M A T H E M A T I C A L  E C O N O M I C S

No. 7(14) 2011

# A COMPARISON OF THE BEHAVIOUR OF MARKET OPTION PRICES IN RELATION TO OPTION PRICES RESULTING FROM THE BLACK-SCHOLES MODEL DURING PERIODS OF A BULL AND BEAR MARKET

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**Abstract.** In continuation to one of my previous papers (*Kształtowanie się cen opcji indeksowych a ich wycena na podstawie modelu Blacka-Scholesa*), where the research problem to what extent pricing of option using theoretical models is tangent to the behaviour of market option prices in Polish conditions was scrutinized, in the present work it is verified whether there exist significant differences between the behaviour of market option prices in relation to option prices resulting from the Black-Scholes model during periods of a bull and bear market. In order to make those comparisons, the parameters of the lines of regression describing the dependence of market prices of options for index WIG20 on their prices resulting from B-S model were calculated. On the basis of these calculations, it is possible to say how market prices resemble prices resulting from the model. The research was conducted for two periods of similar length, assuming that between 20 September 2004 and 18 March 2005 we had a bull market and during the days between 1 August 2008 and 20 March 2009 the market was bearish.

**Keywords:** option pricing, derivatives, Black and Scholes model.

**JEL Classification:** G13.

## 1. Introduction

Correct option pricing is very important for the appropriate functioning of their markets so it was widely discussed during the initial period of their existence. Nowadays one of the most often used methods of option pricing, which was created at the beginning of the 1970s, is Black and Scholes model (Black, Scholes, 1973) and its subsequent modifications.[1]

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[1] For example, the Garman-Kohlhagen foreign currencies option pricing model (Garman, Kohlhagen, 1983), Merton model for options with underlying security paying a dividend (Merton, 1973).

In one of my previous papers (see (Forlicz, 2007)), it was checked to what extent options prices resulting from the application of theoretical models are consistent with the market option prices quoted in Poland. However this analysis was conducted only for data from the bull market. The purpose of the present article is to check whether there exist differences between the behaviour of market option prices in relation to option prices resulting from the Black-Scholes model during periods of a bull and bear market.

## 2. Data

Two periods, each approximately half-a-year, were taken into consideration. The first period, considered a bull market period, began on 20 September 2004 and ended on 18 March 2005. The second period, considered a bear market period, began on 1 August 2008 and ended on 20 March 2009. The observed options were put and call options for WIG20 Index with different nominal.

## 3. Model

The Black and Scholes model used for pricing chosen options is described by the following equations:

$$C = S \cdot N(d_1) - X \cdot e^{-rT} \cdot N(d_2) \quad , \quad P = X \cdot e^{-rT} \cdot N(-d_2) - S \cdot N(-d_1)$$

$$d_1 = \frac{\ln(\frac{S}{X}) + (r + \frac{\sigma^2}{2}) \cdot T}{\sigma \cdot \sqrt{T}}, \quad d_2 = \frac{\ln(\frac{S}{X}) + (r - \frac{\sigma^2}{2}) \cdot T}{\sigma \cdot \sqrt{T}} = d_1 - \sigma \cdot \sqrt{T},$$

where:

$C$ – call option price;
$P$ – put option price;
$S$ – current price of underlying asset;
$X$ – option strike price;
$r$ – risk-free interest rate;
$T$ – time to maturity expressed in years;
$\sigma$ – standard deviation of rate of return;
$N(d_i)$ – standard normal cumulative distribution function for $d_1$and $d_2$.

In this paper the spot price of underlying asset was a value of WIG20 index[2] (expressed in points). Also strike prices and option prices were expressed in points. Risk-free interest rates were calculated as weighted average of WIBBOR rates[3] for the two, closest to maturity, periods for which WIBBOR exists. For example, for four months the applied formula was:

(2xWIBOR_3M+1xWIBOR_6M)/3.

Standard deviation of rate of return was calculated separately for every day of pricing for data reaching one year back.

## 4. Call options results

The exemplary pattern of behaviour of call options prices quoted during a bull and bear market can be seen in Figures 1 to 6.

