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FORECAST OF PRICES AND VOLATILITY ON THE DAY AHEAD MARKET

Abstract: The subject of this paper is the forecast of prices and volatility on the Day Ahead Market (DAM). The analysis was made for two portfolios of four contracts from 30.03.2009 to 28.10.2011 for two fixings on DAM. Four out of 24 contracts noted on DAM were chosen by PCA. Prices were forecast by the SARIMA models incorporating autocorrelation and seasonality. Value-at-Risk calculated through the DCC model was used to forecast volatility. These models describe well the prices and volatility on the DAM and may be used for forecasting purposes. Prices on fixing 2 are characterized by higher volatility than prices on fixing 1.

Keywords: principal component analysis (PCA), SARIMA model, DCC model, Value-at-Risk, portfolio.

1. Introduction

Investors on the Polish Power Exchange may participate in the Day Ahead Market (DAM, spot market), Commodity Derivatives Market (CDM, future market), Electricity Auctions, Property Right Market, Emission Allowances Market (CO_2 spot) and Intraday Market. All these markets differ with respect to the investment horizon's length and the traded commodity. The most popular market is DAM where the horizon of the investment is one day. Contracts for electric energy on DAM are characterized by three types of prices: fixing prices, auction prices and intraday prices. Every contract on DAM is a contract on physical delivery of electric energy the next day. DAM offers: 24 contracts for every hour of the day, four block contracts (BASE, PEAK, OFPEAK, MOR) and four indexes (IRDN, IRDN24, SIRDN, SIRDN24), which represent average prices of electric energy on DAM during a day. The fixing price of electric energy on DAM is established three times a day (at 8:00 – fixing 1, at 10:30 – fixing 2 and from 10.35 to 11.30 – fixing 3 (since 23.02.2011)). Several papers [Ganczarek 2008; Ganczarek-Gamrot 2009; 2010] show that prices on DAM are characterized by seasonality, autocorrelation in mean and variance as well as long memory. In this paper, prices are forecasted and risk of price change on

DAM is estimated. The result of this research was used to build the composite portfolio of contracts on electric energy. All 24 time series of daily linear rates of return of electric energy fixing prices from 30.03.2009 to 28.10.2011 were considered.

2. Multivariate linear and nonlinear models

Multivariate linear model of expected value rates of return is as follows:

$$\mathbf{r}_t = \boldsymbol{\mu}_t + \boldsymbol{\varepsilon}_t, \tag{1}$$

where: $\mathbf{r}_t$ – vector of rates of return $N \times 1$,
$\boldsymbol{\mu}_t$ – vector of conditional expected values $N \times 1$,
$\boldsymbol{\varepsilon}_t$ – vector of residuals $N \times 1$ (white noise),
N – number of time series.

The vector of conditional expected value of rate of return of electric energy prices $\boldsymbol{\mu}_t$ is described by *Seasonal Auto-Regressive Integrated Moving Average* SARIMA *(Eng.)* (p, d, q) × (P, D, Q) according to Brockwell and Davis [1996]:

$$p(B)P_s(B^s)\nabla_s^d \mathbf{r_t} = q(B)Q_s(B^s)\boldsymbol{\varepsilon}_\mathbf{t}, \tag{2}$$

where:

$$p(B) = 1 - \sum_{i=1}^{p} p_i B^i, \qquad P_s(B) = 1 - \sum_{i=1}^{P} P_{si} B^i,$$
$$q(B) = 1 - \sum_{i=1}^{q} q_i B^i, \qquad Q_s(B) = 1 - \sum_{i=1}^{Q} Q_{si} B^i,$$

s – seasonal lag,
d – integrated rank,
B – shift operator $B^s r_t = r_{t-s}$,
∇ – differencing operator $\nabla^s r_t = r_t - r_{t-s} = (1 - B^s) r_t$,

This model describes seasonal trend, autoregression and moving average so it is appropriate to analyze expected value of prices and rates of return of prices from the electric energy market. In empirical research, vector $\boldsymbol{\varepsilon}_t$ very often does not have the white noise property. First of all the vector of residuals has varying variance. These properties may be described by the nonlinear autoregression GARCH model:

$$\boldsymbol{\varepsilon}_t = \mathbf{H}_t^{\mathbf{0,5}} \mathbf{u}_t, \tag{3}$$

where: $\mathbf{H}_t$ – conditional covariance matrix $N \times N$: $\mathbf{r}_t \sim D(\boldsymbol{\mu}_t, \mathbf{H}_t)$, $\boldsymbol{\varepsilon}_t \sim D(\mathbf{0}, \mathbf{H}_t)$,
$\mathbf{u}_t$ – vector $N \times 1$ with zero mean and $D^2(\mathbf{u}_t) = 1$.

