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REBALANCING ISSUES IN TRACKING ERROR VARIANCE MINIMIZATION

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Abstract
The paper focuses on portfolio selection based on the approach to index tracking minimizing tracking error variance. The idea of this approach is to replicate a suitably chosen financial index as a convex linear combination (i.e. portfolio) of pre-selected assets that are components of the given synthetic financial index. In this approach, a tracking portfolio that performs as best as the financial index serving the function of benchmark and follows its path of returns is constructed and sought. However, a question arises whether the replication portfolio found under the tracking error variance minimization strategy should be left without alterations (non-rebalanced) or adjusted in respect to market movements (or rebalanced). This is of practical implications as rebalancing induces undesirable transaction costs, and the investor must choose whether he will rebalance his portfolio on a regular periodic basis. In the paper, under the tracking error variance minimization approach, practical aspects of both the buy-and-hold strategy and the rebalancing strategy are investigated with respect to net returns (returns including transaction costs) and to portfolio volatility.

Key words: *portfolio tracking, tracking error variance minimization, transaction costs, rebalancing, net returns.*

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1. Introduction

Over the course of years, use of benchmark has come to great popularity in portfolio construction and its importance prevailed over its original interpretation as a general indicator of market behaviour and market sentiment. Use of benchmark is becoming a standard practice in investment management and is adopted widely in passive portfolio management strategies for asset allocation and performance measurement (still with some impact on active portfolio management as well). The function of benchmark is carried out usually by market indices that are in fact, by their general construction, market cap weighted indices. It can be even said that all market indices provide almost identical indications of the general trends that are present on the market. Simply by copying a well-constructed index, the portfolio manager can construct a portfolio with satisfactory performance and can achieve a relatively high return including transaction costs (that tend to be very low for these types of investment strategies). Managers and investors use benchmark in risk quantification and they relate the behaviour of their constructed portfolio to the behaviour of their benchmark in terms of the measure called tracking error. Tracking error is formally defined as a difference between benchmark returns and portfolio returns (and the aim is to minimize a suitable function of it – such as standard deviation). Tracking error models found immense application in the past few years in investment management. A classical tracking error task focuses on minimization of deviations of market returns from benchmark returns under a set of certain (economic) restrictions. From the perspective of investors, benchmark represents an objective parameter that may help in assessing the risk profile of an investment or a fund and that may enhance transparency of such an assessment.

In the paper, the approach based on tracking error variance minimization is considered and its properties with respect to rebalancing are explored and examined in a small case study. Every time a portfolio is selected for a certain investment horizon, the investor faces a quandary whether he should rebalance the portfolio when the market unfolds in an unfavourable way or he should leave the portfolio without changes. This decision-making is complicated by the fact that each change in portfolio composition (or even its creation itself) brings on transaction costs and too frequent updates may prove expensive eventually. Having this in mind, under the tracking error variance minimization approach, the goal of the paper is to investigate practical aspects of both the buy-and-hold strategy and several rebalancing strategies are investigated with respect to net returns (returns including transaction costs) and to portfolio volatility.

2. Approaches to portfolio tracking, rebalancing and transaction costs

There are several definitions of tracking error and they suggest diverse portfolio construction models. Rudolf et al. (1999) presented and compared four various linear tracking error models in a static (i.e. one-year) framework. In his article, Roll (1992) constructed tracking error portfolios in the Markowitzian mean-variance space that is typical of active asset management. In this re-formulation of the traditional Markowitzian task, mean is represented by average positive deviation of portfolio returns from benchmark returns and this mean is called expected tracking error (in which consequence only positive excess returns are taken into account), and variance is understood as variance of all deviations of portfolio returns from benchmark returns and goes under the name of tracking error. Under this set-up, the ambition is to minimize tracking error variance for a desired level of expected tracking error. Jansen and van Dijk (2002) focused on tracking error variance minimization with respect to benchmark for portfolios with a small number of shares. Furthermore, there were some other approaches applied in other research works such as the method of scenarios or Monte Carlo simulation methods (Dembo and Rosen, 1999; Dempster and Thompson, 2002, Buc and Klieštik, 2013, Gaivoronski et al., 2005).

