

**DEGRESSIVE PROPORTIONALITY –
SOURCE, FINDINGS AND DISCUSSION
OF THE CAMBRIDGE COMPROMISE¹**

Piotr Dniestrzański

Abstract. The principle of degressive proportionality in relation to the division of indivisible goods in the past few years – especially since the signing of the Treaty of Lisbon – has been one of the topics frequently appearing in studies of economists, mathematicians, lawyers and people involved in politics. It combines a purely theoretical and easily definable question with a very important practical problem that concerns primarily members of the European Union and countries aspiring to become one. The article shall include a debate and discussion of the findings of the Cambridge Compromise concerning proposals to solve the problem of allocating seats in the European Parliament.

Keywords: degressive proportionality, European Parliament, Cambridge Compromise, Treaty of Lisbon.

JEL Classification: D39, D63.

1. Introduction

Degressively proportional divisions (and progressively proportional) have been used successfully by communities and businesses for hundreds of years. Practically, whenever it is impossible (or undesirable) or uneconomic to apply a simple and elegant proportionality, the alternative is a degressive or progressive proportionality. But if proportionality is relatively easy to apply (unless it concerns the allocation of indivisible goods, but in this case the problem has already been substantially scientifically refined – see for example Young (2003)), it is the alternatives that are problematic mainly because of their natural ambiguity. The question of the practical application of divisions other than proportional is still an unsolved problem in the European Union and concerns the

Piotr Dniestrzański

Department of Mathematics, Wrocław University of Economics, Komandorska Street 118/120,
53-345 Wrocław, Poland.

E-mail: piotr.dniestrzanski@ue.wroc.pl

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division of seats in the European Parliament. Due to the very large diversity of the population of the Member States, it is impossible to apply proportional division in this case. A broader elaboration on this topic can be found in Cegielka et al. (2010).

2. History of the issue.

Treaty of Lisbon and the report of Lamassoure and Severin

The first decade of the twenty-first century saw the rapid development of the European Union structures resulting primarily in almost doubling the number of its members (from 15 in the mid 1990s to 27 in 2007). In the near future the number of members of the community – after the accession of the Balkan countries, Iceland and Turkey – may exceed 30. This rapid growth and a significant increase in the internal diversity of the Union made it necessary to introduce changes that would guarantee a satisfactorily proportional share in decision-making.

The Treaty of Lisbon (the Treaty of Lisbon amending the Treaty on the European Union and the Treaty Establishing the European Community, signed in Lisbon, on 13 December 2007) was intended to be a “milestone in the further development of the Union” (Barcz, 2008). It is the most important so-called reform treaty (next to the Treaty of Amsterdam (1999) and the Treaty of Nice (2003)) reforming the Community. With regard to the composition of the European Treaty it provides, *inter alia*,² “The European Parliament shall be composed of representatives of the Union’s citizens. They shall not exceed seven hundred and fifty in number, plus the President. Representation of citizens shall be degressively proportional, with a minimum threshold of six members per Member State. No Member State shall be allocated more than ninety-six seats”.

The phrase “degressively proportional” is, from the practical point of view, extremely ambiguous. Very quickly it became clear that the direct application of this provision is impossible without arduous negotiations. Thus in 2007 the European Council invited the European Parliament to prepare a draft of a new distribution of seats in Parliament based on the principles adopted in the (then still not having legal force) Treaty. This task was entrusted to the Constitutional Affairs Committee of the European Parliament (AFCO). On 3 October 2007 AFCO (rapporteurs: Alain Lamassoure and Adrian Severin) presented a report which provided a project of the adequate resolution of the European Parliament. The resolution was adopted at the meeting of the Parliament on

² Article 9, the new Article 14 of the Treaty on the European Union.

11 October 2007. In support of the report, six principles were presented which, according to the Constitutional Affairs Committee, could clarify the rule of degressive proportionality:

Principle 1. **The principle of effectiveness** – the functioning of the European Parliament is impossible if its composition exceeds the specific number of deputies – hence the restriction to 750 members.

Principle 2. **The principle of national representation and the motivation of the voters** – each Member State should have the minimum number of seats, so it will be able to represent their electorate by motivating them to participate in the elections.

Principle 3. **The principle of European solidarity** – in order to ensure adequate representation of less populous States, countries with a greater number of citizens will receive fewer seats than in the case of application of the principle of strict proportionality.

