VARYING EFFECTIVENESS OF TEACHING *VERSUS* CLASS SIZE[1]

Wiktor Ejsmont

Abstract. In the article, the author attempts to measure the changing effectiveness of learning in different-sized classes in Poland. For this purpose, he uses models for panel data. The application of panel models for measuring effectiveness was launched by M. Aitkins and N. Longford in 1986. The author also introduces the concept of unevenness of teaching effectiveness. In the second part of the article he applies the data to the proper models and draws conclusions from the obtained results.

Keywords: school effectiveness, class size, analysis of panel data.

JEL Classification: A20, I20, I21.

1. Introduction

The aim of this article is to examine the dependency between teaching effectiveness and class size. The problem of class size is widely described in the relevant literature, constituting one of the pillars of the education reform. Contrary to appearances, it is not only an object of research for scientists associated with pedagogy, but also for those who deal with economics. Plenty of both theoretical and empirical research was devoted to the concept of *class size effect*. The theoretical studies include such tools as Markov processes (Gary-Bobo, Mahjoub, 2006) or differential equations theories, which are used in solving problems presented in a purely formal manner (Lazear, 2001).

It might seem that creating smaller classes helps improve teaching effectiveness because it is easier to maintain order and discipline in a smaller class. Parents have better access to and contact with the teacher, which helps to solve any potential problems concerning their child. For example, in

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France the average number of students has fallen from 30 to 20. This implies an increase in costs, which according to some economists may be undesirable. One of the main studies that can be cited here was the STAR project carried out in Tennessee, USA. Students in the experiment were assigned randomly (regardless of prior achievement) to classes of different sizes. The result indicated that efficiency increased with a reduction in the number of students in the classroom (Krueger, 1999, 2003).

Including class size to a model can be seen for example in the work by Hoxby (2000), who accounted for the class size logarithm in constructing a random effects model. Taking into account the natural logarithm of the number of students in the classroom, the equation aims to assess the impact of rate changes in this variable. The obtained results indicated that class size had no relevant impact on future teaching effects. Another methodology was adopted in the study by Angrist, Lavy (1999). They built a function that projected the class size based on the number of students in the school. This study has shown that a greater number of students in the classroom affects the students' results.

Subsequently, e.g. Dobbelsteen, Levin, Oosterbeek (2002) postulated that under certain conditions a bigger number of students in the classroom will improve the learning outcomes. This effect can be explained as follows: in a larger class it is more likely to find students with similar interests, knowledge and abilities. From the point of view of pedagogical theories, such conditions are conducive to the general growth of knowledge in the classroom. Hence, based on the existent literature, it is difficult to indicate clearly the supporting arguments for either an increase or a reduction in the number of students in the classroom.

The aim of this article is to show the performance of the educational value added (EVA), depending on the location and size of class. The model used here follows the model described by Aitkin, Longford (1986), who presented several models to study the effectiveness of education in American schools. This model will be adapted to a form with which it is possible to measure the effectiveness of teaching; however, not within one school but with regard to the size and location type of the class in which the student completes high school (in Poland called *liceum*).

Classes in Polish high schools are formed in different ways. In order to reduce the cost of education, local governments often urge schools to create large classes that exceed 30 students. This causes differences. In rural areas the average class size is smaller than in cities. The Polish educational policy does not follow any specific algorithm as far as class sizes are concerned.

A good example can be classes in physical education. Local educational superintendents oblige schools not to exceed 30 students in PE classes. However in reality this is hardly respected as it entails high costs.

In Poland the size of classes in cities is much larger than the size of classes in villages. Obviously the students' performance in cities is also higher. Better performance of students in urban areas is caused by the fact that their parents on average have a higher education status and are wealthier, as well as students having access to learning aids and a wider contact with culture. Such differences condition the better results of city school students even if the bigger class size affects the quality of education.