The process of asset optimization begins with asset pre-selection that will participate in portfolio tracking. However, there are several problems associated with this pre-selection process, two of them are most pertinent. The first problem is to determine the number of assets that should be in the resulting tracking portfolio. The second problem is the choice of criterion on which assets should be selected for portfolio and their proportion should be determined. It is obvious that the increasing number of assets in portfolio pushes tracking error downwards. On the other hand, holding a high number of assets is associated with additional costs such as transaction costs. As claimed by Vardharaj et al. (2004), an optimally constructed portfolio of about 50 shares can track the underlying market index within the limit of 2 %. Several tracking strategies are utilized in practical portfolio management differing according to investment style and according to risk profile. In partial replication strategies, only a small proportion of assets represented in the index is chosen according to certain criteria (e.g. according to market capitalization) and their weights are determined as they are implied by their representation in the index. Nonetheless, these portfolios may not be optimal. Instead, the weights of individual assets should be determined by means of a suitable optimization approach oriented on tracking error minimization.

Efficient asset allocation is not a one-time decision, but it is an ongoing process that subsumes constant revaluations. The selected tracking portfolio can be optimal on the basis of tracking error minimization several months, but in a longer time horizon, tracking portfolio returns may deviate considerably from benchmark returns or their pre-determined target level – which requires rebalancing of these portfolios. However, every rebalancing associates itself with a certain amount of transaction costs. The problem of including transaction costs into tracking error portfolio selection was treated in several studies. Adcock and Meade (1994) added a linear term for transaction costs to the riskless rate in the traditional Markowitz model and minimized their cost function. Chen et al. (2010) solved a portfolio revision problem with transaction costs that are paid at the end of the investment horizon and offered several analytical solutions in the mean-variance framework. In this, they performed a simple empirical experiment using the up-to-date market data in order to show that influence of transaction costs may be reduced by a convenient rebalancing strategy with a lower frequency of portfolio revision. Konno and Yamamoto (2003) compared transaction costs between various rebalancing strategies on the Japanese share market. They concluded that rebalancing strategy inclusive of transaction costs was best and comparable to those usually used by practitioners.

Conventional approaches to portfolio rebalancing comprise periodic rebalancing and rebalancing based on a certain tolerance threshold, which is called tolerance band rebalancing or threshold rebalancing strategy.

- Periodic rebalancing bases on regular (periodic) revisions of the existing portfolio, e.g. daily, monthly, quarterly, semi-annually, yearly, regardless of the magnitude of deviation of portfolio from the target or benchmark (in the sense of returns or portfolio value) between two rebalancing periods. Setting the rebalancing frequency is contingent on the investor's risk aversion and on rebalancing costs. The chief shortcoming of this approach is that rebalancing decisions are timed beforehand and do not account for the market behaviour, which implies that the investor under this strategy rebalances even if his portfolio is near the target (benchmark) portfolio.
- Threshold rebalancing strategies rests in constant monitoring the deviation between the tracking portfolio and the target (benchmark) portfolio and implies an intervention when this deviation exceeds a pre-determined threshold no matter what is the frequency of rebalancing.

Nesbitt (2010) in his rebalancing research study showed that threshold rebalancing strategies were more successful than periodic rebalancing strategies. Strategies were tested for the data from Sep 1978 until Sep 2009. Strategies based on monthly, quarterly, semi-annual and yearly rebalancing failed because these time-period strategies generated lower returns (including transaction costs) than both the benchmark and the non-rebalanced portfolio with the only exception of annual rebalancing. In threshold rebalancing, several tolerance bands for deviations were used (understood either in terms of returns or portfolio values): 1 %, 2 %, 10

%, 25 %, 30 % and 40 %. Out of these six strategies four of them produced a higher cumulative return than the benchmark did. It is of interest that broader tolerance bands directly increased the cumulative portfolio return up to 30 % of the tolerance band when returns were slowly increasing and the standard deviation was decreasing. This particular study favoured the 25 % tolerance band rebalancing strategy, when the cumulative net return (inclusive of transaction costs) were maximum in comparison to the standard deviation. Dichtl et al. (2013) compared performance of various rebalancing strategies for the historical monthly data of U.S. financial markets and those of Great Britain and Germany from Jan 1982 to Dec 2011. Despite differences between these markets, they provided evidence that too frequent and intensive (i.e. monthly) as well as too infrequent (i.e. annual) rebalancing yields worse risk-adjusted performance measures. Quarterly rebalancing appeared most suitable for each of the three markets.