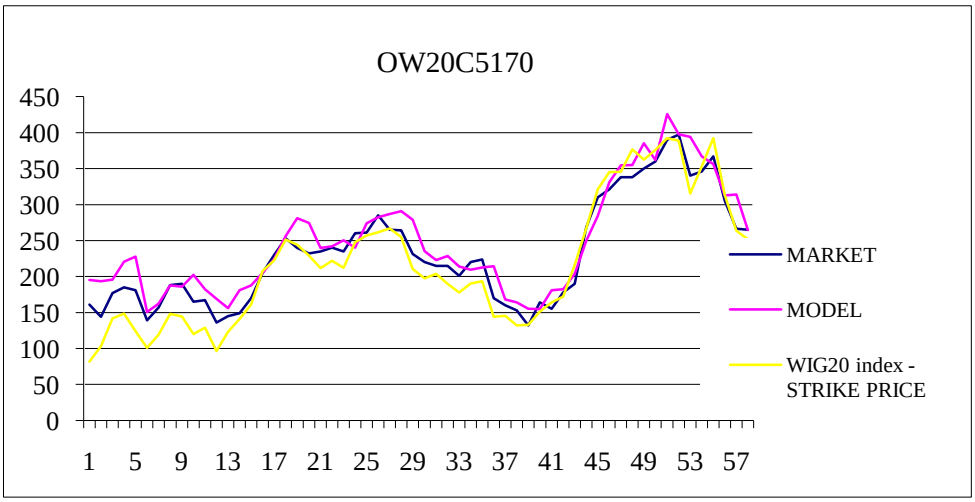


Fig. 1. Market and model prices of call option
with strike price 1700 points expiring in March 2005

Source: author's own study.

By just looking carefully at the figures above, it can be noticed that during a bull market option prices quoted on the market were underestimated relative to model prices and during bear market options quoted on the market are slightly overestimated relative to the B-S model. Another pat-

[2] See: http://www.money.pl/gielda/archiwum/indeksy/.
[3] See: http://www.money.pl/pieniadze/depozyty/zlotowearch/.

tern, which cannot seem strange in any way, is that when getting closer to maturity (moving right in the figure) the market and model prices get closer together. Knowing that visual assessment can be misleading in order to check whether there really appears an under- or overestimation of market prices, regression lines describing dependence of market prices on model prices were computed. The results of those calculations are shown in Tables 1 and 2.

The obtained results most often confirm the conclusions drawn earlier. For the three options quoted at the end of 2004 and the beginning of 2005 the slope of the regression line is significantly lower than 1, or the intercept is lower than 0, or not statistically significantly different from 0; for two options the slope is not significantly different from 1, and the intercept is lower than 0 or not significantly different from 0. In the case of the options quoted when the market was bearish, it is a bit more complicated to draw unequivocal conclusions. For at least five options (depending on the assumed significance level), the slope of the regression line is greater than 1 with the intercept significantly greater than 0. For at least twelve options slope is not significantly different from 1with the intercept most often significantly greater than 0.

