems; 35;
E – water supply, water pollution, waste management; 36-39;
F – construction; 41-43;
G – wholesale and retail trade, repair of motor vehicles and motorcycles; 45-47;
H – transportation and storage; 49-53;
I – accommodation and food service activities; 55-56;
J – information, communication; 58-63;
K – financial and insurance activities; 64-66;

L – real estate activities; 68;
M – professional, scientific, and technical activities; 69-75;
N – administrative and support service activities; 77-82;
O – public administration, defence, compulsory social security; 84;
P – education; 85;
Q – health and social care; 86-88;
R – activities related to culture, entertainment, and recreation; 90-93;
S – other service activities; 94-96;
T – household activities (as an employer), production of goods and household services for own use; 97-98;
U – extraterritorial organizations and bodies; 99.

Table 1. Number of active enterprises

	2022				2023				2024		
	1Q	2Q	3Q	4Q	1Q	2Q	3Q	4Q	1Q	2Q	3Q
Total	**3908848**	**3949844**	**3974520**	**3976514**	**3974072**	**4022237**	**4045637**	**4043243**	**4057392**	**4103297**	**4117233**
S1	358626	360521	361408	359623	357542	359655	360901	358931	357964	360127	359784
S2	533605	541752	547789	542060	539367	546849	551945	546012	546190	555015	558489
S3	852004	853538	852294	848141	840154	843564	842714	837083	831983	834543	831770
S4	233500	233395	233928	233738	232232	233128	233592	232112	231369	232171	232320
S5	120205	129078	124901	119984	119086	128727	125068	120279	120091	130291	126509
S6	195768	203743	211934	219155	225805	231590	236633	240496	243941	247042	249932
S7	894129	902016	908428	912148	915413	928789	936668	941173	945424	957112	963732
S8	721011	725801	733838	741665	744473	749935	758116	767157	780430	786996	794797

Source: own elaboration based on data from the Central Statistical Office.

Table 2. Percentage share of the sector in the entire economy

	2022				**2023**				**2024**		
	1Q	2Q	3Q	4Q	1Q	2Q	3Q	4Q	1Q	2Q	3Q
	100	**100**	**100**	**100**	**100**	**100**	**100**	**100**	**100**	**100**	**100**
S1	9.2	9.1	9.1	9	9	8.9	8.9	8.9	8.8	8.8	8.7
S2	13.7	13.7	13.8	13.6	13.6	13.6	13.6	13.5	13.5	13.5	13.6
S3	21.8	21.6	21.4	21.3	21.1	21	20.8	20.7	20.5	20.3	20.2
S4	6	5.9	5.9	5.8	5.8	5.8	5.8	5.7	5.7	5.7	5.6
S5	3.1	3.3	3.1	3	3	3.2	3.1	3	3	3.2	3.1
S6	5	5.2	5.3	5.5	5.7	5.8	5.8	5.9	6	6	6.1
S7	22.9	22.8	22.9	22.9	23	23.1	23.2	23.3	23.3	23.3	23.4
S8	18.4	18.4	18.5	18.7	18.7	18.6	18.7	19	19.2	19.2	19.3

Source: own elaboration based on data from the Central Statistical Office.

Out of the 21 sectors in the classification, eight were distinguished to represent the subsequent model.

S1 – Industry (sections B, C, D, E);
S2 – Construction (section F);
S3 – Trade; repair of motor vehicles (section G);
S4 – Transportation and storage (section H);
S5 – Accommodation and food service activities (section I);
S6 – Information and communication (section J);
S7 – Services (sections K, L, M, N);
S8 – Remaining sections (P, Q, R, S excluding division 94, A,O).

Figures 2-9 present the quarterly number of active entities for each sector S1 to S8.

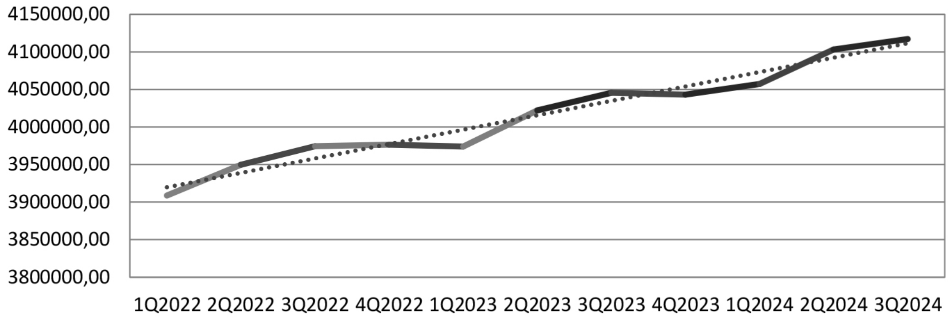

Fig. 1. Total number of active entities with REGON

Source: own elaboration based on data from the Central Statistical Office.

