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Large-anastigmatic-vision-zone progressive addition lens

Yiqun Ji[1,4,5,*], Yanhong Gao[2,*], Hongjun Liu[3], Shujun Sun[1,4,5]

[1] School of Optoelectronic Science and Engineering, Soochow University, Suzhou 215006, China

[2] School of Optical and Electronic Information, Suzhou City University & Suzhou Key Laboratory of Biophotonics, Suzhou 215104, China

[3] National Inspection and Testing Center for Ophthalmic Optics Products, Danyang 212300, China

[4] Collaborative Innovation Center of Suzhou Nano Science and Technology & Engineering Research Center of Digital Imaging and Display of Education Ministry of China, Soochow University, Suzhou 215006, China

[5] Key Lab of Advanced Optical Manufacturing Technologies of Jiangsu Province & Key Lab of Modern Optical Technologies of Education Ministry of China, Soochow University, Suzhou 215006, China

[*]Corresponding authors: jiyiqun@suda.edu.cn (Y.J.), yhgao@szcu.edu.cn (Y.G.)

To address the main challenge of limited vision zone and deteriorated visual clarity caused by residual astigmatism in progressive addition lenses, the study on large-anastigmatic-vision-zone progressive addition lens is conducted. By exploring the influence of optical power along the meridian on the growth rate of astigmatism, the design method for the meridian with tunable optical power is investigated. Subsequently, by extending the tunable optical power from the meridian to the entire lens surface along different shapes of contour lines, the effect of surface tunable optical power on the astigmatism and the vision zone is explored. This leads to the development of a design method for progressive addition lenses with low residual astigmatism, large vision zone, and surface tunable optical power. Based on this design method, progressive addition lens with a usable vision zone area exceeding 84% of the total lens area is optimized, while maintaining a maximum residual astigmatism of only 1.65 D. This holds significant value for the advancement of progressive addition lenses.

Keywords: optical design, progressive addition lens, tunable optical power, large vision zone, astigmatism.

1. Introduction

Progressive addition lenses are lenses with continuously variable surface optical power, which can overcome the shortcoming of traditional single-vision lenses that can only focus objects at a specific visual distance. Progressive addition lenses enable objects at different distances, from far to near, to be clearly imaged on the retina. Additionally, they eliminate the problem of vision jump caused by optical power discontinuities found in bifocal and trifocal lenses [1]. Progressive addition lenses play a crucial role in correcting myopia with presbyopia in middle-aged and elderly individuals [2-7], as well as in myopia control among adolescents [8-11]. However, there is inevitable astigmatism due to the continuously variable optical power of the lenses, which can reduce the area of vision zone, and cause blurring of vision. Consequently, there is an urgent need to reduce astigmatism and expand the vision zone area.

Surface optical power distribution is critical to the successful design of a progressive addition lens. In 2015, QIU *et al.* introduced a hyperbolic tangential function to describe optical power distribution on the lens. Although this method can effectively control the area of the vision zone and the growth rate of astigmatism, the maximum astigmatism of this progressive addition lens is near to 2.50 D [12]. In 2017, TANG *et al.* proposed a numerical method of the Laplace equation with the boundary and link conditions to obtain the surface optical power distribution. Although the maximum astigmatism of this progressive addition lens with this method can be significantly reduced, the width of distance zone is narrow, only 26 mm at $x = -10$ mm [13]. In 2025, PAN *et al.* proposed a differentiable optimization approach based on non-uniform rational B-spline surface to obtain the surface optical power distribution. Although the high-level astigmatism zones near the intermediate zone are pushed toward the edge of the lens, the maximum astigmatism is near to the addition power and the width of distance zone is still narrow [14]. The severer the astigmatism, the worse the visual clarity. The smaller the area of vision zone, the more limited the wearer's effective field of view. Therefore, there still is a challenge in reducing astigmatism and expanding the area of vision zone.

In this paper, we study on the large-anastigmatic-vision-zone progressive addition lens to provide a comfortable visual experience for people. By exploring the relationship between the meridional optical power and the growth rate of astigmatism, the design method for the meridian with tunable optical power is investigated. This contributes to reducing the residual astigmatism near the meridian. Besides, by extending the tunable optical power from the meridian to the entire lens surface along different shapes of contour lines, the effect of surface tunable optical power on the astigmatism and the vision zone are explored. It is helpful to further reduce the residual astigmatism of lens surface and achieve a large vision zone, which has profound implications for the development of progressive addition lenses.

