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Wrocław University of Technology

Production Management

Anna Burduk

METHODS OF PRODUCTION RISK ASSESSMENT

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Introduction

Each field of human's life and activity is related to taking decisions and everybody cares that his undertaken actions are reasonable and bring as much as possible profits or satisfaction. Some of the decisions relate to simple or repeatable actions, or known are consequences of the undertaken actions. In such a case, one says about routine decisions and certainty conditions. When the decisions concern undertaking activities not realised before, complicated or related to further future, then one says about a risk burdening these actions and about the uncertainty conditions. Uncertainty and risk connected with taking decisions related to the future depend on flow of time and rise along with its elongation. The probability also increases that the really obtained value will significantly deviate from the expected value. In similar way is shaped the uncertainty and risk related to taking decisions not realised before, complicated or complex. However, in this case their growth depends on the number of variables considered when taking a decision.

The above statements become particularly important with respect to operating conditions of the present companies. Today, in order to stay in the market, the companies are constrained to develop continuously by setting new goals, adapting to the changing environment and to watching in these changes new opportunities for its activity. Usually, this happens by new investments in technology and products, as well as changes in the way of organisation. Both their environments and the companies themselves become more and more dynamic and complex. Consequently, this results in growing uncertainty and risk of the taken decisions. The planning function also becomes more and more important in the company management process. This is related to taking proper decisions related to development, because on such a taken decision dependent is market position of the company, its competitiveness, level of generated profits and other factors determining the success. The taken decisions influence the company situation both at the moment of their taking and in the future, therefore each decision significant to the company operation should be preceded by analysis, planning and risk assessment.

Although nowadays the risk of a business activity is equally high in the markets of production and services as well as in the financial market, significant differences are visible in the numbers of reference items, trainings and scientific conferences within this scope. This situation is simply transferred to the number of planning, analysis and risk assessment methods to disadvantage of the production sphere, where planning and risk assessment is underestimated and very poorly developed. Available are numerous reference items in the fields of management, analysis and assessment of credit, insurance, commercial, information technology and other risks, but visible is shortage of literature elaborates concerning companies and manufacturing processes.

Peculiarity of production requires different approach to decision-taking than e.g. in finances, where higher risk of an investment is usually connected with a possibility of reaching larger profits. Because of technical and technological limitations, in the production area can not be expected higher results than those possible to be reached with the given technology or work organisation. It seems much more proper to treat the risk in the context of probability of unrealised production goals and unreliability of the manufacturing system. Because of the nature and importance of manufacturing process realisation for industrial companies, use of simulation methods and tools seems to be necessary at planning and assessment of risk.

Chapter 1. Concept of risk and its assessment methods

This Chapter includes a genesis of the present understanding of the term "risk" and the approach to risk in the world literature. It presents selected groups of definitions and risk classification. It presents the methods of risk analysis and assessment most frequently used in the literature. Because of the scope and subject matter of this elaboration, a detailed classification is presented for the quantitative methods only. Described are basic statistical measures of risk, statistical methods and operational research methods used in literature for analysis and assessment of production risk. Since the statistical methods are usually described on selected examples restricting possibilities of their application, applied is the analytical description, illustrated with examples. In the summary of the Chapter, the described methods are compared with respect to the selected criteria.

The word "risk" (in French: *risque*, in German: *Risiko*) originates from old-Italian "risicare" that means "to have the courage", "to dare" or just "to risk", while this concept should be rather associated with free choice than with an unavoidable destiny. As a phenomenon, the risk permanently accompanies each decision related to human business and living activity. It can not be eliminated, because it is connected with taking decisions and is related to the future, with that the concept of uncertainty is inseparably bound. This results in common use of the term "risk", when saying about various activities and fields of human life.

First attempts to making a scientific synthesis of this concept appeared in the work of A.H. Willett in 1901. In his theory, the author assumed that the commonly used term "risk" is an ambiguous concept, strictly related to the concept of uncertainty [51]. Since uncertainty is also an ambiguous concept, the Willett's economic theory of risk was not fully accepted, but aroused several controversies. The subsequent stage of evolution of the risk definition was the theory of F. Knight set forth in 1920s. The concept presented uncertainty as a connection of two components: measurable uncertainty and non-measurable uncertainty. The first one was named the risk and the other one – the uncertainty stricto sensu.

A true breakthrough occurred in the years 1960 – 1970, when many definitions connected with the concept of insurance risk appeared. In

1966, the Committee on Insurance Terminology published an official definition of risk. It defined risk as "*uncertainty concerning appearance of a specific event in the conditions of two or more existing possibilities*" [5]. Till today, practically no universal definition of risk has been created, which should be attributed to its multiaspect meaning.

Identification of risk with danger comes from colloquial language. However, risk should not be perceived this way [43]. First of all because dangers should be avoided and avoiding risk not always proves to be a favourable solution. Observations of the nature of uncertainty and its multisubject range suggest that it is an inherent element of existing and acting, together with a business activity. Would it be reasonable to take up any activity when knowing that it is burdened with risk attributable exclusively to disadvantageous states? Therefore, risk must give some possibilities, so that a human action could be inspired. Risk in a business activity is connected not only with possible losses, but gives also a possibility of a profit. It is basically something positive and makes people try improving their knowledge, so that the taken actions and decisions are innovative and effective. It constitutes a sort of a driving motor of each activity, so it should not be avoided but examined to acquire the knowledge necessary to take right decisions. In practice, one can meet both a unidirectional risk (loss) and a multidirectional risk (loss and profit). The fact that risk is examined only from the viewpoint of possible occurrence of disadvantageous states is not identical with treating risk as danger. This means that risk is intentionally incurred and included in the range of a company activity as its integral part [50].

In practice, risk is often identified with uncertainty of taking decisions. This is related to the fact that these decisions refer to the future. It appears however, that such an approach is not completely true. In literature, one can meet various definitions which try showing relations between uncertainty and risk, based on the F. Knight's theory of measurable and non-measurable uncertainty. According to this theory, risk is understood as both measurable and non-measurable deviations from expected results of the taken decisions and business activities. The main criterion of separating risk from uncertainty (non-measurable risk) is a possibility of using the probability theory by the decision-maker. The former can be measured or exactly estimated by the probability theory, but the latter does not meet this requirement or meets it insufficiently exactly [50,86]. One can say about uncertainty conditions e.g. in the situation where the production manager who implements a new solution has no information about the new manufacturing process. He can ascertain justice of its implementing on the ground of his own intuition only. But if some examinations and simulations were carried-out previously to provide information allowing calculation of probable situations, then the innovative actions would concern the risk conditions.

1.1 Risk factors

Risk in business appears always where goods are created, processed, offered or purchased. Nowadays, instability of the market is growing and the changes occurring there can bring serious results. In each field, the number of new products, services and enterprises with higher and higher degree of technical and technological complexity is growing. Risk can be classified according to the following factors:

- internal (microeconomic) factors,
- external (mezoeconomic, macroeconomic) factors.

Internal factors are related to internal activity of a company. They are determined by situational-financial analysis of the company. In particular, they are related to the performed processes and activity towards suppliers, customers and institutions. The most significant source of risk originating inside the company is time and related to it pressure of deadlines resulting in deterioration of both the manufactured products and human labour. The time frames can be exceeded due to various reasons; the most often occurring are:

- breach of contract terms by suppliers,
- failures of machines and facilities,
- manufacturing defective products and related necessity to repair them or manufacture additional pieces,
- unexpected random causes which can include shortage of production resources caused by disasters, war, epidemic among the staff etc.

Numerous dangers exist in business activity. Apart from those related to the time factor, one can mention also several factors connected with human activity. They include all the activities (intended or unintended) detrimental to the company. In particular, this is shortage of proper abilities, especially dangerous at the decisive level. An often made error is improper evaluation of production volume in relation to the demand. Errors in managing are one of the most frequent causes of a company bankruptcy [116]. Another underestimated aspect connected with human activity is honesty of employees. Probably, one of the most distressing causes of risk is that the danger comes from a person employed in the company, well informed, having well-grounded knowledge about the company activity, recognised as a good and effective worker. Irony of this situation can be even greater if the fault is on the side of the highest-level management [37].

External factors are in literature determined also as macroeconomic factors. They are related to globalisation of economic processes and to general economic situation of the country, as well as to international relations. They concern e.g. condition of the economy (recession, prosperity), gross domestic product and domestic demand, inflation, monetary policy (monetary-credit and currency policy), customs and legislative policy. Degree of free activity is limited, because at a certain moment appear legal obstacles or political objections, which can be detrimental to the project and innovative policy of the company. Globalisation has become one of the most important factors conditioning business activity of a company. Nowadays, the business environment is no longer limited to the country of the company's seat. The South-Eastern Asian economies enter into relations with western markets creating a very complex network. The present companies can be endangered by competition coming equally easily from the other hemisphere and from the neighbouring city. Nowadays, integrative aspirations of many countries with the global economy are intensified and the global risk increases. In the world, markets become more and more homogeneous and demand for capital (mainly the foreign one) increases, because many countries do not have their own capital big enough. More and more companies make their activity to higher and higher degree international. Reasons of internationalisation of the company activity include [80]:

- Searching for new markets – after the company has filled its domestic market demand, more favourable possibilities of further growth are in foreign markets.
- Searching for raw materials – companies locate their affiliates in various parts of the world in order to get access to basic resources facilitating their primary activity.
- Searching for new technologies (scientific ideas and designs), as no country exists dominating in all kinds of technologies.

- Searching for production efficiency, and so for possibilities of moving the business to the countries with lower production costs, mainly lower labour costs.
- Avoiding political and legislative obstacles, and so foreign investors try avoiding delays related to licences and regulatory procedures, omitting import quotas or political-legal and social barriers, e.g. resulting from intensive ecological protests.

Results of global risk concern on one hand the countries of the investment origin (i.e. investing abroad) and on the other hand the countries receiving investors with their foreign capital. This category of risk includes [80]:

- transfer of modern technique and technology,
- transfer of capital,
- transfer of profits,
- new labour markets,
- new outlets,
- international regulatory instruments.

Risk of modern technique and manufacturing technology transfer is related to promotion of scientific knowledge and managerial know-how. It depends on effectiveness of implementing innovations and improving product quality according to the requirements of open international competitiveness and considering uncertainties in investment tenders.

Risk of material-financial capital transfer is identified with flow of such assets as fixed and working assets, financial means and securities. In the situation of international integration of financial capital, risk of transferring cash to a high degree depends on currency risk. The currency risk is related to adjusting currency rates in a given country, and thus to more expensive or cheaper export and import. A measure of this risk is variability of current value of reached incomes and born expenses related in the future to volatility of currency rates.

Risk of profit transfer is identified with effectiveness of protecting the generated incomes and profits, as well as their free or restricted flow between the countries, i.e. from the country that receives foreign capital to that of the investment origin.

Risk of new labour markets – When investing in weakly developed countries, supranational companies must count the risk of losses resulting from employing cheap workers. Their low wages not always mean lower production costs, especially when the manpower is not sufficiently effective.

Risk of new outlets can result from protecting the companies of the capital receiving country against international competition, as well as from maintaining domestic property and related rights in order to protect national sovereignty corresponding with social sensitivity in the given country. Over the world, along with technical-technological and organisational development, cycles of product life become shorter, number of obsolete technologies increases, necessity of improving product quality intensifies and competition for global customers grows. Supranational companies bear risk of losses devoting too much time for introducing new products to foreign markets, because their competitors can quickly duplicate them according to the muster and introduce to the market sometimes even faster than the companies initiating the innovations.

For the purposes of both internal regulation and international economical policy, authorities of individual countries can introduce several barriers and stimuli encouraging or discouraging from investments. These stimuli can be: accelerated depreciation, tax relieves and exemptions, government guarantees at taking credits, low-interest loans, accessibility of infrastructure, investment advisory and information. In turn, the barriers aimed at intensifying risk can be: requirements of obtaining investment permission, reduction of foreign capital participation, difficulties in investment tenders and at obtaining orders for government purchases, tax regulations (e.g. transfer prices), limited access to local authorities and finances [43].

1.2 Literature approach to risk and groups of its definitions

Ambiguity of the concept and lack of a clearly specified definition caused that variable approaches to the subject of risk can be found in literature. The most widespread approaches are: German, American and scientific approach.

The German approach restricts the essence of the concept of risk to obtaining a negative effect as a result of a taken decision, i.e. treats the risk as "*danger of non-performing the assumed goal at taking a specific decision*" [15]. Non-achieving the goal can appear by occurrence of both a loss and a profit lower than assumed. Hereinafter, the German approach will be applied.

This approach is enlarged by the American concept that treats risk not only as a possibility of suffering a loss but also of gaining a profit. Two categories of risk are distinguished in this approach [5]:

- Pure risk (*static risk*). This concerns potential occurrence of loss. This risk is difficult to be overcome and controlled because it is mostly influenced by external factors, irrespective of the undertaken actions. In practice, it is impossible to be guided by this risk. However, it should be reckoned and always taken into account. Its characteristic feature is that it is always present and is not subject to changes. A typical protection against this kind of risk are insurances and some means preventing from losses. This category includes e.g. risk of fire, explosion, illness, death etc.
- Dynamic risk (*speculative risk*). This is a risk that can lead to both positive and negative results. It is undertaken consciously in order to obtain a positive result (profit), but there is a chance that it will lead to a negative result (loss). According to this approach, risk is identified by deviation from the intended effects (the deviation can be positive or negative) [39]. This understanding of the concept can be most frequently found in the literature related to the questions of probability and mathematical statistics [39].

The above-presented approach is shown in Fig. 1.

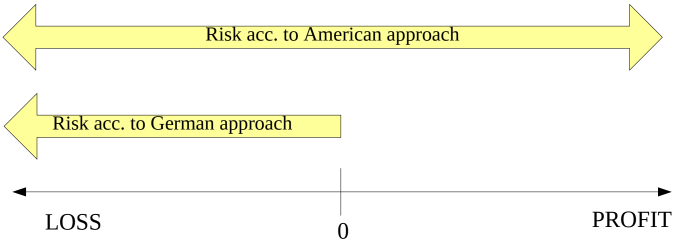


Fig. 1. Concept of risk acc. to American and German approaches

Another approach, equally often met in literature, is the scientific approach. It treats risk as "(…) a situation in that future conditions of managing can not be predicted with full certainty, but known is probability distribution of occurrence of these conditions" [39]. In addition, in this approach it is assumed that [39]:

- risk is related to distribution of a random variable, e.g. sales volume, amount of costs or profit, etc.,

- the measure of risk is dispersion degree of a random variable,
- the higher dispersion degree of a random variable, the higher risk,
- the definition "risk amounts to X %" means that the given variable can change within ± X % from the determined value.

Because of the above-mentioned approaches, numerous definitions of risk can be found in literature. Risk is understood in different ways and can signify [5, 39, 40, 49]:

- possibility of suffering a loss,
- probability of suffering a loss,
- discrepancy between real and expected results,
- probability of a result different than expected,
- possibility of occurrence of undesirable events,
- conditions in which occurrence of a loss is possible,
- uncertainty, danger, possibility of non-achieving the goal.

Since risk is a common phenomenon, it happens in all the fields of life and is of interdisciplinary nature; many scientific domains are engaged in its analysis, e.g. probability theory, statistics, econometrics, image recognition theory, reliability theory, operational research, organisation and management theory, psychology, sociology, philosophy and others. Multitude of approaches to risk and its definitions makes quoting all of them useless, but – on the grounds of analysis and the items [5, 39, 40, 49] – they can be subdivided into 6 groups shown in Fig. 2.

GROUPS OF RISK DEFINITION

Acc. to decisions taken to realise determined goals:
Risk is uncertainty related to future events or results of decisions. Results of decisions lead to occurrence of unexpected value of loss or profit.

Acc. to sources of risk occurrence:
Source of risk is incompleteness of information or a taken decision that is not optimum with respect to the assumed goal.

Acc. to risk symptoms:
Risk is a deviation from expected value of assumed goal.

Acc. to probabilistic or statistical measures:
Risk is a subjective (individual) probability of single events or those which did not happen at all.

Acc. to image recognition theory:
Risk is a discrete measure utilising the image recognition theory and cost associated with this measure. At assessing risk, created or utilised are existing abstract risk patterns, and then the examined risk is placed in the risk space.

Acc. to reliability theory:
Reliability is understood as a property of an object determined by its ability to fulfil the imposed requirements.

Fig. 2. Selected groups of risk definitions

It can be said on the grounds of the above-mentioned approaches and groups of risk definitions that interpretation of the concept depends on the subject interested in its analysis. The above groups of definitions demonstrate that risk is a comprehensive and ambiguous concept and has no unambiguous interpretation, which should be associated with its ambivalence. Risk creates a chance of a success, but at the same time threatens with a failure. Moreover, risk is a universal concept, because it applies to all aspects of life.

Depending on the field, among others the following risks can be distinguished: risk of a disaster, financial risk, credit risk, insurance risk, market risk, risk of exceeding time frames and others. Each of them will be determined by another definition and another mathematical model. Difficulties in precise defining the risk result also from its both unidirectional and bidirectional nature. Whether the risk is uni- or bidirectional, depends on the field of interest and also on the accepted model and the influencing factors.

1.3 Risk and uncertainty Individual attitudes towards risk

Examination of risk in business activity is a quite new phenomenon. Till some time, the businessman himself dealt with risk decision-his business activity. He looked for ways and means to reduce the risk results. Usually, he was interested in risk as much as he had to be insured against it, with use of an insurance agent. Nowadays, increase of risk and its complexity incline to appointing risk managers [101].

A risk manager is a person responsible for detecting and, indirectly, eliminating any irregularities and disturbances acting to the company's disadvantage. A risk manager should be involved in economic, legal and technical problems. He should have at his disposal a wide range of information concerning external and internal conditionings of the company activity. With respect to the area of interest and to superior nature of the taken decisions, the risk manager is located just at the general manager or the company owner [79].

Function of a risk manager requires from the person holding this post knowledge from the borderland of many fields of science. That person must be also distinguished by some psychological predispositions. It is known that risk can affect human behaviour in various ways. On one hand, it can prompt to business activities, and on the other hand – to conservative attitudes. One of the reasons of actions consisting in avoiding risk is the deeply rooted view that risk leads to a disadvantageous state, so it should be avoided. One forgets that taking processrisk can be connected with possibilities of gaining profit. Therefore, risk has two dimensions: positive and negative. The first is a source of enterprising behaviours and the second

– of conservative attitudes [79]. Three basic aspects can be distinguished in risk analysis:

- uncertainty,
- profit,
- inclination of investors to taking risk.

Uncertainty of taking decisions results from ignorance of future state of nature, so it is connected with the time factor. In risk analysis, profit is the basic motive of business activity. When taking a specific decision, an investor expects larger benefits. Since the future is uncertain, the expected profits are also uncertain. Whether the given investor will take a determined decision, to a high degree depends on his individual attitude. Three attitudes of the decision-makers towards risk are mentioned in literature [116, 115,79].

- neutrality towards risk,
- aversion towards risk,
- fondness for risk.

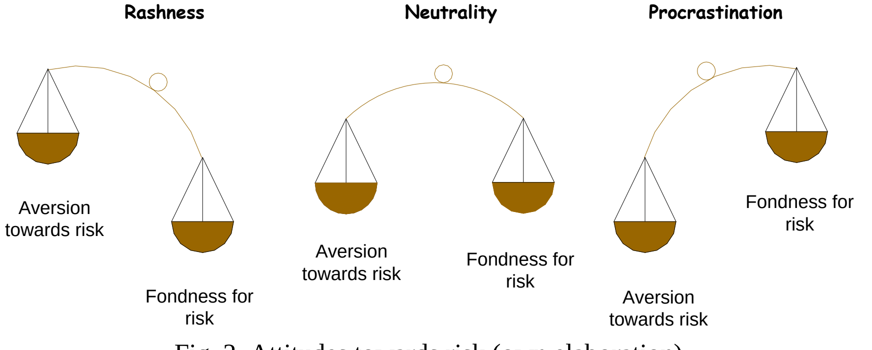


Fig. 2. Attitudes towards risk (own elaboration)

A person neutral towards risk is a person aware of risk, treating uncertainty as a normal element of life and undertaking proper preventive actions. Such people apply a consistent procedure of risk analysis and management in order to choose the best way of operation. Among the units involved in direct risk evaluation, one should mention banks, international funds, engineers carrying-out mechanical testing etc. [116].

The people showing fondness for risk undertake an action even when, according to calculations, probability of loosing is higher than probability of winning. The stronger fondness for risk, the higher must be danger of loss to make the given person give up [115]. The people inclined to take a risk are those who actively invest their savings in financial markets and treat their future in a fatalistic way. They want to act, even if they realise a relatively high probability of a failure.

The persons avoiding risk do not undertake any actions, if chances of reaching positive results are too low. If the chances are high enough, hope for an expected profit wins the natural aversion to risk. The more the given unit avoids risk, the higher must be the chances to gain profit [116]. Another behaviour of the persons with aversion to risk is procrastination. The procrastinators' motto is to postpone taking a decision pending a favourable situation development. Decisions are usually taken by such a person post factum, which often generates additional costs. An example can be calling a service at the moment when a failure appears. This extends repair time, generates additional costs and the repair is made at an unknown time, which can mean a least appropriate moment. A strong argument for rejecting such an attitude is that each company acts in competitive conditions, so postponing a decision can not only deprive the company of profits, but also undermine its market position [79].

Apart from these mentioned, one can distinguish another attitude giving similar results as avoiding risk, that is neglecting risk [116]. The people neglecting risk are those who more or less intentionally live in blissful ignorance of risk at that they are exposed.

It seems that people are usually willing to avoid risk and readily pay for its reduction. This can explain existence of numerous insurance companies. However, realistic attitude towards uncertainty and to potential results is always a better strategy in business and leads to better results, because this means lower investments in insurances and other preventive measures. This attitude permits also perceiving a good opportunity of making a business where the others can see too high risk. ***An example is the company Xerox that developed the technology of electrostatic copying, whereas their competitors came to the conclusion that the market did not justify such an investment. After a few years, that technology became dominant in the market and the risk taken)))by Xerox was converted to considerable profit [53]. Innovations are a necessary element in the activity of production companies. Some entities invest in innovative activity, the others spend considerable amounts for purchasing licences and know-hows. All this in order to be vested with a better, cheaper and more modern technology than the others have. Both the innovative activity and implementing new technologies are connected with bearing enormous expenditures and entail risk of a failure. However, as can be seen on the Xerox example, a risk-burdened decision can result in a good profit.

Proper understanding and respect for risk are a good philosophy even for those who are declared risk-takers [115]. If the investors are well prepared, they are less exposed to failures and painful defeats, according to the saying "earlier warned – better armed". Therefore, attitude of a manager to risk should be reasonable, because aversion to risk results in lost opportunities and wasted resources, but fondness for risk can lead to a disaster.

1.4 Risk and uncertainty at decision-taking

Managing an economic organisation is a series of decision-taking processes and those of creating conditions for effective realisation of the decisions [39]. The decisive situation in a company can be presented in various ways, e.g. in form of a decision matrix, decision tree and a mathematical model. A decision matrix, called also a consequence matrix, is shown in Table 1.

Table 1. Decision matrix [22]

Variants of action	**States of nature**
	S_1 . . . S_2 . . . S_n
A_1	OK_{11} . . . OK_{12} . . . OK_{1n}
.	.
.	.
.	.
A_2	OK_{21} . . . OK_{22} . . . OK_{2n}
.	.
.	.
.	.
A_m	OK_{m1} . . . OK_{m2} . . . OK_{mn}

Designations:
A = decision variants,
S = possible states of nature,
OK = expected benefits.

Taking economic decisions means "*creating or forecasting possible variants of events (so-called states of nature) and actions for the* *purpose of managing, as well as their analysing and selecting*" [49]. To assess, which decision is better and which is worse (at a given state of nature), one should compare benefits resulting from them. So, the expected benefit (*OK*) connected with a given decision must be properly measured. The quantity used for measuring and expressing benefits is named the target variable [22]. This variable reflects the goal of an action, to reach that a solution of the decision problem and a best decision are searched for.

Many classifications of decision problems exist in literature, but from the viewpoint of risk assessment in production systems a classification based on certainty degree is presented below. Decisions are divided as follows [22, 39, 49]:

1. Decisions taken in certainty conditions

   When known are all possible results of actions being a subject of choice or the state of nature consists of one element only, then the decision-maker knows certainly, which state of nature will happen.

2. Decisions taken in risk conditions

   The decision-maker knows the probability distribution of occurrence of individual states of nature, which can result from theoretical assumptions or be an empirical distribution observed in the past. It can also result from subjective assessment of the decision-maker considering chances of occurrence of individual states of nature. This kind decisions are taken most frequently.

3. Decisions taken in uncertainty conditions

   These decisions occur when the decision-maker has no information about probability of individual states of nature or when e.g. the given decision problem is considered for the first time and it is impossible to use previous experiences.

1.5 Classification of risk

When classifying the risk, one should use many criteria, obtaining this way numerous sets of various kinds of risk. On the ground of kinds of risk classifications presented in literature, it can be said that their multitude results from two reasons:

1. multitude of the fields in which the risk occurs, and

2. way of treating the risk:

- as discrepancies between reality and possibility, or
- as relationships between responsibility and result.

Because of extensiveness of the question of risk assessment in manufacturing systems, classification of risk will be limited to economic risk and to the types of risk most often met in literature. Quoting all the possible classifications seems impossible and useless with regard to the purpose of this work. Figure 3 shows a chart of economic risk classification.

TYPES OF ECONOMIC AND BUSINESS RISK ACC. TO VARIOUS AUTHORS

General classification [4,5,29]

- **Inherent risk** - functioning on the ground of the law of large numbers
- **Subjective risk** - related to individual imperfection of a human, subjectively assessing probability of occurrence of specific events
- **Objective risk** - being an absolute form of uncertainty related to impossibility of forecasting development of some phenomena

With regard to possible diversification [4,22]

- **Systematic risk** - unresponsive to diversification
- **Unsystematic risk** - possible to be restricted by means of diversification

With regard to economic system area [4,5,9,29]

- **Constant risk** - covering the entire economic system
- **Inconstant risk** - covering a given investor or company

With regard to decisions of the company development [4,5,37]

- **Project risk** - resulting from technical realisation conditions
- **Company risk** - resulting from incorrect planning assessments
- **Owners' risk** - resulting from non-demonstrating interest in differentiation of the company development directions

With regard to field of occurrence [4]

- **Production risk** - occurring in production sector
- **Commercial risk** - occurring in commercial sector
- **Financial risk** - occurring in financial sector

With regard to strength of influence [4,13]

- **Normal risk** - that undertaking is necessary since it is inherent in the nature of economic processes
- **Acceptable risk** - that the company can take
- **Unacceptable risk** - that the company can not take
- **Necessary risk** - that the company is unable not to take

Fig. 3. Selected types of economic risk met in literature

1.6 Classification of methods of risk analysis and assessment

In literature, one can find many methods of analysis and assessment of risk in a company. As already mentioned, this is because risk is an ambiguous concept and happens in many fields of economy and science. Authors of the publications classify and name individual methods of risk analysis and assessment in different ways (compare [5, 39, 22, 40, 49]), but two classes of methods are most frequently mentioned:

1. verbal methods, called also descriptive or general methods, permitting recognition of risk present in the organisation;
2. measurable methods, called also quantitative methods, permitting assessment of risk magnitude and utilising numerical data.

The above classes include individual methods of risk analysis and assessment. Here again, literature classifies individual groups of methods to the classes in various ways. Figure 4 shows both classes of methods together with the groups distinguished in them.