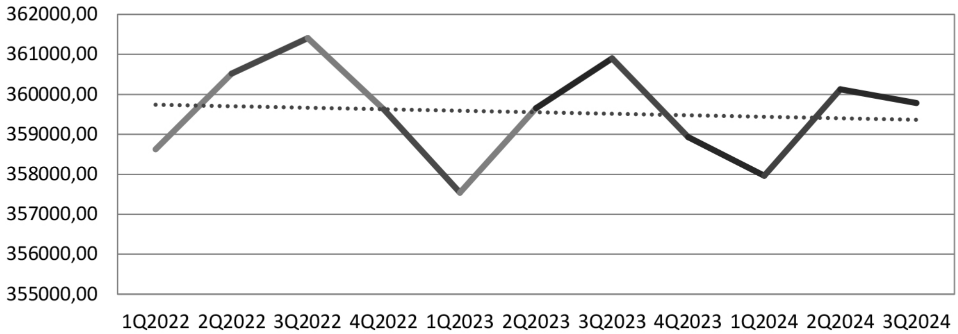

Fig. 2. Total number of entities in the sector S1 – Industry

Source: own elaboration based on data from the Central Statistical Office.

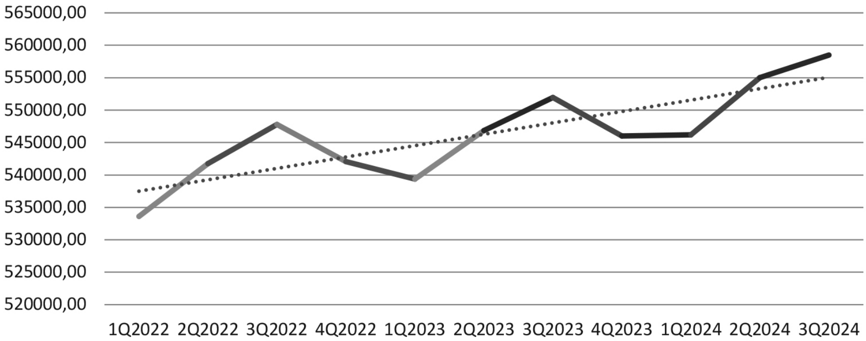

Fig. 3. Total number of entities in the sector S2 – Construction

Source: own elaboration based on data from the Central Statistical Office.

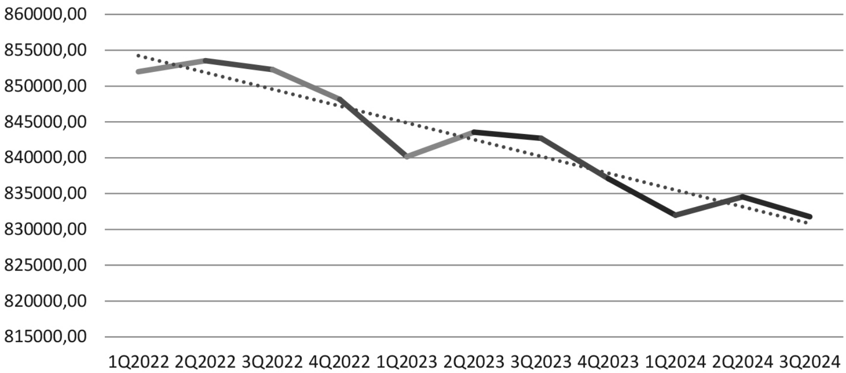

Fig. 4. Total number of entities in the sector S3 – Trade

Source: own elaboration based on data from Central Statistical Office.

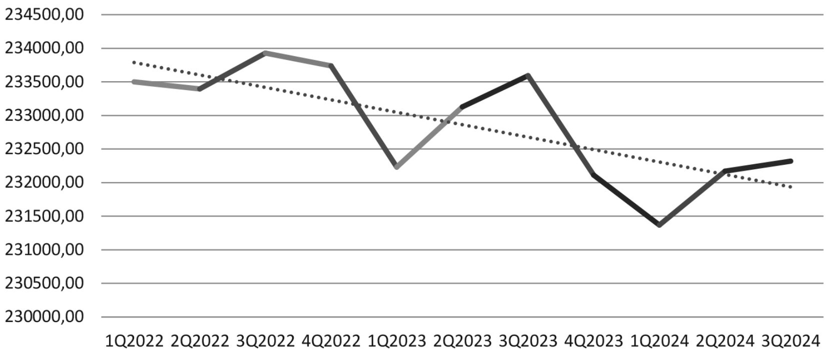

Fig. 5. Total number of entities in the sector S4 – Transportation and storage

Source: own elaboration based on data from the Central Statistical Office.

