M A T H E M A T I C A L   E C O N O M I C S

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ADJUSTMENT FUNCTION AS A TOOL FOR DISTRIBUTION OF SEATS IN THE EUROPEAN PARLIAMENT[1]

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Abstract. The article points out the concept of degressive proportionality, which is defined in the Lisbon Treaty and concerns allotment of seats in the European Parliament. The article introduces the concept of adjustment functions that allows the execution of the transition from the proportional division to degressively proportional. It reminds us of the four methods based on adjustment functions leading to degressively proportional divisions in the weak sense. Methods that are listed are: "shifted proportionality" (by Pukelsheim), parabolic method (by Ramirez), power-type method (by Ramirez), and power-type method (by Haman). The article contains a proposition of a special form of an adjustment function, which is dependent on an increasing, strictly concave function and several parameters. These are the population of the least and most populous country of the European Union (now Malta and Germany), the minimum and maximum number of seats to be allocated (now 6 and 96) and the additional parameter *c*. It is chosen in such a way that, with a fixed method of rounding of the adjustment function (and thus calculating the number of seats per country), the total number of seats does not exceed a certain fixed value (target 750).

Keywords: degressive proportionality, the European Parliament, the adjustment function, the Lisbon Treaty.

JEL Classification: D630.

1. Introduction

One of the most important institutions of the European Union, in which representatives of the Member States sit, is the European Parliament. The rules governing the allocation of seats have changed along with the increasing number of members of the Union. Due to the large difference in the population of the Member States, it is now impossible to use any of the

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proportional methods. Each Member State should have at least minimal representation in the Parliament; hence, if Malta with a population of about 400 thousand inhabitants received 5 seats, Germany with 200 times the population of Malta would receive nearly 1,000 seats, and the whole Parliament would amount to more than 6,000 members (Cegiełka et al., 2010). Therefore it has been decided to develop new rules for the distribution of seats that have been included in the Lisbon Treaty (*Traktat z Lizbony*, 2007).

According to Art. 9a, Paragraph 2 of the LisbonTreaty: "The European Parliament shall be composed of representatives of the Union's citizens. They shall not exceed seven hundred and fifty in number. Representation of citizens shall be degressively proportional, with a minimum threshold of six members per Member State. No Member State shall be allocated more than ninety-six seats".

In 2007 the European Council invited the European Parliament to prepare a draft of a new distribution of seats in the Parliament based on the principles adopted in the Lisbon Treaty, which at that time was not legally binding. The Committee on Constitutional Affairs, which was responsible for this task, presented in October 2007 a report (Lamassoure, Severin, 2007), in which it included a project of an adequate resolution of the European Parliament. The resolution was adopted during a meeting of Parliament on 11 October 2007. In support of the report the six principles were presented, which, according to the Committee on Constitutional Affairs, could clarify the rule of degressive proportionality:

(1) The principle of effectiveness: the functioning of the European Parliament is impossible if its composition exceeds the specific number of deputies – hence the restriction of 750 members.

(2) The principle of national representation and the motivation of the voters: each Member State should have the minimum number of seats, so it will be able to represent their electorate by motivating them to participate in the elections.

(3) The principle of European solidarity: in order to ensure adequate representation of less populous States, countries with a greater number of citizens will receive fewer seats than in the case of application of the principle of strict proportionality.

(4) The principle of relative proportionality: the ratio of the population size to the number of seats is greater the larger the State, and smaller the smaller the State.

(5) The principle of fair distribution: no State will have more seats than a larger Member State or smaller amount of seats than a smaller Member State.

(6) The principle of reasonable flexibility or flexible direct degressiveness: small changes in the allocation of seats may be implemented if other principles are obeyed and the modification aims at the most equitable distribution of seats.

In the above-mentioned reasoning one may find many statements about the problems of precise definition of the principle of degressive proportionality because rules 1 to 6 do not provide any exact formula or algorithm that allows unambiguous determination solutions. This would be particularly important in the context of future enlargements or modifications due to demographic changes.

In consideration of the above, it may be stated that for the current demographic data a number of possible divisions that meet the principle of digressive proportionality can be provided. The most extreme and simplest of these is equal division, i.e. allocating to each Member State the same number of seats, but this division firmly favors small countries. At the other extreme there are proportional divisions, whose defects have been already mentioned. The current composition of the European Parliament does not meet the principles of degressive proportionality since it was approved before the entry into force of the Treaty of Lisbon. However, the proposed distribution of seats contained in the report by Lamassoure, Severin (2007) meets the requirements of degressive proportionality, but the ambiguity of the method raises many questions. The Rapporteurs themselves admit that in the future more precise guidelines should be created and applied to future EU enlargements and thus avoiding political bargaining based on national interests.