For the estimation of the matrix $\mathbf{H}_t$ Engle (2002) and Tse and Tsui (2002) proposed independently Dynamic Conditional Correlation (DCC). Model DCC proposed by Engle is as follows:

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{\Gamma}_t \mathbf{D}_t, \quad (4)$$

$$\mathbf{\Gamma}_t = \mathrm{diag}(q_{11,t}^{-0,5};...;q_{NN,t}^{-0,5})\mathbf{Q}_t \mathrm{diag}(q_{11,t}^{-0,5};...;q_{NN,t}^{-0,5}),$$

where: $\mathbf{D}_t = \mathrm{diag}(h_{1t}^{0,5}, h_{2t}^{0,5}, ..., h_{Nt}^{0,5})$ – diagonal matrix $N \times N$, every element of this matrix is univariate GARCH model,

$\mathbf{\Gamma}_t$ – conditional correlation matrix,

$\mathbf{Q}_t = (q_{ijt})$ symmetric, positive matrix $N \times N$:

$$\mathbf{Q}_t = (1-\alpha-\beta)\bar{\mathbf{Q}} + \alpha \mathbf{u}_{t-1}\mathbf{u}_{t-1}^{'} + \beta \mathbf{Q}_{t-1}, \; u_{it} = \frac{\varepsilon_{it}}{h_{it}^{0,5}}$$

$\bar{\mathbf{Q}}$ – unconditional variance matrix of $\mathbf{u}_t$

α, β – positive parameters, $\alpha + \beta < 1$.

More information about univariate and multivariate GARCH models is given by: Osiewalski, Pipień [2002], Zivot, Wang [2006], Fiszeder [2009], Ganczarek [2008], Trzpiot [2010] and Ganczarek-Gamrot [2010].

3. Risk estimation

On the electric energy market, where sudden and dramatic changes of prices are very frequent, one of the appropriate risk measures is Value-at-Risk (VaR). VaR is given by the formula [Piontek 2001; Weron, Weron 2000]:

$$\mathbf{VaR}_\alpha = (\mathbf{R}_\alpha + \boldsymbol{\mu}_t)\mathbf{P}_0. \quad (5)$$

$$\mathbf{R}_\alpha = \mathbf{F}^{-1}(\alpha)\sqrt{\mathbf{H}_t}. \quad (6)$$

where: $\mathbf{R}_\alpha$ – vector of quantiles of order α for portfolio rates of return,

$\mathbf{P}_0$ – vector of prices,

$\boldsymbol{\mu}_t$ – portfolio expected value,

$\mathbf{H}_t$ – conditional covariance matrix,

$F^{-1}(\alpha)$ – quantile of order α for standardized distribution.

The Kupiec [1995] test is used to estimate the effectiveness of *VaR*. The testing hypotheses are as follows:

$$H_0: \quad \omega = \alpha$$
$$H_1: \quad \omega \neq \alpha,$$

where ω is a proportion of the number of results exceeding VaR_α to the number of all results. Assuming that the null hypothesis is true, the statistic:

$$LR_{uc} = -2\ln[(1-\alpha)^{T-K}\alpha^K] + 2\ln\left\{\left[1-\left(\frac{K}{T}\right)^{T-K}\right]\left(\frac{K}{T}\right)^K\right\}, \quad (7)$$

where: K – a number of excesses,

T – a length of time series,

α – the given probability of the loss of value not exceeding *VaR*,

has an asymptotic χ^2-distribution with 1 degree of freedom.

4. Empirical analysis

In Figure 1, the time series of Index Day Ahead Market (IRDN PLN/MWh) was presented. After 2008, prices on DAM are characterized by a positive trend, and a clearly lower volatility than prices in 2008. So in the analysis the time series from 30.03.2009 to 28.10.2011 were used.