2. Data description

The study on the relationship between EVA and class size has been carried out based on class size and school location. Four types of school location were adopted: village – V;[2] city up to 20 thousand residents – C20; city between 20 and 100 thousand residents – C20-100; city over 100 thousand residents – C100. The class size values have been assumed every three students, starting from 10 and ending at 38 students. The object of the study was "location + aggregate class size" – later in the article referred to as "object". The subjects of the study were high school graduates who qualified for one of the analyzed sections (location + aggregate class size). Two Polish high school curriculum subjects were analyzed – Polish language in 2008-2010, and mathematics in 2010. Table 1 shows the aggregated results at the level of analyzed locations and aggregated class sizes throughout Poland. The values "run through" the class sizes between 10 and 38 students. Each group is numerous, only for villages in the 37-38-students category smaller values have been observed.

Denominations:

- x_{ij} – the number of points obtained by i junior high school (or gymnasium) students within j – this object;
- y_{ij} – the number of points obtained by the i high school students within j – this object;
- n_j – the number of students within j – this object;
- n – the number of all students, i.e. $n = n_1 + \ldots + n_k$;
- k – the number of objects;
- $\bar{x}$ – average final exam score of all junior high school graduates;

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[2] High schools (*licea*) are rare in rural areas although in every Polish voivodship there are such schools. Most of them are located in eastern Poland.

- $\bar{y}$ – average final exam score of all high school graduates;
- $\bar{x}_j$, $\bar{y}_j$ – average final exam score, respectively: junior high and high school at the level of j – this object.

Table 1. Summary of average student's results (scores) depending
on the location and size of class

Object	2008			2009			2010				
	No. of students	Average J-H	Average M-P	No. of students	Average G-H	Average M-P	No. of students	Average J-H	Average M-P	Average J-S	Average M-M
V 10-12	520	66.01	47.95	430	60.27	44.91	309	63.74	50.75	46.43	47.13
V 13-15	716	67.45	48.66	530	62.12	47.28	462	64.68	49.84	46.55	48.55
V 16-18	986	68.09	47.86	956	64.55	51.27	706	67.75	56.89	53.05	57.58
V 19-21	1018	71.12	53.99	914	66.21	54.42	842	67.49	54.57	50.84	55.19
V 22-24	875	70.09	54.67	1354	66.70	53.52	618	68.08	55.05	50.71	54.24
V 25-27	902	69.60	51.89	769	66.59	53.36	752	68.01	57.65	51.54	58.74
V 28-30	731	71.65	55.43	871	68.77	55.99	347	70.28	58.85	52.05	57.82
V 31-33	541	76.95	58.67	609	69.88	58.47	190	69.15	60.03	48.63	53.84
V 34-36	245	75.91	61.52	420	74.62	63.31	138	72.14	62.90	55.33	66.52
V 37-38	111	81.86	64.05	37	61.03	42.70	38	83.74	67.89	63.63	66.63
C20 10-12	1877	68.06	50.30	975	61.29	48.45	867	63.29	51.57	46.47	47.55
C20 13-15	2692	68.22	51.30	1997	64.34	50.82	1671	65.49	54.59	48.27	50.93
C20 16-18	3975	69.22	52.57	3356	64.32	51.77	2868	67.81	55.76	50.62	55.35
C20 19-21	5192	70.59	53.97	4612	66.99	54.49	4788	69.20	58.16	52.33	57.22
C20 22-24	7846	72.37	55.11	7183	68.32	55.33	6665	71.55	60.49	55.27	61.55
C20 25-27	8326	74.37	57.26	9453	69.25	56.02	8730	73.62	61.50	58.85	64.90
C20 28-30	9064	75.98	58.80	10764	71.81	58.72	9869	74.98	63.01	60.20	66.51
C20 31-33	5646	77.56	60.57	8424	73.31	60.09	6687	76.14	63.71	62.23	68.47
C20 34-36	2315	78.25	60.58	2986	74.25	60.89	2463	76.86	63.91	64.39	70.68
C20 37-38	823	77.30	63.33	595	76.10	61.07	600	80.25	65.95	67.02	73.86
C20-100 10-12	2497	68.35	52.39	1245	60.51	46.97	1088	63.47	52.38	46.29	48.72
C20-100 13-15	3179	69.08	54.31	2135	63.12	49.25	2093	65.21	54.55	47.35	50.42
C20-100 16-18	4726	69.68	52.18	3513	64.35	50.63	3515	66.91	55.92	50.54	54.72
C20-100 19-21	6211	71.32	53.53	5315	66.94	54.21	4670	69.73	58.52	53.89	58.22
C20-100 22-24	9472	73.16	55.30	8880	69.28	56.93	8237	71.82	60.99	55.99	61.43
C20-100 25-27	13233	76.40	59.32	13341	71.58	59.34	12328	74.70	62.93	59.93	65.80
C20-100 28-30	18468	78.59	61.29	19878	74.10	61.70	18711	77.17	65.10	63.99	70.33
C20-100 31-38	16308	79.86	63.13	19172	75.56	63.42	18120	78.58	66.40	66.61	72.73
C20-100 34-36	8673	80.70	64.58	11094	76.49	64.45	8001	80.23	66.98	69.69	75.75
C20-100 37-38	1422	82.01	66.18	1383	76.37	64.15	1193	81.07	66.99	72.53	77.15
C100 10-12	3270	70.00	53.20	2414	64.42	50.85	2129	67.88	56.16	50.70	53.98
C100 13-15	4209	70.09	52.72	3752	65.53	51.39	3304	68.23	56.36	50.20	53.82
C100 16-18	5431	71.96	53.83	4720	67.31	53.95	4647	69.52	56.96	52.42	56.09
C100 19-21	6558	73.33	55.12	6269	68.15	54.21	5412	71.48	59.47	55.11	59.47
C100 22-24	8513	76.41	58.73	8148	70.22	56.35	8026	74.75	61.76	59.70	64.58
C100 25-27	13554	78.44	60.89	12984	73.99	60.06	12201	77.84	65.18	65.49	70.07
C100 28-30	18930	81.23	63.88	21357	76.59	63.15	20610	80.06	67.43	69.41	74.03
C100 31-33	17593	82.67	65.71	20496	78.76	65.73	16799	82.05	69.61	73.82	77.34
C100 34-36	7266	82.75	66.86	9178	79.14	66.03	7319	82.52	69.54	74.45	77.77
C100 37-38	1046	81.69	65.08	1494	77.19	64.14	975	82.10	67.54	71.18	74.91
Total	224960	76.48	59.34	234003	72.58	59.60	208988	75.71	63.53	62.38	67.47