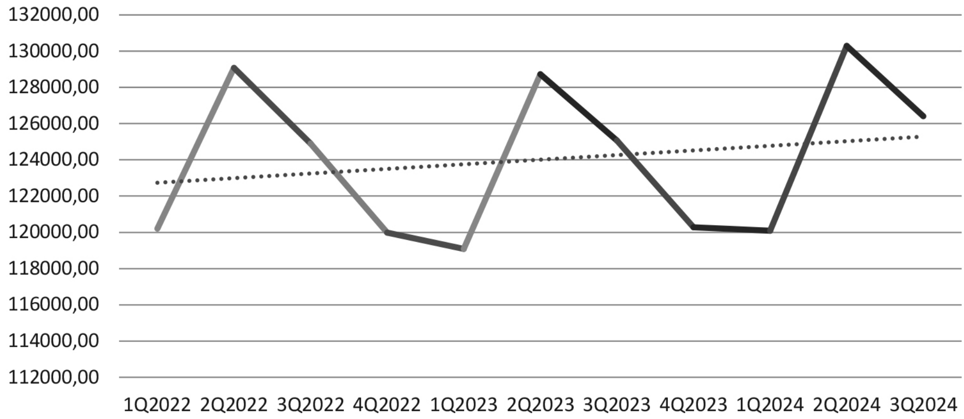

Fig. 6. Total number of entities in the sector S5 – Accommodation and food service activities

Source: own elaboration based on data from the Central Statistical Office.

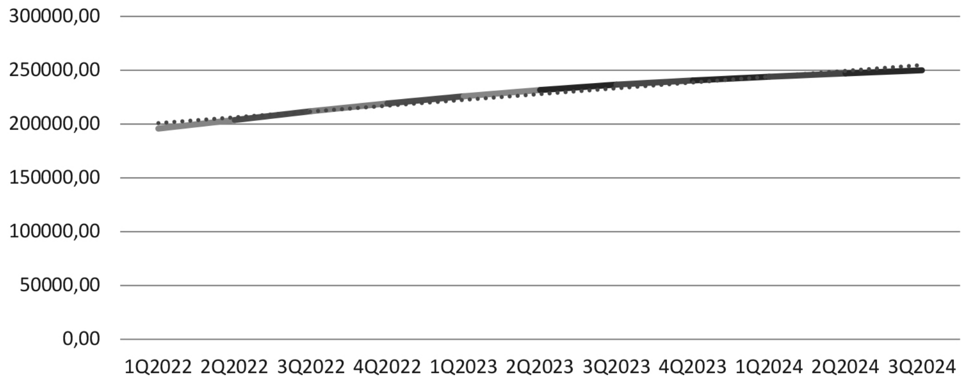

Fig. 7. Total number of entities in the sector S6 – Information and communication

Source: own elaboration based on data from the Central Statistical Office.

In the subsequent part of this study, following the presentation of results in the form of figures and tables, we will conduct a quantitative analysis aimed at using Leontief's input-output model to examine the relationships between different economic sectors. This model, based on an intersectoral flow matrix, allows for a precise assessment of how changes in one sector affect other sectors, facilitating a more comprehensive evaluation of mutual interactions. Using this approach, we will be able to estimate how changes in the number of entities in one sector such as declines due to bankruptcies may impact other sectors through both direct

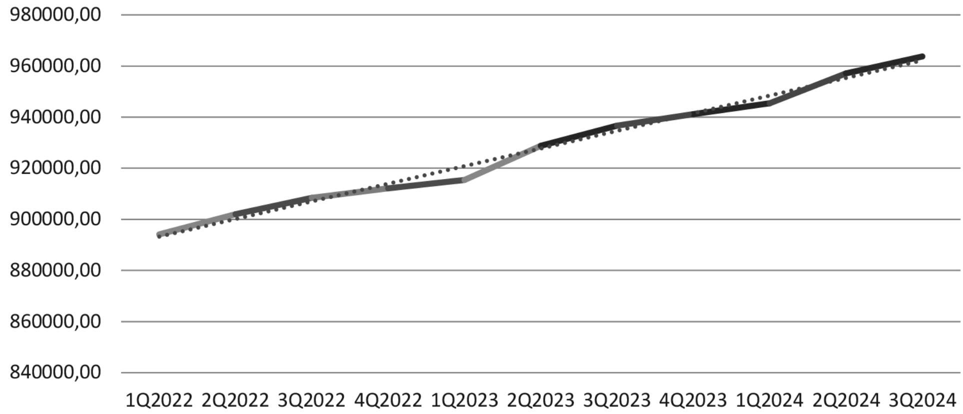

Fig. 8. Total number of entities in the sector S7 – Services

Source: own elaboration based on data from the Central Statistical Office.