In this paper, the study by Jansen and van Dijk (2002) is taken for an inspiration. The classical approach to portfolio construction is presented in the paper footed on tracking error variance minimization with respect to the Standard and Poor's 500 (S&P 500) Index that is – in line with convention – chosen as the benchmark. Several strategies are considered for the sake of comparison and tracking portfolios are constructed out of 40 shares represented in the S&P 500 Index and pre-selected on the basis of their (maximum) market capitalization. Period rebalancing strategies based on regular monthly, quarterly, semi-annual and annual portfolio revisions are compared with threshold rebalancing strategies, in which 2.5 %, 5 %, 10 %, 15 %, 20 % and 25 % tolerance bands are set. The details of the empirical exercise are clarified further – the theoretical aspects are outlined in the next section, and this section is ensued by another section presenting the practical issues.

3. Formulation of tracking error variance minimization task and of rebalancing

In the general formulation, a sample of historical observations on benchmark returns and asset returns for assets pre-selected for portfolio tracking are needed. Hence, assume that a history of T historical observations of logarithmic returns is available and that the tracking portfolio is to be composed of k assets. Let $\mathbf{Y} = (Y_1, ..., Y_T)'$ denote a $(T \times 1)$ vector of benchmark returns, let $\mathbf{x}_t$ denote the vector of asset returns at any time t (whereas $t \in \{1,...,T\}$) with elements $\mathbf{x}_t = (x_{t,1}, ..., x_{t,k})'$, and eventually let $\mathbf{X} = (\mathbf{x}_1 | ... \ | \mathbf{x}_T)'$ denote a $(T \times k)$ matrix of returns of the k assets that are to be represented in the tracking portfolio. The symbol $\boldsymbol{\omega}$ will stand for a $(k \times 1)$ vector of unknown portfolio weights $\omega_1, ..., \omega_k$ that are obtained by minimizing the following quadratic optimization problem

$$\min_{\boldsymbol{\omega} \in \Re^k} (\mathbf{Y} - \mathbf{X}\boldsymbol{\omega})'(\mathbf{Y} - \mathbf{X}\boldsymbol{\omega}) \quad \text{subject to} \quad \boldsymbol{\omega}'\mathbf{1} = 1, \qquad (1)$$

in which $\mathbf{1}$ is a $(k \times 1)$ vector of ones. This general formulation of the optimization task allows an extension and can be complemented by the constraint banning short sales. Tracking error(s) can be written as a $(T \times 1)$ vector and defined with $\boldsymbol{\varepsilon} := \mathbf{Y} - \mathbf{X}\boldsymbol{\omega}$. The quantity $\boldsymbol{\varepsilon}'\boldsymbol{\varepsilon} / T$ is called tracking error variance and represents the objective function under this approach. It is not easy to interpret this quantity, for interpretation, it is more advisable to take the square root of tracking error variance, which produces a measure comparable to standard deviation. At any rate, this measure is a non-central measure and is thus influenced not only by random positive or negative deviations but also by a possible underperformance or outperformance relative to the benchmark. Theoretical details and a further exposition on these issues may be found in Rudolf et al. (1999).

Denote the moment of portfolio construction by the subscript τ (obviously satisfying $\tau \geq T$), denote the prices of individual assets at time τ by the symbols $P_{\tau,1}$, ..., $P_{\tau,k}$ and the price of the benchmark as $P_{\tau,B}$. If the initial investment is Ψ_τ, the following portfolio holdings are suggested: $h_{\tau,1} = \Psi_\tau \cdot \omega_1 / P_{\tau,1}$, ..., $h_{\tau,k} = \Psi_\tau \cdot \omega_k / P_{\tau,k}$. At the same time, a fictional investment into the benchmark is done and the holding $h_{\tau,B} = \Psi_\tau / P_{\tau,B}$ is made. The symbol Ψ will also denote the value of the tracking portfolio at any time denoted carefully in the subscript. Adding "B" in the subscript after the time instance will indicate that the value of the benchmark investment is had in mind. Finally, assume that there is a percentage rate of transaction costs $\varphi \in [0,1)$ that applies to the value of investment changes. Symbols that were introduced for a particular time extend naturally in their validity also for some future times. In consistency with the previous outline, there are several possibilities how to maintain this portfolio by the investor until the end of the investment horizon.