Source: Central Examination Commission in Warsaw (2010).

3. Research methodology

3.1. The Aitkin-Longford model

The data presented in Point 2 is called non-balanced panel data. The panels are non-balanced as the number of observations for each objectis different, i.e. n_j. In the case of equal numbers of observations the obtained data is balanced. In the literature these models are most frequently applied to cross-sectional and time series data, i.e. those where the object is observed during a particular time.

The model applied here is a model of random factors. In econometrics, this model owes its popularity to the article by Balestra, Nerlove (1966) on the demand for natural gas. When the population that we want to describe is not homogeneous, the study model must account for this heterogeneity. If the sample of items comes from a large population, it is better to assume that the individual item effect is a realization of a random variable. In this model there are two random components. The random factors model is also known as a variance components model (VC) or an error component model. This model takes the form of:

$$y_{ij} = \alpha + \beta x_{ij} + \xi_j + e_{ij}.^3 \tag{1}$$

It is assumed for the model that:

- e_{ij} = random variableof the $N(0,\sigma^2)$ distribution;

- ξ_j = random variable of the $N(0,\sigma_I^2)$ distribution;

- the random components for different schools and different students are uncorrelated;

- the individual random component ξ_j is uncorrelated with the random component e_{ij} (i.e. $E(\xi_j, e_{is}) = 0$).

