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DELTA AND GAMMA FOR CHOOSER OPTIONS

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Abstract

The paper deals with calculation and analysis of parameter delta and gamma for the chooser options price. A chooser option is an option that gives its holder the right to choose at some predetermined future time whether the option will be a standard call or put with predetermined strike price and maturity time. Although the chooser options are more expensive than standard European-style options, in many cases they are more suitable instrument for hedging of the portfolio value. For effective using of the chooser option as a hedging instrument there is necessary to check values of the parameter delta and gamma for this options. Typical patterns for the variation of these parameters are shown in this paper, too.

Key words: *chooser option, delta parameter, gamma parameter, hedging.*

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1. Introduction

The development of financial markets and growing uncertainty of its participants are the main motivation to look for the new financial instruments. The level of investment risk is increasing simultaneously therefore everyone who operating in financial markets has to react to market changes and to correct his investment strategy in time. For this reason investors are looking for new investment possibilities that could fit changing situation in the market and generate income from the investment.

Correctly used derivatives can help investors to increase their expected returns and minimize their exposure to risk. For the first time options were traded in 1973. Then the volumes of their trade had risen rapidly all over the world. (Whaley, 2006)

Standard types of options are traded actively but new types of options bring possibilities for investors of hedging their investment portfolios. Over the last decades the size of the transactions of exotic options in financial markets has expanded. A large variety of these instruments is available to investors. Exotic options are a general name to derivative securities that have more complex cash-flow structures than standard put and call options. The main motivation for trading exotic options is that they allow a much more precise views on future market behavior than those offered by “plain vanilla” options. These options are usually traded over the counter and are marketed by sophisticated corporate investors or hedge funds. They have almost unlimited possibilities and can be adapted to the specific needs of any investor. These options are playing a significant hedging role because they meet the needs of hedgers in cost effective ways. Exotic options are usually less expensive and more efficient than standard instruments. Exotic options can be used as attractive investments and trading opportunities. (Whaley, 2006)

2. Theoretical background of chooser options

An option is an instrument giving its owner the right but not the obligation to buy or sell something at advance fixed price. There are two types of options. A call option gives the holder the right to buy the underlying asset at the strike price in expiration time. The writer of the option has the obligation to sell the underlying asset if the buyer of the call option decides to exercise his right to buy. A put option gives the holder the right to sell the underlying asset at the strike price in an expiration time. The writer of the put option has the obligation to buy the underlying asset at the strike price if the buyer decides to exercise his right to sell. Strike price or exercise price is the price at which the option holder has the right to buy or to sell the underlying asset. (Jarrow, Rudd, 1983)

All mentioned aspects of the plain options are the same for the exotic options. Exotic options may have uncertain exercise prices, expiration time and several underlying assets. Exotic options are mainly over the counter instruments.

A chooser option is part of the exotic option family. It is an option on options and is one of path-dependent options. A standard chooser option gives its holder the right to decide at some prespecified time whether the option will finally be a put or a call. This option is sometimes named “as-you-like” option (Rubinstein, Reiner, 1992; Hull 2012).

The strike prices of the call and put option may be the same but it is not necessary. If the strikes are the same, the chooser option is named a simple chooser option. When the strikes or even expiry dates are not the same this option is referred to as a complex chooser option (De Weert, 2008).

A chooser option is suitable for investors who expect strong volatility of the underlying asset but who are not uncertain about direction of the change. The chooser option holding is similar to holding a straddle strategy which consists of a put and call options with the same strike price. But the chooser holder does not have to pay for both options. He is flexible to decide which option to buy later. A straddle holder has to pay for both options immediately. For this reason the chooser options are less expensive than straddles. (Rubinstein, Reiner 1992; Faseruk, Deacon, Strong, 2004)

3. Basic principles of option pricing

Many scientific studies are devoted to analysis of different methods of options pricing because the valuation of options is very complex.