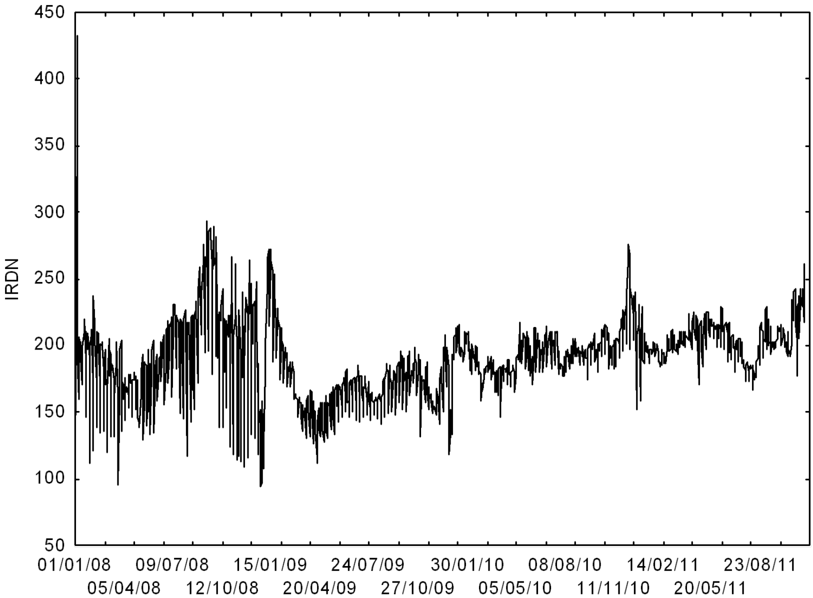

Figure 1. IRDN (PLN/MWh) noted on DAM from 01.01.2008 to 28.10.2011

Source: working papers.

To forecast prices and volatility, daily rates of return of 24 fixing prices from fixing 1 and fixing 2 are used. Prices of electric energy during a day are characterized by strong dependence. Based on the result of Principal Component Analysis (PCA) to forecast contracts of electric energy in hour: 2, 6, 10 and 22 (Figure 2) were used. So on fixing 1 contracts: K1.2, K1.6, K1.10, K1.22 were analyzed and on fixing 2 contracts: K2.2, K2.6, K2.10, K2.22 were analyzed.

For each of eight contracts the SARIMA(1,0,1)$(1,1,1)_7$ model was used to describe the mean of time series for linear rates of return. For every contract the parameters of SARIMA models are significant (on significance level 0.05). In Figure 3 ACF and PACF functions of residuals SARIMA(1,0,1)$(1,1,1)_7$ model for contract K1.2 were presented.

In Tables 1 and2 real fixing prices and forecast fixing prices with Relative Root Mean Square Error (RRMSE) for fixing 1 and 2 during one week are presented.

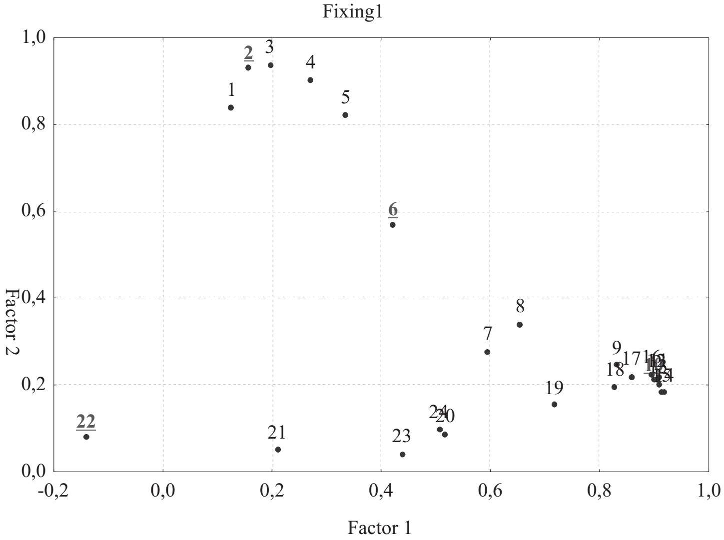

Figure 2. Result of PCA for contracts on fixing 1

Source: working papers.