2. The principle of degressive proportionality in formal terms

Let l_i mean the population of a country i, m_i – the number of allocated seats in the EP, n – the number of Member States of the Parliament. It was assumed that the population of EU countries are set in an ascending sequence:

$$l_1 < l_2 < \ldots < l_n .$$

The principle of degressive proportionality is satisfied when there occur the inequalities:

$$m_1 \leq m_2 \leq \ldots \leq m_n \, , \tag{1}$$

$$\frac{l_1}{m_1} < \frac{l_2}{m_2} < \ldots < \frac{l_n}{m_n} \, . \tag{2}$$

According to the Lisbon Treaty it should be

$$m_1 \geq 6 \, , \; m_n \leq 96 \, , \tag{3}$$

also imposed a limit on the total number of seats *N:*

$$N = \sum_{i=1}^{n} m_i \leq 750 \, . \tag{4}$$

It is worth following J. Haman, who claims that if the total number of seats to be divided is fixed, the monotonicity condition (1) may be inconsistent with the condition (2) defining the degressive proportionality. For example, if 21 seats are divided between two countries, which number respectively 1,095,000 and 1,005,000 inhabitants, it is the first one you cannot allocate less, so it receives 11 seats and the other one 10 (Haman, 2007). Then the coefficients of the people/seats are respectively 99,545 and 100,500; thus, the larger the state, the lower the ratio, which contradicts the principle of relative proportionality. In the case of the European Parliament, this may mean that if the total number of seats will be constant (e.g. 750), then with a larger number of countries such paradoxes may occur. In the case of countries such as Greece, Belgium, Portugal, the Czech Republic and Hungary, whose populations are very similar, it is very probable. Article 9a of the Lisbon Treaty actually mentions only the fact that the total number of seats does not exceed 750, which slightly reduces the risk. In order to get rid of this problem, Haman (2007) proposes to replace the concept of the seat/mandate, which is allocated to a specific country, with a country's quota, i.e. an ideal (generally noninteger) number of seats to which a State is entitled. As the number of seats a State receives is a result of rounding amounts to an integer, it may not satisfy the same conditions that are satisfied by quotas.

Another limitation contained in the Lisbon Treaty is the minimum and maximum number of seats a Member State can get. These numbers are 6 and 96, respectively. The Rapporteurs recommendation is that the numbers are fully exploited, which in the future can be difficult. On the one hand, the Union may be entered by a large number of small states which must be provided with 6 seats, on the other, if it is accompanied by a giant Turkey, the 96 seats allocated to Germany will not be certain. For now, it should be ensured that new methods do not lead to solutions too distant from the current ones, but with the accession of new countries to EU structures, this situation will have to change. It is also important that the methods include a margin of choice for “political agreement”.

3. Adjustment functions and digressive proportionality

Let us assume for now that it is about a proportional division. If in a rectangular coordinate system the horizontal axis describes the number of people, and the vertical axis – the number of seats, the graph of the proportional relation of number of seats to the number of people is a straight line passing through the origin, whose slope corresponds to the coefficient of proportionality. In fact, the points corresponding to this dependence are generally at or above this line, because of that the number of seats must be an integer. As a result, it may occur that depending on how the rounding is performed too many or too few seats were distributed. This problem is solved in two ways. The first one is to change the method of rounding – the fractional part of the amount shall be rounded up, but not classically from 0.5, but arbitrarily in such a way as to allocate the correct number of seats. This is the largest reminder method created by Hamilton. The second approach is to maintain the way of rounding and a small change in slope of the angle (see Figure 1) in such a way as to allocate the correct number of seats. These are the divisor methods (Young, 2003).

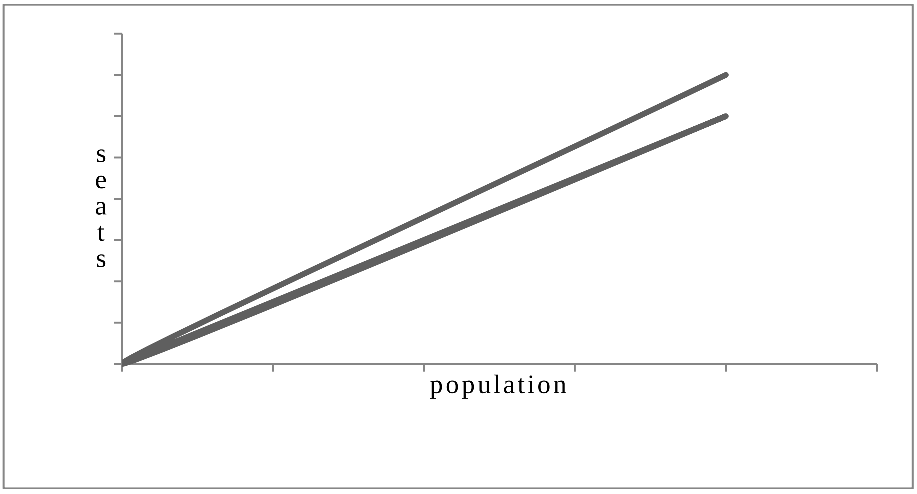

Fig. 1. Proportional dependence of the number of seats to the number of people

Source: Haman (2010).

In the case of degressive proportionality, the seats gain should be slower than growth of the population, i.e. the line representing the relation between the number of inhabitants and the number of seats should, with an increase in the number of residents, be accompanied with a decrease of its inclination to the horizontal axis.