– The investor may choose not to revaluate the composition of the portfolio at all and opt for the buy-and-hold strategy. In such a case, transaction costs are incurred only at the moment of portfolio creation in the amount

$$\varphi \sum_{i=1}^{i=k} | h_{\tau,i} | \cdot P_{\tau,i} , \tag{2}$$

which reduces into $\varphi \cdot \Psi$ when there is a ban on short sales (or when all holdings are positive).

– Another possibility is to rebalance the portfolio at regular time intervals of length, say, $\Delta\tau$ ($\Delta\tau > 0$), no matter what the situation on the market is and how the tracking portfolio copies the index. In this case, at the next time $\tau + \Delta\tau$, the task presented in (1) is re-solved with the updated data stored in **Y** and **X**. This updating is done on a sliding basis, keeping the length of observations to be T. New holdings are thus produced, $h_{\tau+\Delta\tau,1}$,..., $h_{\tau+\Delta\tau,k}$, and the portfolio must be revised accordingly. In addition to the initial transaction costs resulting from the first portfolio construction given by (2), at the moment of revision, $\tau + \Delta\tau$, rebalancing transaction costs arise in the amount

$$\varphi \sum_{i=1}^{i=k} | h_{\tau+\Delta\tau,i} - h_{\tau,i} | \cdot P_{\tau+\Delta\tau,i} . \tag{3}$$

This, of course, goes on a sliding basis at rebalancing times $\tau + \Delta\tau$, $\tau + 2\Delta\tau$, ... until the end of the investment horizon.

– Finally, another possibility is to set a threshold and to monitor discrepancy between the value of the tracking portfolio and the value of the investment into the benchmark. For this, some maximum tolerance threshold δ (with $\delta > 0$) must be set. If at some future time $\tau + \pi$ (with $\pi > 0$) the situation $|\Psi_{\tau+\pi} - \Psi_{\tau+\pi,B}|/\Psi_{\tau+\pi,B} > \delta$ first happens to be the case, this is the impetus for an intervention and the portfolio is rebalanced. With this intervention portfolio, additional transaction costs are associated in the same manner as explained about the formula (3). However, one must bear in mind that to warrant consistency, it is necessary to rebalance also the index to the new value of the intervention tracking portfolio. Only then comparisons of values makes sense.

There is one grave simplification with these strategies in comparison to their practical implementation since they should take into consideration also the fact that, at revision times, transaction costs must be paid and they should decrease the value of the portfolio. It is assumed here in the paper instead that there exists a separate account, from which these transaction costs are covered. Only the final value of the tracking portfolio is confronted with the volume of transaction costs (in an inflation-free world), and the net value of the investment is computed by subtracting the transaction costs total from the portfolio value.

4. Empirical exercise, its practical aspects and results

Insomuch as monthly returns exhibit higher stability over time and their statistical property are finer from a modeller's perspective, the empirical exercise utilized the data observed on a monthly frequency. The in-sample-period spanned 5 years from Jan 2006 to Jan 2011 and included 60 effective observations of monthly returns, and the out-of-sample period represented another 3 years from Jan 2011 to Feb 2014 counting 36 monthly instances in which the tracking portfolio might be rebalanced. A selection of shares represented in the S&P 500 Index was made respecting two criteria: market capitalization and availability of historical data. As to the former, shares with the largest market capitalization represented in the S&P 500 Index (as of March 2014) were selected, but as the composition of the S&P 500 Index is subject to constant changes and some shares were introduced to the market only after 2006 they had to be removed from the list in the latter and replaced by other share. The list of shares participating in the exercise is provided in Table 1 and their categorization under the Global Industry Classification Standard (GICS) taxonomy is indicated.