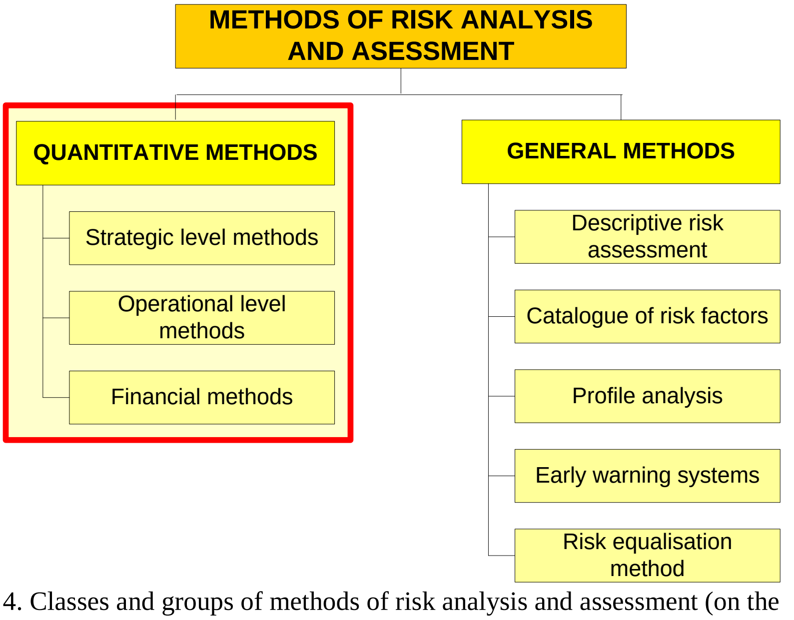


Fig. 4. Classes and groups of methods of risk analysis and assessment (on the ground of [39])

Since managing a manufacturing process is of operational nature and its control requires using detailed numerical data on its course, the subject of further analysis will be quantitative methods of operational level only. Generally, statistical methods and operational research methods can be used on operational level. Figure 5 shows all groups of quantitative methods of risk analysis and assessment with particular respect to the methods of operational level.

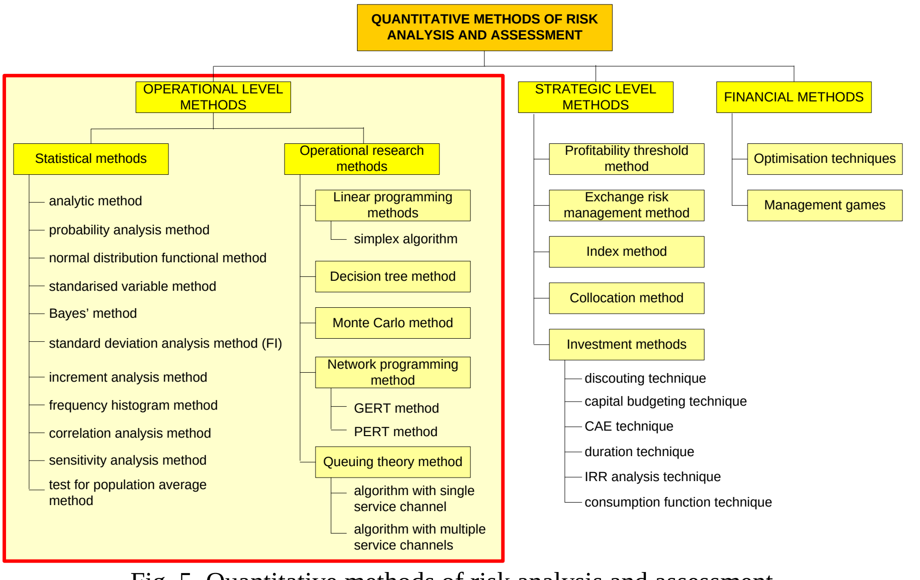


Fig. 5. Quantitative methods of risk analysis and assessment

Classification of the operational research methods does not present any problems, because this field is relatively well developed. A difficulty creates classification of statistical methods. The major problem is small number of literature items in this field, because most of the authors analyse and assess risk on the strategic level or in financial area, omitting importance of risk at managing a production company on the operational level.

Another problem is lack of an objective and scientific description of these methods utilising probability theory and mathematical statistics. Since the literature items are usually directed to managers of higher and medium management levels, it seems that description of the methods should include definitions and mathematical theorems making their base, especially when the definition or theorem used in the method concerns a particular case, refers to a given range or includes limitations. Unfortunately, the literature items describing statistical methods include usually a verbal description only or are presented on selected examples, with omission of special cases of definitions and mathematical theorems.

1.7 Statistical measures of risk

As statistical measures of risk accepted are measures of dispersion. They inform about differences between the really obtained and the expected values (goals). The problem of significant influence of dispersion on actual efficiency of manufacturing systems is known especially in production processes [5]. Most frequently, the measures of dispersion (risk) include [5, 19, 22 ,36, 54]:

- variance,
- semivariance,
- standard deviation,
- coefficient of variation,
- coefficient of asymmetry.

Variance

Variance of statistical variables from a data set $x_1, x_2, ..., x_n$ has the form of arithmetical mean (average) of square deviations of values in the set from their average value, with the restriction that the sum of square deviations is divided not by *n* like at calculating the average, but by *n-1*. The formula for variances is as follows [22]:

$$s^2 = \frac{1}{n-1}\sum_{j=1}^{n}(x_j - \overline{x})^2 \tag{1.1}$$

or in the form more convenient for calculations [22]:

$$s^2 = \frac{\sum_{j=1}^{n} x_j^2 - n(\overline{x})^2}{n-1} , \tag{1.2}$$

where $\overline{x}$ = average value.

In the case of a set of grouped data (interval series), the respective formulae for variances accept the form [22]:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{k}(x_i - \overline{x})^2 n_i \text{ and} \tag{1.3}$$

$$s^2 = \frac{\sum_{i=1}^{k} x_i^2 n_i - n(\overline{x})^2}{n-1} \ , \tag{1.4}$$

where n_i = sample sizes in an empirical distribution.

For a random variable *X*, the following expression is named a variance [24]:

$$D^2(X) = E[X - E(X)]^2 = \begin{cases} \sum_i [x_i - E(X)]^2 p_i & \text{for a step random variable} \\ \int_{-\infty}^{\infty} [x - E(X)]^2 f(x) dx & \text{for a continuous random variable.} \end{cases} \tag{1.5}$$

Variance is a characteristic determining the scatter (diversification) degree of a characteristic value or of a random variable around the average or expected value. The larger diversification of a characteristic or random variable in a set, the larger are deviations of the value from the average and the larger variance [22]. The smallest value that can be accepted by variance is 0, which happens where all values of the characteristic are identical (complete lack of diversification). In such a case, there is no uncertainty about the final result, so the decision will not be burdened with a risk (variance is 0, so the risk is also 0). Variance is often used at calculating expected profits. It results from the variance formula that the larger deviations of attainable profits from the expected (average) profit, the larger is the variance and thus the risk related to taking a specific decision.

Semivariance

In the case of treating risk as an undesirable effect for a decision-maker (German approach to risk), only negative deviations from the expected profit are considered in calculations. In such a case, the measure of risk can be the so-called semivariance determined from the formula [53]:

$$D_s^2(X) = \sum_{i=1}^{n} p_i d_i^2 \ , \tag{1.6}$$

where d_i = negative deviations from the expected profit value, calculated as follows:

$$d_i = \begin{cases} 0 & \text{gdy } x_i \geq E(X) \\ x_i - E(X) & \text{gdy } x_i \leq E(X) \end{cases} . \tag{1.7}$$

Standard deviation

Variance as a characteristic of risk measurement is not very convenient from the viewpoint of interpretation (units of variance are square units of a variable), therefore it is more convenient to employ standard deviation that is the positive square root of the variance. The formula for standard deviation of statistical characteristics is as follows [22]:

$$s = \sqrt{s^2} . \tag{1.8}$$

For a random variable *X*, the standard deviation is:

$$D(X) = \sqrt{D^2(X)} . \tag{1.9}$$

Like in the case of semivariance, for practical reasons it is easier to use the standard semideviation that is square root of the semivariance:

$$D_s(X) = \sqrt{D_s^2(X)} . \tag{1.10}$$

Coefficient of variation

Variance and standard deviation are measures of absolute diversification, i.e. diversification measured in the units in which the given characteristic or variable is measured. In the case when compared should be diversification degrees of two or more distributions, it is better to use a coefficient of variation being a measure of relative diversification [22]. Coefficient of variation is the quotient of standard deviation and average in the given distribution, as expressed by the formula [22]:

$$V = \frac{s}{\bar{x}} . \tag{1.11}$$

For random variables, the coefficient of variation accepts the form [19]:

$$V = \frac{D(X)}{E(X)} . \tag{1.12}$$

Coefficient of variation is often used in risk calculations of production or sale volumes.

Coefficient of asymmetry

In many situations, important is not only the mean level and diversification of a characteristic (random variable), but also asymmetry of its distribution. For this purpose, the measure called coefficient of asymmetry can be used. For empirical variables, the coefficient of asymmetry is determined by the formula [22]:

$$A = M_3^{'} / s^3 \ , \tag{1.13}$$

where $M_3^{'}$ = so-called third central moment, defined as arithmetical average of cube deviations of value of a characteristic from its average [22]:

$$M_3^{'} = \frac{1}{n} \sum_{j=1}^{n} (x_j - \overline{x})^3 . \tag{1.14}$$

For random variables, the coefficient of asymmetry accepts the form [22]:

$$\gamma = \frac{\mu_3}{D^3(X)} . \tag{1.15}$$

If $\gamma > 0$, one says that the asymmetry is positive (right-hand), and if $\gamma < 0$, the asymmetry is called negative (left-hand).

1.8 Statistical methods of risk analysis and assessment

Statistical methods should be used for acquiring, presenting and analysing data on disturbances happening in a manufacturing system, which results from probabilistic nature of the phenomena occurring there [5]. The statistical methods and the operational research methods shown in Fig. 5 are described and presented below. To simplify the description, it was assumed that the random variable has a normal distribution.

Analytical method

The analytical method described in [39] belongs to the methods of statistical description. It consists in determining numerical parameters defining the examined data set. In statistical analysis and assessment of risk of a manufacturing process it can be applied only when a finite and complete data set is at the disposal.

The starting point in this method is determining empirical distribution of a characteristic, i.e. assigning to the increasingly arranged values accepted by the given characteristic the properly defined frequencies of their occurrence, and on this ground drawing-up a diagram of cumulative frequencies. As a result, an empirical cumulative distribution function is obtained.

EXAMPLE 1. <u>APPLICATIONS OF ANALYTICAL METHOD</u>

Observed duration times of the operation of assembling motors to automatic washing machines are given in Table 2. What is the risk that the assembly time will be longer then the 37 minutes designed in the process description?

Table 2. Data to Example 1 of application of analytical method – Motor assembly times

No. of measure-ment	Time [min]	No. of measure-ment	Time [min]	No. of measure-ment	Time [min]	No. of measure-ment	Time [min]	No. of measure-ment	Time [min]
1	31	7	40	13	33	19	40	25	39
2	37	8	36	14	36	20	32	26	36
3	36	9	34	15	39	21	35	27	30
4	38	10	32	16	40	22	36	28	39
5	35	11	38	17	38	23	34	29	39
6	36	12	32	18	30	24	34	30	36

According to the proceeding in the analytical method, prepared is a diagram of cumulative frequencies (sample sizes), shown in Fig. 6.

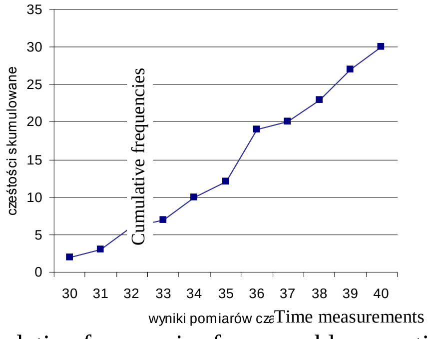


Fig. 6. Cumulative frequencies for assembly operation times

It can be read from the diagram that time of the motor assembly operation equal or shorter than 37 minutes occurred in the examined case 20 times for 30 analysed samples. Therefore, it can be said that the risk of assembly time longer than 37 minutes is 1/3.

Method of probability analysis

This method is applied in investment processes. It serves for comparing and choosing an enterprise that would be characterised by higher level of the achieved goal (e.g. profit, sales volume, production volume etc.), but risk is also considered at taking a decision on implementing the project [5, 39]. For risk analysis of the considered enterprises, standard deviation and coefficient of variation are applied. Stages of the method are as follows:

1. Determining and comparing the average or expected values of the analysed enterprises on the ground of the formula from Annex 1 (Z.1.1) or (Z.1.2);
2. Calculating the variance value from the formula (1.5) for each of the enterprises as a measure of risk with that they are burdened, and then standard deviation from the formula (1.9);
3. Comparing the diversification degrees of distributions of the analysed enterprises by means of the variation coefficient from the formula (1.12);
4. Elaborating probability distributions in tabular of graphical form for each of the enterprises using the expected value and standard deviation. On the grounds of the construed probability distributions it can be determined, which of the considered projects has more chances to obtain higher profit, so it is burdened with lower risk.

EXAMPLE 2. **<u>APPLICATIONS OF PROBABILITY ANALYSIS METHOD</u>**

The company "Alpha" producing plastic packagings considers concluding a long-term agreement with a granulate manufacturer and would like that risk of this decision is the lowest. With respect to the assumed production costs, taken into account are two manufacturers – granulate manufacturer A and granulate manufacturer B. Granulate A is much cheaper, but much larger quantity of it is required for manufacture of a product lot. Decision of the "Alpha" company will depend on costs of raw material necessary to produce a product lot and on probability of passing a strength test by the products. Table 3 shows comparative results of quantities of both raw materials used in production and probability of meeting strength requirements by the products.

Table 3. Data to Example 2 – Comparison of input data about granulate manufacturers A and B

Granulate manufacturer A		**Granulate manufacturer B**	
Quantity of used raw material per lot [kg]	Probability of passing strength test	Quantity of used raw material per lot [kg]	Probability of passing strength test
1000	0.065	1050	0.045
1100	0.085	1120	0.07
1150	0.11	1100	0.25
1250	0.13	1260	0.18
1300	0.18	1350	0.15
1500	0.24	1450	0.13
1650	0.12	1610	0.11
1700	0.07	1780	0.065

According to the stages of the method, determined were expected values, variances and coefficients of variation of probability distributions. Results are given in Table 4.

Table 4. Data to Example 2 – Results of comparison of granulate manufacturers A and B

Granulate manufacturer A			**Granulate manufacturer B**		
Expected value	Variance	Coefficient of variation	Expected value	Variance	Coefficient of variation
1358.5	44127.8	0.15	1311.3	44088.9	0.16

On the ground of the data given in Table 3, diagrams showing probability degree of expected value of used raw material for the granulate manufacturer A (Fig. 7) and the granulate manufacturer B (Fig. 8) were prepared.

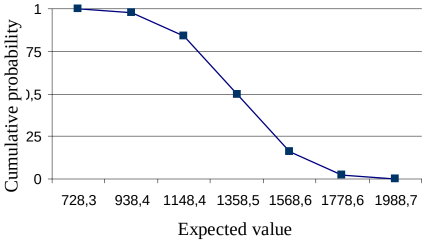


Fig. 7. Probability distribution of expected value for granulate manufacturer A

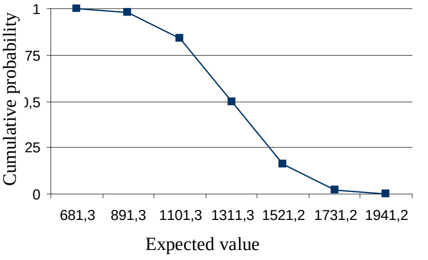


Fig. 8. Probability distribution of expected value for granulate manufacturer B

On the ground of the built probability distributions, it can be established for which of the considered manufacturers the risk of using a determined quantity of granulate is higher. Lower risk occurs in the case of the granulate manufacturer B, since probability for a given expected value is lower.

Method of normal distribution function

By applying the normal distribution function [39] it is possible to determine probability (risk) of occurrence of a value of the examined variable, assuming that the variable has a normal distribution. The method consists in finding a value of the standardised variable (*U*) at a given average value *m* and a given standard deviation σ. Then, the risk will be equal to the area under the curve to the right from the value (*U*). Stages of the method are as follows:

1. Calculating the value of standardised variable (*U*) for the analysed random variable acc. to Annex 1 (Z.1.11);
2. Reading-out the probability value from statistic tables of normal distribution function;
3. In the case when the standardised variable is negative, one should use properties of density function of the random variable with a standard normal distribution (Z.1.9) or properties of distribution function of a random variable with a standard normal distribution (Z.1.10) as presented in Annex 1.

EXAMPLE 3. APPLICATIONS OF NORMAL DISTRIBUTION FUNCTION METHOD

The company plans starting-up production of a new product and to this end it verifies technological assumptions. To verify the times of the tack-welding and welding operations, measurements were taken of the times for a trial series assuming that the average operation time is 40 minutes with standard deviation of 5 minutes. What is the risk that time of the analysed operations will exceed 47 minutes?

According to the formula in Annex 1 (Z.1.11), the standardised variable is $U = \frac{47-40}{5} = 1.4$.

Probability value read-out from statistical tables of the normal distribution function is 0.91. Therefore, the risk that the tacking and welding operations time will exceed 47 minutes is 0.09.

Method of standardised variable

This method is analogous to the method of normal distribution function and by applying it one can determine probability (risk) that value of the examined variable will be contained in an assumed range. It can be applied, when the analysed random variable has a normal distribution. From the viewpoint of statistics, the formulated problem consists in calculating probability that the standardised variable *U* accepts a value from the given range $(a, b>$ at a known average value *m* and standard deviation σ. Stages of the method are as follows:

1. Calculating values of the standardised variables from the formula (Z.1.11) of Annex 1 for extreme values of the range $(a, b>$ and reading-out corresponding values from the tables of standard normal distribution function;
2. Calculating the probability $P(a < X \le b)$ acc. to the formula (Z.1.12).

EXAMPLE 4. APPLICATIONS OF STANDARDISED VARIABLE METHOD

What is the risk that time of the tacking and welding operations of Example 3 will be within 36 to 43 minutes?

The standardised variable U_1 is: $U_1 = \frac{36-40}{5} = -0,8$.

The standardised variable U_2 is: $U_2 = \frac{43-40}{5} = 0,6$.

Probability that the operation time will be contained in the given range is 0.51, so the risk amounts to 0.49.

Bayes' method

This method is based on the Bayes' theorem, acc. to that, for two events named "cause" and "effect", probability of occurrence of the cause can be determined when the effect is already known [37]. It can be applied e.g. for determining risk of a new product design or for choosing a better version of a decision. The entire method consists in making a posterior analysis to help taking a decision connected with the largest expected profit. This analysis uses additional information to determine probabilities of states of nature s_j acc. to the Bayes' theorem [22]:

$$OK_i = \sum_{j=1}^{m} w_{ij}\, p(s_j|x),\ i = 1,...m, \qquad (1.16)$$

where:

w_{ij} = profit related to *i*-th decision and *j*-th state of nature (s_j),

p = posterior probability of occurrence of state of nature s_j on condition that the information x was obtained,

m = number of states of nature.

EXAMPLE 5. APPLICATIONS OF BAYES' METHOD [22]

A company plans production volume of a product on the ground of the demand and sales data of previous years. In this case, the states of nature (s_j) are possible amounts of demand determined for the levels of 200, 300 and 500 pieces of the product. The possible decisions (a_j) related to the production volume are the quantities of 200, 400 and 600 pieces of the product. Table 5 shows profits related to *i*-th decision and *j*-th state of nature (s_j).

Table 5. Data to Example 5 – Possible benefits related to given states of nature [22]

Possible decisions on production volume (a_j) **[pieces]**	**Demand quantity (states of nature** s_j **) [pieces]**			
	s_1: 200	s_2: 300	s_3: 400	s_4: 500
a_1: 200 pieces	400	400	400	400
a_2: 400 pieces	0	600	1200	1200
a_3: 600 pieces	-300	300	900	1500

On the grounds of experiences of previous years it was estimated that demand for 200 pieces of the product (state s_1) will occur with probability $p_1 = 0.1$; state s_2 will occur with probability $p_2 = 0.1$; state s_3 will occur with probability $p_3 = 0.6$; state s_4 will occur with probability $p_4 = 0.2$. In addition, conditional probabilities $p(x_i|s_j)$ were evaluated, as presented in Table 6.

Table 6. Data to Example 5 – Conditional probability

Results from a sample	**States of nature**			
	s_1	s_2	s_3	s_4
x_1	0.70	0.10	0.05	0.01
x_2	0.15	0.80	0.10	0.04
x_3	0.10	0.07	0.75	0.05
x_4	0.05	0.03	0.10	0.90
Total	**1.00**	**1.00**	**1.00**	**1.00**

The expected benefits (OK_i) were calculated from the formula (1.16), as presented in

Table 7. Posterior expected benefits

Decision	**Information from a sample**			
	x_1	x_2	x_3	x_4
a_1	400	400	400	400
a_2	396.6	795	1165.8	1233.6
a_3	107.4	524.4	878.4	1304.4

It results from Table 7 that when a demand for 200 pieces occurs, the decision a_1 should be taken (because is burdened with the lowest risk) and similarly, when a demand for 400 pieces occurs, the decision a_2 should be taken, etc.

Method of standard deviation analysis (FI)

Analysis of standard deviation finds its application in risk assessment because it is one of basic measures of dispersion. This method permits determining the variable level in a given range (x_1, x_2), with assumed probability level. To this end, the following quantities are calculated [5, 36, 39]:

1. average value m of random variable,

2. standard deviation σ,
3. values of standardised variables at the (x_1, x_2) range limits acc. to the formula in Annex 1 (Z.1.11),
4. Φ acc. to the formula [5]:

$$\Phi(x) = \frac{2}{\sqrt{2\pi}} \int_0^x e^{\frac{t^2}{2}} dt . \tag{1.17}$$

5. Probability that the random variable is included in the given range (x_1, x_2) is [5]:

$$P(x_1 \leq X \leq x_2) = \frac{1}{2}\left[\Phi\left(\frac{x_2 - m}{\sigma}\right) - \Phi\left(\frac{x_1 - m}{\sigma}\right)\right]. \tag{1.18}$$

EXAMPLE 6. <u>APPLICATIONS OF ANALYSIS OF STANDARD DEVIATION METHOD (FI)</u>

A production company analyses quality of its products. Inspection results of 10 successive production lots are given in Table 8. What is the risk of occurrence of 5 to 7 rejects in a lot?

Table 8. Data to Example 6 – Inspection results of 10 production lots

Lot No.	No. of rejects	Lot No.	No. of rejects
1	3	6	2
2	9	7	11
3	5	8	8
4	6	9	4
5	7	10	3

On the grounds of data from Table 8, the following values were calculated:

- average value: 5.8;
- standard deviation: 2.79;
- values of standardised variables at the range limits (5,7):
- $U_1 = \frac{5 - 5{,}8}{2{,}79} = -0{,}29$,
- $U_2 = \frac{7 - 5{,}8}{2{,}79} = 0{,}43$.
- Probability of occurrence of 5 to 7 rejects in the inspected production lot is 0.14.

- Therefore, risk of occurrence of 5 to 7 rejects in any production lot is 0.86.

Method of increment analysis

This method serves for solving decision problems. In this case, the decision criterion is maximisation of the expected profit value. Building complete benefit tables, i.e. tables assigning suitable profit values to each combination of decision and results (consequences of a decision), is not required in this method. However, created is a function, having usually one maximum, that determines the expected profit value. Choosing a specific decision means finding the argument for that the function reaches its maximum.

EXAMPLE 7. <u>APPLICATIONS OF INCREMENT ANALYSIS METHOD [39]</u>

A small production plant received the order for five pieces of a special, very precise tool. The order is to be executed by means of automatically controlled machines, where an operator's action is limited to placing a piece of metal in a fixture and choosing a proper machining program. Unit costs of raw material and machining amount to PLN 40. Value of a rejected, scrapped piece is PLN 15. Additional costs related to an additional work shift, necessary in the case of producing insufficient number of correctly manufactured pieces amount to PLN 200. The probability density function of the random variable *x*, being the number of manufactured pieces necessary to obtain five correct products, accepts the following values:

$f(5) = 0.510$ $f(6) = 0.310$ $f(7) = 0.112$ $f(8) = 0.040$ $f(9) = 0.020$ $f(10) = 0.008$

What number of the tools should be produced to make the expected profit value possibly largest (to minimise the risk of suffering a loss)?

- Fixed production costs $T = 200$;
- unit production cost $C = 40$;
- cost of scrapping an additional good product $R = 15$.
- Optimum size of the planned production (burdened with the lowest risk) is the smallest value *i*, at that:

$$\Phi = \frac{f(i+1)}{F(i)} \leq \frac{C-R}{T}, \text{ where } F(i) = \sum_{j=m}^{i} f(j).$$

It results from the above data, that:

$i = 5$	$F(5) = 0.510$	$f(6) = 0.310$	$\Phi = 0.61$
$i = 6$	$F(6) = 0.820$	$f(7) = 0.112$	$\Phi = 0.14$
$i = 7$	$F(7) = 0.932$	$f(6) = 0.040$	$\Phi = 0.04$

The Φ value is for the first time smaller than 0.125 in the third line, so Φ meets the condition for $i = 7$. Therefore, to minimise the risk of suffering a loss, production of seven pieces of tools should be planned.

Method of frequency histogram analysis

This method is a graphic method that consists in preparing frequency histograms, which shape can be a source of information about course of the manufacturing process. Histograms having irregular shapes with a few distinct maximum values, make a basis to suspect presence of significant production disturbances. By analysing shapes of histograms one can also conclude about type of probability distribution of the random variable. Accepting a correct probability distribution guarantees correct estimation of the random variable parameters, which increases accuracy of anticipations and reduces risk of the taken decisions.

EXAMPLE 8. APPLICATIONS OF FREQUENCY HISTOGRAM ANALYSIS METHOD

At a production company, analysed were data about times of repairing defective units (rejects). The acquired data are settled in Table 9. What is the risk that the repair time is longer than 8 minutes?

Table 9. Data to Example 8 – Times of repairing rejects

Reject No.	Time of repair [min]	Reject No.	Time of repair [min]	Reject No.	Time of repair [min]
1	5	6	6	11	7
2	7	7	4	12	5
3	4	8	10	13	8
4	8	9	5	14	7
5	9	10	6	15	6

On this ground, a frequency histogram was built, as shown in Fig. 9.

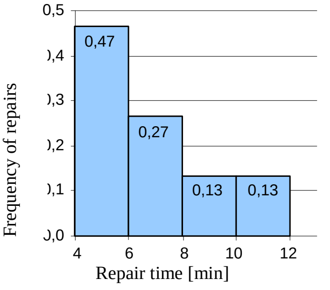


Fig. 9. Frequency histogram

It can be found from the frequency histogram that risk of a repair time longer than 8 minutes is 0.26.

Method of correlation analysis

The correlation analysis method permits determining a relationship between variables and strength of this relationship. This analysis can be used for determining relationships between production factors and parameters [5]. The method is applied in risk analysis because minimisation of risk is strictly related to accuracy of evaluating expenditures of production factors. Correlation is characterised by the correlation coefficient that, e.g. in an empirical distribution of variables *X* and *Y*, is determined by the formula [22]:

$$r = \frac{c_{xy}}{s_x s_y}, \tag{1.19}$$

where c_{xy} = covariance in two-dimensional empirical distribution; s_x and s_y are standard deviations in empirical marginal distributions of variables *X* and *Y*, respectively.

Influence of a random variable on values of another variable is described in analytical way by the so-called regression model. Its main component is the regression function whose analytic form is determined on the ground of results of a random sample. Parameters of this function are subject to estimation based on a random sample by means of procedures established in the correlation and regression theory. Therefore, they are not included in this textbook. Basic characteristics of correlation coefficient and classic linear regression model are presented in Annex 1.

EXAMPLE 9. <u>APPLICATIONS OF CORRELATION ANALYSIS MODEL [22]</u>

In a production plant, 10 measurements were taken of water consumption at manufacture of a product (*X* = production volume in thousands of pieces, *Y* = water consumption in cubic metres). What is the risk of increased water consumption in the product manufacture assuming that the classic linear regression is proper for description of water consumption in relation to production volume?

From the viewpoint of mathematical statistics, this exercise consists in finding parameters α and β, being parameters of a linear regression diagram (compare Annex 1). To determine estimations of these parameters, values of the estimators $\hat{\alpha}$ and $\hat{\beta}$ are calculated. Table 10 presents measurement data and results of calculations necessary for determining these estimators.