2. System design method

2.1. How tunable optical power along the meridian affects the growth rate of astigmatism

The schematic illustration of the progressive addition lens is shown in Fig. 1. The blue dashed line is the meridian, which aligns with y-axis. The points A and B are the far

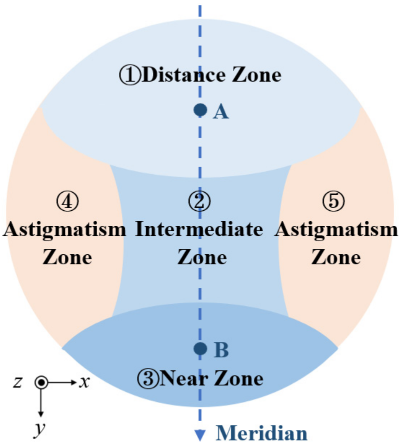

Fig. 1. Schematic illustration of progressive addition lens.

reference point (FRP) and near reference point (NRP), respectively. Progressive addition lens is divided into five zones, including a distance zone, intermediate zone, near zone, and two astigmatism zones. The distance zone with lower optical power is in the upper part, and the near zone with higher optical power is in the lower part. Meanwhile, the intermediate zone where the optical power varies progressively and smoothly is used to connect the distance zone and near zone. There is inevitable astigmatism in two astigmatism zones due to the continuously variable surface optical power, which causes a bad visual experience.

The stubborn astigmatism is mainly influenced by the surface optical power distribution of progressive addition lens. The surface optical power distribution is obtained based on the meridional optical power distribution and the counter line distribution. In order to reduce the negative effect of the astigmatism on the visual clarity and achieve a large vision zone, the tunable optical power along the meridian is firstly studied by exploring the influence of optical power along the meridian on the growth rate of astigmatism.

According to the Minkwitz theorem [15], the relationship between the optical power along the meridian $D(0, y)$ and the astigmatism $C(x, y)$ is as follows:

$$\lim_{x \to 0} \frac{\partial C(x, y)}{\partial x} = 2 \frac{\partial D(0, y)}{\partial y} \tag{1}$$

From Eq. (1), the growth rate of astigmatism near the meridian is proportional to the growth rate of optical power along the meridian. Therefore, the astigmatism near the meridian can be reduced by decreasing the growth rate of the optical power along the meridian.

As shown in Fig. 2(a), the logistic function exhibits a smooth S-shaped distribution, which not only has the characteristic of continuously gradual change throughout the

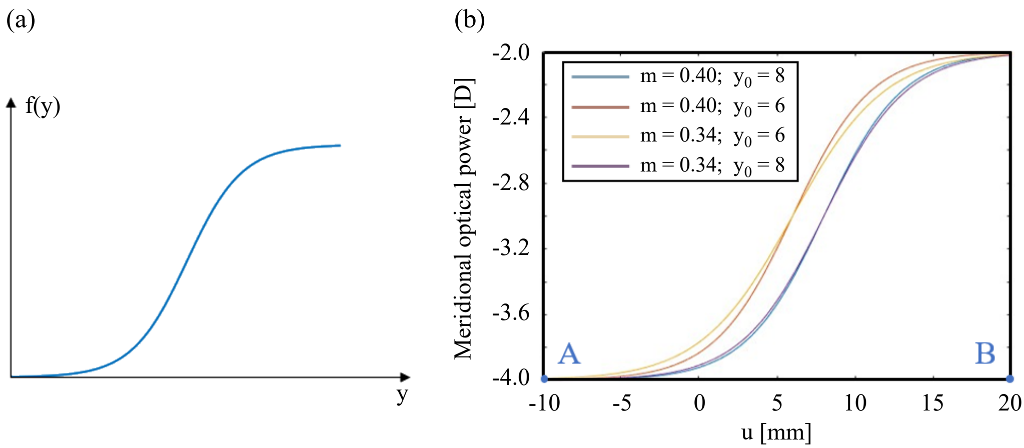

Fig. 2. Schematic of the tunable optical power distribution along the meridian. (a) The distribution of logistic function. (b) Tunable optical power along the meridian.