With the above assumptions, we get:

$$\mathrm{var}(y_{ij}) = \mathrm{var}(\xi_j + e_{ij}) = E(\xi_j + e_{ij})^2 - E^2(\xi_j + e_{ij}) =$$
$$= E(\xi_j^2 + 2\xi_j e_{ij} + e_{ij}^2) = \sigma_I^2 + \sigma^2, \tag{2}$$

$$\mathrm{cov}(y_{ij}, y_{pj}) = \mathrm{cov}((\xi_j + e_{ij}),(\xi_j + e_{pj})) = E(\xi_j^2 + \xi_j e_{ij} + \xi_j e_{pj} + e_{ij}e_{pj}) = \sigma_I^2$$

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[3] In the context of the beneficiaries of education, this model was first described byAitkin, Longford (1986), hence the title of the section.

$$\rho = cor(y_{ij}, y_{pj}) = \frac{\sigma_I^2}{\sigma_I^2 + \sigma^2}. \tag{3}$$

The coefficients of the thus formulated model are estimated by means of maximum likelihood (e.g. Atkin, Longford, 1986) or by the generalized least squares method (e.g. Baltagi, 2005). The estimator of the α and β parameters is of the form:

$$\begin{bmatrix} \hat{\alpha} \\ \hat{\beta} \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{k} w_j & \sum_{j=1}^{k} w_j \bar{x}_j \\ \sum_{j=1}^{k} w_j \bar{x}_j & \sum_{j=1}^{k}\sum_{i=1}^{n_j} (x_{ij} - \bar{x}_j)^2 + \sum_{j=1}^{k} w_j \bar{x}_j^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{j=1}^{k} w_j \bar{y}_j \\ \sum_{j=1}^{k}\sum_{i=1}^{n_j} (y_{ij} - \bar{y}_j)(x_{ij} - \bar{x}_j) + \sum_{j=1}^{k} w_j \bar{x}_j \bar{y}_j \end{bmatrix},$$

where $w_j = n_j \sigma^2 (\sigma^2 + n_j \sigma_I^2)$. More on the estimation can be found in the work by Ejsmont (2009b), where the whole estimation algorithm is described in detail, including an estimation of variance components σ^2 and σ_I^2. Below, the procedure for the learning efficiency treatment is described (as used by Aitkins and Longford). The compilation of objects is done by comparing the expected value of the random component ξ_j (formula 1). This component refers to how much the average result $j-$ of the object $-$ deviates from the average result for the whole population. In Figure 1, the dashed line marks the average result $j-$ of the object, while the continuous line shows the average result across the population (factor e_{ij} is responsible for the deviation from the level of average result $j-$ of the object). If value ξ_j is positive, then we can say that the $j-$ the study object $-$ made progress in relation to the average general population result; whereas if it is negative, then it scored lower than the average general population result.
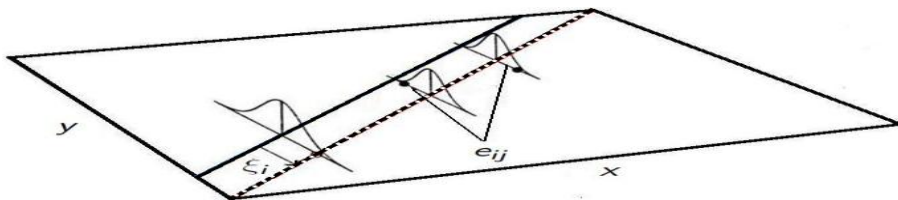

Fig. 1. Diagram showing the concept of measuring the growth of knowledge
with the Aitkin and Longford model

Source: own calculations based on Skrondala, Rabe-Hesketh (2008, p. 96).

To estimatethe value of the ξ_j component (which is not known), we have used the Mean Square Error theorem quoted below (e.g. Jakubowski, Sztencel, 2004, p. 135).

The theorem**.** *Suppose that a random vector* (A, B) *is given where variable* A *is observed and variable* B *cannot be observed. If* $E(B^2) < \infty$, *then the optimal forecast (for* B *) in the sense of mean square error does exist and one can take* $E(B/A)$.

Because components σ^2 and σ_I^2 are known before the estimation of the model, we can use this as *a priori* information. We determine the conditional distribution of the random variable ξ_j under the condition $\bar{y}_j$ (Bayesian approach). From formula (1), the average at the level $j-$ of the given school, can be expressed by:

$$\bar{y}_j = \alpha + \beta\bar{x}_j + \xi_j + \bar{e}_j. \tag{4}$$

With the assumptions thus made, $\bar{y}_j$ is normally distributed: $N(\alpha + \beta\bar{x}_j, \sigma_I^2 + \sigma^2/n_j)$. This distribution has been taken as *a priori* distribution. Because ξ_j is a random variable of the $N(0, \sigma_I^2)$ distribution then the conditional distribution $f(\xi_j/\bar{y}_j)$ will also be a normal distribution.