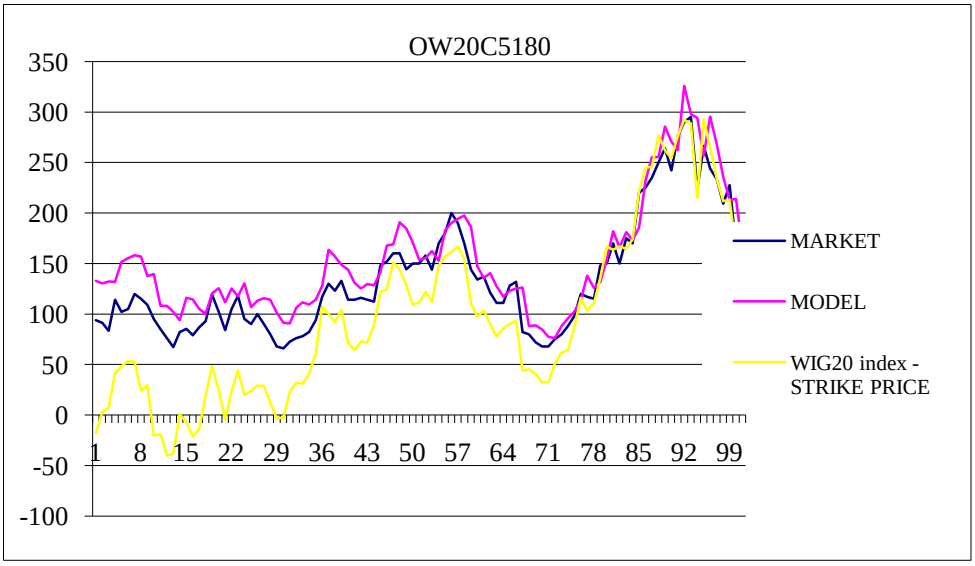


Fig. 2. Market and model prices of call option
with strike price 1800 points expiring in March 2005

Source: author's own study.

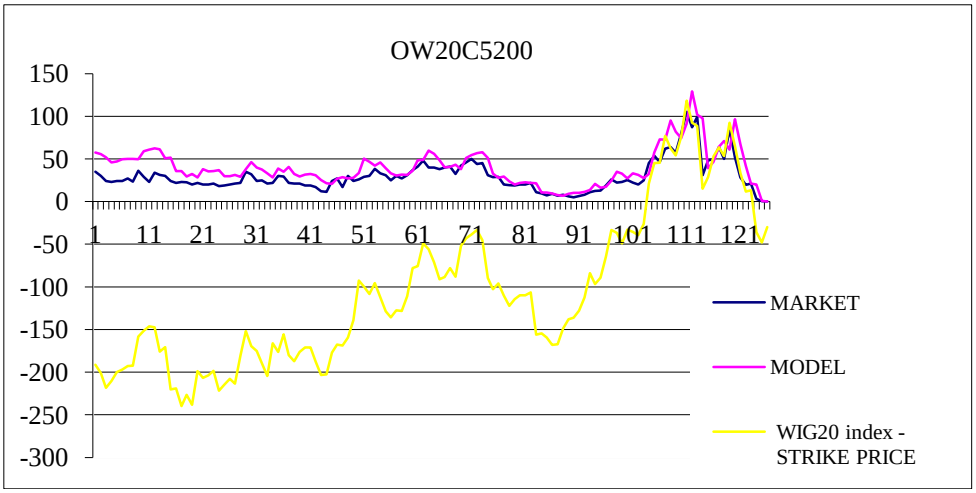


Fig. 3. Market and model prices of call option
with strike price 2000 points expiring in March 2005

Source: author's own study.

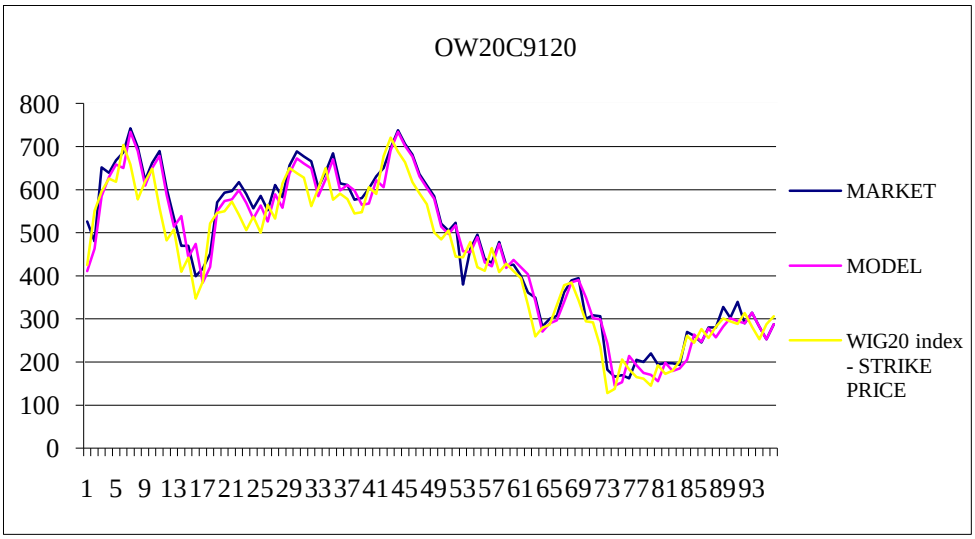


Fig. 4. Market and model prices of call option
with strike price 1200 points expiring in March 2009

Source: author's own study.