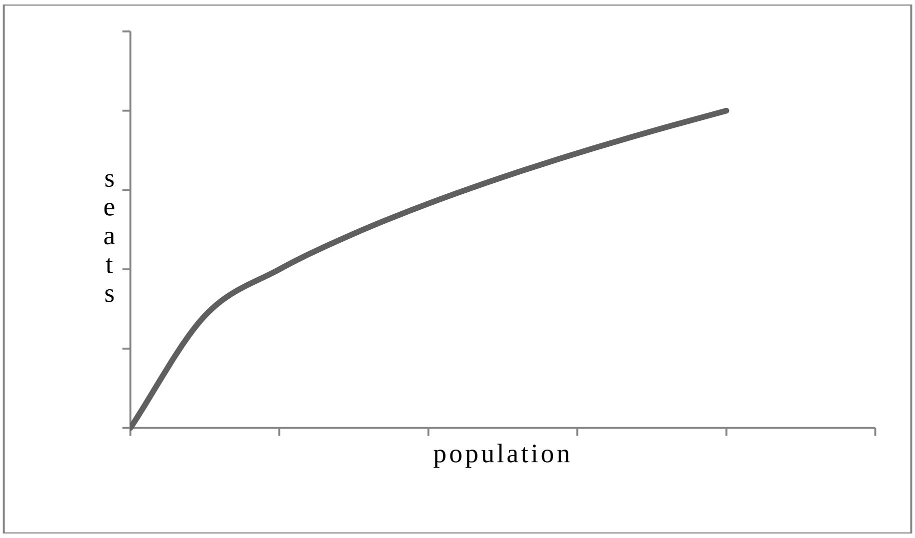

Fig. 2. Degressively proportional dependence of the number of seats on the population

Source: author's own study.

The stronger the curvature of the line, the greater the force of degression (in the case of a horizontal line an equal division would be obtained). The choice of the method of degressive distribution can thus be brought down to (strictly concave and increasing) function of the relation between the number of inhabitants, and the number of seats (concavity here is not a necessary condition, but sufficient). Obviously, after rounding the results to integers the concavity may "spoil".

We introduce the following notations:

l – the population of the smallest countries in the European Union (now Malta);

L – the largest population country in the European Union (now Germany);

n – total number of EU countries in the European Union (currently 27);

N – total number of seats available for distribution (currently 785, target 750);

m – the smallest allowable size of the delegation of a Member State (now 6);

M – the maximum allowable size of the delegation of a Member State (now 96).

We are looking for an increasing and strictly concave function *f*, for which there are conditions:

a) $f(l) = m$,

b) $f(L) = M$,

c) $\sum_{i=1}^{n} [f(l_i)]_W = N$.

Following Ramirez (2006) and Haman (2007), the function was called *adjustment function.* It can be treated as a new measure for populations. If a linear function was allowed (it is concave but not strictly concave), the adjustment function in the form $f(x) = cx$ would measure population "as usual", i.e. instead of degressive, proportionality would be simply proportional.

The symbol $[t]_W$ means rounding of a real t with the use of Webster's method, or to the nearest integer number. Obviously, other rounding methods can be used: Adams' – up, Jefferson's – down. The symbol $[t]_W$ is substituted with symbols $[t]_A$ or $[t]_J$ respectively.

If we recall the requirements imposed by the Treaty of Amsterdam (6 seats for each state + a seat for every 0.5 million residents to 25 million + a seat for every million inhabitants between 25 and 60 million + a seat for each additional 2 million over 60 million inhabitants), it means that the adjustment function was of the form:

$$f(x) = \begin{array}{ll} 6 + 2x & \text{for} \quad x < 25 \\ 31 + x & \text{for} \quad 25 \le x \le 60 \\ 61 + 0.5x & \text{for} \quad x > 60 \end{array}.$$

Thus the graph is a polygonal line consisting of three segments.

4. Overview of known forms of adjustment function

The method of "shifted proportionality" by Pukelsheim: F. Pukelsheim applied (2007) a simpler design than the Amsterdam one. The graph of his adjustment function is a polygonal line composed only of two sections.