Note**:** From the probability theory, the following fact is known: if the random variables are: $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$, and $\rho_{1,2} = cor(X_1, X_2)$, then the conditional distribution X_1/X_2 is in the form of:

$$N\left(\mu_1 + \rho_{1,2}\frac{\sigma_1}{\sigma_2}(X_2 - \mu_2), \sigma_1^2(1 - \rho_{1,2}^2)\right).$$

Thus, given the fact: $\rho' = cor(\xi_j, \bar{y}_j) = \sigma_I^2/(\sigma_I\sqrt{\sigma_I^2 + \sigma^2/n_j})$, we conclude that $f(\xi_j/\bar{y}_j)$ has normal distribution in the form of:

$$N\left(\rho'\frac{\sigma_I}{\sqrt{\sigma_I^2 + \sigma^2/n_j}}(\bar{y}_j - \alpha - \beta\bar{x}_j), \sigma_I^2(1 - \rho'^2)\right), \text{ or reformulated as:}$$

$$N\left(\rho n_j^*(\bar{y}_j - \alpha - \beta\bar{x}_j), n_j^*(1 - \rho)\sigma_I^2/n_j\right), \tag{5}$$

where $n_j^* = w_j /(1-\rho)$. The comparison of schools has been based on the comparison of average values of the conditional distribution given in formula (5). Thus, the effectiveness of teaching or the educational value added (EVA) has been defined as:

$$e_j = \hat{\rho}n_j^*(\bar{y}_j - \hat{\alpha} - \hat{\beta}\bar{x}_j).\qquad(6)$$

In order to verify whether the obtained random effects are significant, let us use the Breusch-Pagan test (e.g. Baltagi 2005). This is a test of Lagrange multipliers with the hypotheses: $H_0 : \sigma_I^2 = 0$.

Alternative hypothesis: $\sigma_I^2 \neq 0$.

Test statistics is of the form:

$$LM = \frac{\left(\sum_{j=1}^{k}n_j\right)^2}{\left(2\sum_{j=1}^{k}n_j(n_j-1)\right)}\left[\frac{\sum_{j=1}^{k}\left(\sum_{i=1}^{n_j}e'_{ij}\right)^2}{\sum_{j=1}^{k}\sum_{i=1}^{n_j}e'^2_{ij}}-1\right]^2 \sim \chi^2(1),\qquad(7)$$

where e'_{ij} are the residuals obtained by applying the OLS method to all data (regardless of the school). The above formula states that the test statistics *LM* has asymptotic chi-square distribution (assuming the null hypothesis) with one degree of freedom. We reject the null hypothesis if the value of the *LM* statistics is in the critical right-hand area.

Table 2. Basic statistical characteristics of the random effects model

Subject	Polish language			Mathematics
Characteristics	2008	2009	2010	2010
Random component variance – σ^2	147.288	135.797	171.555	206.026
Inter-object variance – σ_I^2	0.521	0.479	0.438	1.692
p – value normality	> 0.01	> 0.01	>0.01	> 0.01
beta coefficient	0.622	0.603	0.544	0.721
alpha coefficient	10.981	14.554	21.093	20.711
LM – p – value	< 0.01	< 0.01	< 0.01	< 0.01

Source: own calculations, using Excel and R-project and based on the data from the Central Examination Commission in Warsaw (2010).

Table 2 presents the main statistical characteristics of the estimated models. The resulting models are matched in terms of residual normality. The estimated values of the *LM* test indicate that σ_I^2 is statistically significant at the significance level of 0.01. Therefore the use of a random effects model (associated with σ_I^2) is legitimate. The resulting coefficients β are at a very similar level.