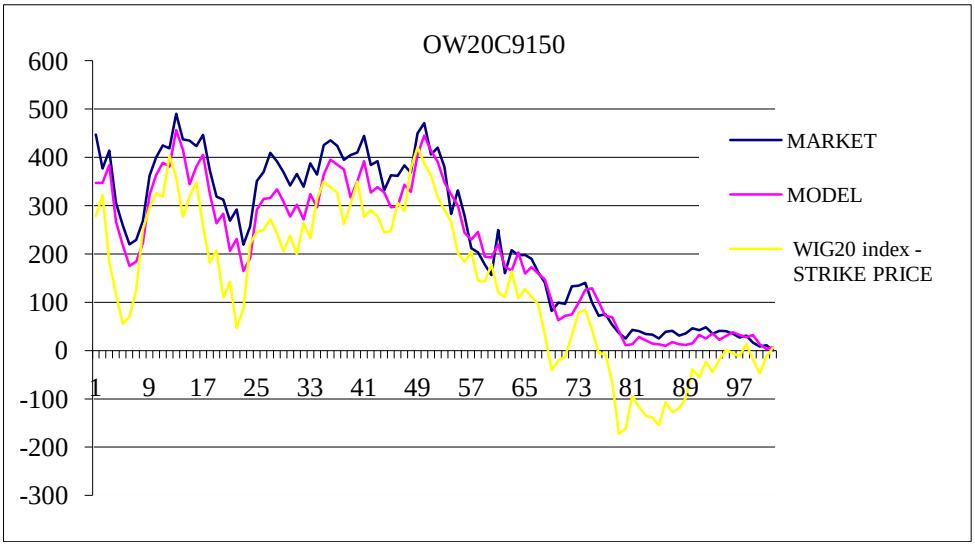


Fig. 5. Market and model prices of call option
with strike price 1500 points expiring in March 2009

Source: author's own study.

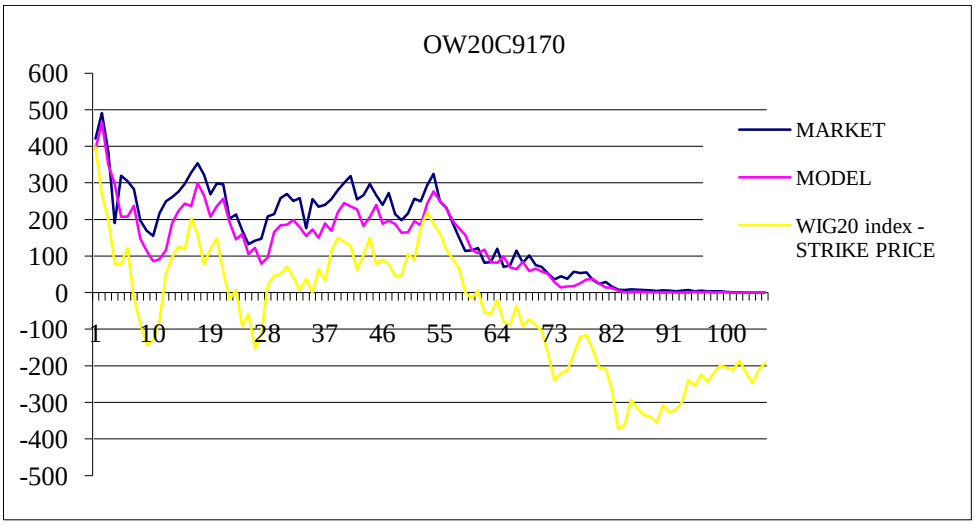


Fig. 6. Market and model prices of call option
with strike price 1700 points expiring in March 2009

Source: author's own study.