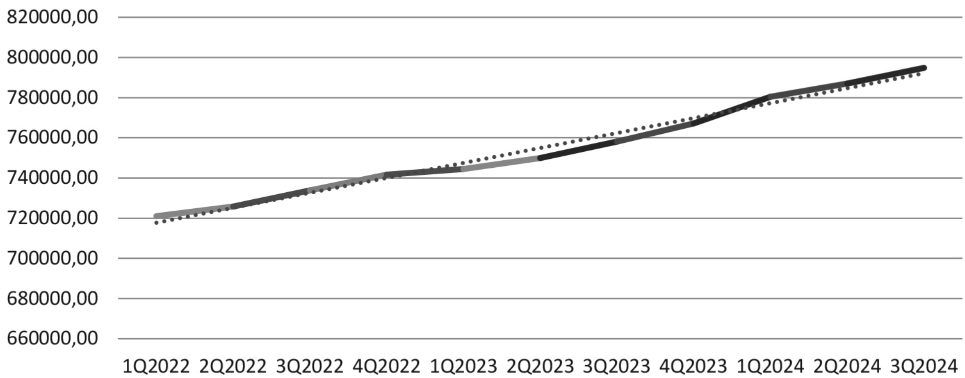

Fig. 9. Total number of entities in the sector S8 – Remaining sections

Source: own elaboration based on data from the Central Statistical Office.

and indirect linkages. Leontief's model allows for an analysis of both the immediate effects and the long-term consequences of these changes.

The results of this analysis will serve as a foundation for further research into the resilience of individual sectors to shocks and their role in shaping supply gaps resulting from the elimination of suppliers or producers due to bankruptcy, industry shifts, or business closures. Utilizing an intersectoral flow matrix will formalize these dependencies and allow for precise estimation of potential economic impacts, providing a basis for predicting future changes and developing effective risk management strategies.

Such an approach is particularly important given the complexity of a market economy, where fluctuations in the number of enterprises can lead to important changes in the flow of goods and services, with far-reaching consequences for many interconnected sectors.

3. Calculation of the Leontief Matrix

3.1. Objective and Theoretical Basis

The Leontief input-output model, used in economic analysis, allows for assessing sectoral dependencies based on interindustry transactions. Its advantage lies in its ability to quantify direct and indirect effects of sectoral fluctuations. However, its static nature and reliance on historical data limit its capacity to capture dynamic economic changes, such as technological shifts or rapid market disruptions. Despite these constraints, it remains a robust tool for structural economic analysis, particularly in evaluating supply chain vulnerabilities.

3.2. Calculation of sectoral shares

To determine the indicators used in constructing the Leontief matrix, I based the analysis on the number of entities in each sector for every quarter. For each sector, I calculated its percentage share of the total number of entities in the economy. The formula used to calculate this share is:

$$a_{ij} = \frac{\text{number of entities in the sector in a given quarter}}{\text{total number of entities in the economy in a given quarter}}.$$

Example: for the industry sector in q1

$$a_{ij} = \frac{358626}{3908848},$$

$$a_{ij} = 0.0917.$$

These calculations were performed for each sector and each quarter.

3.3. Average Sector Coefficients

For each sector, I calculated the average share in the economy based on data from 10 quarters (from the first to the tenth quarter). The average share of each sector serves as the basis for constructing the technical coefficient matrix (**A**).

Example for the industry sector

$$\text{average share of the industry sector} = \frac{0.0917+0.0913+\cdots+0.0874}{11} = 0.0898.$$

3.4. Construction of the Technical Coefficient Matrix (A)

The technical coefficient matrix (**A**) was constructed by creating an 8 × 8 matrix (for 8 sectors), where each element a_{ij} was calculated as the product of the average shares of the sectors. The value a_{ij} represents the influence of sector *i* on sector *j*, under the assumption that all sectors are interconnected within the economy and their percentage shares may indicate mutual interactions.

Example: if the average share of the industry sector was 9% (0.0898), and the average share of the services sector was 23% (0.2307), then the element $a_{\text{industry, services}}$ will be:

$$a_{\text{industry,services}} = 0.0898 \times 0.2307 = 0.0207.$$

The results for each sector are shown in Tab. 3.

Table 3. Results for each sector

	S1	S2	S3	S4	S5	S6	S7	S8
S1	0.0080	0.0122	0.0188	0.0052	0.0028	0.0051	0.0207	0.0168
S2	0.0122	0.0185	0.0286	0.0079	0.0042	0.0077	0.0314	0.0256
S3	0.0188	0.0286	0.0441	0.0122	0.0065	0.0119	0.0485	0.0394
S4	0.0052	0.0079	0.0122	0.0034	0.0018	0.0033	0.0134	0.0109
S5	0.0028	0.0042	0.0065	0.0018	0.001	0.0018	0.0071	0.0058
S6	0.0051	0.0077	0.0119	0.0033	0.0018	0.0032	0.0131	0.0107
S7	0.0207	0.0314	0.0485	0.0134	0.0071	0.0131	0.0534	0.0434
S8	0.0168	0.0256	0.0394	0.0109	0.0058	0.0107	0.0434	0.0353

Source: own elaboration.