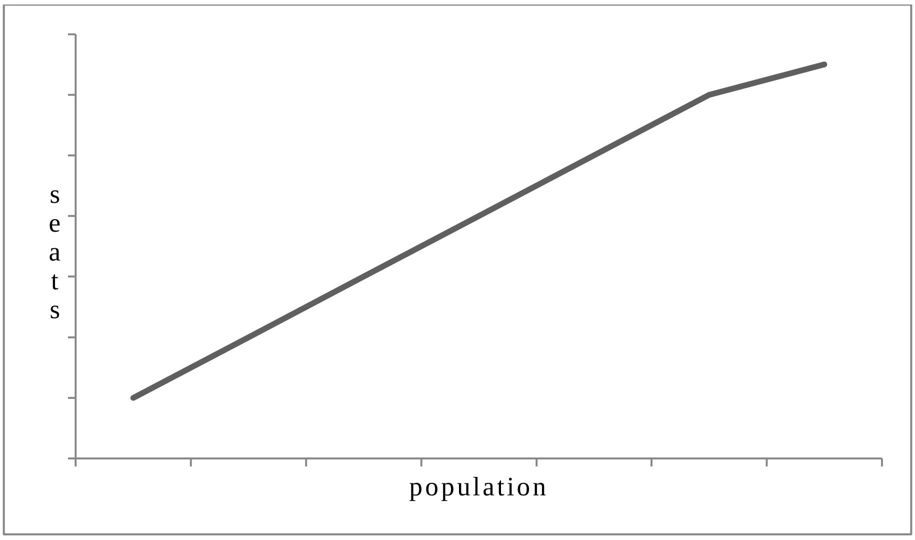

Fig. 3. Pukelsheim's polygonal line composed of two sections

Source: author's own study.

From the conditions a), b) it can be concluded that the first one (left-most) segment has ends (*l*, *m*), (*L'*, *M'*) and second (*L'*, *M'*), (*L*, *M*), where *L'*, *M'* denote the population and the number of seats of the country second in the EU in terms of population after Germany (now France). Out of the six coordinates of the three points being the ends of sections, only *M'* is not a given number. It should be chosen in such a way that condition c) should occur. Approaching the problem from another angle, it can be assumed that an equation of a line containing the first (from left) segment has the form of $y = a(x - l) + m$, and the equation of the second line $y = b(x - L) + M$, but inequality $b < a$ must occur. Since the point(L', M') belongs to both lines, thus:

$$a = (b\,(L' - L) + M - m))/(L' - l).$$

Moreover, the condition $b < a$ is equal to the inequality $b < (M - m)/(L - l)$.

Pukelsheim to his proposal accepted the population data from 2006. In addition, $m = 6$, $M = 96$, $N = 751$ and rounding method by Adams. If we assume $b = 0.716$, it appears that $a = 1.216$. Pukelsheim's distribution is: 96, 83, 80, 77, 59, 52, 32, 26, 20, 19, 19, 18, 18,17, 16, 15, 13, 13, 12, 11, 10, 9, 8, 8, 7, 7, 6 starting from the largest Germany to the smallest Malta.

If Webster's method of rounding was used, it should be $b = 0.600$. Then $a = 1.253$, and the distribution would be as follows: 96, 84, 81, 79, 60, 53, 33, 26, 19, 19, 19, 18, 18, 17, 16, 15, 12, 12, 12, 11, 10, 8, 8, 7, 6, 6, 6.

Finally, with the rounding method by Jefferson, we assume $b = 0.535$. Then $a = 1.273$ and 96, 85, 82, 80, 61, 54, 32, 26, 19, 18, 18, 18, 18, 17, 16, 15, 12, 12, 12, 10, 9, 8, 8 , 7, 6, 6, 6.

Parabolic Method by Ramirez: In this method Pukelsheim's polygonal line was replaced by Ramirez's parabola (Ramirez, Polomares, Marquez, 2006). Ramirez took the same data as Pukelsheim, with the exception of the number of seats N (here $N = 750$). The idea of Ramirez's solution was based on "fixing" parabola in the two end points (l, m), (L, M) and choosing a negative coefficient in such a way as to satisfy condition c). The general form of the equation of such a parabola is:

$$y = a(x - l)(x - L) + b(x - l) + m,$$

where

$$b = (M - m) / (L - l).$$

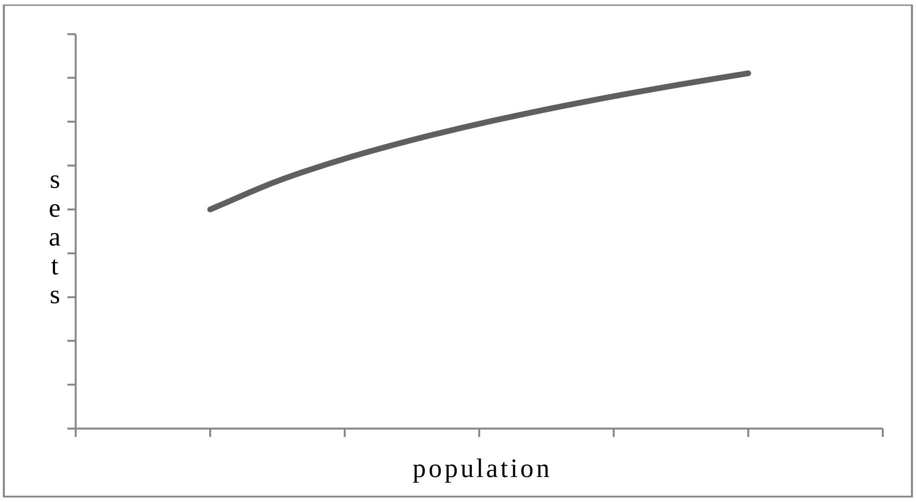

Fig. 4. The ends of the parabola (l, m), (L, M)

Source: author's own study.