Table 1.The shares participating in the empirical exercise

Company name	GICS sector	Company name	GICS sector
Amazon.com Inc	Consumer Discretionary	Wells Fargo	Financials
Chipotle Mexican Grill	Consumer Discretionary	Allergan Inc	Health Care
Ford Motor	Consumer Discretionary	Becton Dickinson	Health Care
Goodyear Tire & Rubber	Consumer Discretionary	Boston Scientific	Health Care
Mohawk Industries	Consumer Discretionary	Medtronic Inc.	Health Care
TJX Companies Inc.	Consumer Discretionary	3M Company	Industrials
Whirlpool Corp.	Consumer Discretionary	Fastenal Co	Industrials
Avon Products	Consumer Staples	Flowserve Corporation	Industrials
Mondelez International	Consumer Staples	Fluor Corp.	Industrials
PepsiCo Inc.	Consumer Staples	Roper Industries	Industrials
Chevron Corp.	Energy	Broadcom Corporation	Information Technology
CONSOL Energy Inc.	Energy	Jabil Circuit	Information Technology
Exxon Mobil Corp.	Energy	Microchip Technology	Information Technology
Noble Corp	Energy	Salesforce.com	Information Technology
Fifth Third Bancorp	Financials	Yahoo Inc.	Information Technology
Huntington Bancshares	Financials	The Mosaic Company	Materials
Northern Trust Corp.	Financials	Owens-Illinois Inc	Materials
Plum Creek Timber Co.	Financials	AT&T Inc	Telecommunications Services
Progressive Corp.	Financials	Exelon Corp.	Utilities
SunTrust Banks	Financials	Pinnacle West Capital	Utilities

Source: The authors.

In portfolio tracking, the initial investment was made at the end of the in-sample period as of 3 Jan 2011 in the amount of U.S. $ 100 000. Shorts sales were permitted as well and the rate of transaction costs was set to $\delta = 0.1$ %. The following strategies were undertaken and considered:

– The buy-and-hold strategy, in which case the portfolio created on 3 Jan 2011 was not rebalanced during the three-year out-of-sample period.
– The periodic rebalancing strategies, under which the portfolio created on 3 Jan 2011 was rebalanced annually, semi-annually, quarterly and monthly.

– The intervention rebalancing strategies, under with the initial portfolio was rebalanced only if it deviated from the benchmark S&P 500 Index by more than 2.5 %, 5 %, 10 %, 15 %, 20 % and 25 %.
– The investment into the index as of 3 Jan 2011 without rebalancing in order to enable comparison of the tracking strategies in terms of their performance.

In computations and preparing graphical presentations, the software R version 3.0.1 (R Core Team, 2013) was employed with several of its libraries, `quadprog`, `timeSeries`, `PerformanceAnalytics` and `tseries`.

The results and the behaviour of the rebalancing strategies considered under tracking error variance minimization are summarized graphically in Fig. 1 and Fig. 2 and numerically in Table 2.

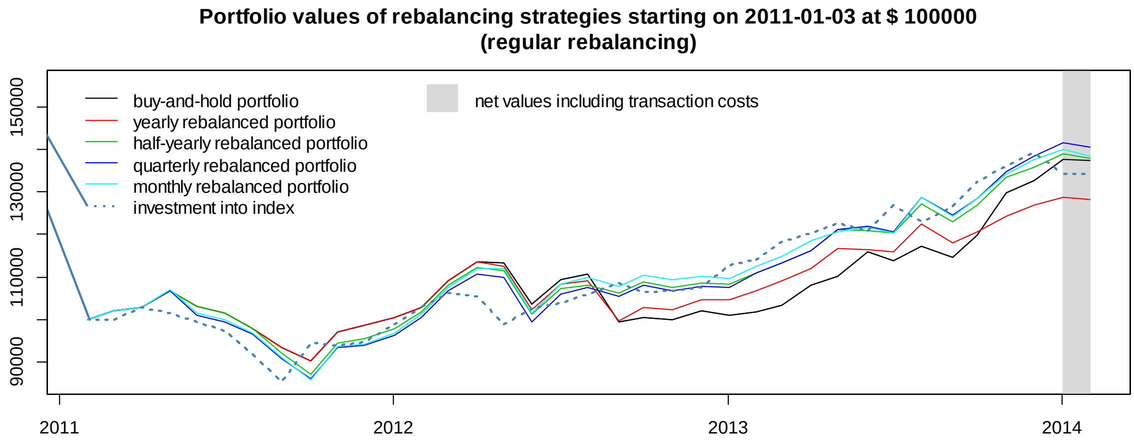

Figure 1 The price development of the periodic rebalancing strategies
Source: The authors.