Table 10. Data to Example 9 – Measurements of water consumption

i	x_i	y_i	$x_i y_i$	x_i^2
1	1	8	8	1
2	2	15	30	4
3	3	8	24	9
4	4	10	40	16
5	5	22	110	25
6	6	14	84	36
7	7	17	119	49
8	8	28	224	64
9	9	22	198	81
10	10	26	260	100
$\sum$	55	170	1079	385

Substituting the data from Table 10 to the formulae (Z.1.22) and (Z.1.23) of Annex 1, one obtains:

$$\hat{\alpha} = \frac{1097 - 55 \cdot 170 / 70}{385 - 55^2 / 10} \cong 1{,}96 \text{ and } \hat{\beta} = \frac{1}{10} 170 - 1{,}96 \cong 5{,}22 .$$

Therefore, the water consumption regression function in relation to production volume, determined from the sample, has the form $\hat{y} = 1{,}96x + 6{,}22$. The regression coefficient obtained from the sample, $\hat{\alpha}$ =1.96, can be interpreted as mean increase of water consumption in thousands of cubic metres related to production increase by a unit (thousand pieces). This coefficient can be treated as risk of increasing water consumption with production increase.

Method of sensitivity analysis

This method was described in the works [5] and serves to examining influence of changes of one (input) variable on value of another (output) variable being together in a functional relationship. By using it, one can find factors the most important for a given enterprise.

However, counting this method by the authors to the group of statistical methods raises doubts. The sensitivity factor can be determined with the formula [5]:

$$W_w = \frac{\dfrac{\Delta W_{WY}}{W_{WY}}}{\dfrac{\Delta W_{WE}}{W_{WE}}} \quad , \qquad (1.20)$$

where:

W_{WY} = output variable,

W_{WE} = input variable,

Δ = increment of variable value.

EXAMPLE 10. <u>APPLICATIONS OF SENSITIVITY ANALYSIS METHOD</u>

A production company is going to check, whether there is a risk of obtaining smaller than assumed production volume caused by too small number of employees. At present, in production employed are 5 workers who manufacture in average 13 pieces per week. Results obtained by experiments made a basis for determining the sensitivity factor W_W (Table 11).

Table 11. Data to Example 10 – Number of employees and obtained production volume

Number of employees	**Number of produced pieces per week**	**Sensitivity factor W_W**
6	15	0.8
7	17	0.8
9	22	0.9
10	23	0.8
11	24,5	0.7

The performed sensitivity analysis indicates that the largest sensitivity factor W_W is obtained when employing nine workers. It can be found on this ground that the number of production workers is the risk factor significantly affecting production volume.

Method of test for average population value

From the viewpoint of mathematical statistics, this method consists in verification of statistical hypotheses of a random sample. The method is used, when formulated are some opinions on a population distribution, which should be verified on the ground of results from a random sample. This verification is carried-out at an assumed significance level α that determines the critical probability value. If the obtained probability is higher than the assumed significance level α, grounds exist for accepting the null hypothesis. Otherwise, the null hypothesis is rejected in favour of an alternative hypothesis.

There exist several tests for determining the average value of a population. They are applied when distribution of the general population or parameters of that distribution are known. If the general population has normal distribution with unknown average value *m* and known standard deviation σ, the following statistics is applied:

$$U = \frac{\bar{x} - m_0}{\sigma}\sqrt{n}, \tag{1.21}$$

where:

$\bar{x}$ = average of the sample,
n = frequency of the sample.

In the next step, the critical value u_α should be calculated, determining a critical area *Q*, where [19]:

$$Q = \{U : |U| \geq u_\alpha\}. \tag{1.22}$$

Applying the significance test with the so-built critical area *Q* consists in calculating the statistics *U* using the results of a specific sample and in checking whether it is contained within the critical area *Q*. If $|u| \geq u_\alpha$, the null hypothesis should be rejected un favour of an alternative hypothesis; otherwise there are no grounds for rejecting the null hypothesis at the assumed significance level α.

EXAMPLE 11. <u>APPLICATIONS OF TEST METHOD FOR AVERAGE POPULATION VALUE [22]</u>

Time of assembling the element T in an automatic washing machine is a random variable with normal distribution. The technical standard provides for 6 minutes for this operation, but in the operators' opinion that standard time is too short and creates a risk of exceeding the production time. This supposition should be checked at the assumption that standard deviation of the assembly time is σ = 1 min. 30 s.

It was measured that in the group of 25 workers the average assembly time is $\bar{x}$ = 6 min 20 s. The significance level α = 0.05 was accepted. Formally, in this case the verified null hypothesis is H_0 :m = 6.

According to the formula (1.22), the value of this statistics is:

$$U = \frac{6{,}33 - 6}{1{,}5}\sqrt{25} = 1{,}1 .$$

The value of normal distribution function N (0,1) read-out from the table indicates that no ground exists to reject the null hypothesis, because the value *U* is outside the critical area *Q*. This means that sample results do not confirm the supposition that the standard is incorrect and creates a risk of exceeding the production time. The difference between the average assembly time obtained in a sample and the average assembly time specified by the standard is not statistically significant, i.e. it can be accidental.

1.9 Operational research methods of risk analysis and assessment

"Operational research" is the name of a scientific discipline that was born during the World War 2. After WW2, its achievements were applied in the economical practice at the lowest level of the economy management (in microscale) [33]. In practice of management, operational researches permit building models, on whose grounds possible is taking efficient decisions with respect to the accepted technical, economic, organisational and social criteria. These models are built on the ground of identification of real situations described by characteristic quantities and parameters. At assessing production risk, applied are selected operational research methods which serve for solving specific decisive situations in order to take an optimum decision.

Simplex algorithm

The simplex algorithm is a universal method of solving linear programs, i.e. such decisive problems where both the limiting conditions and the target function are linear functions. Essence of this algorithm consists in examining successive basic solutions of a linear program in canonical form in such a way that:

a) one finds any (whichever) basic solution of a program,

b) one checks whether it is optimum,

c) if a given solution is not optimum, one finds the next basic solution, better or not worse from the previous one.

Therefore, the simplex algorithm is a stage procedure. In each stage, a basic solution is determined and checked whether it can be still improved. The procedure is finished at the moment of finding that the actual basic solution can not be more improved, which means it is optimum. In practice, the algorithm is often used at choosing variants of the production program.

EXAMPLE 12. <u>APPLICATIONS OF SIMPLEX ALGORITHM [33]</u>

A plant manufactures two products W_1 and W_2. A restriction in the production process are stocks of three raw materials: S_1, S_2 and S_3. In Table 12, given are unit expenditures of raw materials on production of products, stocks of raw materials and prices of products. What are optimum production quantities of W_1 and W_2 products, guaranteeing maximum income on their sale at the existing stocks of raw materials?

Table 12. Data to Example 12 – Expenditures of raw materials for product manufacture and their stocks [33]

Raw materials	***Consumption of raw material for 1 piece of product [kg]***		**Stock of raw material [kg]**
	W_1	W_2	
S_1	2	1	1000
S_2	3	3	2400
S_3	1.5	-	600
Price [PLN]	30	20	

In the mathematical model describing the presented situation, two decision variables exist: x_1 means production quantity of W_1 and x_2 means production quantity of W_2. The model is as follows:

$$
\begin{aligned}
&(1) \quad 30x_1 + 20x_2 && \rightarrow \max, \\
&(2) \quad 2x_1 + x_2 && \leq 1000, \\
&(3) \quad 3x_1 + 3x_2 && \leq 2400, \\
&(4) \quad 1.5x_1 && \leq 600, \\
&(5) \quad x_1, x_2 && \geq 0.
\end{aligned}
$$

Solving the example by means of the simplex algorithm starts from reducing the task to the canonical form by adding the free variables x_3, x_4 and x_5 to the left sizes of the inequalities. And so:

$$\begin{aligned}
&30x_1 + 20x_2 + 0x_3 + 0x_4 + 0x_5 && \rightarrow \max,\\
&2x_1 + x_2 + x_3 && = 1000,\\
&3x_1 + 3x_2 + \quad x_4 && = 2400,\\
&1.5x_1 + \qquad\qquad x_5 && = 600.
\end{aligned}$$

It should be noted that the free variable x_3 can be interpreted as idle stock of the raw material S_1, variable x_4 as idle stock of the raw material S_2 and variable x_5 as idle stock of the raw material S_3. So, the simplex table for each initial basic solution has the form presented in Table 13.

Table 13. Data to Example 12 – 1st simplex table to Example 12 [33]

c_b	c_j	30	20	0	0	0	Solution (b_i)
	Basic variables	x_1	x_2	x_3	x_4	x_5	
0	x_3	2	1	1	0	0	1000
0	x_4	3	3	0	1	0	2400
0	x_5	1.5	0	0	0	1	600
	z_j	0	0	0	0	0	0
	$c_j - z_j$	30	20	0	0	0	

By executing subsequent steps of the simplex algorithm one obtain subsequent simplex tables. The last simplex table is the one shown in Table 14.

Table 14. Data to Example 12 – 4th (last) simplex table to Example 12 [33]

c_b	c_j	30	20	0	0	0	**Solution (b_i)**
	Basic variables	x_1	x_2	x_3	x_4	x_5	
20	x_2	0	1	- 1	2/3	0	**600**
0	x_5	0	0	-1.5	0,5	1	**300**
30	x_1	1	0	1	- 1/3	0	**200**
	z_j	30	20	10	10/3	0	18 000
	$c_j - z_j$	0	0	- 10	- 10/3	0	

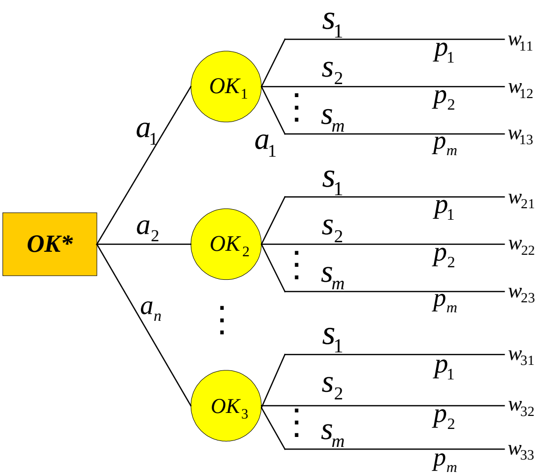
In the analysed example, value of the target function increases subsequently from 0 in the first iteration to 18 000 in the last iteration. So, summarising, the optimum solution of the task is:

$$x_b = \begin{bmatrix} x_2 \\ x_5 \\ x_1 \end{bmatrix} = \begin{bmatrix} 600 \\ 300 \\ 2000 \end{bmatrix}$$

At this solution, S_2 = 300 kg of raw material in stock remains idle.

Decision tree method

The decision tree method is a technique of presenting a decision problem that at the same time permits following the subsequent steps of solving a decision problem. A decision tree is a graphic presentation of all the elements of a decision problem: permissible decisions, states of nature and their probabilities, as well as possible benefits or lost possibilities [24, 22]. The figure below shows general form of a single-stage decision tree.



where: a = actions
s = states of nature
p = probability
w = possible results

Fig. 10. Schematic presentation of decision tree

The decision tree is composed of two kinds of nodes (decision and random ones) and branches (paths). Decision nodes are marked with squares and indicate that a decision must be taken at a given moment of the decision-making process. Paths emerging from a decision node represent various possible decisive variants. Random nodes are marked with circles from which emerge branches representing states of nature. They inform that, at the given moment of the decision process, further course of events is determined with fixed probability by external factors, but not by the decision-maker.

Having at the disposal information about benefits connected with each state of nature and each decision, by moving from right to left side of the decision tree one can determine expected benefits for each variant and write them above random nodes corresponding to individual variants. The decision corresponding with the largest expected benefit is considered the optimum one. Magnitude of that benefit is usually written above the decision node.

This method is counted to the risk analysis and assessment methods, because it allows the decision-maker to recognise structure of the decision problem, especially when the problem is more complicated.

EXAMPLE 13. <u>APPLICATIONS OF DECISION TREE METHOD</u>

To present the decision tree method, used are the data of Example 5 shown in Table 5 and probabilities of demand magnitude. The decision tree for this case is shown in Fig. 11.

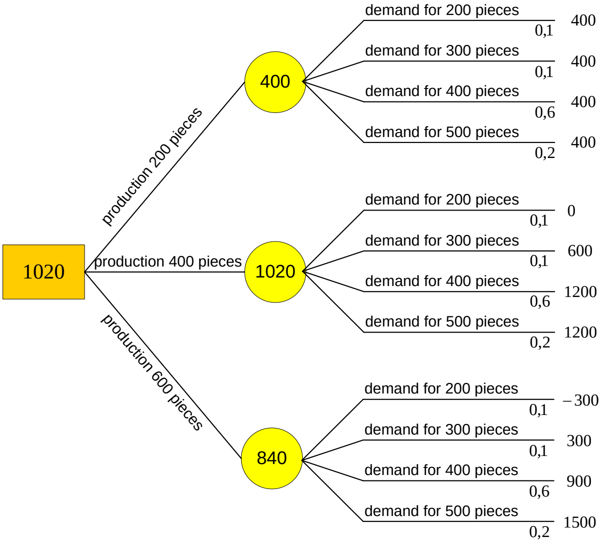


Fig. 11. Decision tree to Example 13

Interpretation of the decision tree indicates that the best decision (minimising the risk of suffering a loss) is to produce 400 pieces of the product. At this decision, the expected profit amounts to 1020.

Monte Carlo simulation method

Simulation using the Monte Carlo method is also called the stochastic simulation. It serves for examining properties of a model or its fragments (parameters, variables, limitations) being sources of uncertainties. To the model inserted are disturbances drawn randomly from a suitable probability distribution, and next the model solution is determined. The operation is repeated *n* times. This method is applied in risk analysis because it permits assessing results of the taken decisions by examining influence of many input variables on magnitude of the estimated parameters. Unlike the sensitivity analysis where variables are considered separately, in the simulation analysis the uncertain variables are examined jointly considering relationships between them [18]. Because of large number of its stages, procedure of the Monte Carlo method is shown in Fig. 12.

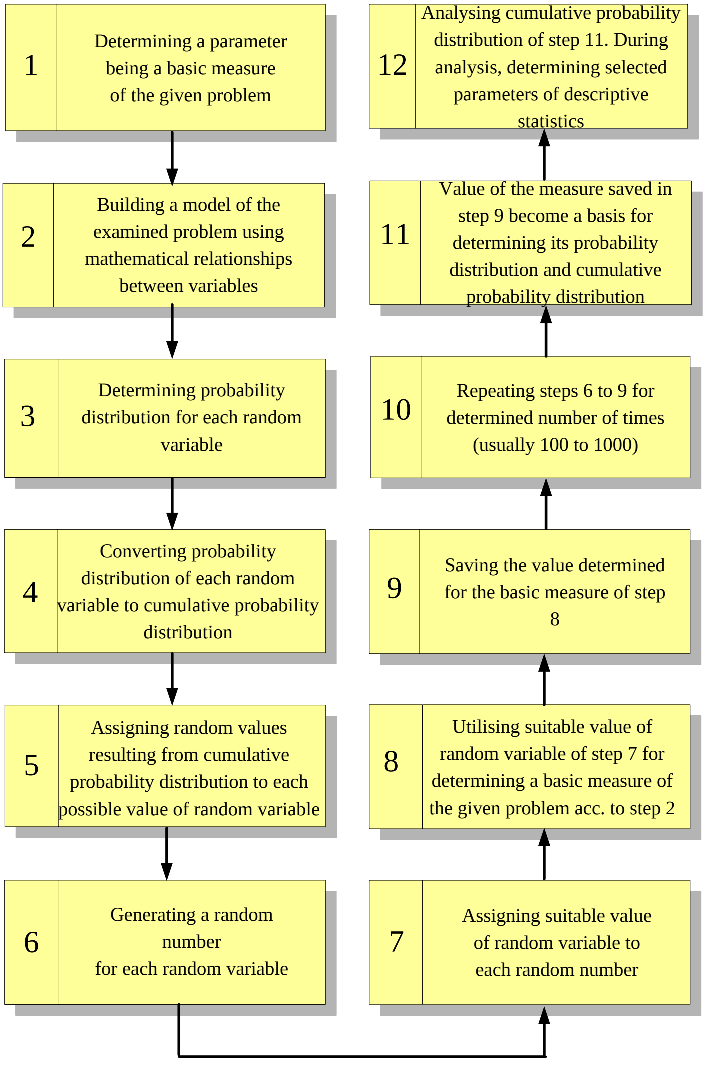


Fig. 12. Stages and procedure of Monte Carlo method [38]

Advantages of this method include [18, 38]:

- possibility of handling large data sets,

- ability of determining logic structure of models and performing mathematical operations on them,
- necessity of adapting the model properly and preparing the simulation course as an integral part of the examination.

EXAMPLE 14. APPLICATIONS OF MONTE CARLO METHOD [39]

In a plant, incomes and expenses related to production of a product were examined. It was noted:

- average incomes = PLN 9000
- average expenses = PLN 8000.

The analysts fixed that the expenses are shaped according to normal distribution with standard deviation of PLN 1000, but expenditures take the form of a normal distribution with standard deviation of PLN 1100. The company management decided that manufacture of the product would be stopped when surplus of incomes over expenses is smaller than 70 % because this would be connected with the risk of suffering a loss.

Table 15. Data to Example 14 – Randomly generated values of incomes and expenses [39]

Random numbers of incomes	Random numbers of expenses
10055	7201
5418	7963
9728	8013
8198	5342
9909	8694
9878	8271
8306	9125
8821	8455
8534	8714
9503	9192

Table 16. Data to Example 14 – Financial results for fixed random numbers [39]

Item	Random numbers of incomes	Random numbers of expenses	Financial result	Item	Random numbers of incomes	Random numbers of expenses	Financial result	Item	Random numbers of incomes	Random numbers of expenses	Financial result
1	10055	7201	2854	34	8198	5342	2856	67	8306	9125	-819
2	10055	7963	2092	35	8198	8694	-496	68	8306	8455	-149
3	10055	8013	2042	36	8198	8271	-73	69	8306	8714	-408
4	10055	5342	4713	37	8198	9125	-927	70	8306	9192	-886

5	10055	8694	1361	38	8198	8455	-257	71	8821	7201	1620
6	10055	8271	1784	39	8198	8714	-516	72	8821	7963	858
7	10055	9125	930	40	8198	9192	-994	73	8821	8013	808
8	10055	8455	1600	41	9909	7201	2708	74	8821	5342	3479
9	10055	8714	1341	42	9909	7963	1946	75	8821	8694	127
10	10055	9192	863	43	9909	8013	1896	76	8821	8271	550
11	5418	7201	-1783	44	9909	5342	4567	77	8821	9125	-304
12	5418	7963	-2545	45	9909	8694	1215	78	8821	8455	366
13	5418	8013	-2595	46	9909	8271	1638	79	8821	8714	107
14	5418	5342	76	47	9909	9125	784	80	8821	9192	-371
15	5418	8694	-3276	48	9909	8455	1454	81	8534	7201	1333
16	5418	8271	-2853	49	9909	8714	1195	82	8534	7963	571
17	5418	9125	-3707	50	9909	9192	717	83	8534	8013	521
18	5418	8455	-3037	51	9878	7201	2677	84	8534	5342	3192
19	5418	8714	-3296	52	9878	7963	1915	85	8534	8694	-160
20	5418	9192	-3774	53	9878	8013	1865	86	8534	8271	263
21	9728	7201	2527	54	9878	5342	4536	87	8534	9125	-591
22	9728	7963	1765	55	9878	8694	1184	88	8534	8455	79
23	9728	8013	1715	56	9878	8271	1607	89	8534	8714	-180
24	9728	5342	4386	57	9878	9125	753	90	8534	9192	-658
25	9728	8694	1034	58	9878	8455	1423	91	9503	7201	2302
26	9728	8271	1457	59	9878	8714	1164	92	9503	7963	1540
27	9728	9125	603	60	9878	9192	686	93	9503	8013	1490
28	9728	8455	1273	61	8306	7201	1105	94	9503	5342	4161
29	9728	8714	1014	62	8306	7963	343	95	9503	8694	809
30	8198	9192	-994	63	8306	8013	293	96	9503	8271	1232
31	8198	7201	997	64	8306	5342	2964	97	9503	9125	378
32	8198	7963	235	65	8306	8694	-388	98	9503	8455	1048
33	8198	8013	185	66	8306	8271	35	99	9503	8714	789
								100	9503	9192	311

Table 17. Data to Example 14 – Probability distribution of financial result [39]

Financial result	Number of events	Probability
>=0	74	0.74
>100	71	0.71
>200	68	0.68
>300	65	0.65
>400	61	0.61
>500	61	0.61
>600	57	0.57
>700	55	0.55
>800	51	0.51
>900	47	0.47
>1000	45	0.45

Probability that the financial result will by positive amounts to 74 %. This is higher than the expected value, and therefore production will not be stopped.

An advantage of this method is its versatility. It permits calculating any values of efficiency indices, obtaining finally the whole range of possible indices together with their occurrence probabilities [14]. In addition, by means of proper computer tools one can determine probability distribution of an index together with its expected value and standard deviation.

Network programming methods

The network programming methods are the techniques of planning enterprises which guarantee their efficient execution course [33]. Such a method is composed of a series of actions connected with each other. The actions must be executed in a determined sequence. Execution of some of them must be preceded by execution of some others, however there are also such actions which can be performed simultaneously [33]. Relationships between events and actions define a logic structure of the network model that can be deterministic if during the enterprise realisation performed are all the actions presented in the network, or stochastic if during the enterprise realisation engaged is only a part of the actions presented in the network with a determined positive probability.

PERT method

Although the PERT method (*Program Evaluation and Review Technique*) belongs to the network methods with determined logic structures, the parameters describing individual parts of a project (or technological process) can be of probabilistic nature. Duration times of individual actions are random variables and their probability distribution corresponds to the beta distribution whose particular case is the normal distribution.

For each action, given are three assessment parameters of duration time of individual actions composing the network:

***a* =** optimistic time, i.e. duration time of an action in the most favourable conditions,

***b* =** pessimistic time, i.e. duration time of an action in the least favourable conditions,

***m* =** modal time, i.e. the most probable duration time that occurs most frequently at multiple repeating an action; the time can be determined as median value of the range between the optimistic and the pessimistic times. With this, fulfilled is the relation $a \le m \le b$.

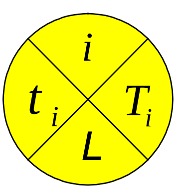
On the ground of these three time estimates, the expected duration time of the activity (t_e) is calculated from the formula [33]:

$$t_e = \frac{a + 4m + b}{6} \tag{1.23}$$

and variance of the expected time, determining real deviation of the activity duration time from the determined expected time, is calculated from the formula [33]:

$$\sigma^2_{i-j} = \left(\frac{b - a}{6}\right)^2 . \tag{1.24}$$

In the next step of the method, one should draw the so-called activity network as a graph. Elements of the graph are circles describing events and including determined elements, as well as arrows determining transitions to subsequent states. Elements of an activity network are shown in Fig. 13.



where:

i = event number,

t_i = the earliest moment of the event occurrence,

T_i = the latest moment of the event occurrence,

L = spare time,

→ = actions

Fig. 13. Basic elements of action network in PERT method

When designing a graph, one should remember that a very important role plays determining relations between all the activities. The activities must be executed in a strictly determined sequence; execution of one of them must be preceded by execution of the others, and some of then can be executed in parallel [33]. When building an activity network, obligatory are some principles (axioms) which should be absolutely followed to avoid errors. They prescribe the following [33]:

- precisely one initial peak and one final peak exist,
- peaks and arcs should be ordered (preceding number is lower than following number) i < j,
- two events are connected with only one arrow, but if the process organisation forces the situation that start and end of two or more activities concern the same peaks (activities performed in parallel), some apparent activities should be introduced, i.e. ones with duration time equal to zero.

Above the arrows, estimates of activity duration times *a*, *m* and *b* are written down, and below them – expected times t_e.

An example of the activity network is shown in Fig. 14.

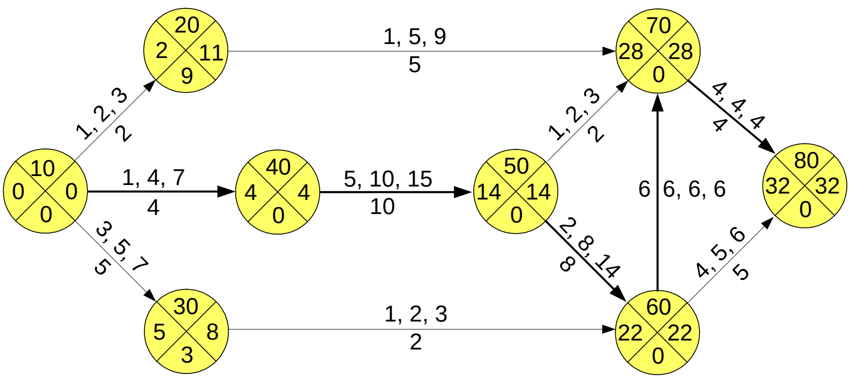


Fig. 14. Exemplary activity network

Both the expected activity duration time and the variance should be calculated for all the activities creating the network or, depending on the analysed problem, for critical activities only (whose spare times are equal to zero). It is assumed that variance of performance time (σ^2_{TW}) is total of variance of activities creating the network or of critical activities. Knowing the expected performance time and its variance, one can also calculate the probability that the enterprise will be completed in a certain, imposed (directive) time limit (t_d). To determine this probability, a statistics is calculated from the following formula [33]:

$$x = \frac{t_d - t_W}{\sqrt{\sigma_{TW}^2}}, \tag{1.25}$$

where:

t_d = imposed time limit,

t_W = expected time of completing the enterprise (the earliest possible time for the final event), and

σ_{TW}^2 = variance of the performance time limit.

For the so calculated coefficient x from the normal distribution function table, probability of maintaining the imposed time limit is read-out:

$$P\{t_d \leq t_W\} = F(x) \tag{1.26}$$

EXAMPLE 15. APPLICATIONS OF PERT METHOD [33]

On the ground of the given activities composing a manufacturing process with low automation degree shown in Fig. 14, as well as their sequence and duration times (Table 18), the following should be determined:

a) the shortest time of production process duration time,
b) risk of failure to produce the product during the assumed time of 30 minutes.

Table 18. Data to Example 15 – Time characteristics of process operations for PERT method

Numbers of process operations ***i-j***	**Duration times of process operations [min]**			**Expected times** t_e
	Optimistic time ***a***	Modal time ***m***	Pessimistic time ***b***	
10 – 20	1	2	3	2
10 – 30	3	5	7	5
10 – 40	1	4	7	4
20 – 50	2	3	4	3
20 – 70	1	5	9	5
30 – 50	3	6	9	6
30 – 60	1	2	3	2
40 – 50	5	10	15	10
50 – 60	2	8	14	8
50 – 70	1	2	3	2
60 – 70	6	6	6	6
60 – 80	4	5	6	5
70 - 80	4	4	4	4

The critical path marked in Fig. 14 with bold arrows runs through the following process operations:

10-40-50-60-70-80

and the shortest time of completing the production process is 32 minutes.

However, in the case of activity duration times of a low-automated production process, its completion time is a random variable and the real time can be more or less different. Thus, necessary is knowing this deflection, i.e. variance of the expected time acc. to the formula (1.24). Variances of the expected time for critical activities are as follows:

$$\sigma^2_{10-40} = \left(\frac{7-1}{6}\right)^2 = 1 \qquad \sigma^2_{40-50} = \left(\frac{15-5}{6}\right)^2 = \frac{25}{9}$$

$$\sigma^2_{60-70} = \left(\frac{6-6}{6}\right)^2 = 0 \qquad \sigma^2_{70-80} = \left(\frac{4-4}{6}\right)^2 = 0$$

$$\sigma^2_{50-60} = \left(\frac{14-2}{6}\right)^2 = 4$$

And so, $\sigma^2_{TW} = 1 + \frac{25}{9} + 4 + 0 + 0 = \frac{70}{9}$, from which $\sigma_{TW} = \sqrt{\frac{70}{9}} = 2{,}78$.

So, the expected deflection value of the real process duration time from the expected time (32 min) determined from the network is ± 2.78 min.

Knowing the expected process duration time and its variance, one can calculate the risk (probability) of completing the process within the pre-assumed time limit. To determine this risk, the statistics is calculated from the formula (1.25):

$x = \frac{t_d - t_W}{\sqrt{\sigma^2_{TW}}} = \frac{30-32}{\sqrt{\frac{70}{9}}} = \frac{-2}{2{,}78} = -0{,}71$, and $F(-0{,}71) = 0{,}236651$.

If the probability value of maintaining the planned time limit is within [33]:

- 0 to 0.25 - there is a slight chance of producing the product within the assumed time,
- 0.25 to 0.6 - meeting the time limit is real,
- 0.6 to 1 - there exist unemployed production capacities (excess of resources).