Principle 4. **The principle of relative proportionality** – the ratio of the population size to the number of seats is greater the larger the State, and smaller the smaller the State.

Principle 5. **The principle of fair distribution** – no State will have more seats than a larger Member State or smaller amount of seats than a smaller Member State.

Principle 6. **The principle of reasonable flexibility or flexible direct degressiveness** – small changes in the allocation of seats may be implemented if other principles are obeyed and the modification aims at the most equitable distribution of seats.

Mathematical analysis of the rules for selecting the composition of the European Parliament with regard to the AFCO report can be found in the work by Cegiełka et al. (2010). The AFCO report and the resolution of the EP adopted on its basis in no way solved the problem. It only clarified the expression “degressive proportionality”, the only possible direction. The report did not change in any way the ambiguity that it contained, and any of the very likely problems with the composition of the next terms of the Parliament. Problems with the ratification of the Lisbon Treaty by some Member States (eventually it came into force on 1 December 2009) were in this situation beneficial to all concerned, i.e. members of the Union. The practical implementation of the provisions of the Treaty were avoided, the allocation of seats in the Parliament of the seventh term of office has remained almost unchanged when compared with the division under the Treaty of Nice. The postponement of the practical application of the solutions of the Treaty gave time to clarify and modify its provisions.

3. Proposed solutions

It is easy to notice that the authors of the part of the Treaty of Lisbon concerning the composition of the European Parliament had in mind such a division of seats among EU members that would allot more to countries with smaller populations (and fewer to countries with larger populations) than would appear from the proportional division. The problem is that it is not yet clarified what these deviations from the normal proportion should look like. Further work of politicians and scientists (after the entry into force of the TL) show that the problem is not easy. The main reason lies in the complexity of the idea of degressive proportionality – a departure from proportional distribution means entering the level of inequality which usually gives a lot more opportunities of a solution than a resolution based on equality.

Among the many proposals of a mathematical solution of the ambiguity in the Treaty of Lisbon, two seem to be the most natural. The first, proposed and analyzed among others by Pukelsheim (2007; 2010) is the idea of so-called “shifted proportionality”. According to it, all States are to receive a fixed number of seats and the rest of the pool is allocated proportionally. Pukelsheim proposes that each member of the Union at the beginning gets six seats (which at 27 countries gives 162 places in the Parliament) and the remaining 589 seats divided by Webster’s method, which is proportionally rounding it to the nearest integer. Because of the provisions of the Treaty, the States that in such a situation would receive more than 96 seats should be deprived of the excess seats. With the current composition of the EU the country that the cut would concern most is Germany, the next would be (in the case of accession) Turkey. Pukelsheim symbolically defines this method as “Fix + Prop”. Table 1 provides a hypothetical distribution of seats with the use of this solution. The allocation of seats with the use of the shifted proportionality is considered in the work of Cegiełka et al. (2010). The work presents a hypothetical composition of the EP by using this method, taking into account three possible methods of rounding: Webster’s, Adams’ and D’Hondt’s.

Another interesting option is to model the distribution of seats in the Parliament by choosing the appropriate quadratic function. A proposal for such a solution was presented and analyzed by Ramirez-Gonzalez, Palomares Batistuta, Márquez Garcia (2006). The authors assumed that at the quadratic function $A(x) = a + bx + cx^2$ (with a negative coefficient a) is flexible enough to model the issue of distribution under consideration. They

made a hypothetical allocation of seats in accordance with the principle of degressive proportionality, based on the Constitutional Treaty (never entered into force), which also mentioned this principle as a mechanism for selecting the composition of the Parliament. The authors allocated 750 seats between the Member States – this was the number of Members provided for in the Constitutional Treaty. Partial results of this research are to be found in the fourth column of Table 1.