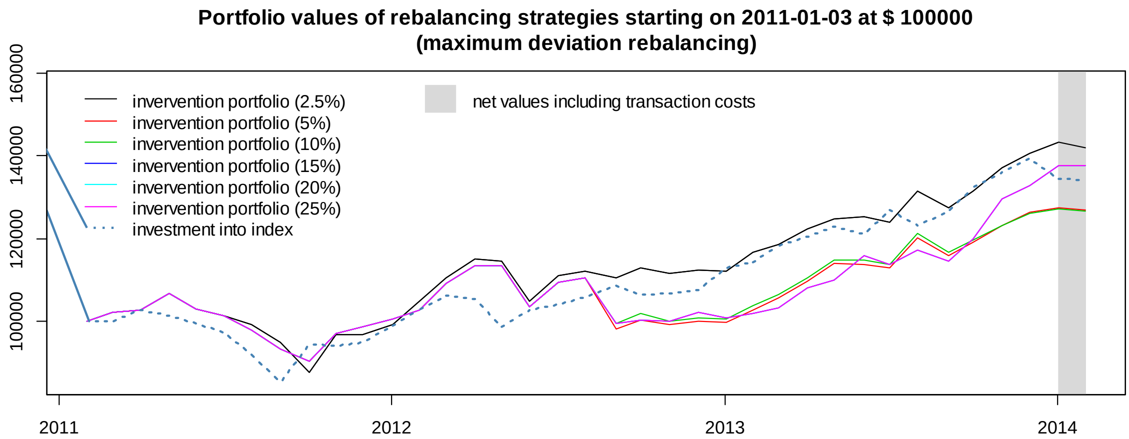

Figure 2 The price development of the threshold rebalancing strategies
Source: The authors.

Whereas Fig. 1 shows how the initial investment of $ 100 000 changed over time for the buy-and-hold tracking error strategy, and the four periodically rebalanced strategies as well as for the fictive investment into the index, Fig. 2 does this for the five six threshold investment strategies and for the investment into the index. The final value of portfolios without the effect of transaction costs in both graphs is shown at the beginning of the gray vertical strip (which is the very start of 2014) and here the strategies quit. At the ending of this gray vertical strip on the right, the net final values of portfolios are displayed inclusive of transaction costs.

Table 2 contains information on the performance of the rebalancing strategies. The first three lines of information for each strategy is the final portfolio value, the total transaction costs and the net final portfolio value. The other three lines informs on the common performance (irrespective of the performance of the benchmark index) and the last three lines shows the performance relative to the index. For the sake of completeness, excess returns are defined as exceedances of portfolio returns over benchmark returns.

Table 2.Performance descriptives of the juxtaposed rebalancing strategies

Periodic rebalancing strategy	**Buy-&-hold**	**Yearly**	**Half-yearly**	**Quarterly**	**Monthly**	**Index**
Final portfolio value ($)	137 793	128 715	138 912	141 666	140 159	**134 310**
Total transaction costs ($)	218	573	841	1 126	1 731	**100**
Net final portfolio value ($)	137 575	128 142	138 071	140 540	138 428	**134 210**
Mean return (p.m.)	0.92%	0.72%	0.94%	1.00%	0.96%	**0.84%**
Standard deviation (p.m.)	4.12%	3.84%	3.79%	3.96%	4.00%	**3.60%**
Mean to standard deviation ratio (p.m.)	0.2226	0.1878	0.2480	0.2512	0.2414	**0.2343**
Mean active return (p.m.)	0.07%	-0.12%	0.10%	0.15%	0.12%	**NA**
Active standard deviation (p.m.)	5.51%	5.28%	5.16%	5.33%	5.41%	**NA**
Information ratio (p.m.)	0.0133	-0.0230	0.0187	0.0286	0.0225	**NA**
Threshold rebalancing strategy	**2.5 %**	**5 %**	**10 %**	**15 %**	**20 %**	**25 %**
Final portfolio value ($)	143 250	127 406	127 207	137 793	137 793	137 793
Total transaction costs ($)	1 289	508	513	218	218	218
Net final portfolio value ($)	141 961	126 897	126 694	137 575	137 575	137 575
Mean return (p.m.)	1.03%	0.69%	0.69%	0.92%	0.92%	0.92%
Standard deviation (p.m.)	3.85%	4.07%	3.97%	4.12%	4.12%	4.12%
Mean to standard deviation ratio (p.m.)	0.2665	0.1699	0.1731	0.2226	0.2226	0.2226
Mean active return (p.m.)	0.18%	-0.15%	-0.16%	0.07%	0.07%	0.07%
Active standard deviation (p.m.)	5.49%	5.45%	5.39%	5.51%	5.51%	5.51%
Information ratio (p.m.)	0.0335	-0.0277	-0.0288	0.0133	0.0133	0.0133

Source: The authors.