Having the normalised values, we proceed to construct the matrix.

3.5. Construction of Matrix A and Matrix L

$$\mathbf{A} = \begin{bmatrix} 0.0080 & 0.0122 & 0.0188 & 0.0052 & 0.0028 & 0.0051 & 0.0207 & 0.0168 \\ 0.0122 & 0.0185 & 0.286 & 0.0079 & 0.0042 & 0.0077 & 0.0314 & 0.0256 \\ 0.0188 & 0.0286 & 0.0441 & 0.0122 & 0.065 & 0.0119 & 0.0485 & 0.0394 \\ 0.0052 & 0.0079 & 0.0122 & 0.0034 & 0.0018 & 0.0033 & 0.0134 & 0.0109 \\ 0.0028 & 0.0042 & 0.0065 & 0.0018 & 0.0001 & 0.0018 & 0.0071 & 0.0058 \\ 0.0051 & 0.0077 & 0.0119 & 0.0033 & 0.0018 & 0.0032 & 0.0131 & 0.0107 \\ 0.0207 & 0.0314 & 0.0485 & 0.0134 & 0.0071 & 0.0131 & 0.0534 & 0.0434 \\ 0.0168 & 0.0256 & 0.0394 & 0.0109 & 0.0058 & 0.0107 & 0.0434 & 0.0353 \end{bmatrix}.$$

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Matrix **L** = (**I** – **A**):

$$(\mathbf{I}-\mathbf{A}) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.0080 & 0.0122 & 0.0188 & 0.0052 & 0.0028 & 0.0051 & 0.0207 & 0.0168 \\ 0.0122 & 0.0185 & 0.286 & 0.0079 & 0.0042 & 0.0077 & 0.0314 & 0.0256 \\ 0.0188 & 0.0286 & 0.0441 & 0.0122 & 0.065 & 0.0119 & 0.0485 & 0.0394 \\ 0.0052 & 0.0079 & 0.0122 & 0.0034 & 0.0018 & 0.0033 & 0.0134 & 0.0109 \\ 0.0028 & 0.0042 & 0.0065 & 0.0018 & 0.0001 & 0.0018 & 0.0071 & 0.0058 \\ 0.0051 & 0.0077 & 0.0119 & 0.0033 & 0.0018 & 0.0032 & 0.0131 & 0.0107 \\ 0.0207 & 0.0314 & 0.0485 & 0.0134 & 0.0071 & 0.0131 & 0.0534 & 0.0434 \\ 0.0168 & 0.0256 & 0.0394 & 0.0109 & 0.0058 & 0.0107 & 0.0434 & 0.0353 \end{bmatrix},$$

$$(\mathbf{I}-\mathbf{A}) = \begin{bmatrix} 0.9910 & -0.0122 & -0.0188 & -0.0052 & -0.0028 & -0.0051 & -0.0207 & -0.0168 \\ -0.0122 & 0.9815 & -0.286 & -0.0079 & -0.0042 & -0.0077 & -0.0314 & -0.0256 \\ -0.0188 & -0.0286 & 0.9966 & -0.0122 & -0.065 & -0.0119 & -0.0485 & -0.0394 \\ -0.0052 & -0.0079 & -0.0122 & 0.9966 & -0.0018 & -0.0033 & -0.0134 & -0.0109 \\ -0.0028 & -0.0042 & -0.0065 & -0.0018 & 0.999 & -0.0018 & -0.0071 & -0.0058 \\ -0.0051 & -0.0077 & -0.0119 & -0.0033 & -0.0018 & 0.9968 & -0.0131 & -0.010 \\ -0.0207 & -0.0314 & -0.0485 & -0.0134 & -0.0071 & -0.0131 & 0.9466 & -0.0437 \\ -0.0168 & -0.0256 & -0.0394 & -0.0109 & -0.0058 & -0.0107 & -0.0434 & 0.9647 \end{bmatrix}.$$

3.6. Calculating the Inverse of the Matrix $(I – A)^{-1}$ and Determinant Calculation

To calculate the inverse matrix, we first need to find its determinant. Inverting a matrix requires that it be square and have a non-zero determinant. The determinant is a numerical value that indicates whether the matrix is invertible; if the determinant is 0, the matrix is singular (non-invertible). We calculate the cofactor matrix: This is a matrix in which each element is the algebraic complement corresponding to an element of matrix **A**. Then, we transpose the cofactor matrix to obtain the transposed cofactor matrix, and divide it by the determinant of **A**.

$$A^{-1} = \frac{1}{\det(A)} * \text{(transposed cofactor matrix).}$$

The determinant is 0.0833, which means we do not need to perform further calculations to eliminate any potential singularity.