3.2. The uneven growth of knowledge and Lazear's theorem

The measuring unit which expresses the progress of students by a non-negative[4] number can be the ratio of points obtained by a given student at the academic high school final examinations (the final "maturity" exams, or the actual student's knowledge) to the averaged result of all students who had the same number of the junior high school graduation points. This result will be represented by the value:

$$p_{ij} = \frac{y_{ij}}{\hat{y}_{ij}}, \tag{8}$$

where $\hat{y}_{ij} = \alpha + \beta x_{ij}$ is a simple linear regression fitted to all the data regardless of the object.

The larger the ratio of the actual result to the average result, the greater the progress made by a given student. Hence, to measure the irregularity of knowledge (which should be understood as uneven teaching effectiveness), it is enough to calculate the Gini coefficient:

$$G(\overline{p}_j) = Gini_j(\overline{p}_j), \tag{9}$$

where $\overline{p}_j = (p_{1j,\dots,}p_{n_jj})$.

Table 3. The coefficients of simple linear regression fitted to the data describing the humanities and science final exam results

Subject	Year	Gradient coefficient – β	Intercept coefficient – α
Polish language	2008	0.677	7.555
	2009	0.659	11.789
	2010	0.585	19.227
Mathematics	2010	0.761	20.001

Source: own calculations, using Excel and based on the data from the Central Examination Commission in Warsaw (2010).

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[4] Because we want to calculate the standard Gini coefficient which is defined on the basis of non-negative data.

Table 3 shows the coefficients of simple linear regression fitted to all the data that represent the results of both junior and academic high school final examinations for individual school subjects and years of students.

Below, an excerpt of the theorem presented by Lazear (2001) is quoted. More on the results obtained by Lazear can be found in the work by Ejsmont (2008, 2009a).

Lazear's theorem. *Let p be the probability that a student does not interrupt his or her own or other students' learning process, i.e. behaves well; then the optimum class size is an increasing function of the probability parameter p.*

A way to interpret the p parameter can be described by the following reasoning: suppose that the class consists of twenty students of which ten are clever and ten less intelligent. Then, by creating one smaller class of the ten less intelligent students we should not expect that the teaching effectiveness will increase. However, if we create a class of five clever students and five less intelligent ones, then the results can be expected to improve. In other words, the p parameter represents the homogeneity of the analyzed class.

4. The obtained results and conclusions

Figure 2 illustrates the change in the educational value added depending on class size and location calculated for the subject of Polish language. A distinct increase in efficiency as the class size increases is clearly visible. The increase is also visible depending on location, i.e. the more densely populated the school location area, the greater the EVA. Moreover, also the formation of certain optimal points is noticeable. Depending on the year, the optima occur at different points. However, some exceptions to this rule can also be seen, such as the 37-38 point for cities of less than 20 thousand people. It is noticeable that smaller class sizes (regardless of the year) generally perform much worse than the bigger sizes. Table 1 shows that smaller classes on average scored lower in junior high and high education. The reason for this could be that the classes were created according to the prior achievement of students. The results obtained may be subject to some error associated with the fact that smaller classes consisted of less able students. The situation is very well explained by Figure 3, which shows that the small classes are taught much more unevenly than larger ones. Figure 3 shows the unevenness for location + aggregate class sizes from Table 1. The unevenness was calculated according to the methodology presented in Section 3.2,

