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DEMAND FORECASTING IN A BUSINESS BASED ON EXPERTS' OPINIONS – AN APPLICATION OF WEIBULL DISTRIBUTIONS

Abstract: Demand forecasts in a business may be constructed by various methods, e.g. by using Type-I formal models, or by using type-II formal models based on experts' opinions. The experts can be the business's managers or persons from outside of the studied business. The experts can not only construct forecasts, but also subjectively specify the probability of them coming true. In this article, the Weibull distribution is described, a distribution that may be applied in the process of constructing product demand forecasts for a business. The methodology for constructing a point forecast is explained, along with the methods for evaluating the chances of the forecast coming true and the methods for judging the probability connected with sales profitability.

Keywords: demand, demand forecasting, experts' opinions, Weibull distribution applications.

1. Introduction

Forecasting is the process of predicting future events, which has as its aim the reduction of risk in the business management process. In any business, an important role is played by forecasts concerning the amount of demand for the business' product, which form the foundation for subsequent sales plans. Sales plans are the starting point for the creation of production plans, which then influence supply and employment strategies, etc.

Demand for a business' products is an unobservable quantity. Because of this, many different variables are used to describe it. These can be, for example, orders placed by buyers or recorded product sales. There is no problem with the choice of variable if the demand for a business' product is fully satisfied by the business. In such a case, the values of both variables are identical. In practice, however, few businesses fulfil all of their orders in full and within the order deadline. In such cases the problem arises of choosing an appropriate variable describing demand. The choice of this variable depends mainly on the purpose for which the forecast is being constructed. It can also be determined by the available forecasting data.

Demand forecasting is based on the dependencies that characterize demand or the dependencies that exist between demand and other phenomena. These may be either retrospective dependencies (those that have occurred up to this point) or prospective dependencies (those that the forecaster assumes to occur in the future). In the first case, type I formal models are used for constructing forecasts [Abraham, Ledolter 2005; Clements 2002; Montgomery et al. 1990; Pardoe 2006]. The parameters of these models are estimated by statistical methods. In the second case, type-II formal models may be used for forecast construction. The form and the parameters of these models are determined on the basis of experts' opinions [Dittmann et al. 2011]. Type II formal models are mainly used in the case of new products being introduced to the market and also when it is believed that retrospective dependencies will change during the time period the forecast covers.

Experts may ascribe subjective probabilities of fulfilment to the judgements (opinions) that they create. A *subjective probability* is a probability of an event occurring that is determined by an expert. It specifies, for example, what is the probability, according to the expert, that the value of the forecasted variable (demand size) during the forecasted time interval will

– be equal to a specified value (point forecast),
– be contained in a specified numerical interval (interval forecast).

By specifying this probability, the expert (forecaster) is basing it on his/her own intuition and experience, and on the available information concerning the forecasted variable. A substantial problem connected with the use of experts' opinions for evaluating probability is the impossibility of verifying the accuracy of such an evaluation. Even if the forecast proves to have been accurate, it does not mean that the subjective probability had been accurately specified. Because of this, it is important that the experts used are people who possess an appropriate amount of knowledge about the forecasted phenomena. Experts may base their judgement on retrospective dependencies that concern the forecasted phenomenon and on preliminary forecasts constructed with the use of formal type I models. In modifying these initial forecasts, experts are basing their predictions on subjective opinions concerning the effect of phenomena not considered in the initial forecasting process. It is worth using such a forecasting process in business, as it allows for an integration of type-I formal models with experts' opinions. However, it requires the involvement of the business' managers in the forecasting process.

Based on experts' judgement concerning the chosen values of the variable or the distribution parameters, it is assumed that the forecasted variable follows a given probability distribution, e.g. a trapezoidal, beta triangular, normal, or a Weibull distribution [Dittmann 2006]. The point forecast is most often assumed to be:

1) the expected value or,
2) the mode of the probability distribution.

Because forecasting errors cause negative consequences for a business, the forecasts that are used in managing a business should be acceptable forecasts. These

are forecasts whose degree of uncertainty is acceptable to their users. An evaluation of a point forecast constructed by experts may be conducted by various methods. The estimated error or the reliability (chances of fulfilment) of the constructed forecast may be determined.