entire curve, but also has a relatively slow growth rate at both ends. These characteristics align with the distribution of optical power along the meridian. The mathematical description of the logistic function $f(y)$ can be expressed as:

$$f(y) = \frac{1}{1 + \exp(-y)} \tag{2}$$

Based on the logistic function, the tunable optical power along the meridian $D(0, y)$ is constructed:

$$D(0, y) = D_A + \frac{D_B - D_A}{1 + \exp\left[-m(u - y_0)\right]} \tag{3}$$

where D_A and D_B asymptotically approach the optical power at the FRP A and NRP B, respectively; u represents the y-coordinate value of any point on the meridian, m represents the modulation factor, and y_0 represents the translation factor.

The tunable optical power distribution along the meridian is shown in Fig. 2(b), which shows that the optical power along the meridian varies progressively and smoothly from the FRP A to the NRP B. On the one hand, the growth rate of the tunable optical power along the meridian can be controlled by tuning the modulation factor m. On the other hand, the growth rate of tunable optical power along the meridian is proportional to the growth rate of astigmatism near the meridian. Therefore, decreasing the value of the modulation factor m contributes to reducing astigmatism near the meridian. The translation factor y_0 contributes to adjusting the vision zone area. The smaller the translation factor y_0, the smaller the distance zone area and the larger the near zone area. The larger the translation factor y_0, the larger the distance zone area and the smaller the near zone area.

2.2. How surface tunable optical power affects astigmatism and vision zone

The surface optical power distribution is obtained by extending the optical power distribution from the meridian to the entire lens surface along the shape of contour lines. In this paper, three different shapes of contour lines are studied, such as circle, ellipse, and parabola. Firstly, the equations for circle, ellipse, and parabola are as follows:

$$x^2 + y^2 = R^2 \tag{4}$$

$$(x^2/a^2) + (y^2/b^2) = 1 \tag{5}$$

$$y^2 = \pm 4px \tag{6}$$

where R represents the radius of the circle, a and b represent the length of semi-major axis and semi-minor axis, and p represents the distance between the focus and the vertex measured along the axis of symmetry. The parameters R, a, b, and p can be utilized to control the shape of the circle, ellipse, and parabola, respectively.

In progressive addition lenses, the optical powers at the FRP A $(0, y_\mathrm{d})$ and NRP B $(0, y_\mathrm{r})$ correspond to the minimum and maximum optical powers of the lens, respectively. The function $\ln(x^2 + y^2)$ approaches $-\infty$ at the point (0, 0). Based on the function $\ln(x^2 + y^2)$, we construct an auxiliary optical power function $M_i(x, y)$ composed of three sets of different curves, where $i = 1, 2, 3$, so that it approaches $-\infty$ at the FRP A $(0, y_\mathrm{d})$ and approaches $+\infty$ at the NRP B $(0, y_\mathrm{r})$. Then, the auxiliary optical power function $M_i(x, y)$ can be expressed as:

$$M_1(x, y) = \ln\left[x^2 + (y - y_\mathrm{d})^2\right] - \ln\left[x^2 + (y - y_\mathrm{r})^2\right] \tag{7}$$

$$M_2(x, y) = \ln\left[\frac{x^2}{a^2} + \frac{(y - y_\mathrm{d})^2}{b^2}\right] - \ln\left[\frac{x^2}{a^2} + \frac{(y - y_\mathrm{r})^2}{b^2}\right] \tag{8}$$

$$M_3(x, y) = \ln\left[p|x| + (y - y_\mathrm{d})^2\right] - \ln\left[p|x| + (y - y_\mathrm{r})^2\right] \tag{9}$$

where, $M_1(x, y)$, $M_2(x, y)$ and $M_3(x, y)$ are the auxiliary optical power functions based on circles, ellipses and parabolas, respectively.

