ARBITRAGE IN ECONOMICS AND ELSEWHERE – FACTS WELL KNOWN AND LESS KNOWN (THREE PAPERS ON ARBITRAGE IDEAS, MODELING AND PRICING – YESTERDAY, TODAY AND TOMORROW) – INTRODUCTION TO THE SERIES

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Abstract. The presented series of articles on arbitrage theories and their methodological aspects consist of three papers entitled as follows: I. The Primer on Arbitrage Conceptions in Economics: Their Logics, Roots and Some Formal Models (Historical and Bibliographical Notes). II. Mathematics of Financial Arbitrage: From Algebraic Geometry at the Turn of the 19th and 20th Centuries to Modern Martingale (Generalized) Considerations III. The Arbitrage in Stochastic Finance, Social Choice Theory and Macroeconomics. The articles are devoted to present – in a historical perspective – the basic ideas and "metamorphoses" of the notion and role of an arbitrage (originated as a kind of clever and rational speculation – the last word is used in a "neutral", not pejorative sense) as well as to point out its various, important connotations (not merely in finance or even economics) and to demonstrate some mathematical inevitable technicalities, reflecting, in fact, the logical essence and the modern view of the arbitrage (and non arbitrage) conditions.

Keywords: arbitrage, stochastic finance, free lunch, Arbitrage Pricing Theory, martingale, cornucopia.

JEL Classification: C02, C60, C70.

1. Description of the subject

The articles are devoted to present – in a historical perspective – the basic ideas and "metamorphoses" of the notion and role of an arbitrage (originated as a kind of clever and rational speculation – the last word is used in a "neutral", not pejorative sense) as well as to point out its various, important connotations (not merely in finance or even economics) and to

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demonstrate some mathematical inevitable technicalities, reflecting, in fact, the logical essence and the modern view of the arbitrage (and non arbitrage) conditions.

The first paper has been just written and submitted to *Mathematical Economics*. It plays the role of a foreword and provides the key notions of the theory (a kind of an informal "vocabulary", which will be developed and formalized in the subsequent parts of the series). Some history of the subject is outlined and the main stream of the modern research studies is indicated. After the seminal work of Miller and Modigliani, two of them are of a special importance: Harrison, Kreps, Pliska "martingale wine" theory of modeling stochastic dynamic, excluding the possibility of arbitrage in finance (modern topics are actually far distant from the origins: include taxes, transaction costs, information asymmetry and non-classical preference), and Ross's Arbitrage Pricing Theory (in a modern, more and more general setting, but grounded in factor analysis ideas – of the pioneering work of Ross). The main text of the article is supplemented by a relatively large, representative bibliography. It is aimed merely to be "the guidance on interesting and constructive historical object" and, at the same time, to reveal the some perspective of the subject, but in any way does not attempt completeness. As a matter of fact, such a task is impossible (almost) to carry out at the moment and is not an aim to place it in this "little trilogy". After the "announcement" of the subject performed in the first article of the series, the author passes to a more detailed discussion of introduced notions and related problems. So in the second part of the series, we review the significant theoretical facts from the arbitrage and "no arbitrage" modeling of stochastic dynamics in finance (mainly those worked in the second half of the 20th century) as well as those of the arbitrage pricing theory. This segment of chain was programmed as a "special mélange" of mathematical notions and topics which actually turned out to be the "proper and efficient" (to some extent) tools for solving "urgent" problems appearing in modern finance. We meet there Stiemke, Farkas, Minkowski's observations, Hahn, Banach's (type) theorems and their developed version in the spirit of the stochastic analysis (Kreps, Yan), modern (very abstract) formulations of fundamental theorems of arbitrage pricing (originated by Harrison, Kreps, Pliska), modern (re)formulation of Ross's APT (in Hilbert space and "beyond"– formalism). Connotations with stochastic dominance, attitudes to risk and efficiency of markets are also mentioned. This part is now in preparation – it is almost completed and is planned to be published in the next issue of *Mathematical Economics*.

The third paper is partly intended to continue and develop the previously posed questions, and to present some current trends of generalization and topics in modern arbitrage theory (i.e. arbitrage with transaction costs and with taxations, arbitrage on incomplete markets, arbitrage on so-called large financial markets or arbitrage valuations of insurance cash flows, premium and others). But the author's main interest here is to check if it is possible to integrate (or "couple") crucial ideas and observations from "martingale arbitrage theory" with "Chichilnisky's social choice-topological arbitrage" models (and, if the answer is not "absolutely not", asking "to what extent?"). The natural, subsequent step is to search for possibilities of implementation of such a "methodological mixture" to problems of intergenerational equity and, more generally, to sustainable development modeling. Generally, it seems very interesting to investigate the macroeconomic version of arbitrage consequences as well as the formulation (and interpretation) of the no-arbitrage conditions – these research studies are relatively new at the moment. It is also worth noticing that an arbitrage equalizing ideas link the problems discussed in the present study with the equilibrium theory, "hidden behind their practical, financial costumes". Finally, let us remember an obvious but important fact: the non-arbitrage-type requirements (as well as exclusion of the "free lunch" possibilities and the very classic law of one price) really constitute the generalizations of "rational" condition appearing in the neoclassical production theory (and, consequently, in the theory of economic growth) concerning the impossibility of producing anything from nothing (commonly known as the "phenomenon of cornucopia").

2. Conclusions

The notion (and the idea) of the arbitrage play an important role in both technical and conceptual spheres of "sub-theories" concerning valuation in the economic theory, finance and the social choice area. Somewhat paradoxically, it enters in the above themes "by negation": mainly – through the characterization of the mathematical, formal constraints and various (equivalent or "almost equivalent") requirements, aimed to guarantee non-arbitrage conditions (or related postulates). Eliminating of arbitrage opportunities or existence of circumstances, when one may get non-risky profits (or gain positive payoff without investing anything), makes up the minimal and, at the same time, "basic and natural" conditions to correct evaluations of goods (the "gaps" are then deleted and equilibrium prices can be determined).

In the present "Introduction" (to the series of three papers) the plan of the author's undertakings (to gather the main ideas, to look at them in some historical perspective, to point out crucial connotations, to reveal the "core of mathematical machinery hidden behind the economic stories" and to picture generalizations and leading modern streams of development of the "arbitrage -no arbitrage dilemmas") is indicated.