Table 1. Regression coefficients for regression lines describing dependence of market prices on model prices and their significance for some call options expiring in March 2005

| OPTION | *n* | Slope $\beta$ | $p(H_0: \beta = 1, H_1: \beta < 1)$ | Intercept $\alpha$ | $p(H_0: \alpha = 0, H_1: \alpha < 0)$ |
|---|---|---|---|---|---|
| OW20C5170 | 57 | 0.967037 | 0.175572 | –6.91517 | 0.223314 |
| OW20C5180 | 101 | 0.973204 | 0.203563 | –14.0783 | 0.003729 |
| OW20C5200 | 126 | 0.690637 | 2.92012E-11 | 1.83635 | 0.937812 |
| OW20C5190 | 115 | 0.886729 | 0.000709 | –4.6411 | 0.084618 |
| OW20C5210 | 75 | 0.577793 | 7.28E-17 | 2.031347 | 0.005336 |

Source: author's own study.

Table 2. Regression coefficients for regression lines describing dependence of market prices on model prices and their significance for some call options expiring in March 2009

| OPTION | *n* | Slope $\beta$ | $p(H_0: \beta = 1, H_1: \beta > 1)$ | Intercept $\alpha$ | $p(H_0: \alpha = 0, H_1: \alpha > 0)$ |
|---|---|---|---|---|---|
| OW20C9120 | 96 | 1.005654 | 0.36575469 | 7.64808 | 0.178992 |
| OW20C9130 | 98 | 1.011587 | 0.248657553 | 12.18763 | 0.036403 |
| OW20C9140 | 101 | 1.041307 | 0.01192739 | 13.44731 | 0.011216 |
| OW20C9150 | 102 | 1.097129 | 2.01176E-07 | 8.182322 | 0.046563 |
| OW20C9160 | 106 | 1.107898 | 1.85179E-06 | 12.2214 | 0.006671 |
| OW20C9170 | 106 | 1.131587 | 4.88808E-05 | 15.34428 | 0.002495 |
| OW20C9180 | 109 | 1.141942 | 8.40893E-08 | 14.29558 | 5.59E-05 |
| OW20C9190 | 124 | 1.008985 | 0.280631231 | 26.55742 | 1.84E-16 |
| OW20C9200 | 125 | 1.022166 | 0.079358229 | 21.85996 | 1.91E-15 |
| OW20C9210 | 156 | 0.983583 | 0.032048679 | 24.07518 | 4.96E-21 |
| OW20C9220 | 156 | 0.991261 | 0.199576829 | 19.75936 | 7.12E-15 |
| OW20C9230 | 156 | 0.994972 | 0.290418681 | 17.43263 | 2.44E-20 |
| OW20C9240 | 156 | 0.994028 | 0.294706186 | 16.62408 | 3.21E-18 |
| OW20C9250 | 156 | 0.990883 | 0.220053501 | 14.39771 | 1.11E-17 |
| OW20C9260 | 156 | 0.992387 | 0.282566728 | 14.5168 | 4.75E-21 |
| OW20C9270 | 156 | 0.985185 | 0.158855071 | 12.13067 | 3.56E-18 |
| OW20C9280 | 156 | 0.974053 | 0.084232135 | 10.08916 | 1.01E-12 |
| OW20C9290 | 156 | 0.980400 | 0.17942246 | 9.163734 | 6.76E-13 |
| OW20C9300 | 156 | 0.978343 | 0.122395965 | 7.039622 | 1.16E-15 |
| OW20C9310 | 156 | 0.974823 | 0.131532254 | 7.627502 | 5.59E-20 |
| OW20C9320 | 156 | 0.944767 | 0.08396103 | 7.635534 | 2.66E-11 |

Source: author's own study.

## 5. Put options results

A similar analysis as conducted for call options was done for put options. The behaviour of some of them is shown in Figures 7 to 10.