In their article, Ramirez, Polomares, Marquez (2006) used older data on populations, but for the sake of this comparison, into the method by Ramirez the same data as in Pukelsheim's method was used. All calculations were performed using EXCEL spreadsheet.

If one was to use the rounding method by Adams, it should be assumed $a = -0.0027$. Then the breakdown is as follows: 96, 78, 76, 74, 59, 52, 33, 27, 20, 20, 20, 19, 19, 18, 17, 16, 13, 13, 13, 11, 10, 9, 9 , 8, 7, 7, 6.

In the case of Webster's rounding method, you must take $a = -0.0034$. Then the breakdown is as follows: 96, 79, 76, 75, 59, 53, 34, 27, 20, 20, 20, 19, 19, 18, 17, 16, 13, 13, 13, 11, 10, 9, 8 , 7, 6, 6, 6.

In turn, the rounding with the method by Jefferson, one must take $a = -0.00405$. Then the breakdown is as follows: 96, 79, 77, 75, 60, 54, 34, 27, 20, 20, 20, 19, 19, 18, 16, 16, 13, 13, 12, 11, 10, 8, 8 , 7, 6, 6, 6.

Table 1. Comparison of methods of Pukelsheim and Ramirez for three ways to rounding

Country	Population (in thous.)	Pukelsheim (Adams)	Ramirez (Adams)	Pukelsheim (Webster)	Ramirez (Webster)	Pukelsheim (Jefferson)	Ramirez (Jefferson)
Germany	82 438	96	96	96	96	96	96
France	62 999	83	78	84	79	85	79
United Kingdom	60 393	80	76	81	76	82	77
Italy	58 752	77	74	79	75	80	75
Spain	43 758	59	59	60	59	61	60
Poland	38 157	52	52	53	53	54	54
Romania	21 610	32	33	33	34	32	34
Netherlands	16 334	26	27	26	27	26	27
Greece	11 125	20	20	19	20	19	20
Portugal	10 570	19	20	19	20	18	20
Belgium	10 511	19	20	19	20	18	20
Czech Republic	10 251	18	19	18	19	18	19
Hungary	10 077	18	19	18	19	18	19
Sweden	9 048	17	18	17	18	17	18
Austria	8 266	16	17	16	17	16	16
Bulgaria	7 719	15	16	15	16	15	16
Denmark	5 427	13	13	12	13	12	13
Slovakia	5 389	13	13	12	13	12	13
Finland	5 256	12	13	12	13	12	12
Ireland	4 209	11	11	11	11	10	11
Lithuania	3403	10	10	10	10	9	10
Latvia	2 295	9	9	8	9	8	8
Slovenia	2 003	8	9	8	8	8	8
Estonia	1 345	8	8	7	7	7	7
Cyprus	766	7	7	6	6	6	6
Luxemburg	469	7	7	6	6	6	6
Malta	405	6	6	6	6	6	6
In total	492 975	751	750	751	750	751	750

Source: author's own study.

Table 1 compares the results obtained by Ramirez's and Pukelsheim's methods for all types of rounding.

Regardless of the method of rounding, it is easy to see that Pukelsheim's method favors large countries, and the parabolic method of Ramirez – medium-sized countries. Both methods can lead to distributions that do not satisfy the condition of degressive proportionality. By comparing the ratio (population/seats), it was found that for both methods and all types of rounding the distributions obtained are not degressively proportional (as we know, this ratio should be smaller for a country with a smaller population; hence, it is suffice to check it with pairs of neighboring countries together in the table). In the case of Pukelsheim's method, wrong inequalities were received for the following pairs of countries:

– the United Kingdom, Italy, Belgium, the Czech Republic, Slovakia, Finland, Luxemburg, Malta with the use of the rounding method by Adams;

– Belgium, the Czech Republic with the use of the rounding method by Webster;

– Greece, Portugal for Jefferson's method.

On the other hand, for the method by Ramirez the incorrect inequalities for the following pairs of countries:

– Belgium, the Czech Republic, Luxemburg, Malta, with the use of the rounding method by Adams;

– Belgium, the Czech Republic with the use of the rounding method by Webster;

– Belgium, the Czech Republic, Sweden, Austria, Slovakia, Finland for Jefferson's method.

This proves that the most appropriate way of rounding is Webster's method. It is also worth noting that in almost every case the pair of States which did not allow fulfilling the inequalities defining degressive proportionality were Belgium and the Czech Republic. The reason is obviously the fact that their populations are only slightly different, and the methods assign a different number of seats.

Power-type Method by Ramirez: Here, following Ramirez, the below form of an adjustment function was assumed:

$$f(x) = m + (M - m)\left(\frac{x - l}{L - l}\right)^{\alpha},$$

where the parameter α from the interval (0, 1) must be chosen in such a way that there occurs the condition c) for the appropriate method of rounding (summing up the adjusted amounts to the total number of seats N). Conditions a), b) are obviously satisfied for arbitrary values of α. Calculations were performed as in the parabolic method for $m = 6$, $M = 96$, $N = 750$, $n = 27$.