- Since in the analysed example $F(-0{,}71) = 0{,}236651 \leq 0{,}25$, it can be said with probability higher than 0.5 that impossible is manufacturing the product within the assumed time, i.e. the risk is $R \geq 0{,}5$.

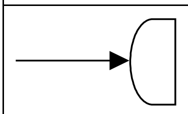
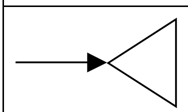
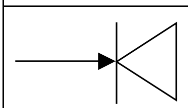
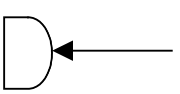
GERT method

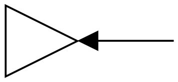
The GERT method (*Graphical Evaluation and Review Technique*), like the PERT method, belongs to the network programming methods. GERT is an analysis procedure of stochastic networks, created as a combination of the following concepts [38]:

- building a PERT-type network,
- using graphs of signal flow,
- using Elmaghraby's graph algebra,
- using logic elements in the networks.

Unlike the PERT method, the network includes the node types "and", "or", "xor", "may follow" and "must follow". Symbols of nodes of stochastic networks are given in Table 19 [38].

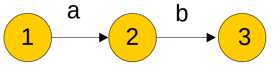
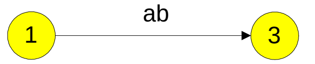
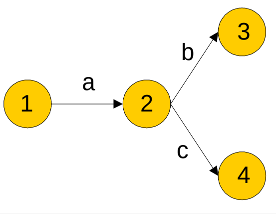
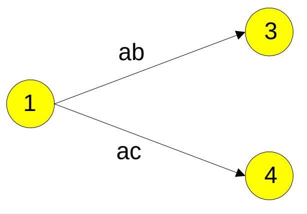
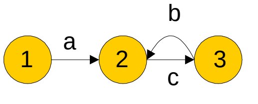
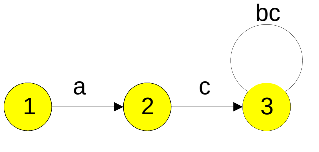
Table 19. Nodes of activity network [38]

Node type	**Node name**	**Node description**
Receiver-type nodes		
	**„and”**	Will be executed if all the arcs leading to it are executed.
	**„or”** (inclusive)	Will be executed if one or several arcs leading to it are executed.
	**„xor”** (exclusive)	Will be executed if one and only one of the arcs leading to it is executed.
Source-type nodes		
	**„must follow”**	Will be executed if executed are all the arcs emerging from it.

	**„may follow”**	Will be executed if executed is one and only one of the arcs emerging from it with known probability less than one.

The procedure of building graphs does not differ from the procedures concerning traditional network plans. An original element of this method is reduction of stochastic networks. A network can be reduced with various methods. One of them is using substitute graphs with the algebra. Table 20 presents basic principles of reducing graphs by drawing substitute graphs.

Table 20. Reduction of graphs using substitute graphs [38]

Original graph	**Substitute graph**
	
	
	

However, this method is rarely used, because its use makes the network diagram loose its original understanding. A more often reduction method is solving a system of linear equations [38]. Peaks of a graph can be determined with the formulae [33]:

$$w_i = f_i(w_1, w_2 ....w_n) . \qquad (1.27)$$

The signal flow graph is illustrated by the following system of equations:

$$[I - T]\, M = N, \qquad (1.28)$$

where:

I = identity (unit) matrix,
T = transmittance (flow) matrix,

M = columnar matrix of dependent variables,
N = columnar matrix of independent variables.

EXAMPLE 16. <u>APPLICATIONS OF GERT METHOD [39]</u>

Given is a production process realised on four workstations marked successively 1, 2, 3 and 4, where the fourth station is the finished product store. On individual workstations performed are the process operations which can create a salvageable reject resulting in withdrawing the product to repeat the previous operations, or an unsalvageable reject that is transferred to the unsalvageable rejects store 5. Determined flows between the stations in the example stand for probabilities of an event occurrence. For example, flow from the workstation 1 to the workstation 2 amounts to 0.7 and probability of making a reject is 0.3, while the chance of repairing the reject is 0.1 and the chance of creating scrap is 0.2, see Fig. 15.

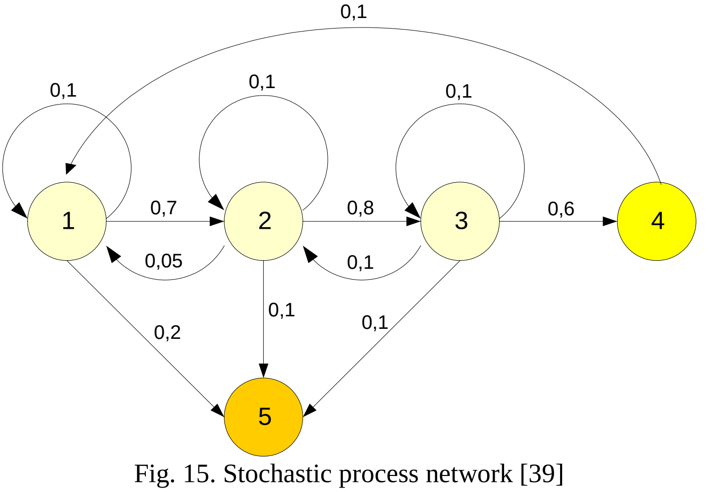


Fig. 15. Stochastic process network [39]

The task consists in determining probability (risk) of obtaining a good product and determining the number of products which should be delivered to the workstation 1 in order to obtain 100 good products after the further process operations.

Probability of obtaining good products is:

$$p_{14} = p_{12}\, p_{23}\, p_{34} = 0{,}7 * 0{,}8 * 0{,}6 = 0{,}336 = 33{,}6\%$$

With the accepted assumptions, the transmittance matrix is:

$$T = \begin{bmatrix} 0{,}10 & 0{,}70 & 0{,}0 & 0{,}00 \\ 0{,}05 & 0{,}05 & 0{,}80 & 0{,}00 \\ 0{,}10 & 0{,}01 & 0{,}10 & 0{,}60 \\ 0{,}00 & 0{,}00 & 0{,}00 & 0{,}00 \end{bmatrix}$$

$$[I - T]M = \left\{ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0{,}10 & 0{,}70 & 0{,}0 & 0{,}00 \\ 0{,}05 & 0{,}05 & 0{,}80 & 0{,}00 \\ 0{,}10 & 0{,}01 & 0{,}10 & 0{,}60 \\ 0{,}00 & 0{,}00 & 0{,}00 & 0{,}00 \end{bmatrix} \right\} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ w_4 \end{bmatrix}$$

After solving the system of equations one obtain:

0.55 $w_1 = w_4$ and

p_{14}=0.55.

Number of the details which should be delivered to the workstation 1 to obtain 100 good products is [33]:

$$w = \frac{100}{p_{14}} = 182 .$$

Method of queuing theory

The queuing theory is applied in solving problems of mass service [5]. The mass service models are useful at obtaining optimum solutions concerning selection: number of service appliances depending on the stream of entries and number of entries from the operators.

Algorithm with single service channel

Analysis of mass-service systems requires acquiring information about the moments of the customers' arrivals, time of the service and possible behaviours of the customers. To this end, the following parameters are accepted [33]:

λ = arrival rate; determines average number of the customers arriving in time unit;

μ = service rate; determines average number of the customers serviced in time unit;

ρ = traffic intensity; determines ratio of the number of the customers arriving to the number of those serviced in time unit, where [33]:

$$\rho = \frac{\lambda}{\mu}. \tag{1.29}$$

It is assumed that all the customers are treated equally. Usually, two cases are considered in this method [33]:

a) when $\lambda < \mu$ and

b) when $\lambda \geq \mu$.

In the case a), with the additional assumption that both rates are equal, it can be considered that the system is aiming at the equilibrium condition [33]. This means that probability of a determined length of the queue is constant in each time unit. In the case b), the system is unstable and probability of a long queue increases. This concerns also the situation when both rates are equal. Then the service channel can not make-up for the time when it was temporarily inoperative (unused).

After fixing values of the basic parameters (λ, μ, ρ), the queue problem can be solved. The solution consists in indicating, in the given conditions, the best (optimum) system of factors controlled by the management of the examined unit.

Algorithm with multiple service channels

The algorithm becomes much more complicated, when many service channels exist. The individual formulae depend on the assumed probability distribution of the customers' arrivals. In addition, the following constants and variables are introduced [5, 33]:

r = number of service channels,

k = length of the queue,

a = maximum efficiency of the service point.

The most important formulae for the systems in which the queue length ranges between one and the number of service channels are given below [5, 33]:

Average time of queuing is (T) [5]:

$$T = \frac{P_m}{r(a-\lambda)}. \qquad (1.30)$$

Probability that all the service channels are occupied (P_m) [5]:

$$P_m = \frac{\Omega^r}{r!(1-\mu)} P_0. \qquad (1.31)$$

Probability that n = 0 units is queuing in the system (no queue) [33]:

$$p(n=0) = \frac{1}{\sum_{i=0}^{r-1} \frac{\rho^i}{i!} + \frac{\rho^r}{(r-\rho)(r-1)!}}. \qquad (1.32)$$

Probability of the system stoppage (P_0) [5]:

$$P_0 = \frac{1}{\sum_{k=0}^{m-1} \frac{\Omega^k}{k!} + \frac{\Omega^m}{m!(1-\mu)}}. \qquad (1.33)$$

Probability that the queuing time is longer than t_0 [5]:

$$p(t > t_0) = p(n > r-1) e^{-\mu t_0 (r-\rho)} \qquad (1.34)$$

Service intensity coefficient (Ω) [5]:

$$\Omega = \frac{\lambda}{a}. \qquad (1.35)$$

In the case of numerous service channels, the service rate is [5]:

$$\mu = \frac{\lambda}{r \times a}. \qquad (1.36)$$

EXAMPLE 17. <u>APPLICATIONS OF QUEUING THEORY [33]</u>

In a production plant, elements for manufacture are transported to the production line by means of carriages. The carriages are loaded in the raw material store by one of the two storekeepers. By estimating average arrival rates and service rates, the following results were obtained: $\lambda = 3{,}8$ carriages per hour and μ = 2 carriages per hour. The company is going to analyse the queue and to obtain answers to the following questions:

a) Is the logistic system heading to equilibrium condition?

b) What is the risk that a carriage will have to wait for loading?

c) What is the risk that a carriage will have to wait for loading longer than 0.5 hour?

Ad a) It should be checked whether valid is the inequality $\lambda < r\mu$, where: $\lambda = 3{,}8$; $\mu = 2{,}0$; $r = 2$;

after substituting 3.8 < 4.0 which proves that the equilibrium condition can be reached.

Ad b) On the ground of the formula (1.32) it can be determined:

$$p(n=0) = \frac{1}{1 + 0{,}95 + \frac{(0{,}95)^2}{1{,}05 \cdot 1}} = 0{,}355932 \approx 0{,}36,$$

so the risk that a carriage will wait for loading is *R* = 1 - *p*(*n*=0) = 1-0.36 = 0.64.

Ad c) On the ground of the formula (1.34) it can be determined:

$$p(t > 0{,}5) = p(n > 1)e^{-2 \cdot 0{,}5(2-0{,}95)} = 0{,}305932 \cdot 0{,}349938 = 0{,}107057 \approx 0{,}11,$$

which means that the risk that a carriage will wait for loading longer that 0.5 hour is low, amounting to 0.11.

1.10 Comparison of quantitative methods of risk analysis and assessment

Each economic activity is accompanied by risk. However, it is rarely planned and assessed at taking production decisions [5, 22, 37]. This situation can be caused by the fact that the quantitative methods proposed by literature are not very useful in manufacturing reality.

The above-presented quantitative methods of risk analysis and assessment relate to single questions, assuming occurrence of suitable factors and conditions, as well as imposing limitations. Analysis of these factors indicates that, to assess risk of a manufacturing process, one should simplify the problem in a way making application of the method possible. With regard to complexity of today's production systems and the number of affecting them external random factors, this approach seems improper, because results obtained from such analysis must be burdened with too large error.

Difficulties in wider and more common use of quantitative methods for planning and assessing risk of manufacturing processes are also caused by the way of their description. Literature items usually present the methods on selected examples with numerous limitations or by verbal description, omitting mathematical theorems and definitions.

Table 21 presents comparison of the described in this Chapter methods of risk analysis and assessment. The comparison criteria were selected to demonstrate their usability at solving practical problems.

Table 21. Comparison of quantitative methods of risk analysis and assessment

METHOD	KNOWN DISTRIBUTION	NUMBER OF INPUT DATA	SEQUENTIA-LITY	GRAPHIC METHOD?	COMPLEXI-CITY
STATISTICAL METHODS					
Analytic	○	○	○	◉	○
Probability analysis	○	◉	○	◉	○
Normal distribution function	●	○	○	◉	○
Standardised variable	●	○	○	◉	○
Bayes'	○	◉	○	○	◉
Standard deviation analysis (FI)	○	◉	○	○	○
Increment analysis	○	◉	○	◉	◉
Frequency histogram	○	○	○	●	○
Correlation analysis	○	◉	○	◉	◉
Sensitivity analysis	○	○	○	○	○
Test for population average value	◉	○	○	◉	◉
OPERATIONAL RESEARCH METHODS					
Simplex algorithm	○	●	●	◉	●
Decision tree	○	◉	◉	●	○
Monte Carlo	●	●	●	○	●

GERT	●	●	●	●	●
PERT	●	●	●	●	●
Queuing theory	○	○	○	○	●
LEGEND					
● - *much / yes*		◉ - *medium / a bit*		○ - *a little / no*	

When analysing the comparison shown in Table 21, it seems necessary to adapt the methods to the current state of technique and manufacturing technology development, as well as to possibilities of data acquiring, collecting and processing.

Chapter 2. Characteristics and modelling of production processes

This Chapter includes characteristics of systems and description of elements composing a production process, possibilities of its decomposition and levels of analysis. Characterised are basic indices of a production process and their role in the management system. Presented are possibilities of modelling and simulating production processes, kinds of models and tasks of modelling, course of a simulation experiment and kinds of simulations, with special consideration of continuous and discrete simulation of production processes. Discussed are advantages and disadvantages of simulation.

The most important task of production management is forming the phenomena occurring in the production process according to the assumed goals of the company and considering all the conditions and circumstances of real manufacturing processes [7]. Any changes within the production processes are preceded by proper production decisions. With respect to the production systems, decisions are always taken in conditions of risk or uncertainty. Considering, in addition, complexity of the production processes and their connections with other company areas and with the environment, taking production decisions should be preceded by analysis of the manufacturing process and assessment of risk related to its taking. In order that a system analysis and risk assessment can be performed quickly and with no intervention in execution and course of the manufacturing process, seems necessary to build simulation models.

One of basic problems in the production system analysis is their proper decomposition into components [7]. This decomposition should be dependent on basic objectives posed to the decision-maker in the management process [25]. High complexity and hierarchical structure of manufacturing processes and systems results in problems with determining proper levels and numbers of levels for which the analysis will be performed. To perform the analysis, it is also necessary to acquire data coming often from numerous company areas. To this end, acquired and analysed are many indices describing the production system. With this respect, more and more frequently created are simulation models of production systems and at selecting a version of a decision employed are such IT tools like integrated computer systems and simulation programs [25].

2.1 *Definition of types and ways of production process decomposition*

As a production (manufacturing) process, determined is "*an ordered set of actions (operations, activities) aimed at making a commodity (product or service)*" [35, 34]. A manufacturing process runs in the environment of a production system, i.e. of a production plant [7]. A production system is "*a system of interconnected material, energy, personnel, capital and information resources. It is intentionally designed and organised in a way to satisfy the customers' needs*" [7, 34]. A production system can not exist without the environment from that the resources originate and generated are profits necessary for further operation [15]. This environment changes and influences the production system, and vice versa.

Depending on the nature of the material stream dominating in a given production, distinguished are:

- **continuous production systems** based on continuous manufacturing processes like those existing in power or chemical industries,
- **discrete production systems** based on discrete manufacturing processes, typical for electromechanical industry including automotive, machine-building or household goods industries.

In the present-day industrial practice, continuous processes related to processing homogeneous or nearly homogeneous streams of solids, liquids, gases, loose materials or their mixtures are automated to a high degree. The systems with discontinuous production, processing streams of heterogeneous materials, connected with machining, plastic working, welding and assembly, are low-automated and just in them observed are changes happening for a few years, strongly affecting present and future industry face [13]. These changes are supported by coexisting development of new manufacturing methods and means, in that employed are possibilities of today's computer technique. In turn, development of computer-aided production systems is also supported, besides current state of computer technique, by development of mathematical programming, modelling and computer simulation, as well as by increasing development of technology, management methods and production organisation.

The selected and above-mentioned definitions and types of production systems do not reflect their specificity completely. With respect to their complexity, it is physically impossible to present their specificity and all the components in individual definitions or schemes and their decomposition should depend on range and field of the performed researches, as well as on the accepted division classifiers [39]. Selected concepts of decomposition of a production system are shown in Fig. 16.

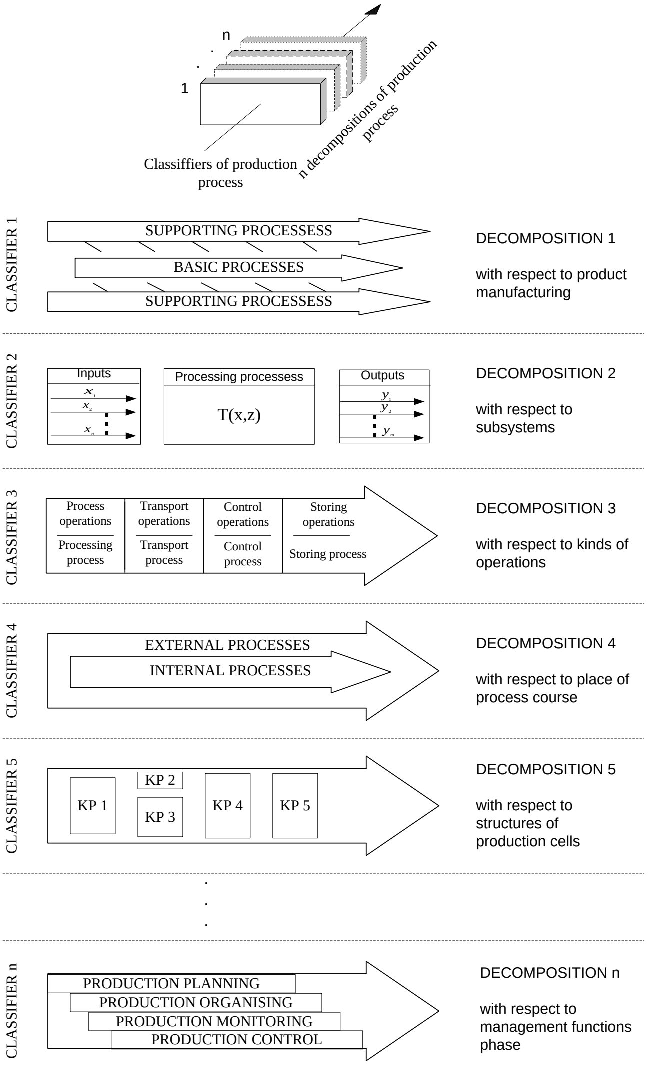


Fig. 16. Multisectional decomposition of production system with respect to selected classifiers

Decomposition 1 divides the processes with respect to their connection with the manufactured product. Basic manufacturing processes achieve directly the planned goals or tasks of a production system in form of products transferred to the system environment. Supporting processes attend all the processes in the production system, supplying them with its products, services or information. A result of their execution is existence of a coupling network between the systems and the environment [7]. Decomposition 2 assumes subdividing the production system into a subsystem of inputs to the system, a processing subsystem and a subsystem of outputs from the system. This is a cybernetic expression of a production system that will be in more details presented in Chapter 3. The next decomposition 3 assumes dividing the production system to processes according to individual kinds of manufacturing operations, i.e. sets of activities heading for transforming the input materials to products [35]. Adequately to the phenomena present in a manufacturing process, manufacturing operations can exist as process, transport, control, storing or maintenance operations. A set of process operations creates a manufacturing process, a set of transport operations creates a transport process, and sets of control, storing and maintenance operations create control, storing and maintenance processes, respectively.

A production system can be subdivided, depending on the places where the processes run, into external processes taking place outside the company and internal processes taking place inside the company. In order to analyse the structure of a production system, one can decompose it to individual work cells but also, although this is not shown in Fig. 16, to work centres, organisational cells, departments etc. A production system can be also analysed with respect to individual management functions and manufacturing stages.

The classifications shown in Fig. 16 and discussed above are not all the possibilities of classification and division of manufacturing processes, but are only the classifications most often met in literature [7, 16, 34, 35]. A way of decomposition should depend on the objective posed to analysis of a production system, so that the decision taken on its ground is optimum and burdened with the lowest risk.

2.2 Elements of production system and process

Besides proper decomposition, equally important in analysis of a production system is selection of the elements to be analysed. At least six significant elements determining structure and features of the realised production processes can be mentioned. They include [11]:

- organisational structure of the company,
- principles of production control and logistics,
- manufacturing processes,
- techniques and methods of product development and manufacturing process development,
- principles of planning and selection of manufacturing systems,
- machines and manufacturing appliances.

These elements can have a form of variable technical and organisational means, techniques, methods and even phenomena [11]. The above-mentioned elements and their most important components are shown in Fig. 17.

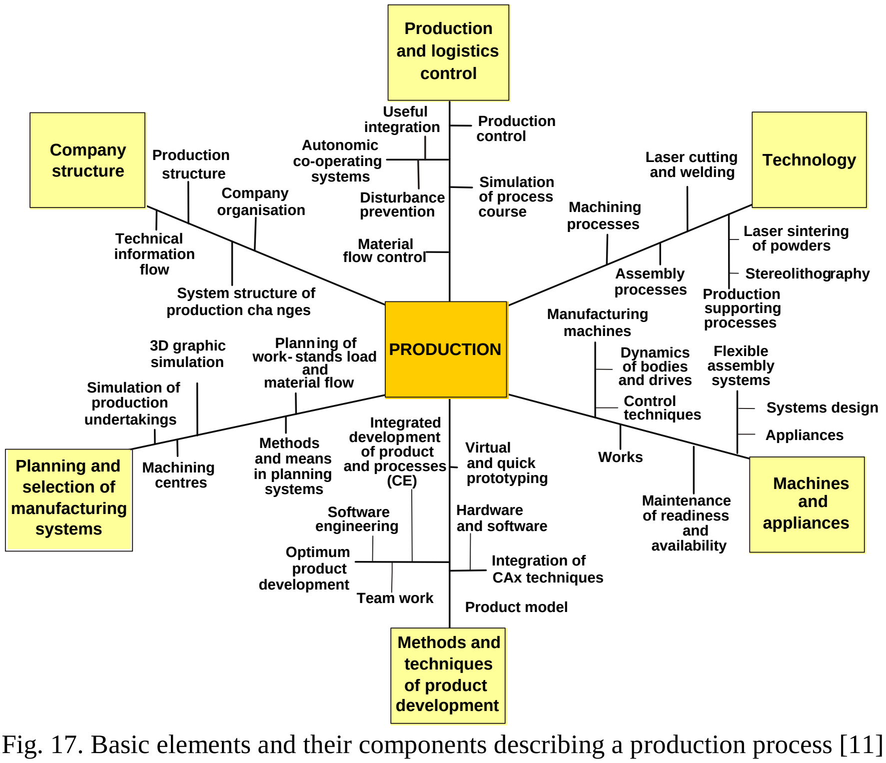


Fig. 17. Basic elements and their components describing a production process [11]

Besides the above-mentioned elements, specificity of a manufacturing process is influenced by individual parameters. Since a manufacturing process runs through various company areas, the parameters describing it come also from many areas. There are extremely many parameters describing a manufacturing process, as well as extremely many ways of its decomposition. Choice of suitable parameters in a process analysis should be dependent on the decision problem. A very simplified structure of parameters describing a manufacturing process is shown in Fig. 18.

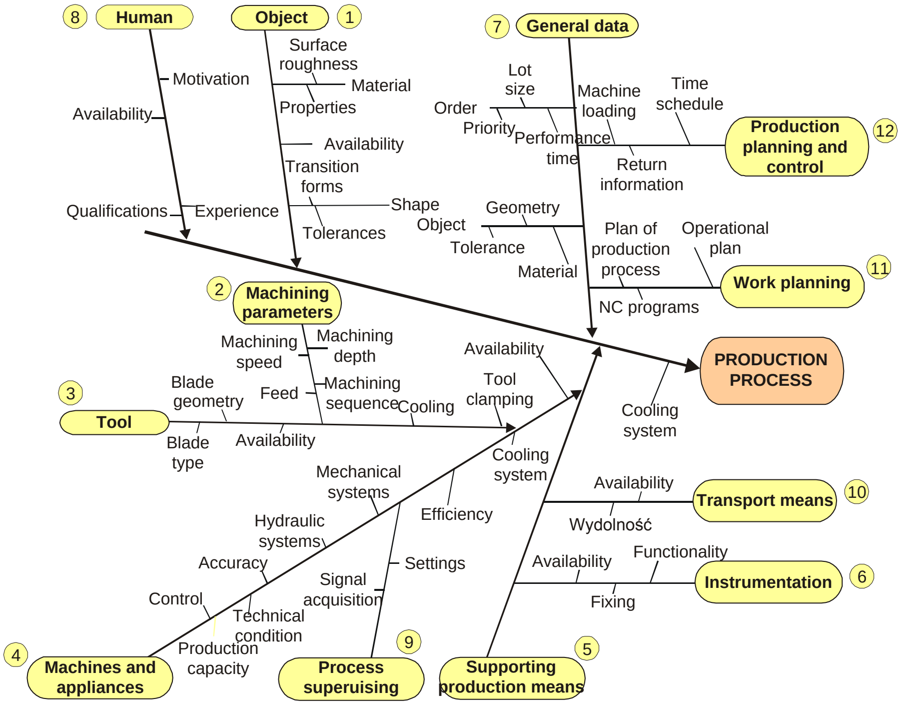


Fig. 18. Structure of basic parameters of a manufacturing process [11]

Hierarchic presentation of a manufacturing process demonstrates also interconnections of individual company areas, as well as functions and parameters of the process.

2.3 Levels of analysis and structure of a production system

When analysing a production system, besides proper decomposition, of a great importance is determining the level or the number of levels on which the system will be analysed. To this end, production structure can be used because of its universal nature, possibility of adapting it to diverse decomposition ways and comprehensive expression of the manufacturing process.

Production structure is an arrangement of work cells and an assembly of cooperative relations between them, specific for a given production system as a whole [7]. The work cells can be of various degree. The smallest (indivisible) cell is a workstation. A workstation is determined as a zero-degree work cell (KP^0) [35]. In real manufacturing processes, workstations as a rule do not exist independently, but are intentionally joined into groups according to determined criteria, which is the merit of designing a production structure.

Production structure is created by workstations (KP^0) properly grouped into first-degree work cells (KP^1), which in turn are joined according to the accepted criteria to second-degree work cells (KP^2) etc., till n^{th}-degree work cells (KP^n) adequately to the given manufacturing unit. If the management process is considered in this structure, arrangement of such cells and their interconnections determine the production-administrative structure and the composing it cells are the production-administrative cells (*KPA*) [7]. Figure 19 shows a schematic presentation of the production-administrative structure accepted in this work.

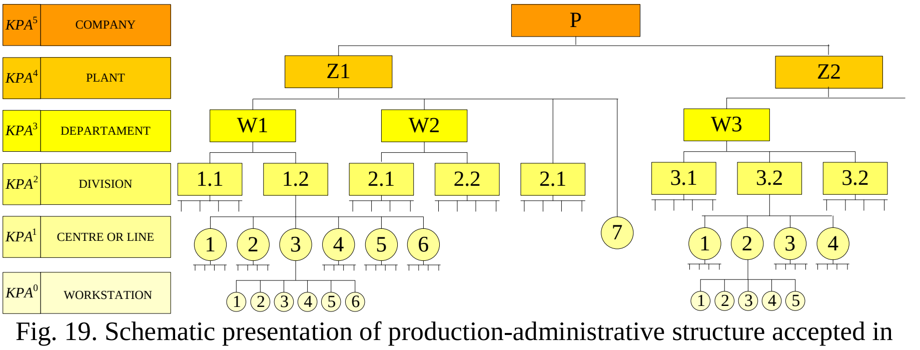


Fig. 19. Schematic presentation of production-administrative structure accepted in this work [7, 35]

With the level on that the production system is considered, strictly connected is the management level i.e. the decision-making level [25]. Decisions corresponding to the highest production structure levels are usually named strategic decisions. They concern achieving the company goals, resources necessary to achieve these goals and ways of achieving them [31]. Decisions taken on middle levels of the structure (divisions or departments) are named tactical decisions. They make a set of the decisions concerning achieving goals and intermediate tasks with respect to strategic goals. The operative decisions taken on the levels of work centres and workstations usually concern executing specific tasks and actions necessary for their performing, to realise the strategy punctually and effectively.