To ensure that the lens's optical power meets patient's prescription, the auxiliary optical power function $M_i(x, y)$ should meet the relationship:

$$M_i(x, y) = M_i(0, u) \tag{10}$$

Through Eqs. (7)–(10), for three different curve shapes, the equations of contour lines $u_i(x, y)$ can be deduced:

$$u_1(x, y) = y_\mathrm{d} + (y_\mathrm{r} - y_\mathrm{d})\frac{\sqrt{x^2 + (y - y_\mathrm{d})^2}}{\sqrt{x^2 + (y - y_\mathrm{d})^2} + \sqrt{x^2 + (y - y_\mathrm{r})^2}} \tag{11}$$

$$u_2(x,y) = y_d + (y_r - y_d)\frac{\sqrt{\frac{x^2}{a^2} + \frac{(y-y_d)^2}{b^2}}}{\sqrt{\frac{x^2}{a^2} + \frac{(y-y_d)^2}{b^2}} + \sqrt{\frac{x^2}{a^2} + \frac{(y-y_r)^2}{b^2}}} \tag{12}$$

$$u_3(x,y) = y_d + (y_r - y_d)\frac{\sqrt{p|x| + (y-y_d)^2}}{\sqrt{p|x| + (y-y_d)^2} + \sqrt{p|x| + (y-y_r)^2}} \tag{13}$$

Three different shapes of contour lines, such as circles, ellipses, and parabolas, are shown in Fig. 3. Based on the contour lines, the surface tunable optical power $D(x, y)$ can be obtained:

$$D(x,y) = D_A + \frac{D_B - D_A}{1 + \exp\left\{-m\left[u(x,y) - y_0\right]\right\}} \tag{14}$$

The surface tunable optical power distributions are shown in Fig. 4. It can be seen that the forms of the surface optical power distributions are the same as the forms of

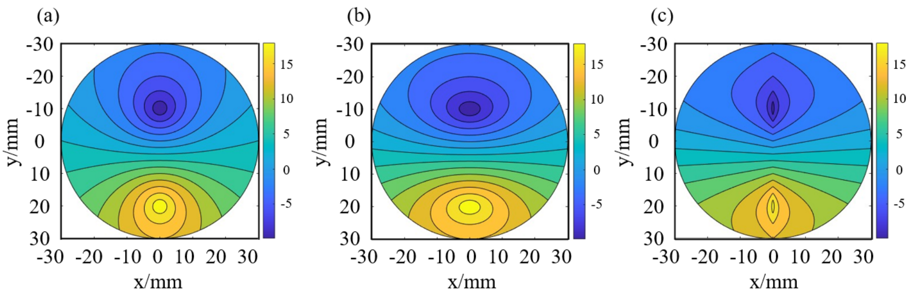

Fig. 3. Three different shapes of contour lines. (a) Circles, (b) ellipses, and (c) parabolas.

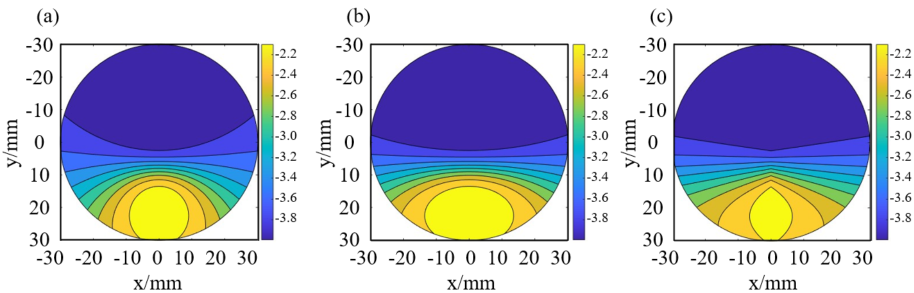

Fig. 4. Surface optical power distributions of progressive addition lens based on different contour lines. (a) Circles, (b) ellipses, and (c) parabolas.

contour lines distributions, and are also distributed along circles, ellipses and parabolas, respectively.

Astigmatism is caused by the fact that the meridional image does not coincide with the sagittal image. In progressive addition lens, the astigmatism $C(x, y)$ is related to the difference between maximum principal curvature k_1 and minimum principal curvature k_2:

$$C(x, y) = (n-1)\left|k_1 - k_2\right| \tag{15}$$

where n represents the refractive index of the lens material.