Table 1. Allocation of European Parliament seats to Member States:
“Fix + Prop” and parabolic methods

Member States	Population 2007	Fix + Prop	Parabolic method	2009-2014
Germany	82,438,000	6 + 90 = 96	96	99
France	62,886,200	6 + 77 = 83	79	72
United Kingdom	60,421,900	6 + 74 = 80	76	72
Italy	58,751,700	6 + 71 = 77	75	72
Spain	43,758,300	6 + 53 = 59	59	50
Poland	38,157,100	6 + 46 = 52	53	50
Romania	21,610,200	6 + 26 = 32	34	33
Netherlands	16,334,200	6 + 20 = 26	27	25
Greece	11,125,200	6 + 14 = 20	20	22
Portugal	10,569,600	6 + 13 = 19	20	22
Belgium	10,511,400	6 + 13 = 19	20	22
Czech Republic	10,251,100	6 + 12 = 18	19	22
Hungary	10,076,600	6 + 12 = 18	19	22
Sweden	9,047,800	6 + 11 = 17	18	18
Austria	8,265,900	6 + 10 = 16	17	17
Bulgaria	7,718,800	6 + 9 = 15	16	17
Denmark	5,427,500	6 + 7 = 13	13	13
Slovak Republic	5,389,200	6 + 7 = 13	13	13
Finland	5,255,600	6 + 6 = 12	13	13
Ireland	4,209,000	6 + 5 = 11	11	12
Lithuania	3,403,300	6 + 4 = 10	10	12
Latvia	2,294,600	6 + 3 = 9	9	8
Slovenia	2,003,400	6 + 2 = 8	8	7
Estonia	1,344,700	6 + 2 = 8	7	6
Cyprus	766,400	6 + 1 = 7	6	6
Luxembourg	459,500	6 + 1 = 7	6	6
Malta	404,300	6 + 0 = 6	6	5
Total	492,881,500	162 + 589 = 751	750	736

Source: from Pukelsheim (2010); Ramirez-Gonzalez, Palomares Batistuta, Márquez Garcia (2006).

The authors also considered the composition of the EP after the accession of Croatia. It can be presumed that the authors of such a solution regarded the principle of degressive proportionality as equivalent to the fact that the function assigning the number of seats depending on the country's population must be increasing and concave. In his paper Dniestrzański (2011) showed that concavity is not a prerequisite for the realization of this principle, but it is a sufficient condition (assuming that we have a decreasing function).

An important element of research is an article by Florek (2011), in which he moved the main considerations to a purely mathematical level. Florek introduced the concept of a sequence degressively proportional to the given sequence and furthermore examined the features of a so-defined object. Mathematical properties of functions that may be useful in the issue of allocation of seats are studied by Słomczyński and Życzkowski (2011). Considerations on the problem of diversification and stability of the division of seats can be found in Łyko (2011).

4. The Cambridge Compromise

In January 2011 the Committee on Constitutional Affairs convened a symposium of mathematicians to develop methods for allocating seats in the EP. The primary objective of the symposium was to present the mathematical foundations which would constitute the basis of the distribution of seats in the EP during consecutive terms. Appropriate arrangements made during the symposium were to allow “the elimination of political bargaining that characterizes the current distribution of seats”; the mathematical formula needs to be “stable, transparent and free from political influence”. The developed model (method) would meet the condition of degressive proportionality. The meeting was held at the Centre for Mathematical Sciences, University of Cambridge on 28-29 January 2011, the participants prepared a report from the meeting (Grimmett et al., 2011). The report of the meeting was adopted by all participants and includes the following findings (points U1.-U3.).

U1. The essence of the principle of degressive proportionality are two conditions:³ DP1. No smaller State shall receive more seats than a larger State: DP2, the ratio population/seats shall increase with an increase of population.

Other conditions (the rules of the AFCO report) are less important (AFCO representatives present at the symposium said that the conditions are not unconditionally applicable) although they cannot be treated too freely. Simultaneous fulfillment of conditions DP1 and DP2 can be difficult and in

³ Determination of a development (Grimmett et al., 2011).

special cases, even impossible. It was recalled (citing the work of Martinez-Aroza, Ramirez-Gonzalez (2008)) that there are hypothetical cases in which the solution satisfying the conditions DP1 and DP2 does not exist. Next the following example was presented: the Parliament has 105 members and the seats should be apportioned between five countries (see Table 2).

Table 2. Division of seats in a hypothetical Parliament

Member States	Population	Seats
Greece	11,305,118	21
Belgium	10,839,906	21
Portugal	10,637,713	21
Czech Republic	10,506,813	21
Hungary	10,014,324	21
Total	53,303,874	105

Source: Grimmett et al. (2011).