3.7. Calculation of the Inverse Matrix in Excel

I used the function (`MINVERSE`) in Excel to calculate the inverse matrix. The result was:

$$(\mathbf{I}-\mathbf{A})^{-1} = \begin{bmatrix} 1.0096 & 0.0146 & 0.0226 & 0.0062 & 0.0033 & 0.0061 & 0.0248 & 0.0202 \\ 0.0146 & 1.0222 & 0.0343 & 0.0095 & 0.0050 & 0.0093 & 0.0377 & 0.0307 \\ 0.0226 & 0.0343 & 1.0529 & 0.0146 & 0.0078 & 0.0143 & 0.0582 & 0.0473 \\ 0.0062 & 0.0095 & 0.0146 & 1.004 & 0.0021 & 0.0039 & 0.0161 & 0.0131 \\ 0.0033 & 0.0050 & 0.0078 & 0.0021 & 1.0011 & 0.0021 & 0.0086 & 0.0070 \\ 0.0061 & 0.0093 & 0.0143 & 0.0039 & 0.0021 & 1.0039 & 0.0157 & 0.0128 \\ 0.0248 & 0.0377 & 0.0582 & 0.0161 & 0.0086 & 0.0157 & 1.0640 & 0.0521 \\ 0.0202 & 0.0307 & 0.0473 & 0.0131 & 0.0070 & 0.0128 & 0.0521 & 1.0424 \end{bmatrix}.$$

Having the Leontief inverse matrix, I proceed with the actual analysis. To evaluate the accuracy of the model, we need to determine the change vector $\boldsymbol{d}'$

$$x' = L * \boldsymbol{d}',$$

where: x' – a prediction of the percentage change in the number of entities in the respective sector, L – Leontief inverse matrix, $\boldsymbol{d}'$ – change vector.

Vector $\boldsymbol{d}'$will represent the percentage change in the number of entities over successive quarters, as shown in Tab. 4.

Table 4. Quarterly percentage change in the number of entities

	(q2–q1)/ q1	(q3–q2)/ q2	(q4–q3)/ q3	(q5–q4)/ q4	(q6–q5)/ q5	(q7–q6)/ q6	(q8–q7) q7	(q9–q8)/ q8	(q10–q9)/ q9	(q11–q10)/ q10
S1	0.0053	0.0025	–0.0049	–0.0058	0.0059	0.0035	–0.0055	–0.0027	0.0060	–0.0010
S2	0.0153	0.0111	–0.0105	–0.0050	0.0139	0.0093	–0.0107	0.0003	0.0162	0.063
S3	0.0018	–0.0015	–0.0049	–0.0094	0.0041	–0.0010	–0.0067	–0.0061	0.0031	–0.0033
S4	–0.0004	0.0023	–0.0008	–0.0064	0.0039	0.0020	–0.0063	–0.0032	0.0035	0.0006
S5	0.0738	–0.0324	–0.0394	–0.0075	0.0810	–0.0284	–0.0383	–0.0016	0.0849	–0.0298
S6	0.0407	0.0402	0.0341	0.0303	0.0256	0.0218	0.0163	0.0143	0.0127	0.017
S7	0.0088	0.0071	0.0041	0.0036	0.0146	0.0085	0.0048	0.0045	0.0124	0.0069
S8	0.0066	0.0111	0.0107	0.0038	0.0073	0.0109	0.0119	0.0173	0.0084	0.099

Source: own elaboration.

Prediction for q11:
Vector $\boldsymbol{d}'$ = (q10 – q9)/q9:

$$d' = \begin{bmatrix} 0.0060 \\ 0.0162 \\ 0.0031 \\ 0.0035 \\ 0.0849 \\ 0.0127 \\ 0.0124 \\ 0.0084 \end{bmatrix},$$

$$x' = \begin{matrix} 1.0096 & 0.0146 & 0.0226 & 0.0062 & 0.0033 & 0.0061 & 0.0248 & 0.0202 \\ 0.0146 & 1.0222 & 0.0343 & 0.0095 & 0.0050 & 0.0093 & 0.0377 & 0.0307 \\ 0.0226 & 0.0343 & 1.0529 & 0.0146 & 0.0078 & 0.0143 & 0.0582 & 0.0473 \\ 0.0062 & 0.0095 & 0.0146 & 1.004 & 0.0021 & 0.0039 & 0.0161 & 0.0131 \\ 0.0033 & 0.0050 & 0.0078 & 0.0021 & 1.0011 & 0.0021 & 0.0086 & 0.0070 \\ 0.0061 & 0.0093 & 0.0143 & 0.0039 & 0.0021 & 1.0039 & 0.0157 & 0.0128 \\ 0.0248 & 0.0377 & 0.0582 & 0.0161 & 0.0086 & 0.0157 & 1.0640 & 0.0521 \\ 0.0202 & 0.0307 & 0.0473 & 0.0131 & 0.0070 & 0.0128 & 0.0521 & 1.0424 \end{matrix} \times \begin{bmatrix} 0.0060 \\ 0.0162 \\ 0.0031 \\ 0.0035 \\ 0.0849 \\ 0.0127 \\ 0.0124 \\ 0.0084 \end{bmatrix} = \begin{bmatrix} 0.0073 \\ 0.0180 \\ 0.0059 \\ 0.043 \\ 0.0854 \\ 0.0135 \\ 0.0155 \\ 0.0110 \end{bmatrix}.$$