2.4 Production system indices

When performing analysis of a production system, besides its proper classification and establishing proper structure and level of analysis, of a great importance is selection of indices which will describe the system state [46]. Analysis of their values provides information and allows taking proper decisions. Like previously, selection of indices describing state of the production system should depend on the goal posed to the analysis [25].

All the company data can be indices. There are the following kinds of indices [28]:

- **simple** – indices recorded directly (e.g. data on orders, data from accountancy) or as an aggregation of numbers recorded directly by adding, subtracting and multiplying (e.g. turnover in a period, costs in a period);
- **complex** – indices obtained by dividing of simple indices; they include:
  - participation indices – numerator and denominator have the same measure; sum of shares in the entity can not exceed 1 (or 100 %), e.g. share of direct material costs of one of the products in total direct material costs in the period;
  - relation indices – numerator and denominator can have different measures (e.g. costs of rooms in PLN/m^2, surcharge rate in calculations); in the case of the same measure the index value can exceed 100 %;

- indices – numerator and denominator have the same measure, but they refer (as a rule) to different periods (e.g. price increase index for planning the sale price).

The above-mentioned groups of indices can create so-called systems of indices being combinations of two or more indices by means of basic mathematical operations [28]. Systems of indices are created by means of one or more basic indices which permit synthetic assessment of the company condition, and are next described by supporting or partial indices.

In common opinion, it is difficult to express complex reality of a production company and manufacturing processes running in it with one synthetic index [10, 30]. According to S. Chajtman [10], to describe complex reality of a production company one should employ relation indices based on relations of 12 parameters affecting results of business activity of the company. In literature exist several various measures used for assessing functionality of production systems. At present however, in high-developed industrial countries, productivity is almost commonly thought to be one of basic criteria of assessing functionality of production systems [16].

2.5 Characteristic of selected indices of production system assessment

Productivity indices [35] of production systems are determined by ratio of input vectors to output vectors, i.e. ratio of results of production activities to expenditures used for achieving them:

$$P = \frac{\sum_{1}^{m} Y}{\sum_{1}^{n} X} \tag{2.1}$$

Total productivity of a production system is reflected by the technological manufacturing level, methods of production organisation and management, skills of employees, as well as changes in capital expenditures and other relations in the design area and in the sphere of the production system operation.

However, it can be impossible to determine the so defined productivity for the entire production system because of diversity and thus incomparability of both input resources and products. A solution of this problem is using weighted sums by means of unit costs of appropriate kinds of resources and prices of individual kinds of products [30] or other conventional units or natural measures [16].

Apart from total productivity of production systems, often applied are partial productivity indices focused usually on a single expenditure. From here originate partial productivity indices of work, capital, energy, machines and appliances, work area, resources and many others. Table 22 shows selected examples of partial productivity indices.

Table 22. Exemplary partial productivity indices [16]

Kind of partial index	**Exemplary expression**
Productivity of work	- number of products, mined tons of coal per man-hour of all the groups employed in the plant (company), - value-added production in PLN per man-hour calculated as above, - production value in PLN per unit labour cost of all the employees, - sale value in PLN per unit labour cost calculated as above
Productivity of machines and appliances	- number of products, tons of steel per machine-hour at the disposal, - number as above per worked machine-hour, - production value in PLN per unit cost of downtime and work of machines
Productivity of capital	- number of product units (tons) per unit expenditure in PLN, - sale value per unit expenditure in PLN, - value of finished products coming to store per unit of current assets frozen in materials and stocks in the given period, - value of finished products coming to store per unit of fixed assets frozen in buildings, machines, appliances and installations

Productivity of energy	- number of product units (tons) per 1 kW (of installed power), - number of product units (tons) per 1 kW (of consumed power), - number of product units per unit energy cost, - production value per unit of consumed heat or solid/liquid fuel

Total productivity indices are generally more reliable than partial productivity indices [16]. Concentration on improving one productivity index can finally result in reducing total productivity by impossibility of using other factors, e.g. "improvement of labour productivity of direct production employees can decrease total productivity at least by unused disposable work hours of very costly production machines and appliances, unused materials, production area or power infrastructure of the department, industrial plant etc." [16].

A very important advantage of the productivity index is its universality, i.e. possibility of referring it to any kind of activity (production, service, administration), to various level systems (national economy, branch, sector, company, department, centre, workstation etc.), as well as to different types of elements of the input and output vectors (energy, labour, area, capital etc.). A disadvantage of the productivity index is its changeability, lack of connection with the set time interval and necessity of using many partial productivity indices for assessment of the entire production system.

An index frequently identified with productivity is **efficiency**. These indices can be identified with each other only with respect to labour resources (e.g. people, machines) and only with respect to the assumed time interval [30]. Efficiency is often named also **production capacity** or **production power** [32, 48]. Capacity is the quantitatively determined maximum production volume that can be manufactured in a certain time by individual production systems (plants, departments, production lines, centres, workstations etc.) in optimum conditions [32].

Two other indices are strictly connected with production capacity: resource utilisation rate and performance. Resource utilisation rate is the value of available production capacity that is really utilised [48]. This index determines percentage of production capacity to that the resource is really utilised in the manufacturing process. Performance is the relation between actual production volume and attainable production volume [48]:

$$performance = \frac{actual\ production}{actual\ capacity} \qquad (2.2)$$

Another index, more and more often analysed by production companies, is **efficiency**. Economic efficiency is quotient of the obtained result and expenditures born to its achieving [30]:

$$efficiency = \frac{result\ value}{expenditure\ value}. \qquad (2.3)$$

From the viewpoint of this very general definition, the above-defined concepts, like productivity and production capacity, are indices of economic capacity.

Another index of basic manufacturing activity is **effectiveness**. This is defined as rate of achieving the assumed results by the system:

$$effectiveness = \frac{results\ obtained}{results\ planned}. \qquad (2.4)$$

2.6 Modelling production systems

With respect to the system complexity, taking a decision related to a production system requires analysing large number of data. This complexity, integration of the system with its environment and disturbances always present in a production system make it difficult and often even impossible to the decision-maker to identify directly the cause-and-effect relationships in the system [26]. Taking decisions on the ground on intuition only is often deceptive and burdened with very high risk.

In this situation, it is often necessary to build models (modelling) and to simulate production systems. The term "*modelling and simulation*" means a set of activities related to building models of real systems and further simulating them on a computer [52]. A computer is not a necessary tool for performing a simulation, but it speeds it up significantly and permits taking into account a large number of data, so it is used commonly. Relation between modelling and simulation in production systems is schematically shown in Fig. 20.

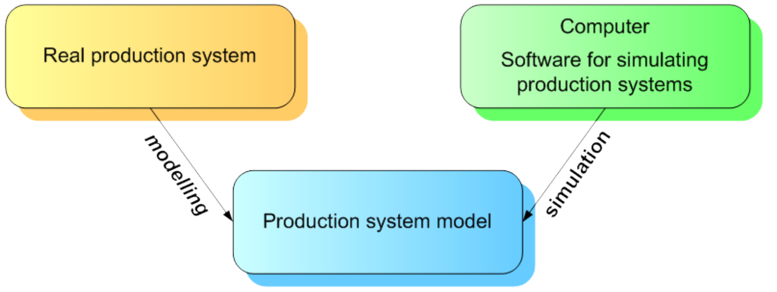


Fig. 20. Basic elements and relations between modelling and simulation of production systems

"*A model is a reflection of the most significant features of an examined or designed object from the viewpoint of the task that it serves in a determined reality or abstraction*" [16]. On the other hand, modelling means "*an activity consisting in matching the original with an acceptable substitute called a model, i.e. this is an approximate reproducing the most important features of the original*" [19].

Diversity of problems which are solved by means of modelling leads to the existence of numerous criteria used for classifying the models. The most general classification made on the ground of literature data [16, 18, 34] is given in Table 23.

Table 23. General classification of models

GROUP	TYPES OF MODELS	DESCRIPTION AND EXAMPLES
I	**PHYSICAL**	Represent, in a proper scale and with a proper accuracy degree, certain features which permit performing analyses and examinations of e.g. a model of an airplane or a building.
	SCHEMATIC	Diagrams, maps, schemes
	SYMBOLIC	Are based on an algorithmic mathematical notation. Mathematical models can be divided to numerical and analytical models.
II	**STATIC**	Ignore part of time or describe temporary state of the system at a certain moment.
	DYNAMIC	Emphasise lapse of time.

III	**DETERMINISTIC**	All the objects have unambiguously determined mathematical or logical interconnections; give completely unequivocal solutions.
	STOCHASTIC	A part of changeability is of random nature; only averaged solutions can be obtained.

A model can be a formal presentation of a theory or a formal description of observations, but most often is a combination of both. In particular, a model [18, 20, 21]:

- permits a researcher checking his/her theoretical opinions about the system, making experiments and observations on it, as well as drawing logical conclusions,
- facilitates understanding the system,
- prompts to carrying-out detailed researches in the future,
- speeds-up making analyses,
- determines methods of testing desired modifications of the system,
- permits easier handling in comparison to the system itself,
- permits controlling much larger number of variability sources than it would be possible at direct examining the system,
- is less costly.

Thanks to getting acquainted with production processes one can solve practical problems [16], which in consequence leads to minimising the risk connected with making a decision. In industrial practice a group of questions exist, which require direct assessment of operation of complex systems working in the conditions of uncertainty or possibility of choosing alternative solutions. In the present conditions, problems are solved by computer simulation of virtually created processes or production systems with use of specialised software. Computer simulation in design of manufacturing processes can relate to the questions formulated in "micro" scale, referring to manufacture and to analysis of machine processes controlling operation of independent, autonomic machines, or in "macro" scale, referring to design and organisational analysis of production centres or lines [43]. Thanks to modelling [16]:

- the object of examination is diminished or magnified to any size,

- analysed are processes difficult to be caught because of too fast or too slow course,
- examined is one selected aspect of the problem, and the others are omitted.

As main goals of production process modelling, the following can be accepted [2]:

- getting acquainted with structure and operation of production objects,
- elaborating algorithms permitting improvement of quality indices of manufacturing processes,
- forecasting kinds of disturbances and their influence on production course.

With respect to the merit of activities directed to taking a decision or to determining unknown features or parameters of a production system, two kinds of modelling tasks can be distinguished [34]:

- analysis-type tasks,
- synthesis-type tasks.

The analysis tasks are such ones in which – knowing characteristics of subsystems and individual elements of a production system – one can determine parameters of the entire production system. They are formulated and solved mainly in order to assess different versions at different stages of designing production systems. In turn, the synthesis tasks are such ones in which – on the ground of set criteria and requirements concerning the entire production system – determined are parameters of its subsystems and elements. They are usually formulated and solved in order to establish a production plan and to find various possible versions of the production system solutions. Diagrams of both types of tasks are shown in Figs. 21 and 22.

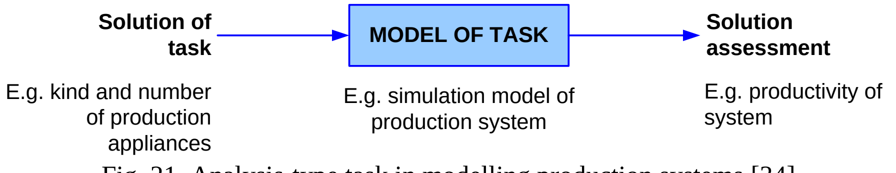


Fig. 21. Analysis-type task in modelling production systems [34]

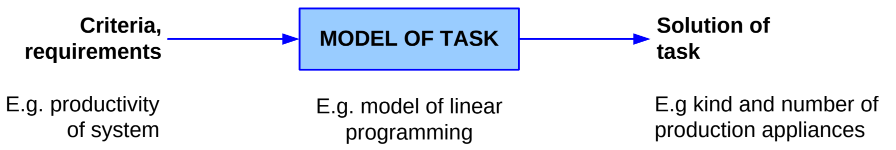


Fig. 22. Synthesis-type task in modelling production systems [34]

At taking decisions or analysing and designing production systems, it is necessary to consider numerous data. In addition, large number of possible solution variants and their complexity often make taking an optimum decision impossible. Simulation analysis of production systems gives such a possibility.

2.7 Simulation analysis of production systems

Most of dynamic phenomena observed in a production system is of synthetic nature, i.e. they are a cumulative result of many partial events happening in various structure areas and at various moments, often significantly distant from each other [45]. If a decision is taken only on the ground of direct observation of such a system, high probability exists that this decision will be burdened with an error and its taking – with high risk.

In this situation helpful can be integrated computer systems (ZSI) and software for computer simulation and optimisation of production systems. The ZSI are mostly used for supplying information concerning state of the production system, but simulation packages permit analysing results of the taken decisions and thus minimise the risk.

„*Simulation is a technique that serves carrying-out experiments on some kinds of models, which describe behaviour of complex systems in certain periods.*" [17]. Another definition presents simulation as „*a form of experimenting on a computer model (simulation model) that gives an answer to the question, how will the analysed system behave in a determined situation*"[29]. Treating a model as a duplicate of a real system permits, among others, transferring conclusioFns of a research performed on a computer model to the system, according to the model construction principles. Therefore, simulation can be also understood as „*examining future effects of the decisions taken today, in the given, specific conditions*" [29]. This means that, on the ground of the developed model, one indirectly obtains an answer to the question, to which degree justified are expectations based on theoretical assumptions concerning determined types of the decisions/activities/actions.

In the simulation process, reproduced are successive fragments (states) of the modelled process course (system behaviour) in the sequence consistent with the time lapse. Diagram of a simulation experiment course is shown in Fig. 23.

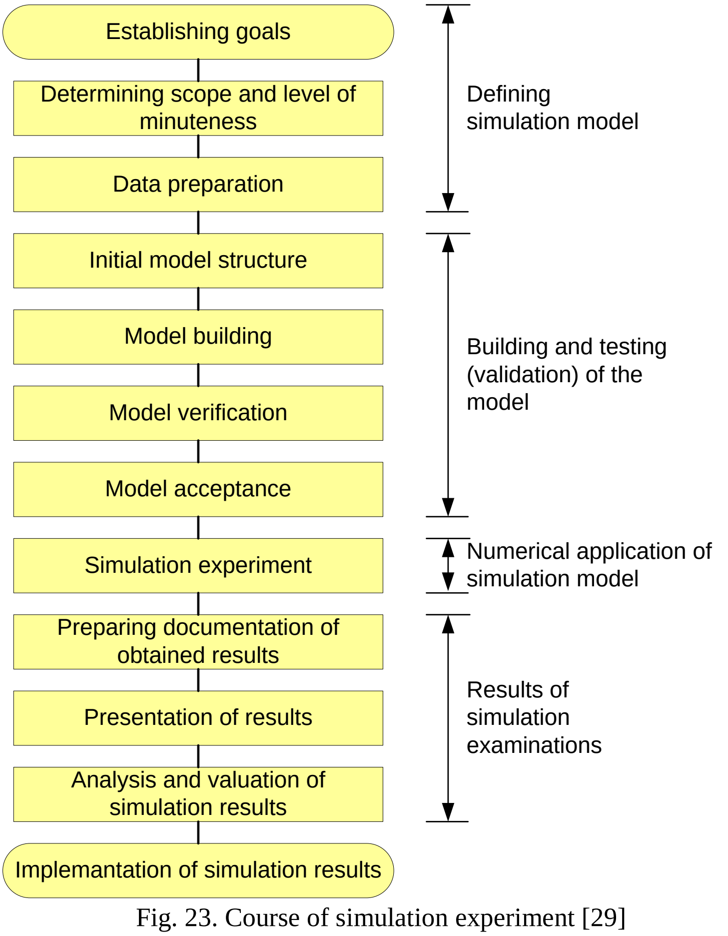


Fig. 23. Course of simulation experiment [29]

A simulation works in the way depending, first of all, on its kind. Three classifications of simulation, most frequently met in literature [1, 2, 8, 21, 46], are:

- static or dynamic simulation,
- stochastic or deterministic simulation,

➢ discrete or continuous simulation.

Static simulation (named also the Monte Carlo method, described in Chapter 1) does not consider course of phenomena in time. Data for a simulation often come from statistic examinations. As opposed to the static one, a **dynamic simulation** considers course of phenomena in time. It describes behaviour of the modelled system, changing in time [1].

Stochastic simulation is based on stochastic processes. i.e. ones built of a random sequence of generated values [21]. Stochastic simulation refers to a simulation in that one or more input variables are random. A stochastic simulation generates a result that is accidental itself, so, in order to make a proper evaluation of the results, several random tests must be carried-out, because each test is statistically different from the others. In turn, **deterministic simulation** does not use random events [21]. This means that course of a simulation experiment is not subject to probability. A deterministic simulation will always give en exactly identical result, no matter how often it is performed. Differences between a stochastic and a deterministic simulation are shown in Fig. 24.

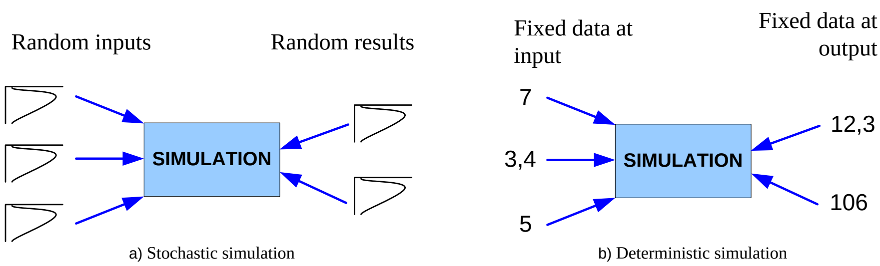


Fig. 24. Differences between stochastic (a) and deterministic simulation (b) [35]

In a **discrete simulation** changes appear at determined points of time [21]. Changes in the model appear at the moment when certain events happen. Most of the manufacturing systems are modelled by means of discrete simulation. Exemplary changes occurring in a model during an experiment performed by means of a discrete simulation are shown in Fig. 25.

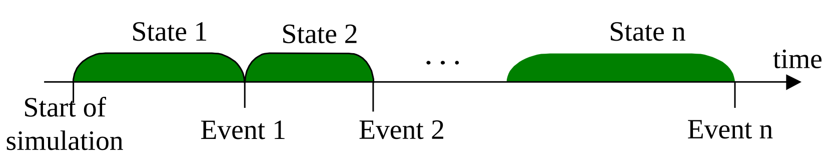


Fig. 25. Exemplary changes during an experiment by means of discrete simulation [21]

In **continuous simulation**, changes happen continuously in time. A simulation is continuous, when values accepted by descriptive variables can be presented by real numbers or their ranges. Continuous models are described by differential equations. In practice, it is difficult to find a system, whose events would be completely continuous or completely discrete. However, usually it can be found, which of the characteristics (continuous or discrete) dominates in the examined system. Exemplary results obtained by means of a continuous and a discrete simulation are shown in Fig. 26.

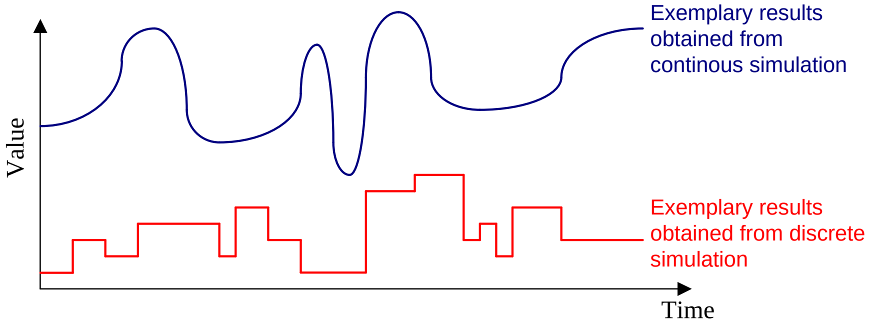


Fig. 26. Comparison of results obtained from a continuous and a discrete simulation [21]

Production processes are of continuous or discrete nature. Because of this, most of simulation packages combine both simulation methods: continuous and discrete. Figure 27 shows comparison of the simulations in the context of application in analysis of production systems.

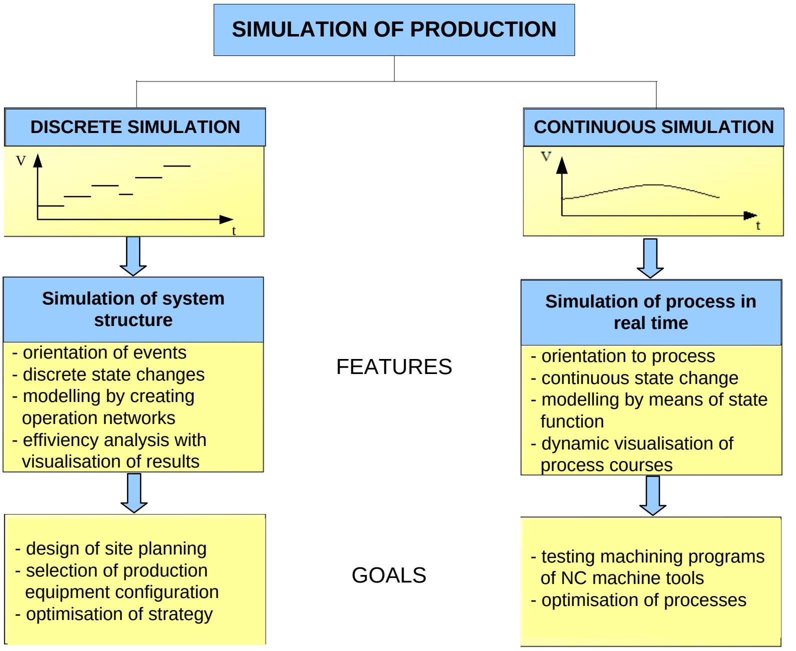


Fig. 27. Discrete and continuous simulation modelling of production systems [46]

Simulation models facilitate complex approach to examining properties of production systems, because they permit combining various modelling techniques and methods. In comparison with analytical methods, they permit examining production systems in complex situations, found in reality. Thanks to that, the results obtained this way are of greater practical meaning.

On the other hand, one should accept the fact that a created model will never describe exactly a complicated real system. Therefore, creating a model is a great skill and requires big experience, helpful at choosing proper elements and parameters for modelling. Table 24 presents advantages and disadvantages of applying simulation [1, 2, 16, 45].

Table 24. Advantages and disadvantages of using simulation

ADVANTAGES OF USING SIMULATION	**DISADVANTAGES OF USING SIMULATION**
1. Simulation permits determining the form of a decisive model by means of experiments carried-out directly on an examined process.	1. Good simulation models are expensive and their preparation takes a long time. 2. Each simulation model has unique nature. Its solutions can

2. Can be used for analysing large and complex decisive problems, which can not be solved with other methods (e.g. operational research). 3. Permits quick preparing a decision thanks to analysis of the effects of experiments carried-out for many successive periods. 4. Gives an answer to a question type "what, if …?". Simulation experiments permit examining various decisive alternatives. 5. Permits analysing interdependences between the model variables influencing the decision-taking in extreme conditions.	not be used for analysing other decision problems. 3. Permits preparing alternative decisive solutions in subsequent experiments, but they are not optimum solutions for all the conditions. 4. Simulation models generate responses to the questions referring to specific and changing conditions. The decision-maker preparing a decision must consider all conditions and limitations of the analysed decision versions. 5. Characterised is with high labour consumption of building, programming and correcting the simulation programs. 6. Necessary is laborious processing the results. 7. Necessary is proficient use of simulation languages and employing experts thoroughly knowing the objects to be modelled.

Chapter 3. Reliability and risk as features of a production system

In this Chapter, a production system is defined in terms of the systems theory and basic questions of reliability theory are presented. Proposed is implementing basic reliability characteristics to the system of analysing manufacturing processes. This permits demonstrating the relationship between reliability and risk as a synonym of unreliability of a production system. Basic types of reliability structures of systems are discussed and illustrated with examples suitable for production systems.

The presented in Chapter 1 state of the art of the risk analysis and assessment methods shows that it is not sufficient for determining the risk of a production system. The methods suggested in literature require adapting a problem to their possibilities, limitations and requirements. Moreover, traditional definitions of risk do not completely reflect specificity of risk of a production system. A solution of these problems can be representation of a production system in terms of the systems theory and applying the general reliability theory in risk assessment.

According to the systems theory, *„A system is a certain entirety in that isolated components cooperate. Operation of a system depends on the function of its components and relations between them. Relations between the components determine the system structure. One can say about systems only when the entirety is organised and should achieve a defined goal divided to sub-goals for individual parts*.” [8]. The components are often named elements, subsystems or segments. Each system can be defined with a model. In turn, the reliability theory is *„a field of applied science, involved in examining and designing objects (elements, systems) from the viewpoint of achieving by them the set requirements (within the given period, in determined conditions)*” [36]. Transferring the used definitions of the concepts "object", "element", "system" and "meeting requirements" to the field of production systems gives new possibilities in using the reliability theory to planning and assessing risk of executing manufacturing processes. The purpose of each economic system, including also a production system, is meeting the set requirements (goals, functions), and then the risk of a production system can be treated as unreliability of "meeting requirements", i.e. reverse of reliability.

Such approach to assessing risk of realising manufacturing processes and a production system seems to be much more advantageous with respect to possibility of building a model considering research needs by any decomposition of the system or dividing it to selected phases. As a result, apart from determining its value, it will be also possible to locate the risk in the system.

3.1 *System in terms of the systems theory*

The proposed in literature models of systems are in different ways defined by individual authors [5, 8, 16, 17, 35, 49], but always the following three components are present in a system model:

- inputs [WE],
- outputs [WY],
- transformation process [T] running in the system, transforming input elements to output elements in a determined sequence.

Diagram of a system is shown in Fig. 28.

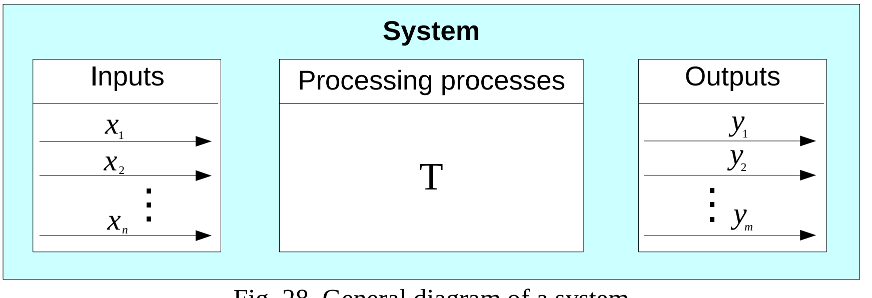


Fig. 28. General diagram of a system

A system can have more than one input and output. States of all *n* inputs and all *m* outputs can be determined by parameters or vectors:

$$x = [x_1, x_2, ..., x_n] \tag{3.1}$$

$$y = [y_1, y_2, ..., y_m]. \tag{3.2}$$

Operation of a system consists in the following: introduced to the system is a stimulus, i.e. a certain input state determined by the parameter (or vector) *x*, and emerges a reaction, i.e. another state determined by the parameter (or vector) y. This change of the state (conversion) is caused by the transformation T, which can be written as follows:

$$y = T(x) \qquad (3.3)$$

Additional components of a system, met in literature, are:

- function (goal) or task making a ground of the system existence [8, 35],
- environment being a set of objects not belonging to the system, but influencing it [35],
- supply, i.e. material, energetic and informative feedbacks between the system components [5, 16, 49],
- disturbances [8],
- management process of the system [5, 16].

A diagram of a system broadened with the above-described elements is shown in Fig. 29.