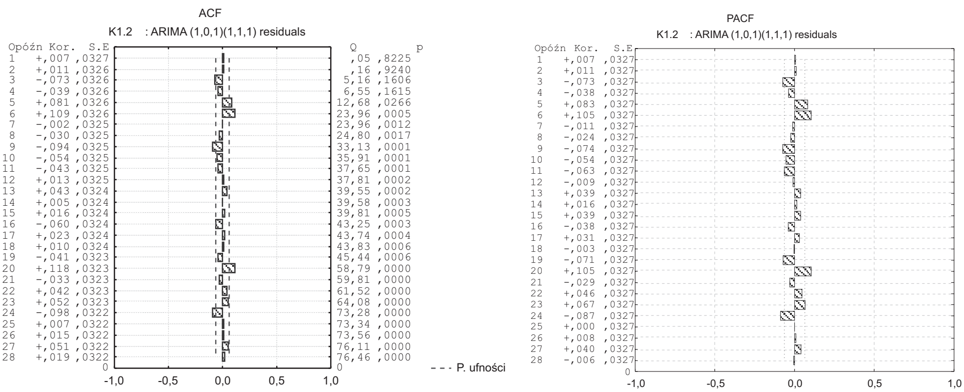

Figure 3. ACF and PACF for SARIMA residual of contract K1.2

Source: working papers.

RRMSEs on fixing 1 are lower than on fixing 2. Firstly, this is the result of better fitting SARIMA model for fixing 1 than for fixing 2, and secondly the greater volatility at fixing 2 (Figure 3) [Ganczarek-Gamrot 2009]. The lowest errors are obtained for contracts in hour 2 (prices of electric energy by night are low, and have low volatility), the highest errors are obtained for contracts in hours 6 and 10 (prices of electric energy during the day are high and are characterized by very high volatility). The contracts K1.22 and K2.22 represent the evening peak, and were prices which exhibit somehow lower errors then contracts for an hour of a day peak.

Table 1. Real and forecast prices on fixing 1 from 29.10.2011 to 4.11.2011

Date	Real prices				Forecast prices (RRMSE)			
	K1.2	K1.6	K1.10	K1.22	K1.2	K1.6	K1.10	K1.22
2011-10-29	169.01	166.70	257.45	207.00	170.71 (0.0453)	169.92 (0.0662)	194.76 (0.0646)	208.85 (0.0486)
2011-10-30	155.86	152.61	196.81	205.00	160.78 (0.0464)	161.09 (0.0686)	166.39 (0.0682)	223.89 (0.0515)
2011-10-31	153.36	159.40	220.02	187.46	156.38 (0.0469)	168.39 (0.0688)	196.32 (0.0692)	204.31 (0.0523)
2011-11-01	149.15	142.08	162.71	177.72	156.55 (0.0471)	168.10 (0.0688)	204.24 (0.0694)	203.61 (0.0525)
2011-11-02	156.01	170.00	240.01	199.99	156.41 (0.0472)	165.24 (0.0688)	207.44 (0.0694)	202.25 (0.0525)
2011-11-03	163.01	177.17	229.17	189.73	160.47 (0.0472)	175.20 (0.0688)	198.77 (0.0694)	203.71 (0.0525)
2011-11-04	163.85	171.58	237.54	183.87	159.80 (0.0472)	175.03 (0.0688)	196.36 (0.0694)	196.89 (0.0525)

Source: working papers.

Table 2. Real and forecast prices on fixing 2 from 29.10.2011 to 4.11.2011

Date	Real prices				Forecast prices (RRMSE)			
	K2.2	K2.6	K2.10	K2.22	K2.2	K2.6	K2.10	K2.22
2011-10-29	176.51	175.12	265.18	204.02	163.36 (0.0581)	166.34 (0.1038)	266.15 (0.0778)	227.84 (0.0664)
2011-10-30	156.95	145.98	191.47	210.00	152.09 (0.0618)	167.25 (0.1097)	208.75 (0.0839)	201.84 (0.0712)
2011-10-31	159.83	168.00	217.14	181.99	155.07 (0.0623)	179.24 (0.1103)	322.87 (0.0848)	201.12 (0.0718)
2011-11-01	158.71	142.03	185.78	197.75	156.70 (0.0624)	170.58 (0.1103)	214.38 (0.0849)	204.32 (0.0719)
2011-11-02	156.48	167.74	225.00	198.13	158.16 (0.0624)	177.92 (0.1103)	309.24 (0.0849)	198.77 (0.0719)
2011-11-03	170.01	184.03	255.01	195.73	160.83 (0.0624)	183.08 (0.1103)	207.24 (0.0849)	203.00 (0.0719)
2011-11-04	166.15	169.69	232.47	194.52	154.86 (0.0624)	172.34 (0.1103)	322.54 (0.0849)	205.23 (0.0719)

Source: working papers.