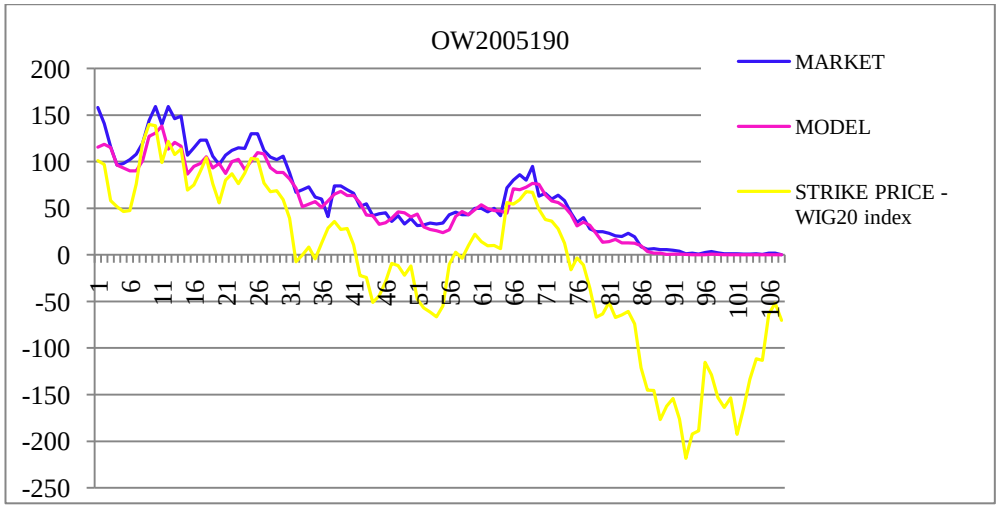


Fig. 7. Market and model prices of put option
with strike price 1900 points expiring in March 2005

Source: author's own study.

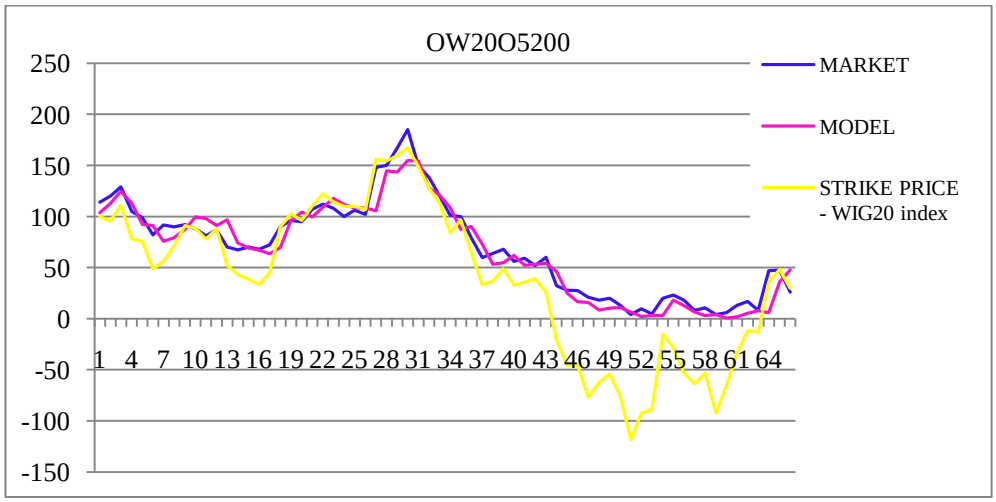


Fig. 8. Market and model prices of put option
with strike price 2000 points expiring in March 2005

Source: author's own study.