For Adams' method, it was assumed that $\alpha = 0.907$. The resulting distribution is: 96, 77, 74, 73, 57, 51, 33, 27, 21, 20, 20, 20, 19, 18, 17, 17, 14, 14, 13, 12, 11, 9, 9 , 8, 7, 7, 6.

For the method by Webster, $\alpha = 0.880$. Then: 96, 77, 74, 73, 57, 51, 33, 27, 21, 20, 20, 20, 20, 18, 17, 17, 14, 14, 13, 12, 11, 9, 9, 8, 7, 6, 6.

For the method by Jefferson $\alpha = 0.861$. The distribution is as follows: 96, 77, 74, 73, 57, 52, 34, 27, 21, 20, 20, 20, 20, 18, 17, 17, 14, 14, 13, 12, 11, 9, 9, 7, 6, 6, 6.

For all the ways of rounding the pair Slovakia and Finland was the one which "spoiled" degressive proportionality. Fourteen seats for Slovakia seem to be unjustified against 13 seats in Finland, while the population of Slovakia is only about 2.5% larger than the population of Finland.

Power-type Method by Haman: As was mentioned earlier, Haman suggests that the word "seat" was replaced by the word "quota". Then a stronger condition (if $l_i > l_j$, then $l_i / m_i > l_j / m_j$) can be substituted by a weaker but easiest condition in the form of:

$$\text{if } l_i > l_j, \quad \text{then} \quad l_i / f(l_i) > l_j / f(l_j),$$

where f is an adjustment function, and the indices i, j elicit all the Member States of the Union.

Instead of talking about degressive proportionality, one can speak of degressive quota proportionality or weak degressive proportionality. In the situation in which the adjustment function is strictly concave and increasing, and takes positive values, the method based on it is weak degressively proportional.

In support of his own method, Haman (2010) puts extra pressure on the fact that the adjustment function satisfies its assumption – that proportion between the quotas of two countries was a function of the ratio of their size. This function is a power-type function (without displacements as in Ramirez), so $f(x) = x^a$. Obviously, if it is concave, it has to be assumed additionally $\alpha \in (0, 1)$. Haman (2010) also notes that because divisor methods are based on priority functions, one can copy the procedure that is used for example in the method of Sainte-Laguë (i.e. Webster's but in divisor version). One just needs to apply it in a limited version of the revised measures for the size of the country.

The exact procedure is as follows:

1. For each country an adjusted measure of country size is designated.
2. Adjusted measure of size of each country is divided in turn by:

$$m - 0.5,\ m + 0.5,\ m + 1.5,\ ...,\ m + M - 0.5.$$

3. Each country will be given m amount of seats plus as many seats as there are largest consecutive numbers attributed to it from the sequence of quotients among initial $(N - nm)$ quotients created in point 2.

If one uses the numbers listed in Article 9a of the Lisbon Treaty to the current situation, then $n = 27$, $m = 6$, $m = 96$.

Table 2. Comparison of the divisions of seats by different methods (data from 2006)

Member State	Population (in thous.)	Treaty of Lisbon	Population / seats	Power-type method $\alpha = 0.702$	Population / seats	Parabolic Method	Population / seats
Germany	82 438	96	858.7	91	905.9	96	858.7
France	62 999	74	851.3	76	828.9	79	797.5
United Kingdom	60 393	73	827.3	73	827.3	76	794.6
Italy	58 752	72	816.0	72	816.0	75	783.4
Spain	43 758	54	810.3	59	741.7	59	741.7
Poland	38 157	51	748.2	53	719.9	53	719.9
Rumania	21 610	33	654.8	36	600.3	34	635.6
Netherlands	16 334	26	628.2	29	563.2	27	605.0
Greece	11 125	22	505.7	22	505.7	20	556.3
Portugal	10 570	22	480.5	22	480.5	20	528.5
Belgium	10 511	22	477.8	22	477.8	20	525.6
Czech Republic	10 251	22	466.0	21	488.1	19	539.5
Hungary	10 077	22	458.0	21	479.9	19	530.4
Sweden	9 048	20	452.4	19	476.2	18	502.7
Austria	8 266	19	435.1	18	459.2	17	486.2
Bulgaria	7 719	18	428.8	17	454.1	16	482.4
Denmark	5 427	13	417.5	14	387.6	13	417.5
Slovakia	5 389	13	414.5	13	414.5	13	414.5
Finland	5 256	13	404.3	13	404.3	13	404.3
Ireland	4 209	12	350.8	11	382.6	11	382.6
Lithuania	3 403	12	283.6	10	340.3	10	340.3
Latvia	2 295	9	255.0	7	327.9	9	255.0
Slovenia	2 003	8	250.4	7	286.1	8	250.4
Estonia	1 345	6	224.2	6	224.2	7	192.1
Cyprus	766	6	127.7	6	127.7	6	127.7
Luxemburg	469	6	78.2	6	78.2	6	78.2
Malta	405	6	67.5	6	67.5	6	67.5
In total	49 2975	750	–	750	–	750	–

Source: Haman (2010).