Small errors suggest the good fit of the SARIMA model to the empirical time series, but residuals of these models indicate the presence of volatility clustering effect and fat tail effect. This means that RRMSEs changes in time. So GARCH models should be used to analyze risk and to forecast volatility.

The risk analysis and volatility forecast were made for two portfolios. Prices and rates of return from fixing 1 and 2 are strongly correlated, so portfolios consisting of contracts from both fixings were omitted in the analysis. Based on PCA, two portfolios were proposed. The first one for fixing 1 and the second for fixing 2. The share of contracts in portfolios was calculated based on relative profit measure. For every contract and for portfolio Value-at-Risk was estimated using the Dynamic Autocorrelation Model (DCC). This model is estimated in two steps. In the first step univariate residual variances for the SARIMA model were estimated. These variances were modeled by IGARCH(1,1) models with *t*-Student distribution. In Figure 3 univariate variances of K1.2 and K2.2 were presented. They were modeled by IGARCH(1,1) models with *t*-Student distribution.

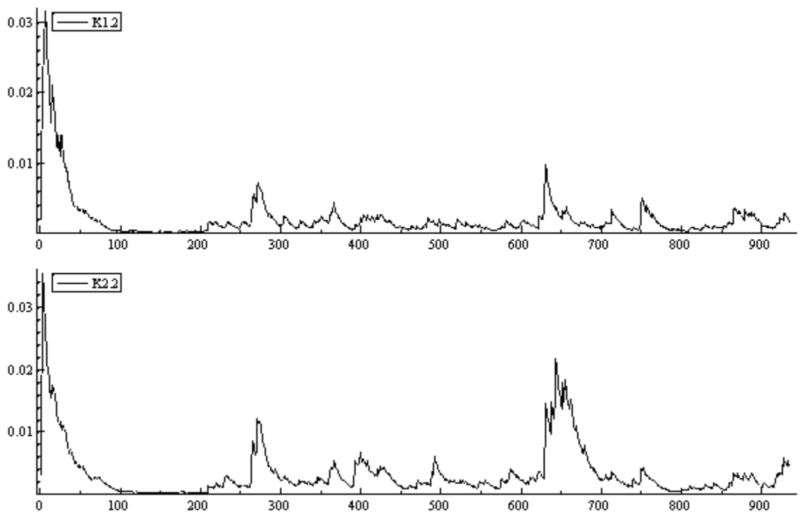

Figure 4. Conditional univariate variances for contracts K1.2 and K2.2

Source: working papers.

In the second step of DCC model estimation, conditional correlation matrix $\mathbf{\Gamma}_t$ was calculated. In Figure 4 elements of matrix $\mathbf{\Gamma}_t$ for contracts on fixing 1 were presented. Next, VaR for every contract and for the portfolios were estimated using equation (4). In Table 3, the results of VaR estimation and results of the Kupiec test are presented. Based on the Kupiec test, the number of excesses of VaR is not significant (on significance level 0.01) for contracts K1.2, K1.22, K2.22 and for the portfolio of contracts on fixing 1. The high values of VaR on fixing 2 are the consequence of higher volatility on fixing 2 than on fixing 1.