based on the models from Table 1. Creating smaller classes in Polish high schools (*liceum*) is often due to the fact that such classes are formed of students who did not qualify in the recruitment process for the types of classes about which they dreamt. In times of demographic decline the head teachers of high schools, wishing to keep the job posts for teachers, either create smaller classes for those students who in the recruitment process received a smaller number of points or place such students in other classes. Obviously, this does not apply to all the surveyed high schools. Nevertheless, such a scenario would certainly be confirmed by the average final examination results from Table 1. The average results increase with the class sizes; however, they are actually falling in many of the classes. The above reasoning explains why teachers do not put much effort to teach thus assorted young people. A considerable unevenness in the increase of students' knowledge can be observed. The situation would be quite different if the students were analyzed not by segregating them according to their achievements, but at random, as in the studies conducted within the STAR program.
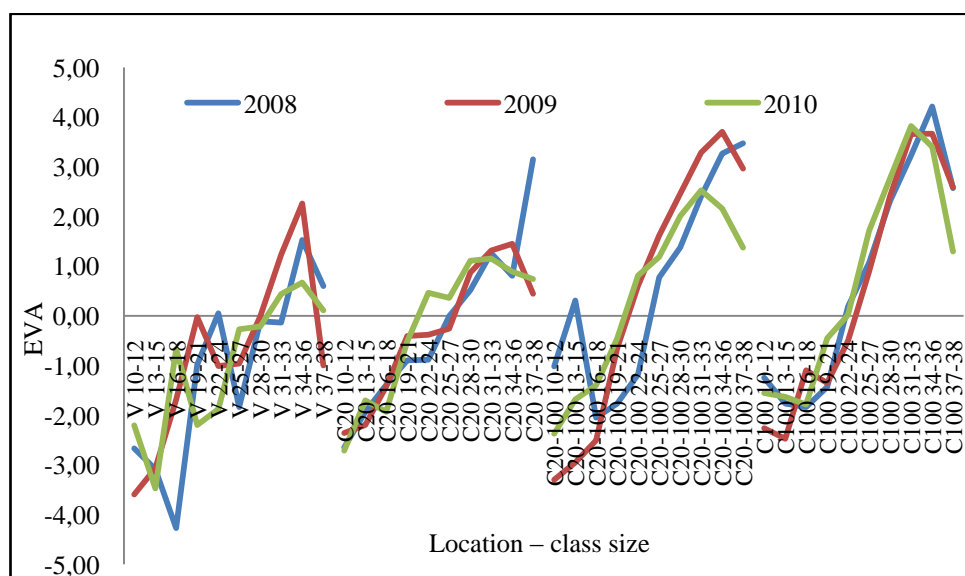

Fig. 2. Educational value added, depending on the location and size of the class designated for the subject of Polish language

Source: own calculations, using Excel and based on the data from the Central Examination Commission in Warsaw (2010).

In the analyses on the effects of class size, it is worthwhile to refer to the work by Akerhielm (1995).The theme of segregating students according

to their ability appears there. To avoid errors associated with, e.g. a strong correlation between class size with other factors that influence student performance, they took into account the average number of students in the classes of the school to prevent the overloading resulting from the segregation of students with different abilities between the classes (within the school).
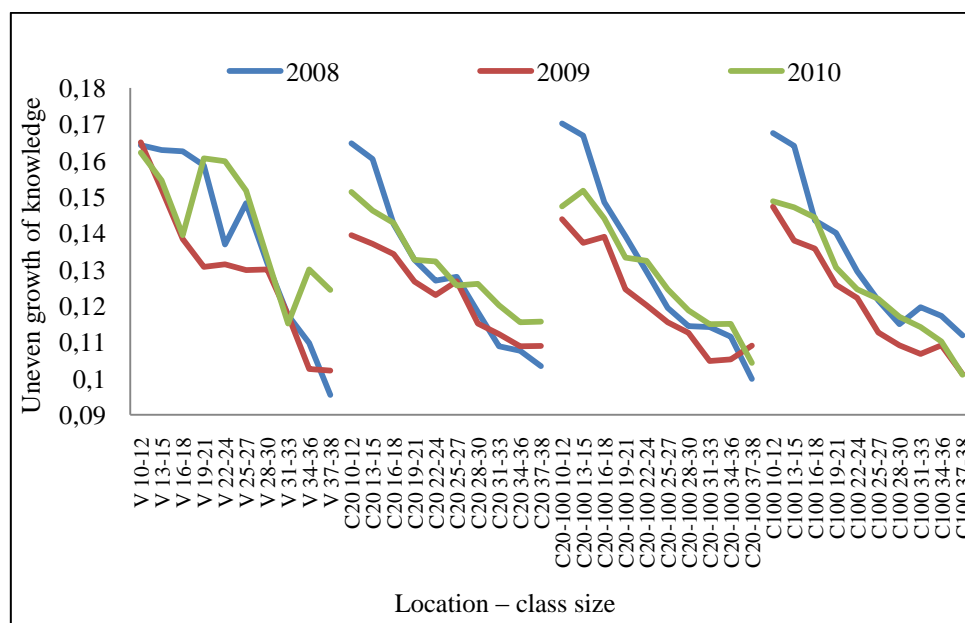

Fig. 3. Unevenness, depending on the location and size of the class designated for the subject of Polish language

Source: own calculations, using Excel and based on the data from the Central Examination Commission in Warsaw (2010).