The aim of this paper is to demonstrate that the use of the Weibull probability distribution in the construction of forecasts based on experts' opinions allows for:
– constructing sales forecasts in a business,
– estimating the chances of forecast fulfilment,
– estimating the risk associated with sales profitability.

2. The Weibull distribution

The Weibull distribution may find widespread use in the construction of forecasts based on experts' opinions. This results from the asymmetry of this distribution and also from the fact that the domain of the distribution has a lower bound. For many variables, a lower bound may be determined with a high degree of certainty (for example for the amount of sales we can assume zero to be the lower bound). On the other hand, it is seldom possible to possess such a certainty about the maximum value.

The probability density function of the Weibull distribution has the form:

$$f(x)=\frac{\beta}{\gamma}\left(\frac{x-a}{\gamma}\right)^{\beta-1}\exp\left[-\left(\frac{x-a}{\gamma}\right)^{\beta}\right] \quad \text{dla } x\in[a,+\infty),\ \beta>0,\ \gamma>0, \tag{1}$$

where: a – minimum value of the variable,
b – shape parameter,
g – scale parameter.

The expected value (m) and the standard deviation (s) are a function of the distribution parameters

$$m=\gamma\cdot\Gamma\left(1+\frac{1}{\beta}\right)+a, \tag{2}$$

$$s=\gamma\cdot\sqrt{\Gamma\left(1+\frac{2}{\beta}\right)-\Gamma^{2}\left(1+\frac{1}{\beta}\right)}. \tag{3}$$

where: G – the Euler gamma function.

3. Point forecasts

Let us consider two cases of applying the Weibull distribution for constructing sales forecasts in businesses.

Case I

An expert specified the minimal value of the forecasted variable a, the most probable value of the variable w and the α – percentile of the distribution q_α.

An α percentile of a probability distribution of the forecasted variable Y (q_α) is such a value for which α per cent of variable values is not greater than that value.

The Weibull distribution parameters γ and β may then be found by solving a system of equations:

$$\begin{cases} \left[\dfrac{-\ln(1-\alpha)}{\dfrac{\beta-1}{\beta}}\right]^{\frac{1}{\beta}} = \dfrac{q_\alpha - a}{w-a} \\ \gamma = (w-a)\left(\dfrac{\beta}{1-\beta}\right)^{\frac{1}{\beta}} \end{cases} . \tag{4}$$

Subsequently, from equations (2) and (3), we can determine the expected value of the distribution and the standard deviation.

Examples of applications of the presented procedure for determining a point sales forecast, the forecasting interval, and the probability of claim fulfilment are shown in the paper by Dittmann 2007.

Case II

The Weibull distribution parameters may also be determined based on percentiles [*Analiza systemowa*… 1985, p. 698; Chen, Hu 1991, pp. 26–28; Dittmann 2008].

The procedure for this is as follows.

An expert specifies the values of a certain number of distribution percentiles, or the order of a certain number of percentiles (the values of the probability distribution function). These may be answers to questions concerning:

1) the value of the percentile – for example for what value are the chances (probability) that sales will be no smaller (no larger) than this value will be 0.05?
or

2) the percentile order – for example what are the chances (probability) that sales will be no smaller (no larger) than ten thousand units?

With this method we obtain a certain number of values of the empirical subjective probability distribution function. Next, we look for such a value of the Weibull

distribution parameters γ and β, for which the theoretical distribution function $F(x)$ has the best fit to the obtained values (based on experts' answers to the posed questions) of the empirical distribution function. As a measure of fit we may use s^2:

$$s^2 = \frac{(\alpha_i - F(x_i))^2}{n}, \tag{5}$$

where: α_i – the value of the empirical distribution function at the point x_i (the order of the i-th percentile),
$F(x_i)$ – the value of the theoretical distribution function at the point x_i,
n – the number of percentiles.