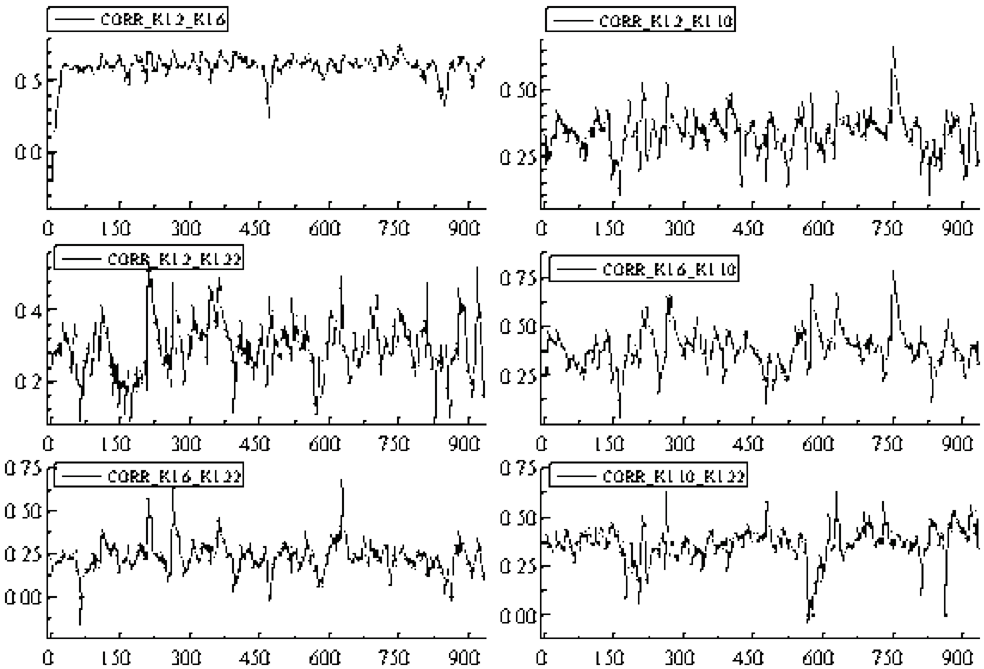

Figure 5. Conditional correlation matrix $\mathbf{\Gamma}_t$

Source: working papers.

Table 3. Value of VaR and result of Kupiec test

Parameters/contracts	K1.2	K1.6	K1.10	K1.22	Portfolio
MIN [PLN/MWh]	3.48	4.16	9.75	7.02	5.19
MEAN[PLN/MWh]	14.31	18.68	28.88	18.78	14.77
MAX[PLN/MWh]	40.01	108.12	78.50	92.70	48.34
p-value of Kupiec test	0.53	0.00	0.00	0.01	0.08
Share of portfolio	0.25	0.25	0.23	0.27	
Parameters/contracts	K2.2	K2.6	K2.10	K2.22	Portfolio
MIN [PLN/MWh]	3.48	4.09	9.61	6.84	5.19
MEAN[PLN/MWh]	14.35	18.66	28.98	18.98	14.87
MAX[PLN/MWh]	40.38	116.24	83.30	131.70	56.50
p-value of Kupiec test	0.00	0.00	0.00	0.63	0.00
Share of portfolio	0.25	0.23	0.24	0.28	

Source: working papers.

5. Conclusion

Prices on fixing 2 of DAM are characterized by higher volatility than prices on fixing 1. Consequently, the transactions on fixing 2 are characterized by a greater level of risk than transactions on fixing 1. Hence investors should prefer to execute transactions on fixing 1.

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PROGNOZOWANIE CEN I ZMIENNOŚCI NA RYNKU DNIA NASTĘPNEGO

Streszczenie: Celem pracy jest prognozowanie cen i zmienności cen na Rynku Dnia Następnego. Analizę przeprowadzono na portfelach zbudowanych z czterech spośród 24 kontraktów notowanych od 30.03.2009 do 28.10.2011 wyłonionych za pomocą analizy głównych składowych niezależnie na dwóch aukcjach. Stopy zwrotu opisano za pomocą modeli SARIMA uwzględniających autokorelację i sezonowość szeregów. Ryzyko zmiany ceny oszacowano w oparciu o wartości VaR z uwzględnieniem zmiennej w czasie warunkowej korelacji modelem DCC. Podsumowując wyniki, można stwierdzić, że zastosowane modele są dobrze dopasowane do szeregów z wybranego okresu badań, ponadto kontrakty na aukcji drugiej charakteryzują się wyższym ryzykiem zmiany ceny niż kontrakty aukcji pierwszej.

Słowa kluczowe: analiza głównych składowych (PCA), model SARIMA, model DCC, Value-at-Risk, portfel.