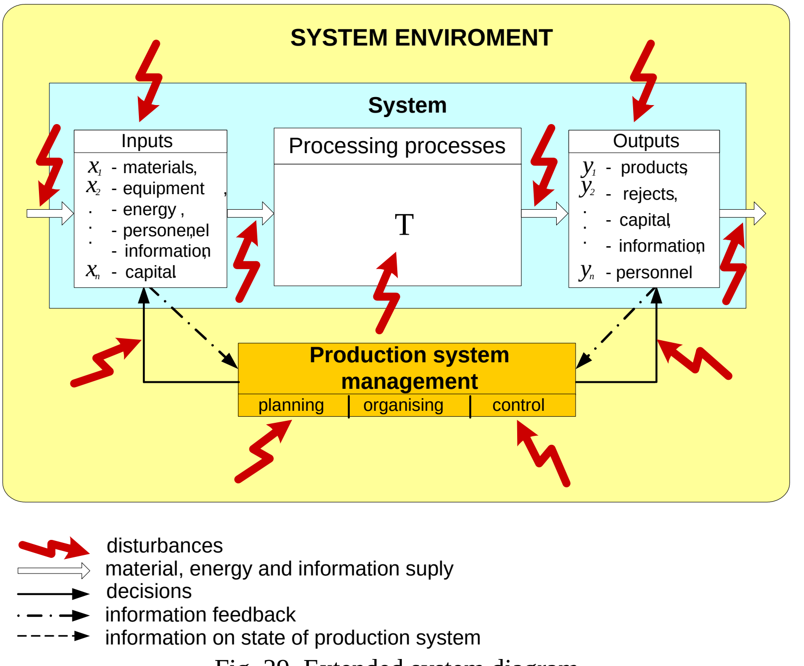


Fig. 29. Extended system diagram

Structure of a model as well as number and minuteness level of individual components should depend on a research problem for that such a system is analysed [8].

3.2 Production system in terms of the systems theory

According to the assumptions of the systems theory, a production system (process) can be presented as „... *intentionally designed and organised material, energetic and informative system employed by humans and used for manufacture of specific goods (products or services) in order to satisfy diversified needs of consumers*.” [16]. This means that a system can be an arrangement of production-administration cells of any level of production structure, e.g. workstations, first-level production cells, second-level production cells etc. till *n*-th level production cells [49]. A system can also consist of individual manufacturing processes, arbitrarily decomposed [5]. A diagram of a system in terms of the systems theory is shown in Fig. 30.

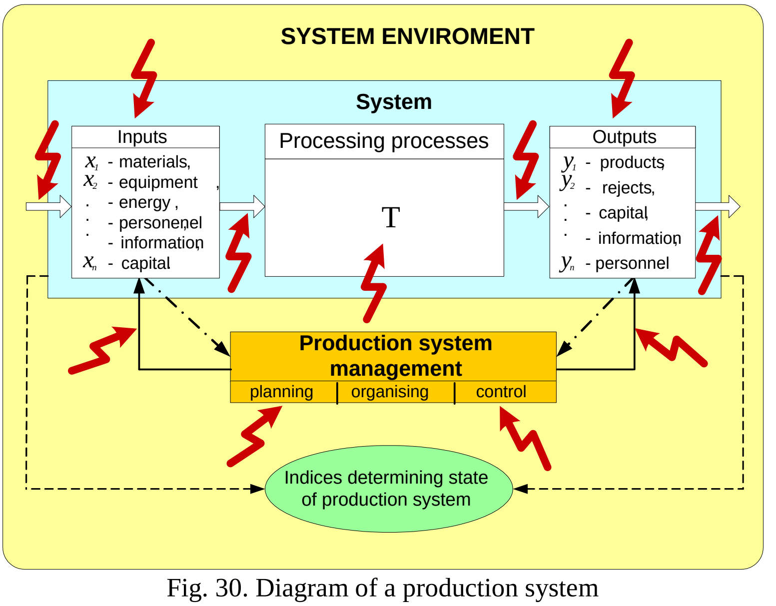


Fig. 30. Diagram of a production system

In a so defined production system, the following elements can be distinguished:

1. **input vector *x* [WE]**, composed of all the production factors;

Elements of an input vector include [16]: technical production means, workpieces, human factor, information, capital.

2. **output vector *y* [WY]**, composed of all the elements leaving the system, so in the case of a manufacturing process they are products, services and production wastes;

   Elements of an output vector include [16]: industrial products, production services, production rejects and secondary raw materials, noxious wastes contaminating the environment, noise, information about product quality, about actual prime cost, about state of manufacturing process, production experience of staff and other output information from the system or remaining in the system for the next production cycles.

3. **transformation (conversion) process [T]**, transforming input vector to output vector;

   In production systems, transformation process is a manufacturing process consisting of process operations, as well as control, transport and store operations [16]. Transformation can be also treated as a set of transitions, conversions or changes performed in a certain set of objects or, more generally, elements being parts of the system [49].

4. **function (goal)** or **„posed requirements”** for whose achieving the system exists;

   The present-day literature mentions three basic goals of organisations and production systems [16]:

- quality and modernity of products,
- increase of productivity,
- reduction of prime manufacturing cost.

5. **environment** that can be composed of other objects or systems not belonging to the considered system, but influencing the production system;

6. **material, energy and information feedbacks (supply)** between individual elements of the system and its environment;

   Feedbacks in a production system should permit effective flow of materials and semiproducts from input stores to finished product stores and even to users of produced goods [16]. Supply, irrespective of its form, is always a subject of control, because information is always a factor that allows controlling the system [49].

7. **disturbances**;

   Like in the case of other objects, each element of a production system is also affected by disturbances. Most frequently, disturbances are unknown inputs to the system, with unknown values. Because of their unpredictable nature, they are of random nature. Magnitude of influence of disturbances on the degree of "meeting requirements" posed to the system will stand for the system reliability degree.

8. **system control process**;

   As management, usually understood is „*controlling or managing complex activities in whose execution people and various material means are engaged* [8]".

   The merit of a production process are all intentional activities aimed at bringing (or maintaining) the production process to a determined standard (pattern) that can be a vector of various parameters [51]. These standards or patterns are determined in the managing process by quantitative defining various types of indices.

3.3 Production system management

From the viewpoint of this work, the question of a production system management consisting in decision-making, as well as of permanently present disturbances, require a more extensive discussion. In each correctly designed system, if inputs to the system are correct and the system is correctly managed, outputs from the system should meet the requirements imposed to the system [5]. It can be said in other words that outputs from the system are inconsistent with expectations and the system is unreliable. This should be counteracted by the system management.

In functional approach, management is composed of the following functions: planning, organising, controlling (motivating) and inspecting. In production practice, management of a production system consists in establishing plans of activities (planning function) which permit meeting the appointed goals and organising the production system so that these goals could be attainable (organising function). In the plans, determined are values of indices which should characterise correct course of manufacturing processes. Comparison of achieved results with those planned (inspection function) makes a base for an adjusting intervention consisting in taking decisions (control function). This way, deviations of the determined indices from the accepted plan are corrected. This process runs continuously. Figure 31 schematically shows the concept of controlling parameters of a production process.

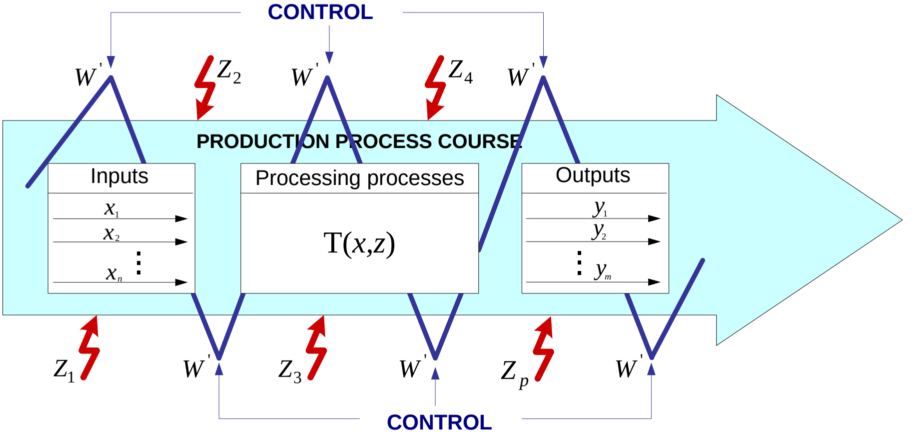


Fig. 31. Schematic presentation of the concept of controlling a production process

In terms of the systems theory, management consists in taking decisions based on the values characteristic for the model. These values can be divided into three groups [8]:

- managing (decisive) quantities, i.e. input vales $x_1, x_2, ...x_n$, which affect the output values. Taking a decision means selecting suitable values of these quantities and executing these decisions, i.e. "supplying" the determined values *x* at the object input, serves achieving the goal of management.
- output values $y_1, y_2, ...y_m$ connected with the goal of management,
- disturbances $z_1, z_2, ...z_r$, i.e. values which also affect the outputs *y* and characterise influence of the environment on the production process. Changes of these quantities make a necessity of counteracting in form of suitable changes of the inputs *x*.

In the case when single values *x*, *y*, *z* exist, i.e. $n = m = r = 1$, the static model of such a process can be described by the function determining influence of *x* and *z* on *y* [8]:

$$y = T(x, z) \tag{3.4}$$

3.4 System reliability

The term **reliability** has many interpretations. However, the most frequently used in literature definition of an object reliability [5, 6, 42, 44] means „*ability of an object to remain in the state that allows executing the required functions in the given conditions and in the given period, on the assumption that delivered are the necessary external means*”. The term "executing the required functions" means the goat or task which the object should achieve. The degree of achieving the goal, i.e. reliability degree of the object, should be determined by selecting suitable indices and numerical determining their quantity. It should be also adapted to the goal of the object reliability analysis.

Probability that an object is in the state of **reliability** (*N*), i.e. ability to perform the required functions in the given conditions and in the given period (0,t), is determined by the general relationship [44]:

$$P_N(t) = \exp\left[\int_0^t \lambda(u)du\right] \qquad (4.5)$$

where $\lambda(u)$ = intensity of damages at the moment $t = u$.

In this definition, both terms "object" and "executing required functions" can be understood in different ways depending on the object of analysis, which causes that the definition of reliability can be treated universally. This results from the definition of an **object** standing for „*any simple or complex system, e.g. element, device, system, subsystem, functional unit or appliance being the consideration object related to solving a determined problem*” [42]. Thanks to such definition, "executing required functions" by an object is determined by type of this object, which results in universality of the reliability definition and possibility of applying it to other objects than technical ones, e.g. production systems and processes [36, 42].

Reliability (*N*) is often identified in literature with readiness [6, 42] and stands for „*a set of properties which describe readiness of the object and influence it: durability, undamageability, serviceability and ensuring operation means*” [42]. The basic measure of readiness is the readiness function $K_g(t)$, defined as "probability that the object is able to achieve the required function in the given conditions in a freely selected period” [42]. The readiness coefficient in function of time is determined as [42]:

$$k_g(t) = P\,, \tag{3.6}$$

which means that the object is ready at the moment *t* or, in different way, that [5]:

$$k_g(t) = P[L(t) = 1]\,, \tag{3.7}$$

where:

L(t) = 1 means that at the moment *t* the object is in operating condition,

L(t) = 0 means that at the moment *t* the object is in failure condition.

In practice, the so-called stationary value of the readiness coefficient k_g^s is often defined and then it is determined as [5]:

$$k_g^s = \frac{T_1}{T_1 + T_2}\,, \tag{3.8}$$

where:

T_1 = average time of the object operation,

T_2 = average time of the object renovation.

Durability (*TR*) [42] of an object id determined by „*time that passes from the beginning till the end of its operation*". This means that ability of an object to performing the required functions can be interrupted by defects and restored by repairs and end of its usability period is determined by reaching the limit condition. **Undamageability** (*NU*) [42] „*characterises ability of the object to executing the required functions (to correct operation not interrupted by a defect) in the given conditions and in the given period*". In turn, **serviceability (***OB***)** [42] determines „*ability of the object to maintaining or reproducing in the given conditions the state in that it can execute the required functions on the assumption that the object is operated in the accepted conditions maintaining the established procedures and means*".

Measures of an object readiness are functionally linked with undamageability measures [42]:

- If the object is unrepairable:

$$k_g(t) = NU(t)\,. \tag{34.9}$$

- If the object is repairable, readiness of the object in the period $\left[t, t + \Delta t\right]$ means probability that at the moment *t* it is usable and will not fail within $[t, t+\Delta t]$ or that the object is damaged at the moment t and will be repaired within $[t, t+\Delta t]$. On the assumption that intensity of failures λ and intensity of repairs μ are constant, it is:

$$k_g(t+\Delta t) = (k_g(t)(1-\lambda\Delta t) + (1-k_g(t))\mu\Delta t . \qquad (3.10)$$

Therefore, average readiness coefficient is equal to the part of the usability time in the entire operation time:

$$k_r(\infty) = \frac{\mu}{\lambda+\mu} . \qquad (3.11)$$

3.5 Reliability of production systems

Transferring the reliability theory to the field of production systems can bring many advantages in planning and assessing risk of production systems, but it requires defining a specific approach to manufacturing process and to reliability.

The reliability theory itself considers basically the states 0/1, i.e. action or no-action [36]. Such approach refers to technical objects, but is not suitable for describing biotechnical objects or operating systems, and thus the production systems (processes). These facts led to creation of the so-called "general reliability theory" whose concepts can be met among others in [5, 6, 5, 27, 36].

The general reliability theory assumes that for each object reliability $\vec{N}$ is a vector with components $\overline{N}$ meaning I-type reliability and $\overline{\overline{N}}$ means II-type reliability. Analogously, unreliability $\vec{Z}$ is also a vector of I-type unreliability with components $\overline{Z}$ and $\overline{\overline{Z}}$ means II-type unreliability. I-type reliability means that the object executes a desirable activity, i.e. consistent with the user's intentions. II-type reliability means the no-action state, desired by the user. Accordingly, absence of a desired activity is named I-type unreliability, while an undesirable action (inconsistent with the user's intentions) means II-type unreliability. Similarities and differences between conventional reliability and general reliability are shown in Fig. 32.

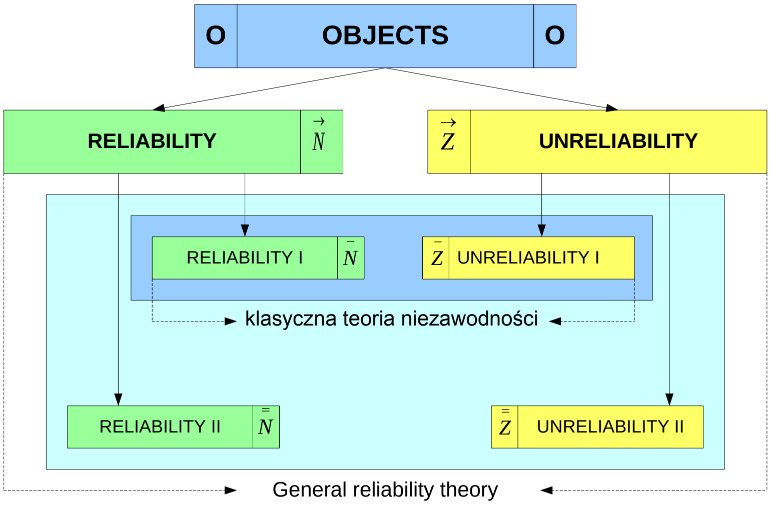


Fig. 32. Relations between conventional and general reliability theories [36]

The general reliability theory defines reliability of an object in different way than the conventional theory: „*A reliable object is such an object that acts in accordance with the user's intentions, but an unreliable object is any object that acts inconsistently with the user's intentions*” [36]. The so defined reliability and unreliability of an object can be transferred to the field of a production system or process. Then one can find that the production system (process) is reliable, when it achieves the indices, parameters and other characteristics assumed by the user, and otherwise one can say about unreliability of a production system (process).

Specificity of today's production systems, and in particular their complexity, permits treating them as operational systems and then their reliability is one of their features measured by degree of achieving the imposed indices, parameters and characteristics. On the other hand, production systems must operate in the environment that continuously influences the system and causes its disturbances. Because of this, reliability in real conditions is of random nature [5, 36].

With respect to the above, reliability of a production system can be determined in a more flexible way. Depending on the needs of the analysis, as "the action consistent with the user's needs" can be accepted the value of any index characterising the process according to "the user". The most often analysed indices of a production process include [16, 28, 30, 35]:

- time of the process duration (*t*),
- efficiency (*W*),

- productivity (P).

In this case, probability that the analysed parameter will not be smaller than that planned, i.e. reliability of the production process, can be similarly determined from the formulae:

$$N = P(t_{pl} \leq t_{rz}), \tag{3.12}$$

$$N = P(W_{pl} \leq W_{rz}), \tag{3.13}$$

$$N = P(P_{pl} \leq P_{rz}), \tag{3.14}$$

where:

W_{pl} = planned value of the analysed index,

W_{rzecz} = real value of the analysed index.

3.6 Relation between reliability and risk

Traditionally, questions of reliability are related to operation of technical objects and this term very rarely refers to economic systems. However, interest in the risk is growing in the economic literature. Since, according to the systems theory, the term "system" can refer both to technical and economic objects, it seems to be justified to transfer the general reliability theory to the field of economy, as well as applying it in planning and assessment of risk. Especially suitable seems to be the area of production systems in which a determined technology makes impossible treating the risk according to the American approach, but only as a possibility of occurrence of results (profits) smaller than the expected ones (German approach).

Transferring the general reliability theory to the ground of production systems can take place by treating unreliability (Z) – in contrast to reliability – as a synonym of risk (R) [5]:

$$R = Z \tag{3.15}$$

The so interpreted risk (unreliability) of a system (e.g. production system) will be probability of the fact that the system will not execute the functions to which it was designed or will stand for probability of occurrence of losses in this system. For such an interpretation, the following equation should be valid:

$$N + Z = 1. \tag{3.16}$$

This equation means that probability of the system being in state of reliability or unreliability is 1. With this respect, valid is also the following:

$$N + R = 1, \text{ and thus} \tag{3.17}$$

$$R = 1 - N. \tag{3.18}$$

Therefore, risk analysis and assessment permits determining reliability of a system operation and vice versa. Although the reliability approach in risk planning and assessment gives more possibilities, it does not situate the risk factors in the system. This is why it is necessary to develop a suitable methodology.

3.7 Reliability structure of systems

Unreliability (risk) or reliability of a system is influenced also by its structure. Since the considered system can be a simple or complex system, its reliability is affected by the reliability structure determining relation between reliability status of the system and reliability status of its objects [6].

Depending on feedbacks existing between objects in the systems, including production systems, various kinds of structures of these systems can be distinguished. Kind of a feedback between individual objects in the system defines the way of its operation, control and adjustment. Analysis of reliability structure of a system should be preceded by dividing the system into individual components, i.e. decomposition of the system, reflecting logical interconnections in the system so that its individual parts are statistically independent and possibly largest [44]. Depending on kind of the feedback between the system objects, the reliability structures most frequently found in literature are [6, 8, 44, 49]:

- series structures,
- parallel structures, and
- series-parallel structures.

4.7.1. Series structure

A system has **series structure**, when its operation requires operation of all the objects/subsystems. This means that a system acts correctly, when all its components also act correctly, but at the moment when any object/subsystem gets damaged, the entire system is subject to damage [36]. In a series structure, feedback of two objects/subsystems consists in transforming an output vector of one object/subsystem to an input vector of another object/subsystem, but it is not necessary that all components of the input vector of one system become components of the input vector of the other system. An exemplary series structure with *n* objects/subsystems is shown in Fig. 33, where "I" stands for input to the system and "O" means output from the system.

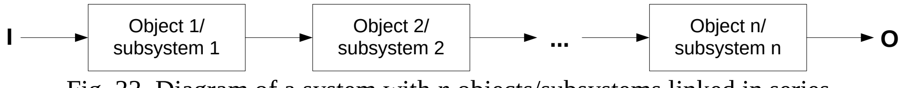


Fig. 33. Diagram of a system with *n* objects/subsystems linked in series

At this type systems, reliability of a system is a product of reliabilities of its objects, thus along with increasing number of objects in a system, its reliability decreases. Reliability of a system (N_S) with series structure is determined by the formula [36]:

$$N_S = N_1 N_2 ... N_n, \quad (3.19)$$

where N_1, N_2, N_n = reliability of individual objects/subsystems of the system.

Using the formulae (3.18) and (3.19), total risk (R_c) of the system will be:

$$R_c = 1 - [(1 - R_1)(1 - R_2)...(1 - R_n)], \quad (3.20)$$

where R_1, R_2, R_n = risk present in individual objects/subsystems of the system.

Series structure is the most common kind of structure in production systems and processes [32]. For example, defect of one machine in a manufacturing process results in incorrect operation or inoperativeness of the entire production system. An example of series reliability structure of a production system on the level KP^0 is shown in Fig. 34.

Fig. 34. Example of series reliability structure of a production system: P – saw, T – lathe, F – milling machine, S - grinding machine

According to the formulae (3.19) and (3.20), the formulae for reliability and risk of this production system are as follows:

- total reliability: $N = N_P N_T N_F N_S N_T$,
- total risk:
  $R_c = 1 - [(1 - R_P)(1 - R_T)(1 - R_F)(1 - R_S)(1 - R_T)]$.

4.7.2. Parallel structure with redundancies

According to the conventional theory, reliability is based on the assumption that the system is in the state of usability, if at least one of its objects is in the state of usability [6], which means that for correct operation of a system sufficient is correct operation of one of its elements. Reliability of this system rises with increasing number of objects coupled in parallel. An exemplary diagram of parallel reliability structure of a system with *n* objects is shown in Fig. 35.

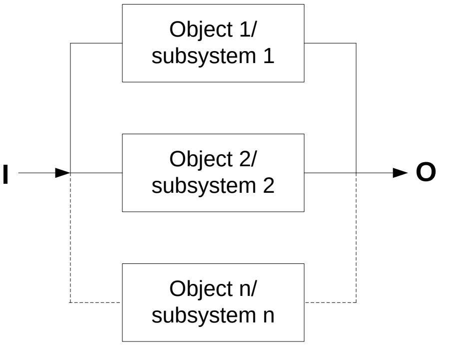


Fig. 35. Diagram of a system with *n* objects/subsystems coupled in parallel

Reliability of the system (N_S) shown in Fig. 35 is determined by the formula [36]:

$$N_S = 1 - [(1 - N_1)(1 - N_2)...(1 - N_n)] \qquad (3.21)$$

where N_1, N_2, N_n = reliability of individual objects/subsystems of the system.

Total risk R_c of the system can be determined with the formulae (3.18) and (3.21):

$$R_c = R_1 R_2 ... R_n = \prod_{i=1}^{n} R_i \ , \qquad (3.22)$$

where R_1, R_2, R_n = risk present in individual objects/subsystems of the system.

In production practice present are parallel structures, but nature of a production process does not permit such interpretation of reliability structure. The conventional reliability theory considers states 0/1 of technical appliances. This means that, in interpretation of conventional reliability theory, a production system is considered reliable if at least one of its elements works correctly. In production systems, such situation happens only in the so-called redundant systems [5, 32, 36], i.e. systems with surplus elements functioning in them. In reality, redundant systems occur very rarely, because surplus of elements (e.g. machines, workers, means of transport etc.) means unused resources, which results in increased costs. An example of a parallel structure with redundancies at KP^0 level is shown in Fig. 36.

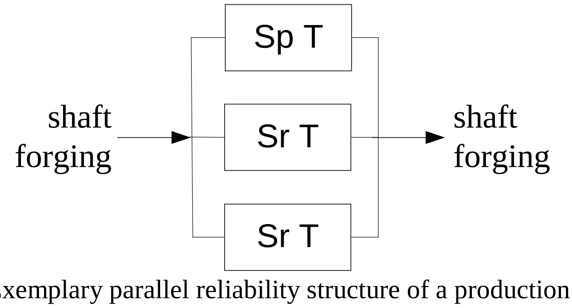


Fig. 36. Exemplary parallel reliability structure of a production system with redundancies: Sp T – operating workstation "lathe", Sr T – redundant workstation "lathe".

According to the formulae (3.21) and (3.22), the formulae for reliability and risk of the production system of the Fig. 36 will be as follows:

- total reliability: $N_S = 1 - [(1 - N_{SpT})(1 - N_{SrT})(1 - N_{SrT})]$,
- total risk: $R_c = R_{SpT} R_{SrT} R_{SrT}$.

4.7.3. Parallel production structure

Since, as mentioned before, redundant production systems are found in practice extremely rarely, in this work proposed is another way of interpretation and determination of risk for **parallel production structures**. For an *n*-element structure shown in Fig. 35, unreliability risk of one element R_i should increase total risk of the system R_c by the value R_i. So, total risk should be the total of risks of individual elements of the system:

$$R_c = R_1 + R_2 + \ldots + R_n = \sum_{i=1}^{n} R_i \ , \qquad (3.23)$$

where R_1, R_2, R_n = risk present in individual objects/subsystems of the system.

An example of a parallel production structure of KP^1 level may be the structure of a production system shown in Fig. 37.

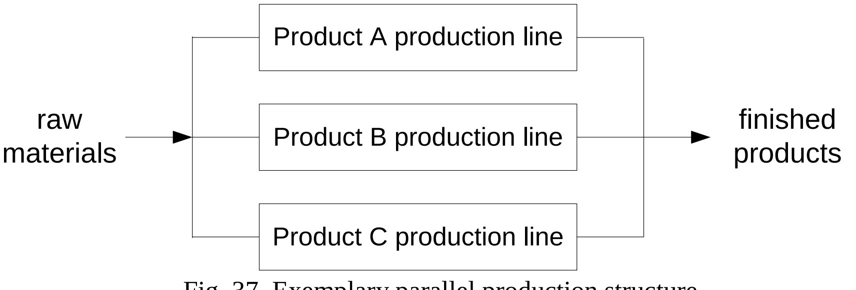


Fig. 37. Exemplary parallel production structure

According to the formula (3.23), risk of the system of Fig. 37 will be expressed as:

$$R_c = R_{lpwA} + R_{lpwB} + R_{lpwC} = \sum_{1}^{3} R_{lpwi} \ ,$$

where *lpwA*, *lpwB*. *lpwC* = individual production lines.

4.7.4. Series-parallel structure with redundancies

This structure is a mixture of a series and a parallel structures. An exemplary diagram of such a system with 6 objects/subsystems is shown in Fig. 38.

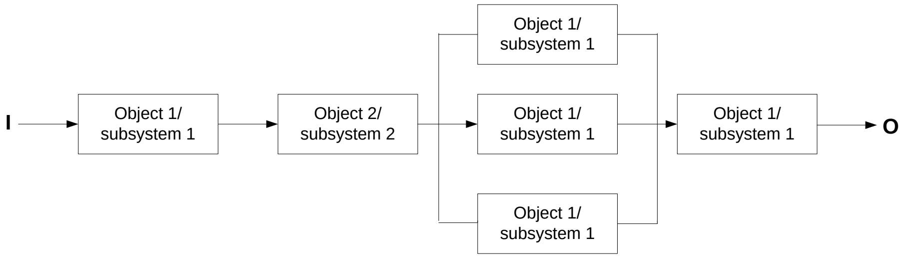


Fig. 38. Exemplary diagram of a series-parallel system

For reliability calculation (N_s) of a system with this type structure, used are the previously presented formulae (3.19) and (3.21). Thus, reliability of the system of Fig. 38 will be determined by the formula:

$$N_s = N_1 N_2 [1-(1-N_3)(1-N_4)(1-N_5)] N_6 ; \qquad (3.24)$$

$N_1, N_2, ..., N_6$ = reliability of individual objects/subsystems of the system.

According to interpretation of the conventional reliability theory or in series-parallel systems with redundancies, unreliability (risk) is expressed as follows:

$$R_c = 1-[(1-R_1)(1-R_2)(1-R_3 R_4 R_5)(1-R_6)] ; \qquad (3.25)$$

$R_1, R_2, ..., R_6$ = risk existing in individual objects/subsystems of the system.

An operational example of a series-parallel structure with redundancies is shown in Fig. 39.