By minimizing the value of this statistic, it is possible to calculate the values of the Weibull distribution parameters γ and β. Assuming that the minimum sales amount is zero, the remaining distribution parameters: the expected value (m), the mode (Mo), the median (Me) and the standard deviation, may be calculated from equations (6), (7), (8), (3):

$$m = \gamma \cdot \Gamma\left(1 + \frac{1}{\beta}\right), \tag{6}$$

$$Mo = \beta\left(1 - \frac{1}{\gamma}\right)^{\frac{1}{\gamma}}, \tag{7}$$

$$Me = \beta(ln2)^{\frac{1}{\gamma}}. \tag{8}$$

Example 1

In order to construct a sales forecast in a certain business, an expert was asked the following questions (percentile order)

1. What is the chance that the sales will be no larger than 10 000 units?
2. What is the chance that the sales will be no larger than 15 000 units?
3. What is the chance that the sales will be no larger than 20 000 units?
4. What is the chance that the sales will be no larger than 45 000 units?
5. What is the chance that the sales will be no larger than 60 000 units?
6. What is the chance that the sales will be no larger than 65 000 units?
7. What is the chance that the sales will be no larger than 70 000 units?

Based on the answers to these questions, we get:

0.05 – from question 1,
0.1 – from question 2,
0.2 – from question 3,
0.7 – from question 4,
0.8 – from question 5,
0.9 – from question 6,
0.95 – from question 7,

A plot of the empirical distribution function values given by the expert was prepared (Figure1). In order to obtain the best fit of the Weibull distribution function to this plot, the Solver tool in the Excel spread sheet program was used, minimizing the s^2 statistic. The distribution parameter values are contained in Table 1, while the plots of the distribution function and the density function are shown in Figures 1 and 2.

We can assume that the point forecast is the calculated expected value of the distribution, 38 060 units or the modal value, 30 040 units.

Table 1. The Weibull distribution parameters[1]

Parameters	Symbol	Values
Shape	*b*	42.94
Scale	*g*	1.98
Expected value	*M*	38.06
Modal	Mo	30.04
Median	Me	35.67
Standard deviation	*S*	20.12

Source: author's own.

4. Interval forecasts. The chances of achieving a sales level contained in a numerical interval

In managing a business, it is often more useful to construct an interval forecast (an interval of forecast) rather than a point forecast.

Let us note that by accepting an assumption about the form of the probability distribution, we may, apart from a point forecast, obtain answers to the following questions which are significant in the process of managing a business:

What is the probability that the value of the forecasted variable (amount of sales) will be smaller or larger than the predicted value?

What is the probability that the amount of sales will be contained in a given interval?

What is the probability of sales not being profitable?

What should the threshold value for the forecasted variable be, so that the probability of obtaining a larger or smaller value (for example, at what level should sales be planned so that the probability of not fulfilling or exceeding the sales plan) is appropriately small?

Answers to the first, second and third questions may be obtained by finding the orders of the corresponding percentiles, meaning the values of the distribution

[1] A program made by K. Poradowska was used for the calculations.

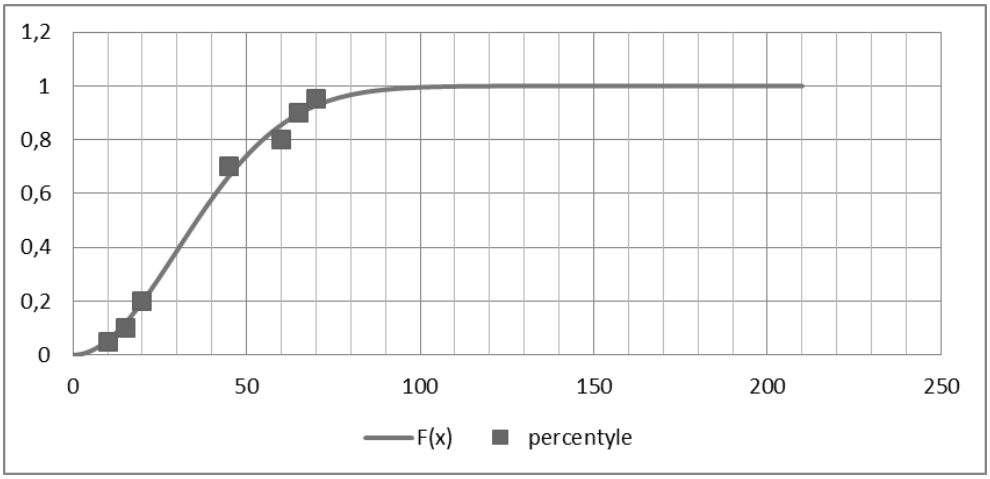

Figure 1. Distribution function $F(x)$

Source: author's own.