The fulfillment of both conditions results in giving each Member State 21 seats. If the size of the Parliament amounted to 106 instead of 105 seats, the condition DP1 would necessitate allocation of one extra seat for Greece. This solution, however, violates a condition of DP2 as the ratio of population to the number of seats is then smaller for larger Greece ($11305118 : 22 = 513,869$) than for smaller Belgium ($10839 : 21 = 516,186$), thus a Member of Parliament of a larger country represents fewer citizens than an MEP of a smaller State.

U2. Hypothetical example of the impossibility of the simultaneous fulfillment of the DP1 and DP2 conditions caused that in the later section the work was focused on two possible solutions:

A. Adopt a method whose outcomes satisfy Condition DP2 but with a possibly reduced Parliament-size.

B. Propose a change to the Lamassoure-Severin definition of degressive proportionality that lies within existing law and allows greater flexibility and transparency.

Alternative A was rejected on the grounds of the vague methods of execution – iterative interpolation. Therefore, finally it was agreed that a better solution is presented by the alternative B, requiring only minor changes to legislation. It was proposed that Paragraph 6 of Resolution⁷⁴ was as follows (changes highlighted in bold): “6[The European Parliament] Considers that

⁴ European Parliament Resolution on “Proposal to amend the Treaty provisions concerning the composition of the European Parliament”, adopted on 11 October 2007 (INI/2007/2169). See Young (2003, p. 81).

the principle of degressive proportionality means that the ratio between the population and the number of seats of each Member State before rounding to whole numbers must vary in relation to their respective populations in such a way that each Member from a more populous Member State represents more citizens than each Member from a less populous Member State and conversely, but also that no less populous Member State has more seats than a more populous Member State”.

U3. A method was recommended for allocating seats in the European Parliament called “base + prop”. It is expressed as follows.

Let m be the minimum and M the maximum number of seats per Member State, and H the target Parliament-size. Write b for the base and d for the free parameter called the divisor. The allocation function A is given effectively by

$$A(p) = \min \left\{ b + \frac{p}{d}, M \right\},$$

where p denotes the population of a State. The total number of seats allocated to n Member States with populations p_1, p_2, \dots, p_n is

$$T(d) = [A(p_1)] + [A(p_2)] + \dots + [A(p_n)],$$

where $[x]$ is a rounding of the fraction x , and the divisor d is chosen in such a way that $T(d) = H$.

The European Parliament has currently $m = 6$, $M = 96$, $H = 751$. It is recommended to set the base number $b = 5$ and use rounding upwards. In Table 3 seats were provided consistent with the Cambridge Compromise.

In the case of four countries (the UK, Italy, Portugal and Czech Republic) the definition of degressive proportionality was unfulfilled (Lamassoure, Severin, 2007), i.e. the ratio of population/mandates (seats) is with these countries greater than the corresponding ratio for a larger state. Although the discrepancy is minor, it enforces a legal sanction and hence the proposed changes to the EP resolution. Obviously at the time of realization of the relevant law (i.e. elections for the next term of the EP) the violation can affect different states.

In the further part of the paper the authors of the compromise discuss how the change of the number of Member States will affect the minimum number of seats. They state that the need to determine the number of seats clearly is essential to maintain full transparency and they propose for consideration two potential solutions.

1. Number of seats allocated as the base division (currently $27 \times 6 = 162$) should be closest but (should not exceed) 25% of the total number of seats. Currently the ratio is $162/754 \approx 0.21$.