The maximum and minimum principal curvatures are given by [16]:

$$\begin{cases} k_1 = H + \sqrt{H^2 - K} \\ k_2 = H - \sqrt{H^2 - K} \end{cases} \tag{16}$$

where,

$$\begin{cases} H = \dfrac{EN + GL - 2FM}{2(EG - F^2)} \\ K = \dfrac{LN - M^2}{EG - F^2} \\ E = 1 + Z_x^2(x, y) \\ F = Z_x(x, y)Z_y(x, y) \\ G = 1 + Z_y^2(x, y) \\ L = \dfrac{Z_{xx}(x, y)}{\sqrt{1 + Z_x^2(x, y) + Z_y^2(x, y)}} \\ M = \dfrac{Z_{xy}(x, y)}{\sqrt{1 + Z_x^2(x, y) + Z_y^2(x, y)}} \\ N = \dfrac{Z_{yy}(x, y)}{\sqrt{1 + Z_x^2(x, y) + Z_y^2(x, y)}} \end{cases} \tag{17}$$

and $Z(x, y)$ is the surface sag, which is given by

$$Z(x, y) = \zeta(u(x, y)) - \sqrt{r(u(x, y))^2 - \left[x - \xi(u(x, y))\right]^2 - \left[y - \eta(u(x, y))\right]^2} \tag{18}$$

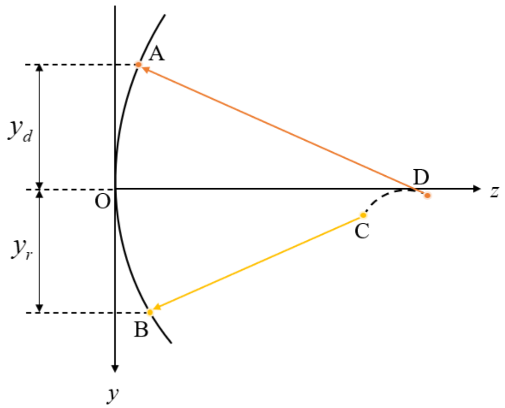

Fig. 5. Construction diagram of sag curve for progressive addition lens.

where $(\xi(u(x,y)), \eta(u(x,y)), \zeta(u(x,y)))$ is the curvature center coordinate of each point. As shown in Fig. 5, the origin O is at lens's center. The arc AB is the sag curve describing the sag variation from the FRP A to the NRP B, while the arc CD is the set of the curvature centers of the points on arc AB. The curvature center $(\xi(u(x,y)), \eta(u(x,y)), \zeta(u(x,y)))$ is given by:

$$\begin{cases} \xi(u(x,y)) = u(x,y) - r(u(x,y))\sin\theta \\ \eta(u(x,y)) = 0 \\ \zeta(u(x,y)) = r(u(x,y))\cos\theta + \int_0^{u(x,y)} \tan\theta \, \mathrm{d}u(x,y) \\ \sin\theta = \int_0^{u(x,y)} \frac{1}{r(u(x,y))} \, \mathrm{d}u(x,y) \end{cases} \tag{19}$$

where, $r(u(x,y))$ is the radius of curvature at each point on the surface. It is given by:

$$r(u(x,y)) = \frac{n-1}{D(x,y)} \tag{20}$$

From the above theoretical study, when D_A is –4 D, D_B is –2 D, we chose m as 0.4 and y_0 as 8 mm to ensure that progressive addition lenses with different contour shapes have the same clear vision zone areas. At this point, the impact of the different contour shapes on lens performance can be evaluated by comparing the maximum astigmatism of the lens and the area of the usable vision zone. The astigmatism distributions under three different shapes of surface optical power distributions are shown in Fig. 6. When the shapes of surface optical power distributions are circular, elliptical and parabolic, the maximum values of astigmatism are 1.90, 2.76 and 2.88 D, respectively.

It is known that the neural system adapts to a certain level of astigmatism [17-19]. When residual astigmatism is less than 0.5 D, it does not affect human visual perception. Therefore, the region with astigmatism less than 0.5 D can be defined as a clear

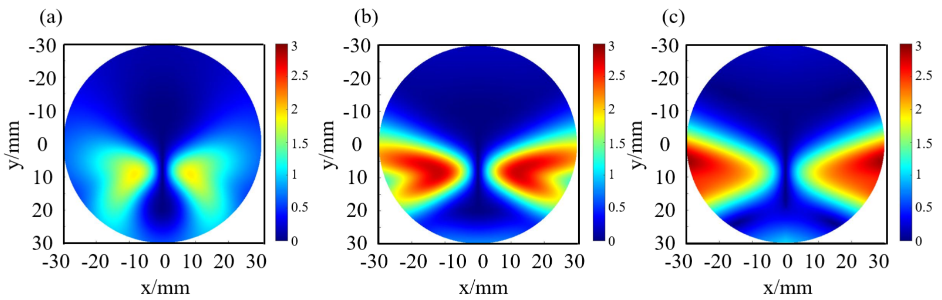

Fig. 6. Astigmatism distributions under three different shapes of surface optical power distributions. (a) Circles, (b) ellipses, and (c) parabolas.