Haman gives the calculated distribution as close as possible to the distribution proposed by Lamassoure and Severin (in the table as the Treaty of Lisbon), according to data for the population as of 2006.

Comparing the columns of Table 2 containing the coefficients of the people/seats, it can be seen that the division proposed by Lamassoure and Severin is degressively proportional, while the division of power series by Haman does not meet the appropriate condition for Belgium and Denmark to the Czech Republic with Slovakia. The parabolic distribution method does not work for the pair of Belgium and the Czech Republic.

Moreover, it can be concluded that the Haman method prevents from using exceeded number of seats required in the Lisbon Treaty limits. However, due to the low number of parameters, it cannot be guaranteed that the simultaneous achievement of both of these restrictions is possible. Here the minimum number of seats 6 is completed, but instead of the maximum number of 96 there is 91. According to Haman, this is not too great a disadvantage, because after the successive enlargements of the Union the number of seats which Germany has now will have to be reduced. Otherwise, because of the limitations of the total number of seats (750) a humorous scenario threatens with the division of: 96, 6, 6, 6, ... On the other hand, what hinders Haman, e.g. in Ramirez's parabolic distribution – the strict adherence to the numbers 6 and 96 needs not to be a weakness because they can be treated as parameters so, if necessary, they can be changed. One can also suggest other increasing functions and strictly concave as functions of the adjustment.

5. Other methods of degressively proportional allocation

Let the function g defined on the set of nonnegative real numbers has the properties:

a) is strictly increasing,

b) is strictly concave,

c) $g(0) = 0$, the graph passes through the origin.

In addition, a positive parameter c was introduced and the adjustment function was defined by the formula:

$$f(x) = m + (M - m) \cdot g\left(\frac{c \cdot (x - l)}{L - l}\right) / \, g(c),$$

where x is the number of population, and other marks as before. Then the function f satisfies the following properties: is strictly increasing, is strictly concave, $f(l) = m$ and $f(L) = M$.

Table 3. The division of seats in the EP by logarithmic method (Webster + Jefferson)

Country	Population (in thous.)	Quota for W	Seats for W	Population /seats	Quota for J	Seats for J	Population /seats
Germany	82 438	96.00	96	858.7	96.00	96	858.7
France	62 886	78.20	78	806.2	78.86	78	806.2
United Kingdom	60 422	75.83	76	795.0	76.55	76	795.0
Italy	58 752	74.20	74	793.9	74.97	74	793.9
Spain	43 758	58.94	59	741.7	59.99	59	741.7
Poland	38 157	52.90	53	719.9	54.00	53	719.9
Romania	21 610	33.79	34	635.6	34.78	34	635.6
Netherlands	16 334	27.26	27	605.0	28.11	28	583.4
Greece	11 125	20.57	21	529.8	21.23	21	529.8
Portugal	10 570	19.85	20	528.5	20.47	20	528.5
Belgium	10 511	19.77	20	525.6	20.39	20	525.6
Czech Republic	10 251	19.43	19	539.5	20.04	20	512.6
Hungary	10 077	19.20	19	530.4	19.80	19	530.4
Sweden	9 048	17.84	18	502.7	18.39	18	502.7
Austria	8 266	16.80	17	486.2	17.31	17	486.2
Bulgaria	7 719	16.07	16	482.4	16.55	16	482.4
Denmark	5 428	12.97	13	417.5	13.32	13	417.5
Slovakia	5 389	12.92	13	414.5	13.27	13	414.5
Finland	5 256	12.74	13	404.3	13.08	13	404.3
Ireland	4 209	11.31	11	382.6	11.58	11	382.6
Lithuania	3 403	10.19	10	340.3	10.41	10	340.3
Latvia	2 295	8.66	9	255.0	8.80	8	286.9
Slovenia	2 003	8.25	8	250.4	8.37	8	250.4
Estonia	1 344	7.33	7	192.0	7.40	7	192.0
Cyprus	766	6.51	7	109.4	6.54	6	127.7
Luxemburg	460	6.08	6	76.7	6.08	6	76.7
Malta	404	6.00	6	67.3	6.00	6	67.3
In total	492 881	–	750	–	–	750	–

Source: author's own study.

Thus, a graph of such a function "holds" extreme points. Then it is enough to choose the parameter c in such a way that $\sum_{i=1}^{n}\left[f(l_i)\right]_W = N$, if the rounding method used is Webster's. For other rounding methods one shall proceed in similar fashion and the result will usually be a different value of parameter c. The functions g, which were selected for testing, are the "shifted logarithm", i.e. $\ln(1 + x)$ and one of the cyclometric functions $\operatorname{arctg}(x)$. Obviously, both functions satisfy all the necessary assumptions, so the adjustment functions f made with their use lead to distributions degressively in quota terms proportional.

Logarithmic Method: Let $g(x) = \ln(1 + x)$, then for Webster's method we have $c = 0.63$, and for the method of Jefferson $c = 0.8$.