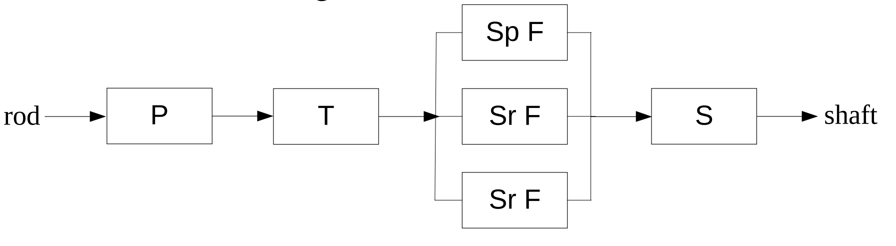


Fig. 39. Exemplary structure of a series-parallel production system with redundancies: P – saw, T – lathe, Sp F – working milling machine, Sr F – reserve milling machine, S - grinding machine

According to the formulae (3.24) and (3.25), total reliability and risk for the production system of Fig. 39 is:

- total reliability of the system:
  $N = N_P N_T [1-(1-N_{SpF})(1-N_{SrF})(1-N_{SrF})] N_S$
- total risk:
  $R_c = 1-[(1-R_P)(1-R_T)(1-R_{SpF} R_{SrF} R_{SrF})(1-R_S)]$.

4.7.5. Series-parallel production structure

When treating the system of Fig. 38 as a **series-parallel production structure**, with the same symbols and on the ground of the formula (3.23), the formula (3.25) accepts the form:

$$R_c = 1-[(1-R_1)(1-R_2)(1-(R_3+R_4+R_5))(1-R_6)] \qquad (3.26)$$

An exemplary series-parallel production structure of KP^1 level is shown in Fig. 40.

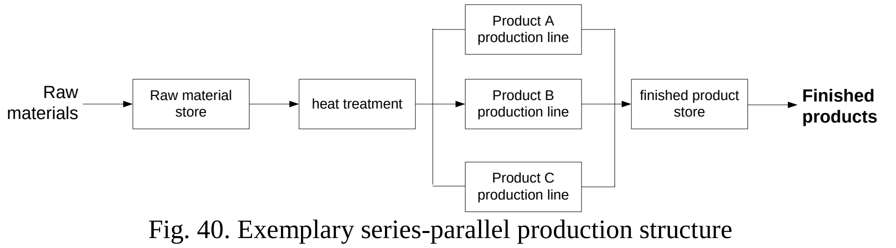


Fig. 40. Exemplary series-parallel production structure

The formula (3.26) for total risk of this system is as follows:

$$R_c = 1-[(1-R_{ms})(1-R_{oc})(1-(R_{lpwA}+R_{lpwB}+R_{lpwC}))(1-R_{mwg})]$$

where:
ms = raw material store,
oc – heat treatment,
lpwA, *lpwB*, *lpwC* – production lines of the products A, B and C,
mwg – finished product store.

Transferring the reliability theory to the ground of production systems gives new possibilities at planning and assessing risk of manufacturing processes. Treating a production system as a system in terms of the systems theory and its decomposition to subsystems permits building reliability structures. Kind of a structure will depend on the goal posed to analysis, decomposition and flow logic of the production system.

The only shortcoming of this approach is rare occurrence of parallel structures with redundancies in production reality. Much more often are found the structures called and characterised as parallel production structures. Because also for this type systems possible is building a reliability structure and determining the risk, this approach will be continued and developed in further parts of this work.

Chapter 4. Assessment of producing system risk by PERT method

In this Chapter, presented is risk assessment of the production system of frames of the passenger wagon bogie MD 523 using the operational method PERT.

4.1 Characteristics of product and its manufacturing process

Frames of the passenger wagon bogie MD 523 are produced in Wroclaw division of an international industrial group. This division is the third biggest in the group and it is involved in manufacture of electric locomotives, goods wagons and passenger wagons, regional trains, trams and underground wagons. It includes the following plants:

- bogie production plant,
- locomotive production plant,
- wagon body production plant,
- service plant.

Manufacture of the bogies MD 523 is carried-out on a separated production line. Most of the process operations are welding operations. The elements for assembly are brought to the production shop with trucks, and transport within the production shop is performed by overhead cranes. Structure of the MD 523 frame is shown in ANNEX 2. Layout of the production line of the MD 523 frame is shown in Fig. 41. Marked are workstations and material flows in the manufacturing process of sub-assemblies and the frame.

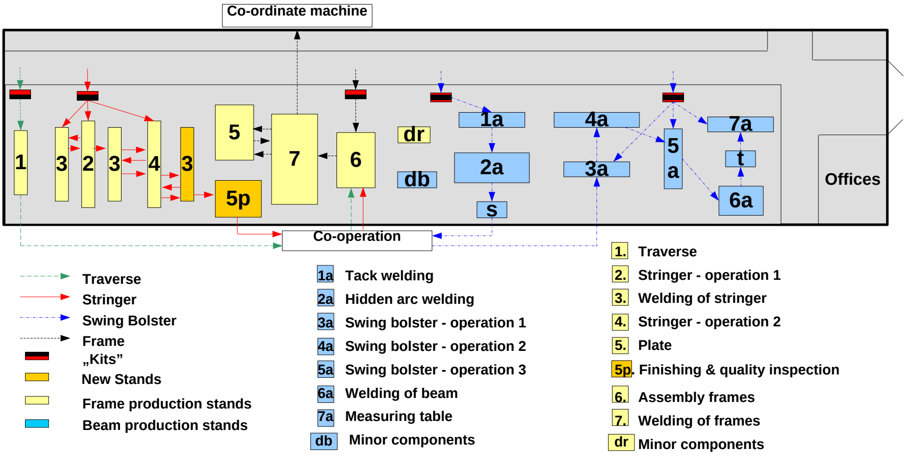


Fig. 41. Initial layout of production line of MD 523 frame

4.2 Application of PERT method for assessing production risk of MD 523 bogie frame

In order to apply the PERT method for determining risk of non-producing a MD 523 bogie frame in the assumed period, three time characteristics of the process operations are required: optimistic time, pessimistic time and modal time. The following values of these times were accepted:

- Optimistic time [*a*] – is a standardised time, consistent with the planned time of operation according to the process engineering. This time can be reached only in the most favourable conditions.
- Pessimistic time [*b*] – is the operation time in the least favourable conditions. In the example, as the pessimistic time accepted is the optimistic time (technological time) increased by the difference between technological and real times, as well as by average correction time. Since probability of occurrence of a repairable reject is 7 % and of a scrapped reject is 3 %, the pessimistic times of operations to be corrected are increased by the repair time in proportions equal to probabilities of producing rejects in these operations.
- Modal time [*m*] – is a median value of the range between the optimistic and the pessimistic times.

On the grounds of these three times, the expected time (t_e) was calculated from the formula (1.23):

$$t_e = \frac{a + 4m + b}{6}$$

and variances of the expected time, determining deflections between the real times and the expected operation time, were calculated from the formula (1.24):

$$\sigma^2_{i-j} = \left(\frac{b-a}{6}\right)^2 .$$

Calculated values of the times are given in Tables 25 to 30. As the PERT method analyses the operation times, the practical example for this method is also divided to process operations, and not to workstations.

Table 25. Time characteristics in production process of cross-beam of MD 523 bogie frame

Actions i-j	Operation No.	Optimistic time [min]	Modal time [min]	Pessimistic time [min]	Expected time [min]	Variance
10-20	10	17	18.0	19	18.0	0.1
20-30	20	10	10.5	11	10.5	0.0
30-40	30	85	102.5	120	102,5	34.0
40-50	40	5	6.5	8	6,5	0.3
50-60	50	5	6.5	8	6,5	0.3
TOTAL:		**122**	**144.0**	**166**	**144.0**	**34.7**
Lead time			**144.0**	**Deflection from lead time**		**5.9**

Figure 42 shows the activity network for the manufacturing process of cross-beam of the analysed product.

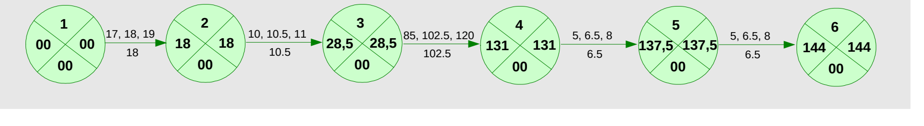


Fig. 42. Activity network for manufacturing process of cross-beam of MD 523 bogie frame

Table 26. Time characteristics in production process of solebar of MD 523 bogie frame

Actions i-j	Operation No.	Optimistic time [min]	Modal time [min]	Pessimistic time [min]	Expected time [min]	Variance
1-2	10	40	75.0	110	75.0	136.1
2-3	20	20	43.0	66	43.0	58.8
3-4	30	90	94.5	99	94.5	2.3
4-5	40	15	24.0	33	24.0	9.0
5-6	50	5	13.5	22	13.5	8.0
6-7	60	90	94.5	99	94.5	2.3
7-8	70	30	31.5	33	31.5	0.3
8-9	80	10	10.5	11	10.5	0.0
9-10	90	30	31.5	33	31.5	0.3
10-11	100	35	37.0	39	37.0	0.4
11-12	110	12	13.0	14	13.0	0.1
12-13	120	120	67.0	14	67.0	312.1
13-14	121	25	34.5	44	34.5	10.0
14-15	122	5	8.0	11	8.0	1.0
15-16	123	15	15.5	16	15.5	0.0
16-17	130	50	52.5	55	52.5	0.7
TOTAL		**592**	**645.5**	**699**	**645.5**	**541.4**
Lead time			**645.5**	**Deflection from lead time**		**23.3**

Figure 43 shows the activity network for the manufacturing process of solebar of the analysed product.

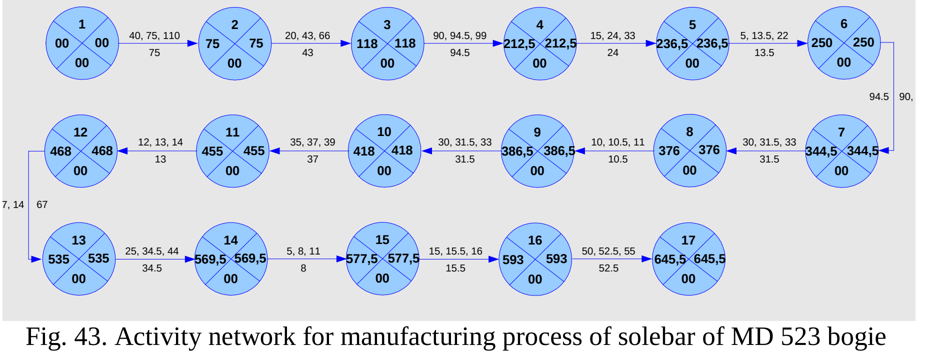


Fig. 43. Activity network for manufacturing process of solebar of MD 523 bogie frame

Table 27. Time characteristics in production process of swing bolster of MD 523 bogie frame

Actions i-j	Operation No.	Optimistic time [min]	Modal time [min]	Pessimistic time [min]	Expected time [min]	Variance
1-2	10	25	29.0	33	29.0	1.8
2-3	20	30	34.0	38	34.0	1.8
3-4	30	80	102.5	125	102.5	56.3
4-5	40	28	29.5	31	29.5	0.3
5-6	50	100	117.5	135	117.5	34.0
6-7	55	15	16.0	17	16.0	0.1
7-8	60	15	16.0	17	16.0	0.1
8-9	80	25	26.5	28	26.5	0.3
9-10	91	25	26.5	28	26.5	0.3
10-11	92	20	21.0	22	21.0	0.1
11-12	93	90	151.0	120	135.7	25.0
12-13	94	45	55.5	66	55.5	12.3
13-14	95	20	24.0	28	24.0	1.8
14-15	96	90	105.0	120	105.0	25.0
15-16	100	30	33.5	37	33.5	1.4
16-17	102	10	10.5	11	10.5	0.0
17-18	104	20	21.0	22	21.0	0.1
18-19	106	10	10.5	11	10.5	0.0
19-20	107	10	10.5	11	10.5	0.0
20-21	108	40	42.0	44	42.0	0.4
TOTAL		**728**	**882.0**	**944**	**866.7**	**160.9**
Lead time		**866.7**		**Deflection from lead time**		**12.7**

Figure 44 shows the activity network for the manufacturing process of swing bolster of the analysed product.

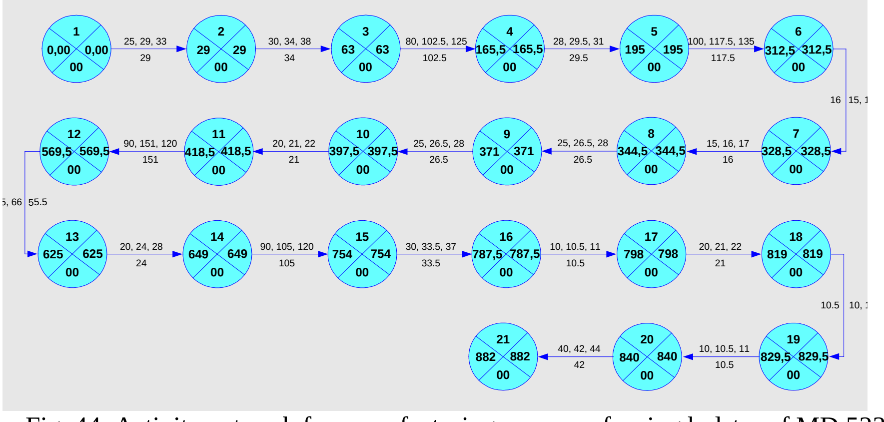


Fig. 44. Activity network for manufacturing process of swing bolster of MD 523 bogie frame

Table 28. Time characteristics in production process of swing bolster body of MD 523 bogie frame

Actions i-j	**Operation No.**	**Optimistic time [min]**	**Modal time [min]**	**Pessimistic time [min]**	**Expected time [min]**	**Variance**
1-2	10	41	43.0	45	43.0	0.4
2-3	20	30	31.5	33	31.5	0.3
3-4	30	210	264.5	319	264.5	330.0
4-5	50	25	26.5	28	26.5	0.3
5-6	60	33	35.0	37	35.0	0.4
TOTAL		**339**	**400.5**	**462**	**400.5**	**331.4**
Lead time			**400.5**	**Deflection from lead time**		**18.2**

Figure 45 shows the activity network for the manufacturing process of swing bolster body of the analysed product.

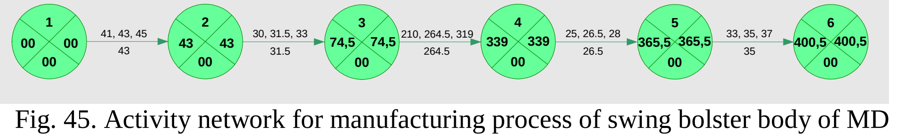


Fig. 45. Activity network for manufacturing process of swing bolster body of MD 523 bogie frame

Table 29. Time characteristics in production process of MD 523 bogie frame

Actions i-j	Operation No.	Optimistic time [min]	Modal time [min]	Pessimistic time [min]	Expected time [min]	Variance
1-2	10	130	133.8	137.5	133.8	1.6
2-3	20	30	33.8	37.5	33.8	1.6
3-4	30	360	368.8	377.5	368.8	8.5
4-5	40	240	246.4	252.75	246.4	4.5
5-6	50	25	31.4	37.75	31.4	4.5
6-7	60	120	122.8	125.5	122.8	0.8
7-8	70	200	204.8	209.5	204.8	2.5
TOTAL		**1105**	**1141.5**	**1178**	**1141.5**	**24.0**
Lead time			**1141.5**	**Deflection from lead time**		**4.9**

Figure 46 shows the activity network for the manufacturing process of bogie frame of the analysed product.

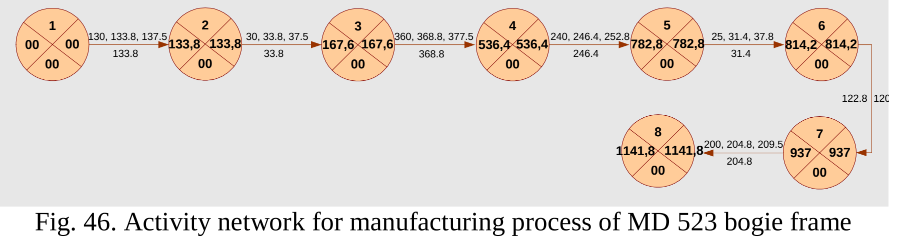


Fig. 46. Activity network for manufacturing process of MD 523 bogie frame

On the ground of analysis of diagrams for individual components of the bogie, one can create a PERT model for the entire product. Fig. 47 shows a diagram based on the manufacturing process of the bogie frame MD 523. Gaps in the diagram represent activities related to manufacturing components of the bogie (complete manufacturing processes). Nodes represent their beginning and finish.

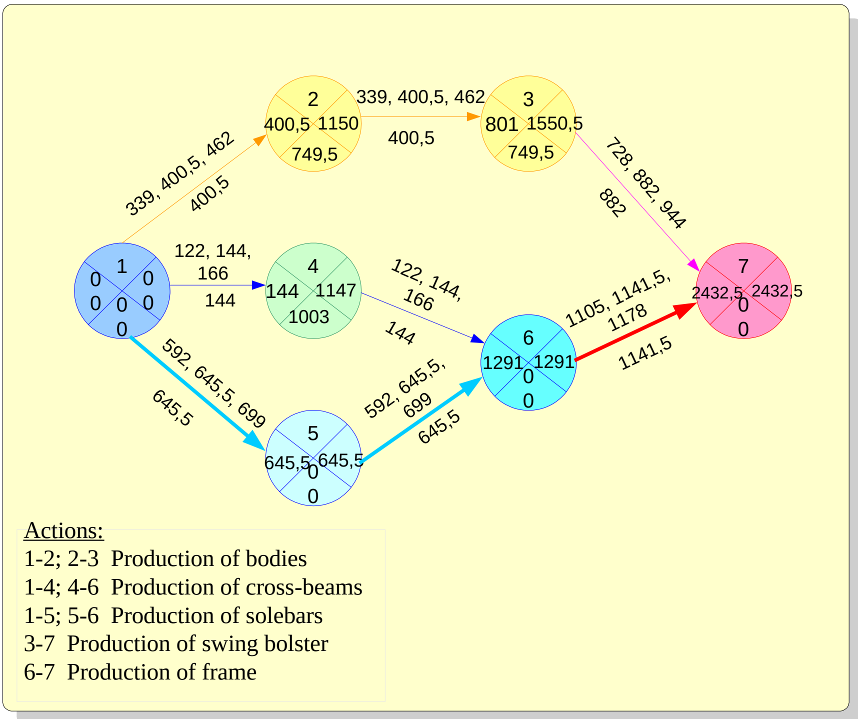


Fig. 47. Diagram PERT of manufacturing process of bogie frame MD 523

In Table 30 settled are time characteristics for the manufacturing process of the bogie frame MD 523.

Table 30. Time characteristics for manufacturing process of bogie frame MD 523

Actions i-j	**Manufacturing process**	**Optimistic time [min]**	**Modal time [min]**	**Pessimistic time [min]**	**Expected time [min]**	**Variance**
		a	**m**	**b**	t_e	σ^2
1-2	**Swing bolster body**	339.00	400.50	462.00	400.50	18.20
2-3	**Swing bolster body**	339.00	400.50	462.00	400.50	18.20
1-4	**Cross-beam**	122.00	144.00	166.00	144.00	5.90
6-6	**Cross-beam**	122.00	144.00	166.00	144.00	5.90
1-5	**Solebar**	**592.00**	**645.50**	**699.00**	**645.50**	**23.30**

5-6	**Solebar**	**592.00**	**645.50**	**699.00**	**645.50**	**23.30**
3-7	**Swing bolster**	728.00	882.00	944.00	882.00	12.70
6-7	**Bogie frame**	**1105.00**	**1141.50**	**1178.00**	**1141.50**	**4.90**
TOTAL	**(critical path)**	**2289.00**	**2432.50**	**2576.00**	**2432.50**	**51.50**
Lead time		**2432.50**	**Deflection from lead time**			**7.18**

Having at the disposal the data given in the Table 30, one can determine the probability.

Since production time in the analysed company is calculated only on the ground of the assumed process time, the required lead time t_d of manufacturing the frame will be total of the assumed operation times, and the expected time will be the expected lead time of performing the task t_W. Employing the formula (1.24), one can calculate probability *x* of performing the production in the set time:

$$x = \frac{t_d - t_W}{\sqrt{\sigma_{TW}^2}} = \frac{2289 - 2432,5}{7,18} = -19,9 .$$

For the so calculated coefficient *x*, one can read-out from the tables of normal distribution function the probability of meeting the imposed deadline, i.e.:

$$P\{t_d \le t_W\} = F(x) = 0 .$$

Since the probability is 0, manufacturing the product till the imposed deadline is impossible.

As can be seen in the above-described example, the PERT method permits determining probability of manufacturing the product in the assumed time, i.e. risk of the analysed production system. However, this method is much more labour-consuming in comparison with the methodology suggested in this work, and introducing any change requires repeated calculations for the entire production process.

Unlike in the methodology of applying simulation models in planning and assessing risk of manufacturing processes, the PERT method gives no answer to the question, which risk factors and to what extent influence the process negatively. The only answer is, whether manufacturing the product in the assumed time is possible or not.

Chapter 5. Summary and conclusions

Formulating the merit of risk, characteristic for a business activity, is a complex problem. This is visible in multitude of approaches, definitions and classifications of risk, presented in Chapter 1. Unfortunately, with respect to technical nature of a manufacturing process, most of them are not suitable for use in the area of production systems. In the area of manufacturing processes, risk manifests itself in the context of loss or probability of non-achieving the assumed indices characteristic for these processes. In the financial area, by undertaking higher risk, an investor can also expect higher profits from the investment.

Even more complicated is determining the risk value. The quantitative methods proposed in literature can be applied for planning and assessing risk of manufacturing processes, but the obtained results are very much estimated only or burdened with high risk, which reduces their practical usability. Their small practical usability results also from their limited possibilities. The quantitative methods suggested by literature refer to individual questions, assume occurrence of determined factors and conditions or impose restrictions on complex reality of manufacturing processes.

On the other hand, risk permanently accompanies every business activity and situation of present production companies forces taking quick decisions concerning manufacturing processes. In order to guarantee safe operation and development of a company, its decision-makers should have a possibility to plan and assess risk both during taking the production decisions and in course of performing manufacturing processes.

Complexity and dynamics of manufacturing processes and their environment requires from decision-makers considering and continuously analysing numerous data and variables. An additional difficulty are randomly appearing disturbing factors which constitute a permanent element of all production systems. As things are, it seems necessary to apply IT tools, and in particular simulation systems. Advantages of simulation and modelling of manufacturing processes, described in Chapter 2, caused that the methodology suggested in this work utilises simulation models for planning and simulating risk of performing manufacturing processes.

In order that risk planning and assessing is usable in practice, it should treat the manufacturing process in a systemic manner. For this reason, in Chapter 3 suggested was defining the production process structure as a reliability structure and using the general reliability theory for risk determination. Thanks to this, the developed method reached great practical importance, because – by assigning risk factors to areas of a production system – it allows determining the areas with the highest risk level. This gives possibilities of eliminating it later.

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ANNEX 1

BASIC CHARACTERISTICS OF RANDOM VARIABLE

- Introduction to probability theory and mathematical statistics
- Basic statistical measures
- Basic characteristics of continuous random variables
- Relationships between features

1.1 *Introduction to probability theory and mathematical statistics*

Strict relation between probability theory and statistics introduces several difficulties in the methods of risk evaluation, since it should be exactly determined, whether the analysed problem refers to a random variable or to a statistical feature (variable). Measures of risk will be different for each of these cases.

Mathematical statistics was created on the ground of probability theory and is an integral part of probability calculus. It is engaged in mathematical models (first of all the probability theory) which serve for examining random events [9]. The relationship between statistics and probability theory can be best demonstrated by citing the statistic definition of probability: ”If, in multiple experiments which can result in occurrence of the event A, frequency of this event demonstrates a clear regularity, oscillating around an unknown number p, and if frequency fluctuations demonstrate a decreasing tendency with increasing number of experiments, the number p is named probability of the event A” [22].

The ground for statistical analyses of an examined feature is determination of the so-called empirical distribution of the feature, which consists in assigning properly defined occurrence frequencies to increasingly arranged values accepted by this feature. In the case of a continuous feature, distribution is determined by assigning numbers (frequencies) to corresponding intervals of the feature values and not to its specific values, as it was for discontinuous (stepped) variables [24]. Such intervals for continuous variables are named class intervals.

In mathematical statistics, the entire statistical population having at least one feature common for all its elements and at least one feature distinguishing these elements is named a *general population* (or just a *population*) [5]. A part of a population on that ground conclusions about the entire population are drawn, is named a *sample* [5]. A sample well reflecting properties of the population is called a *representative sample*.

Treating distribution of a feature in a general population as a random variable distribution causes in consequence that some characteristics of the given feature are treated as distribution parameters of a corresponding random variable. A *random variable* is a feature of a random event, expressed in proper quantitative units. This is for example labour time, consumption of basic materials etc. (because they are subject to variability resulting from several various factors). In course of a research on a production system, one observes *random events*, i.e. such whose result can not be predicted, in spite of specifying conditions in which they occur. As the number of observation increases, frequency of a specific event oscillates around a certain number that becomes probability for significantly large number of experiments (compare the law of large numbers). Comparison of basic concepts of probability theory and mathematical statistics is given in Table 31.

Table 31. Basic concepts used in probability theory and mathematical statistics

PROBABILITY THEORY	**MATHEMATICAL STATISTICS**
random variable	statistic feature
probability	frequency
distribution of a random variable	(interval) series of a feature frequency
diagram of a random variable distribution	frequency histogram of an examined feature
expected value	arithmetic mean (average)
variance	variance
standard deviation	standard deviation

1.2 Basic statistic measures

Arithmetic mean (average)

The basic statistic measure often used in practice of planning and control, but not being a risk measure, is arithmetic mean (average) [39]. It belongs to the so-called "traditional distribution measures". The average ($\overline{x}$) is total of the feature values in the entire set, divided by cardinality of this set. The formula for the average is as follows [24]:

$$\overline{x} = \frac{x_1 + x_2 + ... + x_n}{n} = \frac{1}{n}\sum_{j=1}^{n} x_j \ , \qquad \text{(Z1.1)}$$

where:

x_j (j = 1,...,n) = individual observations in the data set,

n = number of observations.

The average has the following properties [39, 24, 53]:

- all values of the set are considered in the calculations and a change of any value in the set entails a change of the average;
- the average value can not be less than the smallest value of the examined feature and can not be larger than the largest value;
- total of deviations (differences) of the examined feature values from their average is equal to zero, i.e. $\sum_{j=1}^{n}(x_j - \bar{x}) = 0$;
- total of square deviations of the examined feature values from the average is minimum; this means that total of square deviations of individual variables from any other variable different than the average will be always larger:

$$\sum_{j=1}^{n}(x_j - C)^2 = \min . \qquad \text{for } C = \bar{x} ,$$

- product of the average and the cardinality is equal to total values of the feature in the set, i.e. $\bar{x}n \sum_{j=1}^{n} x_j$.

Advantages of the average are as follows [3, 24, 39, 53]:

- to its calculation employed are all the data characterising distribution of the random variable;
- its value does not depend on cardinality of the set, but on mutual proportions of the examined feature, which in practice means a possibility of calculating it from the series, where in place of cardinalities, known are structure indices only;
- it can be used in further statistical calculations.

As disadvantages of the average, the following can be considered [3, 24, 39, 53]:

- possibility of accepting values absent in the examined population or not assumed by the examined feature;
- possibility of misleading in the case when one or more results are "abnormally large or small".

Depending on distribution of the statistic feature and necessity of assigning properly bigger importance to certain measured values, one can also use geometric or harmonic mean, median, dominant or other means [53].

Expected value

For random variables, an equivalent of the arithmetic mean is the expected value. This quantity is characterised by mean value accepted by the random variable. The formula for the expected value of a random variable is as follows [24]:

$$E(X)=\begin{cases}\sum_i x_i p_i & \textit{for discontinuous random variable}\\ \int_{-\infty}^{\infty} xf(x)dx & \textit{for continuous random variable}\end{cases} \qquad \text{(Z.1.2)}$$

where:

p_i = probability function of random variable *X*, accepting values x_i (*i = 1,2,...*),

f(x) = density function.

Expected value is also called mean value, average and also mathematical expectation.

Mean measures determine the mean level of a phenomenon, however they do not inform about variability of the examined feature. The characteristics describing distribution of a feature include *dispersion measures*, called also *measures of dispersion* or *measures of diversification*. They permit measuring diversification of the variable value within the examined population, and thus inform, how big are differences (deviations) between individual values of the population units and the measure of mean [53].