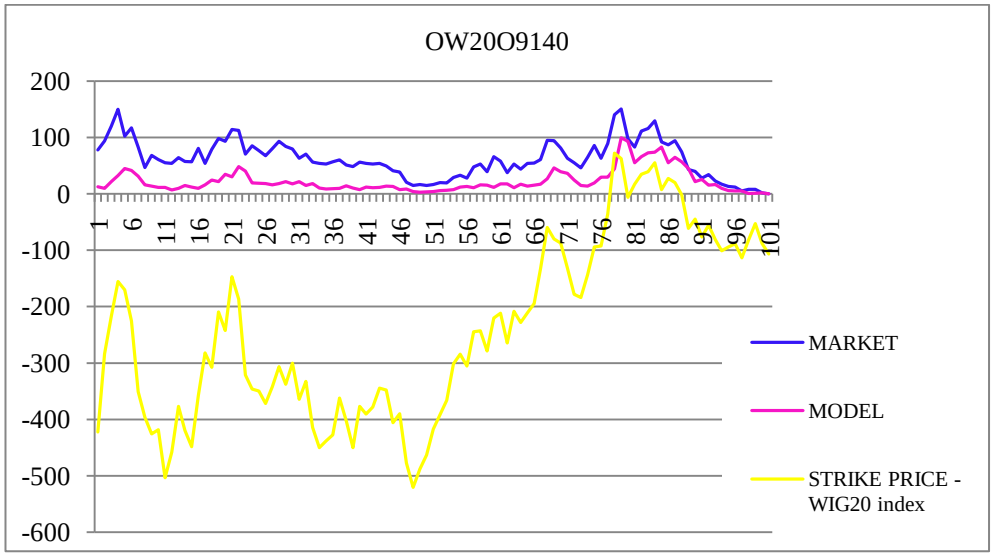


Fig. 9. Market and model prices of put option
with strike price 1400 points expiring in March 2009

Source: author's own study.

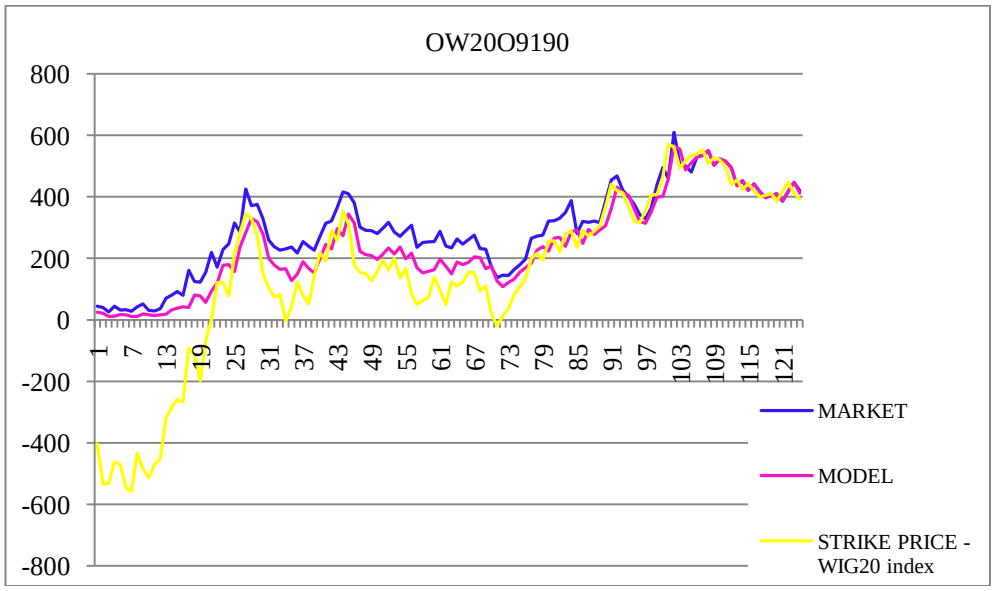


Fig. 10. Market and model prices of put option
with strike price 1900 points expiring in March 2009

Source: author's own study.

Table 3. Regression coefficients for regression lines describing dependence of market prices on model prices and their significance for some put options expiring in March 2005

| OPTION | *n* | Slope$\beta$ | $p(H_0: \beta = 1, H_1: \beta > 1)$ | Intercept$\alpha$ | $p(H_0: \alpha = 0, H_1: \alpha > 0)$ |
|---|---|---|---|---|---|
| OW20O5170 | 99 | 1.265963 | 1.63E-26 | 1.096717462 | 0.003339 |
| OW20O5180 | 109 | 1.155558 | 4.73E-10 | 0.53026098 | 0.275774 |
| OW20O5190 | 108 | 1.140573 | 1.34E-10 | 0.051034148 | 0.486004 |
| OW20O5200 | 66 | 0.963552 | 0.86148 | 5.81990869 | 0.015866 |

Source: author's own study.