The rebalancing strategies considered in the study can be viewed and their performance can be assessed from several perspectives: (a.) performance evaluation of the rebalancing strategies in comparison to the index neglecting transaction costs *on the basis of mean returns and volatility*, (b.) performance evaluation of the rebalancing strategies in comparison to the index with regard given to transaction costs *on the basis of net portfolio values*, and (c.) mutual performance comparison of the rebalancing strategies *on the basis of mean excess returns and excess volatility.*

Ad (a.). In the case of periodic rebalancing, with the exception of the yearly rebalancing strategy, all the periodic rebalancing strategies outperformed the index in terms of mean returns. Nonetheless, all the periodic rebalancing strategies revealed higher volatility of returns than the index did. Comparing mean returns to volatility, the highest performance was found with the rebalancing strategies rebalanced periodically every six months, three months as well as every month. As far as the threshold rebalancing is considered, the mean returns for thresholds 2.5 %, 15 %, 20 % and 25 % were higher than the mean return of the index. Comparing mean returns to risk captured by volatility, the rebalancing strategy with threshold 2.5 % appears most successful.

Ad (b.) With periodic rebalancing, the net portfolio value of the index was exceeded by the portfolio constructed under each periodic rebalancing strategy save the one with yearly rebalancing. Even the buy-and-hold strategy achieved a higher net portfolio value than the index; this is in the face of the fact that it is clear that, for the most part, in the out-of-sample period the non-rebalanced portfolio exhibited worst portfolio values in comparison to all the periodically rebalanced portfolio. An improvement in these trends presented itself at the end of the out-of-sample period. In the case of threshold rebalancing, as before, the threshold rebalancing strategies with thresholds 2.5 %, 15 %, 20 % and 25 % performed better than the index, now in terms of net portfolio values.

Ad (c.). With period rebalancing, the highest excess returns was achieved for the quarterly rebalanced portfolio. This strategy proved best in terms of relation of mean excess return to excess volatility captured by the Information ratio. The lowest volatility of excess returns was manifested with yearly rebalancing. For threshold rebalancing, the highest mean excess return was produced by the rebalancing strategy with threshold 2.5 % and this strategy was found best even from the view of the Information ratio. Yet, the smallest volatility was attributable to the portfolio rebalanced with threshold 10 %.

5. Conclusion

The ambition of the paper was to deepen the understanding of rebalancing issues that are pertinent for any investment strategy, for which reason the buy-and-hold investment strategy is juxtaposed with four periodic rebalancing strategies and six threshold rebalancing strategies in terms of net returns (returns including transaction costs) and to portfolio volatility. The results obtained in a small case study are original in the sense clarified in the next paragraph.

On the footing of the achieved results it turns out that the most suitable universal approach to rebalancing is based on periodic rebalancing as the periodic rebalancing strategies outperformed the S&P 500 Index in three out of four cases in terms of mean return to volatility relationship. On the other hand, out of the six threshold rebalancing strategies, only the one using threshold 2.5 % managed to outperform the S&P 500 Index with respect to this reward indicator. This suggests that periodic rebalancing may offer more chances to obtain a higher expected return than the expected return of the benchmark. Having said that, amongst all the rebalancing strategies overviewed in the paper, the best rebalancing strategy seems the 2.5 % threshold rebalancing strategy and this recommendation holds with respect to all the criteria outlined in Table 1 (except volatility). Outstandingly, the findings of the paper are contrary to the empirical results of Nesbitt (2010) who concluded positively in favour of threshold rebalancing strategies as four of six threshold strategies generated a positive mean excess return, whilst only one of four periodic rebalancing strategies displayed such a pattern of behaviour.

There is no possibility to state firmly which rebalancing strategy the investor should choose. One of the most important factors is transaction costs and their effect on performance. In the paper, no distinction was drawn between transaction costs for purchase and for sale, which might slightly bias the general validity of conclusions.

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