The question remains about what the situation looks like in the case of science subjects. The answer can be found in the biaxial Figure 4. The EVA is marked on the right-hand vertical axis and the uneven distribution of knowledge is marked on the left-hand axis – the axes being of different scales. They were drawn in this manner on purpose: to emphasize the relationship between the EVA and the educational unevenness. It can be seen that the differences between the educational value added and the school location are blurred. The differences between village and city locations are not as distinct as in the case of Polish language. Thus the high school location has a bigger impact in the case of humanities. It is clearly visible that the resulting unevenness is always strongly associated with the EVA.

A "soaring growth" (or decrease in the EVA) causes a decrease in educational unevenness (sometimes "slower growth").
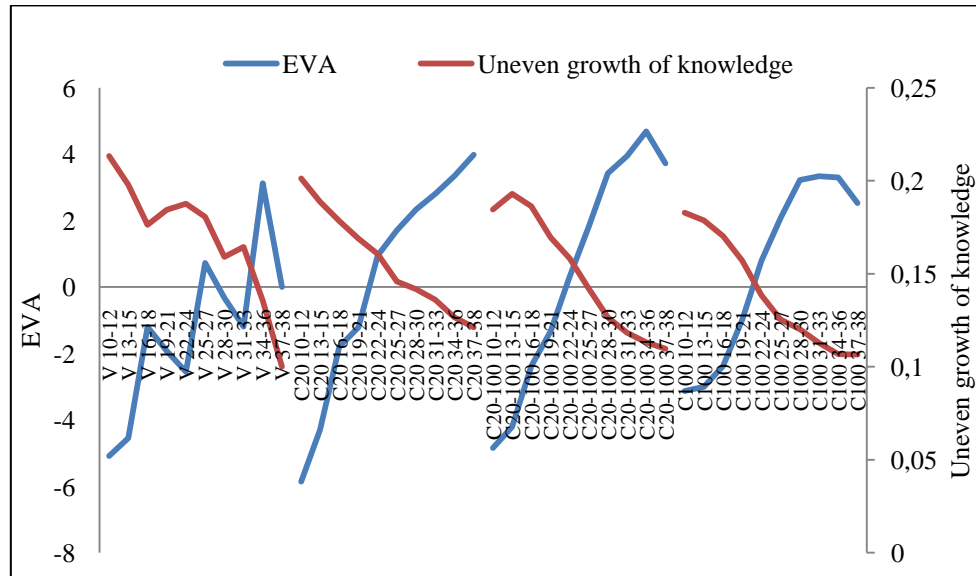

Fig. 4. Educational value added and unevenness of teaching, depending on the location and size of the class designated for mathematics

Source: own calculations, using Excel and based on the data from the Central Examination Commission in Warsaw (2010).

The presented results have shown that the effectiveness of learning increases with the class size (in most of the analyzed cases). At first glance it might seem that the obtained results are inconsistent with the theoretical results obtained by Lazear. Nothing could be more misleading, though, as the presented results also show that with increasing class size the unevenness of teaching effectiveness increased. The unevenness can also be associated with the behavior of students. Greater diversification of the population affects the performance of students. It has been shown that educational unevenness is decreasing, which can be interpreted as $1 - p$ from Lazear's model. So p is growing, thus also increasing the effectiveness of teaching.

The presented reasoning shows that it is not possible to observe in a one-dimensional manner that the increase in class size increases the effectiveness of teaching. With the increase in class size, the educational unevenness was decreasing, showing that the surveyed classes were not homogeneous in terms of students who attended them. The author would like to emphasize also that in the relevant international literature (mentioned in the

introduction), the researchers also noticed that one cannot see the relationship between the EVA and class size in a unidirectional manner, either. The author only demonstrates a certain new way of interpreting (by an uneven distribution of knowledge) the increase of teaching efficiency depending on the size of class, i.e. due to the fact of the unevenness of teaching in smaller classes.

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