1.3 Basic characteristics of continuous random variables

Density function

Density function of a continuous random variable *X* is named the function $f(x)$ determined as follows [24]:

$$f(x)=\lim_{\Delta x\to 0}\frac{P(x<X\le x+\Delta x)}{\Delta x}, \qquad \text{(Z.1.3)}$$

where *P* = probability.

Cumulative distribution function

Distribution of a continuous random variable X can be also characterised by the cumulative distribution function (or just distribution function) determined as follows [24]:

$$F(x) = \int_{-\infty}^{x} f(t)dt\,, \tag{Z.1.4}$$

where $f(t)$ = density function of random variable X .

Distribution function $F(x)$ of a continuous random variable X has the following properties:

- $0 \le F(x) \le 1$ for $x \in R$,
- $\lim_{x \to -\infty} F(x) = 0$ and $\lim_{x \to +\infty} F(x) = 1$,
- $F(x)$ is a monotonically non-decreasing and right-continuous function.

Probability density function

Probability density function of a continuous random variable is the function $f(x)$ determined on the set of real numbers, with the following properties [24]:

- $f(x) \ge 0$,
- $\int_a^b f(x)dx = P(a < X \le b)$ for any $a < b$.

Graphic interpretation of the integer $\int_a^b f(x)dx$ is the area limited by diagram of the function $f(x)$, axis of abscissae and the lines $x = a$ and $x = b$. This is shown in Fig. 48.

Normal distribution of random variable

Normal distribution is the most important distribution of a continuous random variable, both in theoretical considerations and in practical applications of mathematical statistics [19]. Most of the phenomena occurring in the nature demonstrate a normal distribution. It is one of the most common distributions used in the case of industrial processes.

A random variable has the normal distribution with the parameters m and σ, which in short is written as X: $N\ (m, \sigma)$, if its density function is the following form [24]:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-m)}{2\sigma^2}}, -\infty < x < \infty \qquad \text{(Z.1.5)}$$

where $\sigma > 0$.

Diagram of this function, usually determined as the normal curve (Gaussian or bell curve), has the characteristic form as shown in Fig. 48:

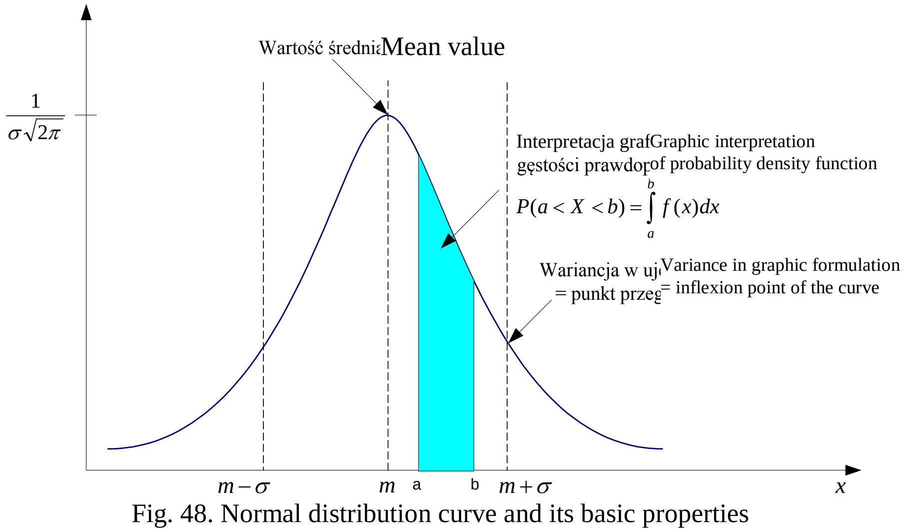

Fig. 48. Normal distribution curve and its basic properties

Normal distribution function

According to the formula (Z.1.4), the normal distribution function has the form:

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{(t-m)^2}{2\sigma^2}} dt \qquad \text{(Z.1.6)}$$

Diagram of distribution function of a random variable X: $N(m,\sigma)$ is shown in Fig. 49.

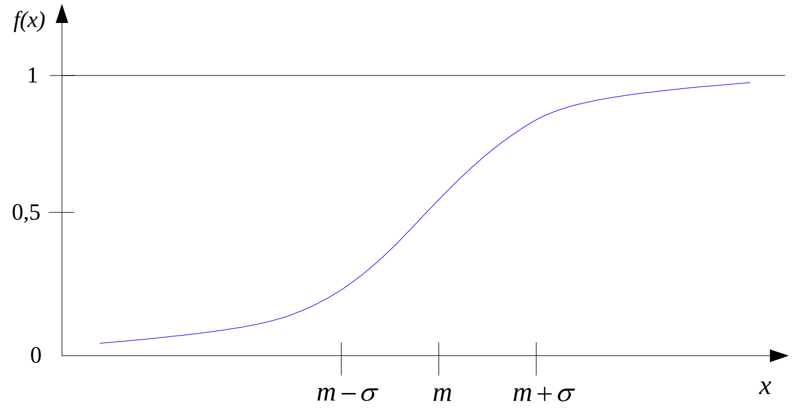

Fig. 49. Diagram of normal distribution function

The formula (Z.1.6) encloses two parameters: m and σ, unequivocally determining form of the density function (π and e are constants). Meaning of these parameters is revealed after calculating the expected value and variance in the normal distribution:

$$E(X)=\int_{-\infty}^{\infty} x\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-m)}{2\sigma^2}}dx=m \qquad \text{(Z.1.7)}$$

$$D^2(X)=\int_{-\infty}^{\infty}(x-m)^2\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-m)^2}{2\sigma^2}}dx=\sigma^2. \qquad \text{(Z.1.8)}$$

So, the parameter m is the average of the random variable X with normal distribution, and the parameter σ is its standard deviation.

The density curve of a normal distribution has the following properties [24]:

- is symmetrical with respect to the line $x=m$,
- reaches its maximum of $\frac{1}{\sigma\sqrt{2\pi}}$ for $x=m$,
- its arms have inflexion points for $x=m-\sigma$ and $x=m+\sigma$.

Standardisation of random variable

Calculating probability on a given range $(a,b>$ of normal distribution is very troublesome, because this probability is equal to the area under the curve and would require integrating the density function with proper parameters within the limits *a* and *b*. In such cases, any normal distribution is reduced to the so-called standard normal distribution for that the density function and the distribution function are tabularised. The normal distribution with mean value $m = 0$ and standard deviation $\sigma = 1$ is called standard normal distribution *N (0,1)*. It is conventional that a random variable having standard normal distribution is designated by *U*, its density function as $\varphi(u)$ and its distribution function as $\Phi(u)$.

Because the function $\varphi(u)$ is symmetrical in relation to the line $u = 0$, the tables frequently include values of both functions for positive values of *u* only. When determining the values $\varphi(u)$ and $\Phi(u)$ for negative *u*, one employs the following properties of these functions [24]:

- $\varphi(u) = \varphi(-u)$, and (Z.1.9)
- $\Phi(u) = 1 - \Phi(-u)$. (Z.1.10)

The operation named standardisation of a random variable *X* permits using the tables of standard normal distribution to calculate probabilities like $P(a < X \leq b)$ for the random variable *X* with distribution parameters *m* and σ. It can be proved that, if the random variable *X* has the distribution *N* (m, σ), the standardised variable is [24]:

$$U = \frac{X - m}{\sigma} \qquad \text{(Z.1.11)}$$

On this ground one obtains, that [24]:

$$P(a < X \leq b) = P\left(\frac{a-m}{\sigma} < \frac{X-m}{\sigma} \leq \frac{b-m}{\sigma}\right) = \dots .$$

$$\dots P\left(\frac{a-m}{\sigma} < U \leq \frac{b-m}{\sigma}\right) = \Phi\left(\frac{b-m}{\sigma}\right) - \Phi\left(\frac{a-m}{\sigma}\right)$$

The values $\Phi\left(\frac{b-m}{\sigma}\right)$ and $\Phi\left(\frac{a-m}{\sigma}\right)$ should be read from the tables of distribution function of standardised normal distribution.

Bayes' theorem

When calculating probability of complex events or streams of events, one usually uses known probabilities of more elementary events based on basic computational rules of probability theory. A natural reaction of everybody who faces the problem of taking a decision in the situation including an element of uncertainty is striving to remove this uncertainty by revealing the truth about the actual status quo. Having such knowledge at the disposal, one calculates updated a posteriori probabilities on the ground of the earlier determined a priori probabilities. One of the methods making possible calculating a posteriori probabilities is the Bayes' method. The Bayes' theorem is a slight extension of the conditional probability [24].

Let $A_i (i=1,2,...n)$ be the only possible and mutually exclusive events, and B is the event that can happen only on the condition of occurrence of one of the events A_i. Then, probability of the event E_i on the condition that the event B happened, is given by the formula [47]:

$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{\sum_{i=1}^{n} P(A_i)P(B|A_i)} \qquad \text{(Z.1.12)}$$

1.4 Interrelations between features

When examining a two-dimensional population (X,Y), important is whether and how much the variables X and Y are correlated. This is performed by analysing properties of the correlation coefficient that in an empirical distribution of the variables X and Y is determined by the formula [24]:

$$r = \frac{c_{xy}}{s_x s_y}, \qquad \text{(Z.1.13)}$$

where c_{xy} = covariance in two-dimensional empirical distribution described by the formula:

$$c_{xy} = \frac{1}{n-1} \sum_{i=1}^{k} \sum_{j=1}^{k} (x_i - \bar{x})(y_j - \bar{y}) n_{ij}, \qquad \text{(Z1.14)}$$

and

s_x, s_y = standard deviations in empirical marginal distributions of the variables X and Y, respectively.

The correlation coefficient determines direction of the relationship, because it measures linear correlation of the variables.

The way how a random variable is formed under influence of another one, can be shown in analytic way by means of the so-called **regression model**. The main component of each regression model is the regression function, whose analytic form is most often determined on the ground of results of a random sample. Parameters of this function are subject to estimation based on the random sample using procedures established according to the correlation and regression theories.

From among many possible forms of the regression model, of the basic importance is the so-called **classic linear regression model**. In the first model, for each fixed value of the variable X, the other variable Y has a conditional distribution with the expected value:

$$E(Y \mid X = x) = \alpha x + \beta \qquad \text{(Z.1.15)}$$

and variance:

$$D^2(Y \mid X = x) = \sigma^2 . \qquad \text{(Z.1.16)}$$

The notation (Z.1.15) means that the 1st type regression function of Y in relation to X is linear and the notation (Z.1.16) means that variance of the random variable Y in its conditional distributions is constant (does not depend on the x values).

In the classic linear regression model, the random variable Y plays the role of the dependent variable, and X is the independent variable. The parameter α of the 1st type regression line is described as the linear regression coefficient, with the following interpretation: this is the value by that the conditional expected value of the variable Y changes, when x increases by one unit.

The classic linear regression model can be also presented by means of a sequence of pairs (x_1,Y_1), (x_2,Y_2), ..., (x_n,Y_n) of an n-element random sample from a two-dimensional population making ground for estimating parameters of the examined relationship. Assuming that maintained are conditions of the classic linear regression, forming the values Y_i in the random sample can be presented as follows [24]:

$$Y_i = E(Y \mid X = x_i) + \varepsilon_i = \alpha x + \beta + \varepsilon_i \text{ (i=1, 2, ... n)}, \qquad \text{(Z.1.17)}$$

where ε_i = such random variables, that:

$$\mathrm{E}\,\varepsilon_i = 0, \qquad \text{(Z.1.18)}$$

$$D^2(\varepsilon_i) = E\varepsilon_i^2 = \sigma^2, \qquad \text{(Z.1.19)}$$

$$\text{cov}(\varepsilon_i, \varepsilon_j) = E\varepsilon_i\varepsilon_j = 0 \text{ for } i \neq j. \qquad \text{(Z.1.20)}$$

Estimation of parameters α and β

Parameters of the classic linear regression model are estimated by means of the least squares method. This problem is reduced to selecting such values of coefficients of the line equation, that its diagram possibly well "fits" to the points representing scatter of individual observations of the sample. The criterion of fitting the line to the sample data by the least squares method is minimisation of sum of squared "vertical" lengths linking the empirical points with the line. This criterion can be written in the following way [24]:

$$S = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} [Y_i - (\alpha x_i + \beta)]^2 \to \min. \qquad \text{(Z.1.21)}$$

The expression S is a function of the unknowns α and β. After differentiating S with respect to α and β, equating the derivatives to zero and solving the set of normal equations, one obtains:

$$\hat{\alpha} = \frac{\sum_{i=1}^{n} (x_i - \overline{x})(Y_i - \overline{Y})}{\sum_{i=1}^{n} (x_i - \overline{x})^2} \text{ and} \qquad \text{(Z.1.22)}$$

$$\hat{\beta} = \overline{Y} - \hat{\alpha}\overline{x} \qquad \text{(Z.1.23)}$$

ANNEX 2

DETAILED STRUCTURE OF PRODUCT AND MANUFACTURING PROCESS OF BOGIE FRAME MD 523

2.1. Cross beam

2.2. Solebar

2.3. Swing bolster

2.4. Swing bolster body

2.5 Bogie frame

1.1. ***Cross-beam***

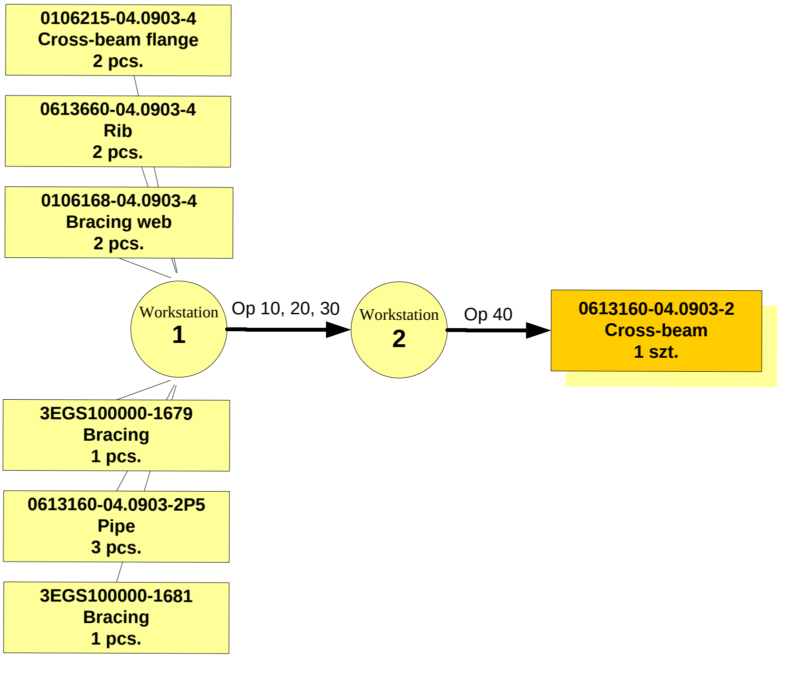

Fig. 50. Product structure and manufacturing process structure of cross-beam

2.2. *Solebar*

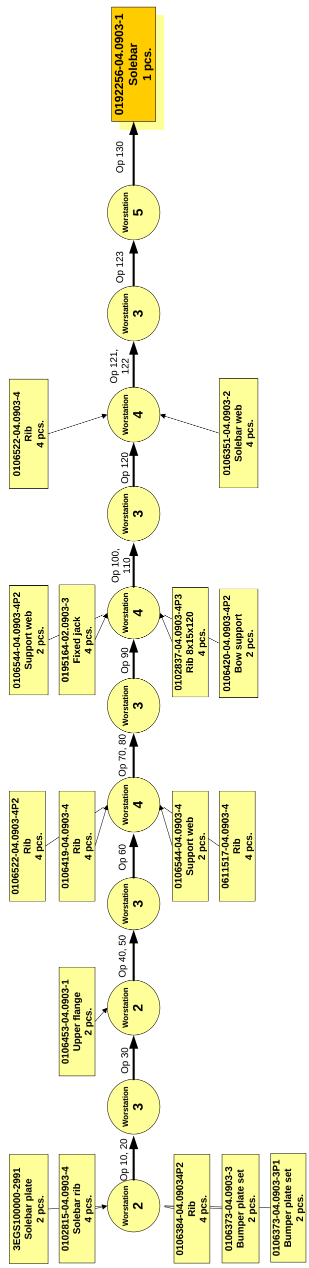

Fig. 51. Product structure and manufacturing process structure of solebar

2.3. *Swing bolster*

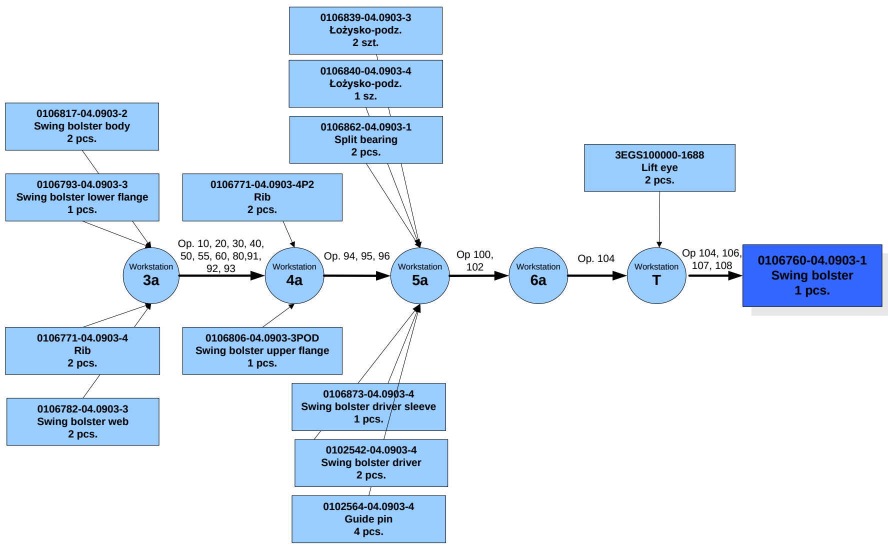

Fig. 52. Product structure and manufacturing process structure of swing bolster

2.4. *Swing bolster body*

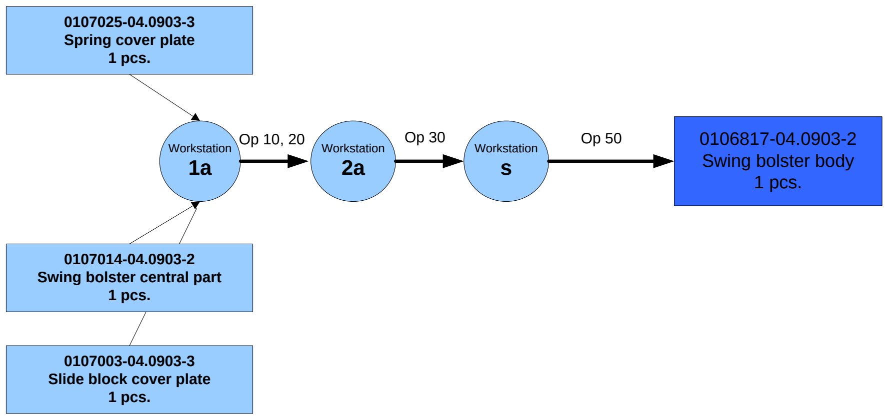

Fig. 53. Product structure and manufacturing process structure of swing bolster body

2.5. *Bogie frame*

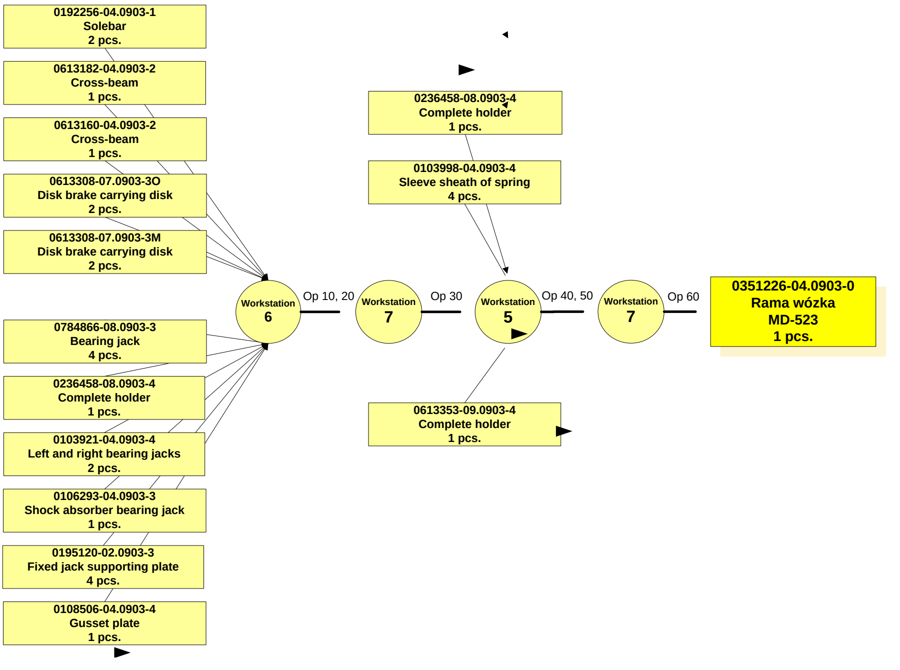

Fig. 54. Product structure and manufacturing process structure of MD 523 bogie frame

ANNEX 3

GLOSSARY

Operational research [8] – is involved in issues of analysis, decision-making and control of complex operational systems, and thus covers the optimisation and decision-making theory, as well as issues related to performing calculation and decision-making operations in computer systems. In wider sense, it is engaged in the problems of controlling and managing complex systems composed of people, machines, materials and money in industry, economy, administration and defence.

Statistical features (or ***features***) [24] – properties of elements in a general population.

Measurable features [24] – features of quantitative nature (e.g. age, weight).

Non-measurable features [24] – features of qualitative nature (e.g. sex, colour of eyes).

Continuous features [24] – features assuming real values, and so values of an uncountable set. If the feature assumes a great number of values, it is treated as continuous one, irrespective whether it is continuous or discrete by definition (so-called quasi-continuous feature).

Step features (or ***discrete features***) [24] – features assuming values from a finite or countable set on a given numerical scale; most often this is a set of positive integers.

Risk factor – factor that appears in the system in random way and can not be simply eliminated by organisational changes.

Organisational factor – factor that causes disturbances in the production system but can be eliminated by simple organisational changes.

Dispersion – difference between the values obtained and expected.

Economical efficiency [30] – index of a production system, determining quotient of the achieved result and expenditures spent on its achieving.

Empirical distribution of a feature [24] – set of properly defined frequencies of occurrence assigned to increasingly ordered values accepted by the given feature.

Histogram [24] – set of rectangles, whose bases are determined on the axis of abscissae by class ranges, and heights are determined on the axis of ordinates by multiplicities corresponding to individual class ranges.

Measure [16] – measure determining size, quality, value etc.

Model [16] – representation of the most important features of an examined or designed object from the viewpoint of the task to that it serves in the determined reality or abstraction.

Modelling [19] – activity consisting in matching to the original an acceptable substitute called a model, i.e. approximate representation of the most important properties of the original.

Undamageability [42] – ability of the object to performing required functions (i.e. correct operation not interrupted by a failure) in a given conditions, in a given period.

Reliability [42] – ability of the object to remaining in the condition that permits performing required functions in a given conditions and at a given moment, assuming that all the required external means are delivered.

Object [42] – any simple or complex system. e.g. element, instrument, system, subsystem, functional unit, appliance) being the subject of consideration related to solving a specific problem.

Maintainability [42] – ability of the object to maintaining or reproducing, in given operation conditions, the state in that it can perform required functions, assuming that the operation takes place in given conditions with maintaining established procedures and means.

Critical area [24] – area of rejecting the hypothesis.

Parameter [16] – value characteristic for a given process or appliance.

General population (or just ***population***) [24] – statistical population on that the statistical examination is performed; its elements are material objects or phenomena having at least one distinguishing property.

A posteriori probability [38] – probability updated a priori, modified on the ground of new information.

A priori probability [38] – ordered probability assigned to a certain event at a specific moment.

Production process [35] – ordered set of activities (operations, actions) aimed at manufacturing a product (good or service).

Productivity [35] – index of production systems that determines ratio of output vectors to input vectors, i.e. ratio of results of production activities to expenditures used to this end.

Sample [24] – subset of elements of a population subject to examination.

Undertaking [33] – organised human's activity aimed at achieving a determined goal, included in a finite time interval, with distinguished start and end, and executed by finite number of persons, finite quantity of technical means, energy, financial means and information.

Class interval [53] – an interval created thanks to determining distribution of a continuous feature by assigning multiplicities (frequencies) to corresponding intervals of the feature values and not to its specific values.

Distribution a posteriori [24] – a priori distribution corrected by additional information from the sample.

Distribution a priori [24] – accepted probability distribution of individual states of nature at the beginning of a decisive analysis and before the experiment.

Distribution of a feature [24] – differences between values of a considered feature and features in the population.

Semivariance [47] – equivalent of variance, considering only negative deflections from the mean or expected value.

Effectiveness [48] – index of a production system, determining degree of achieving the assumed goal by the system.

Performance [48] – index of a production system, determining ratio of actual production volume to the achievable volume.

Standard normal distribution [24] – normal distribution with mean value $m = 0$ and standard deviation $\sigma = 1$.

State of nature [24] – factor that is not controlled by the decision-maker but influences the result of taking the decision.

Workstation [35] – the smallest (indivisible) production cell; a workstation is determined as a zero-degree production cell (KP^0).

Production structure [7] – arrangement of production cells and a set of cooperative relations between them.

Parallel structure of production systems [36] – reliability structure in that risk of unreliability of one element increases risk of the entire system by risk value of this element.

Parallel structure in terms of classic reliability theory [36] – reliability structure of a system whose operation requires operation of at least one object/subsystem.

Series structure [36] – reliability structure of a system whose operation requires operation of all the objects/subsystems.

Series-parallel structure [36] – mixture of a series and a parallel structures.

Simulation [16] – technique of performing experiments on some kinds of models, which describe behaviour of complex systems in certain periods.

Continuous simulation [21] – simulation in that changes occur continuously in time, if values accepted by descriptive variables can be presented by real numbers or their intervals.

Deterministic simulation [21] – simulation whose course is not subject to probability, has predictable inputs and gives predictable results.

Discrete simulation [21] – simulation in that changes appear in determined points of time; changes in the model appear at the moments when some events occur.

Stochastic simulation [21] – simulation based on stochastic processes, i.e. ones built of a random sequence of generated values.

System (in terms of the systems theory) [8] – certain entirety in that selected components cooperate. Operation of a system depends on functioning its components and relations between them. Interrelationships between the components determine the system structure. One can say about a system only when the entirety was organised and is to realise a determined goal divided to sub-goals for individual parts.

Production system [7] – system of interconnected material, energy, personnel, capital and information resources. It is intentionally designed and organised in order to meet the customers' needs.

Structural series – see ***stem-and-leaf graph***.

Arithmetical mean (***average***) ($\bar{x}$) [24] – sum of values of a feature in the entire set, divided by multiplicity of that set.

Reliability theory [36] – field of applied science, involved in examining and designing objects (elements, systems) from the viewpoint of meeting by them the imposed requirements (during a given period and in determined conditions).

The systems theory (technique) [8] – called also the systems engineering or systems science – field involved in theoretical and technical problems of modelling, identifying, analysing and controlling in relation to systems of various nature. In particular, it includes the theory and technique of decision-taking, as well as the theory and technique of optimisation.

Durability [42] – time that passes from the beginning to the end of an object operation.

Variance [24] – statistical measure of dispersion; it determines scatter degree of a feature or a random variable value around its mean or excepted value.

Expected value [24] – equivalent of arithmetical mean; it characterises mean value of a random variable.

Asymmetry coefficient [53] – measure of symmetry of a feature or random variable.

Variation coefficient [24] – measure of relative diversification of a feature or random variable.

Management [8] – science about managing in the narrower sense that includes taking decisions and controlling objects in which executed are complex activities with participation of people. Most often they are economical processes and the goal of controlling them considers economical indices. In the wider sense, it includes entirety of problems related to managing complex undertakings.

Elementary event [24] – result corresponding to the considered random experiment (e.g. dice throw).

Random event [24] – event whose result can not be forecast in spite of specifying conditions in which the event happens.

Goal variable [24] – variable reflecting goal of an activity. To achieve it, one seeks a solution of the decisive problem and the best decision.