From observation of the coefficients of the population/seats (columns 5 and 8 of Table 3), one can draw a conclusion that the divisions obtained by the logarithmic method with both methods of rounding in exactly one place do not meet a condition of degressive proportionality. In Webster's system the pair is: Belgium and the Czech Republic, and Jefferson's system does not work for the couple made by the Czech Republic and Hungary. The reason in both cases is the same, that is, little difference in the populations of these countries. Belgium has only about 2.5% larger population than the Czech Republic, and the difference between the Czech Republic and Hungary is even smaller, only 1.7%.

Table 4. The division of seats in the EP with the use of cyclometric method

Population (in thous.)	Quota for W	Seats for W	Population /seats	Quota for J	Seats for J	Population / seats	Population (in thous.)
Germany	82 438	96.00	96	858.7	96.00	96	858.7
France	62 886	79.53	80	786.1	80.36	80	786.1
United Kingdom	60 422	77.22	77	784.7	78.12	78	774.6
Italy	58 752	75.62	76	773.1	76.57	76	773.1
Spain	43 758	60.12	60	729.3	61.36	61	717.3
Poland	38 157	53.82	54	706.6	55.06	55	693.8
Romania	21 610	33.77	34	635.6	34.71	34	635.6
Netherlands	16 334	27.01	27	605.0	27.76	27	605.0
Greece	11 125	20.21	20	556.3	20.74	20	556.3
Portugal	10 570	19.48	19	556.3	19.98	19	556.3
Belgium	10 511	19.40	19	553.2	19.90	19	553.2
Czech Republic	10 251	19.06	19	539.5	19.55	19	539.5
Hungary	10 077	18.83	19	530.4	19.31	19	530.4
Sweden	9 048	17.48	17	532.2	17.90	17	532.2
Austria	8 266	16.44	16	516.6	16.83	16	516.6
Bulgaria	7 719	15.72	16	482.4	16.08	16	482.4
Denmark	5 428	12.68	13	417.5	12.93	12	452.3
Slovakia	5 389	12.63	13	414.5	12.88	12	449.1
Finland	5 256	12.45	12	438.0	12.70	12	438.0
Ireland	4 209	11.06	11	382.6	11.25	11	382.6
Lithuania	3 403	9.99	10	340.3	10.14	10	340.3
Latvia	2 295	8.52	9	255.0	8.61	8	286.9
Slovenia	2 003	8.13	8	250.4	8.21	8	250.4
Estonia	1 344	7.25	7	192.0	7.30	7	192.0
Cyprus	766	6.48	6	127.7	6.50	6	127.7
Luxemburg	460	6.08	6	76.7	6.08	6	76.7
Malta	404	6.00	6	67.3	6.00	6	67.3
In total	492 881	–	750	–	–	750	–

Source: author's own study.

Once again the thesis is confirmed (see applications for power-type method and parabolic) that the countries with only a few percent of difference in population should receive the same amount of seats.

Cyclometric Method: Let $g(x)$ = arctg(x), then for the method by Webster we have $c = 0.866 = \sqrt{3}/2$, and for Jefferson's method $c = 0.97$.

This time the degressive proportionality condition is not fulfilled by Hungary (19 seats) together with Sweden (17 seats). The difference in populations is quite high this time, 11.4%, but a surplus of two seats of Hungary over Sweden proved to be unfounded. Moreover, if the rounding method used is Webster's, the principle is broken additionally by Slovakia and Finland for reasons already known (only 2.5% of the difference in populations with different numbers of seats).

6. Summary

The article attempts to use such methods of allocation of seats which would lead to degressively proportional divisions additionally satisfying the condition that the least and most populous state of the European Union receives a predetermined number of seats, and the total number of seats does not exceed the specified value. The tool that was used was adjustment function that allows one to move from proportional to degressively proportional divisions. A certain class of corrective functions was indicated to implement the boundary conditions posed, while (due to its concavity) it could lead to degressively proportional divisions. The method was tested for two examples of adjustment functions, logarithmic and one of the cyclometric functions, using for each of them two methods of rounding – Jefferson's and Webster's. Unfortunately, in both cases it was found that the population/seats ratio used for checking degressiveness of the distributions assumed in several places an invalid value. For the logarithmic function in one case, for each method of the rounding, and once for cyclometric function with Jefferson's method and twice with Webster's method of rounding. Thus, the principle of degressive proportionality was satisfied only in the weaker sense, where the "seats" in the ratios of population/seats are substituted with "adjustment function value". It is worth noting that with the increasing number of EU Member States, regardless of the methods used, difficulties in determining the degressively proportionate PE configurations will increase, which will retain the current boundary conditions (number of seats for the state from 6 to 96, the total number of seats no more than 750), since there are a lot more countries waiting to be a part of the EU. Especially if the attempts were made to keep the 96 seats for Germany, then there would occur a "flattening" of adjustment function, resulting in almost proportional distributions. It also worth noting that the conditions governing the degressive proportionality can in conjunction with a request that the total number of seats was fixed, in some cases lead to contradictions, so one may (as proposed by J. Haman), for a period of time, need to adopt a weaker version of degressive proportionality.

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