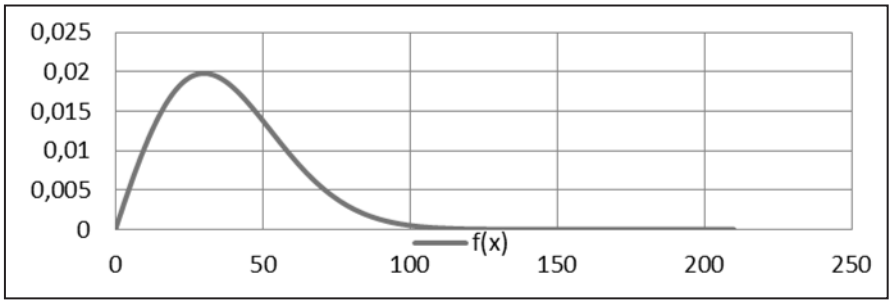

Figure 2. Density function $f(x)$

Source: author's own.

function of the assumed probability distribution. By using an inverse distribution function, or in other words, by finding the corresponding percentile of the distribution, we may obtain the answer to the fourth question.

Example 2

Let us assume that the sales amount follows the Weibull distribution described in Example 1. Based on the chosen percentile values contained in Table 2, we can claim, among other things, that the probability that the business's sales will be larger than 9600 units is 0.95, and that it will be larger than 90 000 units – 0.01. By finding the orders of the appropriate percentiles, we may also determine the probability that the sales amount will be contained in the interval of, for example, 75 000–90 000 units. This probability has the value of 0.04.

If we know the form of the distribution function, we may use this to determine the risk of sales not being profitable for the business. For example, if in the example being considered, the level of sales that ensures profitability is 9000 units, then the chances of achieving at least such a level are 95.5%. Therefore, the risk associated with this venture is rather small. However, if the condition of achieving profitability were sales of at least 100 000 units, then the risk of failure would be quite large. This is because the probability of obtaining such a level is only 0.5%.

Table 2. The chosen percentile values

Percentile		Percentile	
order	value	order	value
0.01	4.3	0.95	75.0
0.02	6.0	0.96	77.0
0.03	7.4	0.97	80.0
0.04	8.5	0.98	85.0
0.05	9.6	0.99	90.0

Source: author's own.

5. Conclusion

Constructing forecasts in a business may be based on experts' opinions. Depending on the judgements formulated by experts, concerning the chosen values of the forecasted variable or the parameters of its distribution, it is assumed that demand has a given subjective probability distribution. One such distribution is the Weibull distribution. The parameters of this distribution may be calculated based on experts' answers to question about the value or order of certain percentiles of the distribution. The distribution function determined from these answers allows us to obtain answers to the question of what are the chances that the demand for a business' products will be higher or lower than that assumed in the sales plan or will be encompassed by a specified interval. It is also possible to estimate the probability of sales not being profitable.

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PROGNOZOWANIE POPYTU W PRZEDSIĘBIORSTWIE NA PODSTAWIE OPINII EKSPERTÓW – ZASTOSOWANIE ROZKŁADU WEIBULLA

Streszczenie: Prognozy popytu w przedsiębiorstwie mogą być budowane różnymi metodami, m.in. przy użyciu modeli formalnych I rodzaju oraz modeli formalnych II rodzaju opartych na opiniach ekspertów. Ekspertami mogą być menedżerowie przedsiębiorstwa lub osoby z zewnątrz przedsiębiorstwa. Eksperci, oprócz budowy prognoz, mogą także określić subiektywne prawdopodobieństwo ich realizacji. W artykule przedstawiono jeden z rozkładów prawdopodobieństwa subiektywnego – rozkład Weibulla – mogący znaleźć zastosowanie w procesie konstrukcji prognoz popytu na produkty przedsiębiorstwa. Pokazano sposób konstrukcji prognozy punktowej, sposób oceny szans realizacji prognozy oraz sposób oceny ryzyka związanego z opłacalnością sprzedaży.

Słowa kluczowe: popyt, prognozowanie popytu, opinie ekspertów, zastosowanie rozkładu Weibulla.