The obtained change vector is multiplied by the number of entities in q10, and then q10 is added to obtain the prediction for q11.

$$\boldsymbol{x'q11} = \begin{bmatrix} 362743 \\ 565013 \\ 839500 \\ 233159 \\ 141412 \\ 250373 \\ 971961 \\ 795635 \end{bmatrix}.$$

Table 5. Prediction results and accuracy rates

	Actual value	Prediction *L*	Measurement error (%)
S1	359784	362743	0.82
S2	558489	565013	1.17
S3	831770	839500	0.93
S4	232320	233159	0.36
S5	126409	174745	11.87
S6	249932	280373	0.18
S7	963732	971961	0.85
S8	794797	795635	0.11

Source: own elaboration.

3.8. Results and Interpretation

The mean predictive absolute error is 2.04% from the last sector. From testing all quarters, the error is 1.29%.

The predictive model demonstrates a high level of accuracy in most cases, with prediction errors below 1% for 6 out of 8 examples. This indicates the model's strong ability to forecast outcomes effectively. However, the analysis reveals significant discrepancies in certain instances, particularly for sample S5 which exhibited the highest prediction error at 11.87%. This outlier suggests the possibility of inaccuracies in the input data, specific anomalies, or the model's mismatch with this particular case. It highlights the need for further investigation, potentially involving the refinement of input data or the inclusion of additional variables in the modelling process.

Conversely, sample S6 showed a minimal prediction error of just 0.18%, underscoring the model's exceptional accuracy for this observation. In the remaining cases, including S1, S2, S3, S4, S7, and S8, the model performed consistently, achieving errors in the range of 0.11% to 1.17%, which is a satisfactory outcome in predictive modelling.

To improve the model's performance, particularly in outlier cases like S5, it is essential to conduct a thorough analysis to identify the root causes of the high error. These could stem from the quality of input data, such as erroneous quantitative data sourced from public registries like Central Statistical Office, or unique patterns in the dataset that the model is not equipped to handle. Employing more advanced modelling techniques or optimizing existing parameters could also enhance overall results.

In conclusion, the model can be considered a reliable predictive tool for the majority of cases. Nevertheless, specific anomalies, such as S5, must be carefully addressed to prevent significant deviations in future predictions and to maintain overall accuracy and robustness.

4. Summary and Conclusions

This study contributes to economic research by applying a modified Leontief input-output model to analyse sectoral dependencies based on the density of entities within specific PKD classifications. By focusing on fluctuations in the number of firms rather than transaction flows, the study offers a new perspective on structural economic shifts. While the research is rooted in economics, its findings have managerial applications, particularly in restructuring advisory and strategic risk assessment for firms operating in highly saturated or vulnerable sectors.

Due to the lack of directly comparable studies, the findings cannot be benchmarked against traditional input-output analyses, which primarily focus on transaction flows rather than structural firm dynamics. While the study is based on data from the Polish economy, similar methodologies could be developed for other countries with well-documented business registry systems. Future research should explore cross-country comparisons to determine whether sectoral dependencies follow similar patterns in different economic environments.

The analysis shows that different sectors vary in their resilience to fluctuations in other parts of the economy. In particular, sectors such as accommodation and food services (S5) exhibit greater variability, resulting in higher prediction errors. This suggests the need for further model optimization, taking into account the specific characteristics of the sectors and additional variables, such as verifying the accuracy of data obtained from the Central Statistical Office with REGON, KRS, and CEIDG databases. Dividing enterprises within a given sector into micro, small, medium, and large firms (based on the number of employees) will allow for more precise determination of correlation coefficients necessary for constructing the Leontief matrix. The conducted research also identified potential supply gaps that may arise as a result of significant declines in the number of entities in some sectors. Analysing such gaps is important for developing effective risk management strategies and supporting sectors most vulnerable to disruptions.

The conclusions drawn from the analysis can serve as a valuable basis for making management and policy decisions aimed at strengthening economic stability.