vision zone. For human eyes, when residual astigmatism is less than 1.0 D, it can be considered as the mild astigmatism, which has a minimal impact on visual perception. Therefore, the region with astigmatism less than 1.0 D can be defined as a usable vision zone [20].

In Fig. 7, when the areas of clear vision zones are comparable, the usable vision zone area of progressive addition lens with circular surface optical power distribution is larger than those of the elliptical and parabolic distributions. Therefore, the optimal shape of surface optical power distribution is determined to be circular.

The growth rate of astigmatism near the meridian is proportional to the growth rate of optical power along the meridian. The optical power distribution along the meridian is first determined utilizing the modulation factor m and the translation factor y_0. Subsequently, the surface optical power distribution is obtained by extending the optical power distribution from the meridian to the entire lens surface along different contour shapes. At this point, the astigmatism distribution under different contour shapes can be obtained by utilizing Eq. (15), and then obtaining the size of the astigmatic field for different contour shapes.

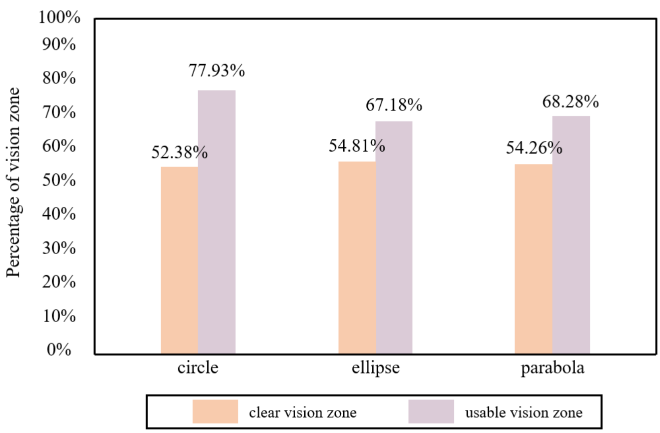

Fig. 7. Vision zones under three different shapes of surface optical power distributions.

Circular contours exhibit a more gradual optical power distribution compared to elliptical and parabolic contours. The slower the growth rate of optical power, the smaller the astigmatism. Therefore, the size of the astigmatic field of the progressive addition lens with circular contour shape is smaller than those of the elliptical and parabolic contour shapes.

3. Result and discussion

Based on the theoretical study in Section 2, for a patient suffering from myopia with −4.00 D and presbyopia with 2.00 D, a progressive addition lens with −4.00 D BASE and 2.00D ADD is necessary. It means that the progressive addition lens should has optical power of −4.00 D in the distance zone and −2.00 optical power in the near zone. Taking into account the wearer habits, the distance between FRP A and the center of the lens should be 10 mm, and the distance between NRP B and the center of the lens should be 20 mm. The specifications of the progressive addition lens are shown in Table 1. We chose m as 0.35 and y_0 as 7 mm to reduce the astigmatism zones, and obtain a large vision zone.

Table 1. Specifications of the progressive addition lens.

Parameter	Value	Units
Myopia	–4.0	D
Presbyopia	2.0	D
PAL prescription (BASE)	–4.0	D
PAL prescription (ADD)	2.0	D
Refractive index n	1.53	/
Modulation factor m	0.35	/
Translation factor y_0	7	mm
Distance y_d between FRP A and center of lens	10	mm
Distance y_r between NRP B and center of lens	20	mm

Optical power distribution map and astigmatism distribution map are commonly used to assess the imaging quality of progressive addition lenses. The optical power distribution map is utilized to evaluate whether the optical power of the progressive addition lens aligns with the patient's prescription. The astigmatism distribution map is utilized to evaluate whether astigmatism of the progressive addition lens has a terrible impact on the visual clarity. The optical power distribution map, as depicted in Fig. 8(a), indicates that the optical power varies from –3.97 to –2.04 D, which falls within the international tolerance range of ±0.12 D. Therefore, the optical power of the progressive addition lens meets the patient's requirements. As shown in Fig. 8(b), the astigmatism distribution map reveals that astigmatism mainly concentrates in two small areas, with a maximum astigmatism of only 1.65 D.