2. The base b is determined as a function n of Member States. The discussion was on the formula of the type $b = 135/n$.

Table 3. The Cambridge Compromise with 27 Member States

Member States	Population ⁵	Bas3 + Quot. → Seats	2009-2014
Germany	81,802,257	5 + 99.90 → 96	99
France	64,886,200	5 + 79.02 → 85	74
United Kingdom	60,714,074	5 + 75.70 → 81	73
Italy	60,340,328	5 + 73.70 → 79	73
Spain	45,989,016	5 + 56.20 → 62	54
Poland	38,167,329	5 + 46.60 → 52	51
Romania	21,462,186	5 + 26.20 → 32	33
Netherlands	16,574,989	5 + 20.20 → 26	26
Greece	11,305,118	5 + 13.80 → 19	22
Belgium	10,839,905	5 + 13.20 → 19	22
Portugal	10,637,713	5 + 12.99 → 18	22
Czech Republic	10,506,813	5 + 12.80 → 18	22
Hungary	10,014,324	5 + 12.20 → 18	22
Sweden	9,340,682	5 + 11.40 → 17	20
Austria	8,375,290	5 + 10.20 → 16	19
Bulgaria	7,563,710	5 + 9.20 → 15	18
Denmark	5,534,738	5 + 6.80 → 12	13
Slovak Republic	5,424,925	5 + 6.60 → 12	13
Finland	5,351,427	5 + 6.50 → 12	13
Ireland	4,467,854	5 + 5.50 → 11	12
Lithuania	3,329,039	5 + 4.10 → 10	12
Latvia	2,248,374	5 + 2.70 → 8	9
Slovenia	2,046,127	5 + 2.50 → 8	8
Estonia	1,340,127	5 + 1.60 → 7	6
Cyprus	803,147	5 + 0.98 → 6	6
Luxembourg	502,066	5 + 0.60 → 6	6
Malta	412,970	5 + 0.50 → 6	6
Total	501,103,425	751	754
Each State receives one non-base seat for every 819 000 citizens or part thereof			

Source: from Grimmett et al. (2011).

With the current populations of members of the EU, the Cambridge Compromise can be best described as follows: each Member State receives 5 seats plus an additional seat for every 819 000 citizens, or part of this

⁵ Data on populations taken from Eurostat (Journal of Laws 22.12.2010 L 338/47).

number or (as viewed from a different perspective): 25% of seats are divided equally, the rest proportionally.

5. Discussion of the Cambridge Compromise

The compromise from Cambridge, which is basically a development of Pukelsheim's (2007; 2010) proposal is one based on classical methods of proportional division. So far this has seemed to be the only solution with a chance of success. The main argument in favor of the compromise is (significantly emphasized by its authors) the great simplicity of the algorithm and (which in the context of the multicultural and diverse interests of the Member States could be important) agreement as to its reasonableness among mathematicians representing the scientific centers of many countries. Relying on classic designs also has a scientific value – all the imperfections of these methods and the accompanying so-called paradoxes (Alabama,⁶ population⁷) are already known. Full awareness of the inadequacy of the proposed solution allows controlling the possible realization of its undesirable effects. The great value of the proposals is also a guarantee of the fixed size of the Parliament. In this case, however, this is not the only solution with such an effect. Despite the undeniable advantages of the compromise, its weak point is a fairly strenuous justification for its uniqueness. This includes two elements. The authors cite a hypothetical example in which they show that it is impossible to satisfy the key conditions for the principle of degressive proportionality, DP1 and DP2. They argue that this example proves that theoretically there can happen such a composition of the Union which excludes their mutual realization. However, this example demonstrates something else. That there may be a situation in which it is impossible to satisfy the three conditions simultaneously: DP1, DP2, and the set size of the Parliament. But the Lisbon Treaty in this respect is less strict. It states only that the Parliament cannot have more than 750 members (plus the President). Thus, the example in no way forces legislative changes – in particular, there is no need to reinterpret the principle of degressive proportionality. The second point also relates to the presented example. The authors wonder whether there always exists a divisor d value thanks to which we could calculate the exact number of seats in the Parliament. Next, following the literature

⁶ The paradox of Alabama: with an increase in the total number of seats to be divided, with unchanged volumes by countries, one of them receives fewer seats than at a lower total number of seats. See Young (2003, p. 81).

⁷ The paradox of the population: the country which has reduced the number of people gaining a seat at the expense of the state whose population has increased (Young, 2003, p. 95).

(Balinski, Young, 2001), they claim that this problem can arise “as a result of unlikely but conceivable coincidences in the factors of population-sizes, but these are very rare in an apportionment problem of the scale of the European Parliament. We feel that this possibility can be disregarded”. Indeed, such a possibility is unlikely. However, the probability is comparable to the likelihood of the situation, which the authors consider as the example of possible conflict conditions DP1 and DP2. The two could refer to the flattening differences in the population of the Member States. The example given by the authors is at the same time a description of such a possibility. There is no degressively proportional distribution (in the meaning of the Lisbon Treaty), which divides the 106 seats between 5 countries listed in Table 2. Accordingly, there exists no value of divisor d , thanks to which 106 seats can be obtained in the Parliament – the equality $T(d) = 106$ does not occur for any d . Thus, the presented example serves the authors as an argument to propose changes to the definition of degressive proportionality. On the other hand, it is believed that the situation which it describes is essentially unrealistic; therefore, such a possibility can be ignored.