The most popular valuation model for options is the Black-Scholes-Merton model. The model is based on the theory that markets are arbitrage free and assumes that the price of the underlying asset is characterized by a Geometric Brownian motion. An analytic solution of this model for pricing European options is commonly used. (Black, Scholes, 1973; Merton, 1973)

Monte Carlo simulation is a numerical method for pricing options. For pricing of the option we need to find the expected value of the price of the underlying asset on the expiration date. Since the price is a random variable one possible way of finding its expected value is by simulation. This model can be adapted to pricing almost any type of option. (Boyle, 1977; Boyle, Broadie, Glasserman, 1997)

Another technique for pricing options is the binomial tree model. It is a simplification of the Black-Scholes-Merton approach as it considers the fluctuation of the price of the underlying asset in discrete time. This model is typically used to determine the price of European and American options. (Cox, Ross, Rubinstein, 1979)

Another alternative method for options pricing is a finite difference method, which is based on the Black-Scholes-Merton model. This numerical method consists of reduction of the field of solution to a finite number of points, and the replacement of partial derivations in Black-Scholes-Merton model by given differences. This approach leads to an equation or to the system of equations whose solution is the numerical estimate of the option price. This numerical method could be very useful for the option pricing especially for some more complex types of exotic options, whose price have to be set numerically. (Hull, White, 1990)

3.1 Black-Scholes-Merton model

It is not easy to determine the right price of an option in practice. A big number of pricing models were generated to solve this problem. The Black-Scholes-Merton model and the Cox, Ross and Rubinstein binomial model are the most used pricing models. The Black-Scholes-Merton model is used to calculate a theoretical price of the option using the five key factors of an option's price – underlying stock price, strike price, volatility, time to expiration and risk-free interest rate. Some assumptions were added to derive Black-Scholes-Merton model – there are no transaction costs, there are no dividends during the life of the option, there are no riskless arbitrage opportunities, security trading is continuous, the stock price follows geometric Brownian process with constant mean and standard deviation and the risk-free rate of interest is constant and the same for all maturities. (Whaley, 2006; Hull, 2012)

Although Black-Scholes-Merton model was derived for valuing European call and put options on a non-dividend-paying stock this model can be extended to deal with European call and put options on dividend-paying stocks, American options or options with different underlying assets: (Whaley, 2016; Hull, 2012)

$$c_{BSM}(S,X,T) = Se^{-qT}N(d_1) - Xe^{-rT}N(d_2), \tag{1}$$

$$p_{BSM}(S,X,T) = Xe^{-rT}N(-d_2) - Se^{-qT}N(-d_1), \tag{2}$$

$$d_1 = \frac{\ln(S/X) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}, \tag{3}$$

$$d_2 = d_1 - \sigma\sqrt{T}; \tag{4}$$

where c_{BSM} is a value of European call option calculated by Black-Scholes-Merton model; p_{BSM} is a value of European put option calculated by Black-Scholes-Merton model; S is underlying stock price; X is exercise price; T is time to maturity; r is risk-free interest rate; q is dividend yield; σ is volatility of stock price; and N is the cumulative normal distribution function.

3.2 Chooser options pricing

As it was mentioned before, a chooser option gives to the investor the right to choose at a prespecified time whether the option will be call or put. Once this choice was made the option stays as a call or a put to time of maturity. At this moment the chooser option has the same payoff as the straddle strategy, but it will be cheaper. The reason for the lower price is that the straddle always has a positive payoff and the chooser can end in out-of-the-money. (Whaley, 2006)

An analytical solution for pricing the chooser options is possible because if the options underlying the chooser are both European and have the same strike price, put-call parity can be used to provide a valuation formula. This was proven by Rubinstein in 1991. (Rubinstein, 1991a; Rubinstein, 1991b)

The value of the simple chooser option at the choice time *t* is

$$\max\left[c_{BSM}(S_t, X, T-t), p_{BSM}(S_t, X, T-t)\right], \tag{5}$$

where c_{BSM} is the value of the call option underlying this option and p_{BSM} is the value of the call option underlying this option. Suppose that S_t is the asset price at choice time *t*. Put–call parity implies that (Whaley, 2006)

$$\begin{aligned} &\max\left[c_{BSM}(S_t, X, T-t), p_{BSM}(S_t, X, T-t)\right]= \\ &= \max\left[c_{BSM}(S_t, X, T-t), c_{BSM}(S_t, X, T-t) + Xe^{-r(T-t)} - S_t e^{-q(T-t)}\right]= \\ &= c_{BSM}(S_t, X, T-t) + \max\left[0, Xe^{-r(T-t)} - S_t e^{-q(T-t)}\right] \end{aligned} \tag{6}$$

To value a chooser option, consider the value at time *t* of a replacing portfolio consisting of: (Whaley, 2006)

1) An European call option with strike price *X* and maturity time *T*.
2) An European put option with strike price $Xe^{-r(T-t)}$ and maturity time *t* whose underlying asset price is $S_t e^{-q(T-t)}$.