Table 4. Regression coefficients for regression lines describing dependence of market prices on model prices and their significance for some put options expiring in March 2009

| OPTION | *n* | Slope $\beta$ | $p(H_0: \beta = 1, H_1: \beta > 1)$ | Intercept $\alpha$ | $p(H_0: \alpha = 0, H_1: \alpha > 0)$ |
|---|---|---|---|---|---|
| OW20O9120 | 96 | 4.270792 | 5.55E-18 | 12.1807980 | 1.91E-21 |
| OW20O9130 | 98 | 1.962478 | 2.18E-07 | 22.3169758 | 9.51E-24 |
| OW20O9140 | 101 | 1.153362 | 0.095012 | 36.1156407 | 6.25E-25 |
| OW20O9150 | 102 | 0.832815 | 0.013007 | 51.43490362 | 2.52E-28 |
| OW20O9160 | 106 | 0.765179 | 2.75E-06 | 69.13541097 | 3.53E-35 |
| OW20O9170 | 106 | 0.781495 | 3.12E-07 | 83.67927087 | 2.77E-30 |
| OW20O9180 | 109 | 0.853116 | 5.59E-07 | 83.81212539 | 2.12E-33 |
| OW20O9190 | 124 | 0.896977 | 1.05E-06 | 71.8293617 | 2.63E-31 |
| OW20O9200 | 125 | 0.93375 | 7.08E-06 | 62.90779298 | 4.8E-31 |
| OW20O9210 | 156 | 0.990342 | 0.180624 | 34.3305986 | 4.08E-17 |
| OW20O9220 | 156 | 0.993418 | 0.242667 | 31.85597191 | 1.42E-13 |
| OW20O9230 | 156 | 0.985421 | 0.024754 | 31.13627294 | 3.21E-15 |
| OW20O9240 | 156 | 0.986203 | 0.010877 | 29.51862365 | 8.17E-16 |
| OW20O9250 | 156 | 0.98741 | 0.00755 | 28.00526272 | 4.27E-15 |
| OW20O9260 | 156 | 0.988029 | 0.003275 | 25.57339532 | 4.4E-14 |
| OW20O9270 | 156 | 0.986074 | 0.001432 | 26.68797674 | 1.75E-11 |
| OW20O9280 | 156 | 0.991137 | 0.009797 | 20.37262441 | 7.69E-09 |
| OW20O9290 | 156 | 0.99365 | 0.043002 | 15.70272228 | 2.06E-05 |
| OW20O9300 | 156 | 0.993141 | 0.028681 | 16.10800238 | 3.57E-05 |
| OW20O9310 | 156 | 0.995598 | 0.108962 | 12.68295038 | 0.001719 |
| OW20O9320 | 156 | 0.997621 | 0.265225 | 9.806607338 | 0.023664 |

Source: author's own study.

On the basis of the results shown in Figures 7 to 10 and Tables 3 and 4, it can be said that generally both in bull and bear market periods put options were overestimated relative to prices resulting from the Black and Scholes model. Even though it is possible to find some differences – most of the options quoted during the bear market that were issued in the first few days of August 2008 were underestimated in the first days of quotation, then they became overestimated to finish almost equal model prices at the beginning of February (the process was faster for options with higher strike prices). Observing the behaviour of put options' prices during the bull market, it is more difficult to notice the existence of such sub-periods. In times of a bull market, the days of overestimation were mixed almost perfectly with the days of underestimation although days with overestimation happened more often.

## 6. Summary

To sum up, there are noticeable differences between the behaviour of market option prices in relation to option prices resulting from the Black--Scholes model during periods of a bull and bear market, but it is difficult to say if these differences behave always in the same way.

## Literature

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