Based on the astigmatism distribution shown in Fig. 8(b), when using 0.50 D astigmatism contour line as a reference, the astigmatism distribution of this lens is shown in

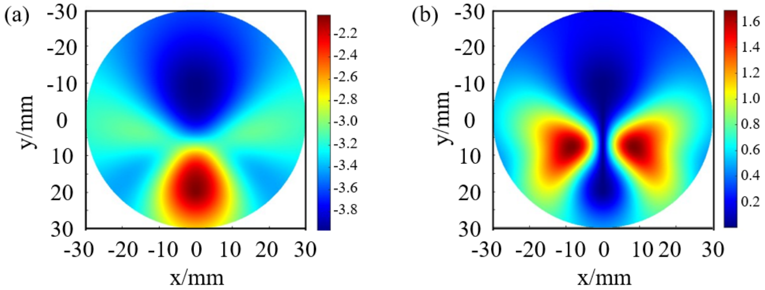

Fig. 8. Imaging quality evaluation. (a) Optical power distribution, and (b) astigmatism distribution.

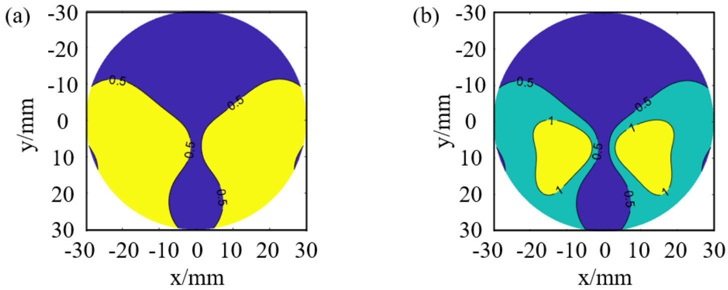

Fig. 9. Astigmatism distribution when using different astigmatism contour lines as reference standards. (a) 0.50 D astigmatism contour line, and (b) 1.0 D astigmatism contour line.

Fig. 9(a). At this point, the area of the clear vision zone in this lens accounts for 47.03% of the total lens area. When using 1.0 D astigmatism contour line as a reference, the astigmatism distribution of this lens is shown in Fig. 9(b). At this point, the area of the usable vision zone in this lens accounts for 84.06% of the total lens area. The astigmatism zones that negatively affect visual clarity concentrate in two small areas, accounting for only 15.94% of the total lens area. The design results demonstrate that the astigmatism zones have been significantly reduced, which contributes to achieving a large vision zone and providing clear vision for users.

In this paper, the progressive addition lens successfully achieves a large vision zone and effectively reduces surface astigmatism. By controlling the modulation factor m and the translation factor y_0, individual customization can be achieved. In the future, the binocular vision quality can also be improved by introducing the temporal offset. Furthermore, the optimized progressive addition lens can be combined with a human eye model to analyze the impact of factors such as distortion on wearing performance.

4. Conclusion

Addressing the issue that the astigmatism of progressive addition lenses reduces vision zone area and causes visual blur, this paper carries out a study on large-anastigmatic -vision-zone progressive addition lens. Firstly, by exploring the influence of optical power along the meridian on the growth rate of astigmatism, the design method for the meridian with tunable optical power is studied. It is found that the astigmatism near the meridian can be reduced by reducing the growth rate of tunable optical power along the meridian. Then, by exploring how surface tunable optical power affects the astigmatism and vision zone, a design method for progressive addition lenses with low residual astigmatism and large vision zone is obtained. It is found that when the surface tunable optical power is distributed along the circular contour lines, the residual astigmatism of the lens is the smallest and the usable vision zone area is the largest. Finally, a progressive addition lens with a large usable vision zone occupying 84.06% of the total lens area and a maximum residual astigmatism of only 1.65 D is obtained. This paper paves the way for the development of progressive addition lenses capable of enlarging a large vision zone and reducing the residual astigmatism.

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