6. Conclusions

It is notable that the populations of the EU Member States in Table 1 are different to the ones in Tables 2 and 3 – due to the fact that the data in Table 2 concerns the case of the earlier period. Although the differences in relative terms are not large, it is worth noting that Belgium and Portugal have swapped places in the hierarchy of the size of countries in terms of population. If the compromise from Cambridge would be adopted to achieve this consequence, will Portugal have one seat less than Belgium? This is an example of how demographic changes in EU Member States may affect the differences in the composition of the Parliament in the following terms. If we add to this the prospect of future enlargements of the EU (at the end of 2011 Croatia signed the accession treaty, several countries have the status of a candidate, of which closest to accession seems to be Iceland), there is no doubt that agreeing on an algorithm that could survive more than one term would be very useful. The Cambridge Compromise is relatively well-received in the political environment of many countries (obviously, there are voices of opposition, especially in countries where implementation of such solutions would deplete representation. For example, Hungary would lose four seats, and voices of discontent are heard from there), but its future will be decided by the Intergovernmental Conference, convened in case there occurs the need to change the Treaty. Elegance,

simplicity and its roots in classical solutions appear to be the assets giving the Cambridge Compromise a chance for its implementation.

Literature

- Balinski M., Young H.P. (2001). *Fair Representation*. Brookings Institution Press. Washington, DC.
- Barcz J. (2008). *Poznaj Traktat z Lizbony*. Urząd Komitetu Integracji Europejskiej.
- Cegielka K., Dniestrzański P., Łyko J., Misztal A. (2010). *Skład Parlamentu Europejskiego w kontekście podziałów proporcjonalnych*. In: A. Barczak (Ed.). *Badania ekonometryczne w teorii i praktyce*. Prace Naukowe Uniwersytetu Ekonomicznego w Katowicach. Katowice. Pp. 159-170.
- Dniestrzański P. (2011). *Dywersyfikacja podziału mandatów w Parlamencie Europejskim*. In: *Ekonometria. Zastosowania metod ilościowych*. Prace Naukowe Uniwersytetu Ekonomicznego we Wrocławiu. Wydawnictwo UE. Pp. 32-41.
- Florek J. (2011). *A numerical method to determine a degressive proportional distribution of seats in the European Parliament*. Mathematical Social Science. Doi: 10.1016/j.mathsocsci.2011.07.003.
- Grimmett G., Laslier J.-F., Pukelsheim F., Ramirez-González V., Rose R., Słomczyński W., Zachariasen M., Życzkowski K. (2011). *The Allocation between the EU Member States of Seats in the European Parliament Studies*. PE 432.760.
- Lamassoure A., Severin A. (2007). *European Parliament Resolution on "Proposal to amend the Treaty provisions concerning the composition of the European Parliament" adopted on 11 October 2007* (INI/2007/2169).
- Łyko J. (2011). *Diversification and stability of distribution of seats in the European Parliament*. Journal of International Scientific Publication: Economy & Business. Vol. 5. Part 2. European Union. Pp. 175-180.
- Martinez-Aroza J., Ramirez-Gonzalez V. (2008). *Several methods for degressively proportional allocation. A case study*. Mathematical and Computer Modelling. Vol. 48. Pp. 1439-1445.
- Pukelsheim F. (2007). *A Parliament of Degressive Representativeness?* Institut für Mathematik. Universität Augsburg. Preprint Nr 015/2007.
- Pukelsheim F. (2010). *Putting citizens first: Representation and power in the European Union*. In: M. Cichocki, K. Życzkowski (Eds.). *Institutional Design and Voting Power In The European Union*. Ashgate. London. Pp. 235-253.
- Ramirez-Gonzalez V., Palomares Batistuta A., Márquez Garcia M. (2006). *Degressively proportional methods for the allotment of the European Parliament seats amongst the EU member States*. In B. Simeone, F. Pukelsheim (Eds.). *Mathematics and Democracy. Recent advances in Voting Systems and Collective Choice*. Springer Verlag. Berlin. Pp. 205-220.
- Słomczyński W., Życzkowski K. (2011). *Mathematical Aspects of Degressive Proportionality*. Arxiv:1110.1468v1 [physics.soc-ph].
- Young H.P. (2003). *Sprawiedliwy podział*. WN SCHOLAR. Warszawa.