In the context of the modern economy, where technological innovations and the ability of enterprises to adapt are fundamental for building a competitive advantage, an approach based on forecasting future changes and actively managing uncertainty becomes a factor for long-term success. The dynamically changing market environment requires the use of advanced analytical models that not only enable the prediction of future trends but also allow for the adjustment of operational strategies to evolving conditions. Such an approach, integrating the analysis of historical data with current risk assessment, promotes increased resilience and flexibility of both enterprises and the entire economy, which, in the long term, is crucial for maintaining their stability and growth.

Ultimately, as Adam Smith emphasised whose theories we reference in relation to general equilibrium models it “is not from the benevolence of the butcher, the brewer, or the baker that we expect our dinner, but from their regard to their own interest” (Smith, 1776, p. 26). Efforts to minimize the risk of bankruptcy are not merely a moral issue but, above all, an economic necessity. This study, by indicating a course of action and the importance of predictive models, makes a significant contribution to building a stable and resilient economy.

5. Limitations and Future Research Directions

This study has certain limitations that should be acknowledged. First, the time frame (2022-2024) may not fully capture long-term sectoral dependencies, as it includes a post-pandemic recovery period that could distort structural patterns. Additionally, this period coincides with the implementation of the KRZ (Krajowy Rejestr Zadłużonych – National Register of Debtors) portal which came into effect on December 1, 2021, streamlining bankruptcy and restructuring proceedings. The increased efficiency in handling insolvency cases may have influenced the observed fluctuations in firm numbers, making it necessary to account for this institutional change in future studies. Second, the model relies on firm registration and deregistration data, which, while valuable, does not account for other factors influencing sectoral dynamics, such as technological shifts, policy changes, or external macroeconomic shocks. Further studies could enhance the model by integrating additional economic indicators to refine predictions and improve robustness. Third, a more precise assessment of intersectoral correlations requires further refinement, particularly in terms of the scale of interactions between entities. The model does not currently differentiate between the impact of small sole proprietorships (JDGs) and large corporations, nor does it account for transaction volumes between sectors. Future research should incorporate these factors to better quantify the strength and asymmetry of intersectoral dependencies. Lastly, the study is limited to the Polish economy. While the approach can be adapted to other countries with similar business registry systems, cross-country comparisons have not yet been conducted. Future research should explore the applicability of the model in different economic environments to validate its broader relevance.

References

Drucker, P. F. (1993). *Post-Capitalist Society*. Harper Business.

Leontief, W. (1986). *Input-Output Economics*. Oxford University Press.

Mankiw, N. G. (2016). *Macroeconomics*. Worth Publishers.

Marshall, A. (1920). *Principles of Economics* (8th ed.). Macmillan.

Sahani, S. K., & Prasad, K. S. (2023). Study and Analysis of Input-Output Analysis to Non-Linear Science. *ACCST Research Journal*, *21*(1), 22-30.

Schumpeter, J. A. (1942). *Capitalism, Socialism and Democracy*. Harper & Brothers.

Shoven, J. B., & Whalley, J. (1992). *Applying General Equilibrium*. Cambridge University Press.

Smith, A. (1776). *An Inquiry into the Nature and Causes of the Wealth of Nations* (Vol. I). W. Strahan and T. Cadell.

Taleb, N. N. (2007). *The Black Swan: The Impact of the Highly Improbable*. Penguin.

The Noble Prize. (1973). *The Nobel Prize in Economic Sciences 1973* – Press Release. Retrieved from https://www.nobelprize.org/prizes/economic-sciences/1973/leontief/facts/

Wpływ wahań liczby podmiotów gospodarczych w sektorach na zależności międzysektorowe: zastosowanie prognoz w modelu przepływów międzysektorowych Leontiefa

Streszczenie: Celem niniejszego artykułu jest zbadanie wpływu wahań liczby podmiotów gospodarczych w różnych sektorach na zależności międzysektorowe. Badania oparto na danych Głównego Urzędu Statystycznego oraz zastosowaniu modelu przepływów międzygałęziowych Leontiefa. Celem badania jest rozpoznanie zależności międzysektorowych i określenie, które sektory są najbardziej podatne na zmiany w innych częściach gospodarki. W analizie wykorzystano zaawansowane narzędzia analityczne, w tym model przepływów międzysektorowych, który pozwala na ich precyzyjną ocenę. Wyniki badań wskazują istotne różnice w odporności poszczególnych sektorów na zmiany, a także występowanie potencjalnych luk podażowych, które mogą wynikać ze zmniejszenia liczby podmiotów w niektórych sektorach. Wnioski wyciągnięte z analiz mogą stanowić podstawę podejmowania decyzji zarządczych i opracowywania skutecznych strategii zarządzania ryzykiem.

Słowa kluczowe: wahania liczby podmiotów gospodarczych, model Leontiefa, analiza międzysektorowa, ryzyko gospodarcze, luka podażowa, prognozowanie