As such, it can be readily valued. At time *t*, the call option has a value of $c_{BSM}(S_t, X, T-t)$, and a put option has a value of 0, if $Xe^{-r(T-t)} \le S_t e^{-q(T-t)}$, and $Xe^{-r(T-t)} - S_t e^{-q(T-t)}$, if $Xe^{-r(T-t)} > S_t e^{-q(T-t)}$. The value of a chooser option is therefore

$$\begin{aligned} &c_{chooser}(S, X, t, T) = \\ &= Se^{-qT}N(d_1) - Xe^{-rT}N(d_2) - Se^{-qT}N(-d_1^*) + Xe^{-rT}N(-d_2^*), \end{aligned} \tag{7}$$

where

$$d_1 = \frac{\ln(S/X) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}, \; d_2 = d_1 - \sigma\sqrt{T}, \tag{8}$$

$$d_1^* = \frac{\ln(S/X) + (r-q)T + (\sigma^2/2)t}{\sigma\sqrt{t}}, \; d_2^* = d_1^* - \sigma\sqrt{t}. \tag{9}$$

If $t = T$, then the chooser option is the same as a European-style straddle, i.e. equals the sum of the values of an European call and put option with strike price *X* and maturity time *T*. (Whaley, 2006; Hull, 2012)

3.3 Delta-hedging

The option traders use sophisticated hedging schemes for hedging of their portfolio. As a first step, they attempt to make their portfolio immune to small changes in the price of the underlying asset in the next small interval of time. This is known as delta-hedging. They then look at what are known as gamma. Gamma is the rate of change of delta with respect to the price of the asset. By keeping gamma close to zero, a portfolio can be made relatively insensitive to fairly large changes in the price of the asset. (Hull, 2012)

The delta of the derivative is defined as the rate of change of its price with respect to the price of the underlying asset. It is the slope of the curve that relates the derivative price to the underlying asset price. (Hull, 2012)

Assume that Δ is the delta of the call option. This means that when the stock price change by a small amount, the option price changes by about 100.Δ% of that amount. Imagine an investor who has sold one option contract. The position of investor could be hedged by buying Δ shares of the stock. The gain (loss) on the option position would tend to be offset by the loss (gain) on the stock position. A position with a delta of zero is referred to as being delta-neutral. (Hull, 2012)

It is important to realize that the position of investor remains delta-neutral for only a relatively short period of time. This is because delta changes with both changes in the futures price and the passage of time. In practice, when delta hedging is implemented, the hedge has to be adjusted periodically. This is known as rebalancing. Hedging schemes such as this that involve frequent adjustment are known as dynamic hedging schemes. (Hull, 2012)

4. The delta of a chooser option

The delta of a chooser option is the rate of change in value of the chooser option with respect to the change in the underlying asset price. This change can be calculated using the partial derivative of a chooser option price with respect to the variable *S*. Therefore the delta of a chooser option is

$$\Delta_{chooser} = \frac{\partial c_{chooser}}{\partial S} = e^{-qT}N(d_1) + e^{-qT}\left[N(d_1^*) - 1\right], \qquad (10)$$

where d_1 is defined in equation (3) and d_1^* in equation (9). Since the value of a chooser option is the sum of a European call and put option, the same formula we get if we use well known fact that the delta of a European call option is

$$\Delta_c(S, X, T) = \frac{\partial c(S, X, T)}{\partial S} = e^{-qT}N(d_1), \qquad (11)$$

and the delta of a European put option is

$$\Delta_p(S, X, T) = \frac{\partial p(S, X, T)}{\partial S} = e^{-qT}\left[N(d_1) - 1\right], \qquad (12)$$

where d_1 is defined as in equation (3). (Rubinstein, Reiner, 1992; Whaley, 2006; Hull, 2012)

The variation of delta of a chooser option with various times to choose (180 days, 90 days, 30 days and 1 day) is shown in Figure 1. Time from choose to maturity time is 180 days. Strike price of this chooser option is \$80, risk-free interest rate is 5%, volatility is 29% and dividend-yield is 4%.

As Figure 1 shows, if $S \approx X$, then the value of this option is most sensitive to the changes in the price of the underlying asset and this sensitivity increases as the time to choice decreases. This fact could be used by the investor who wants to hedge his portfolio using a chooser option with given parameters. Since the exercise price *X* is predetermined in the beginning of option life, the investor needs to pay attention to development of changes in the underlying asset price. If the asset price is the same or very close to the exercise price, the investor has to be careful and adjust his portfolio more often, especially in case that the time to choice is very close. This is because the value of delta parameter shows that the option value is very sensitive to the changes of asset price, so that the neutrality of the portfolio must be adjusted more frequently.

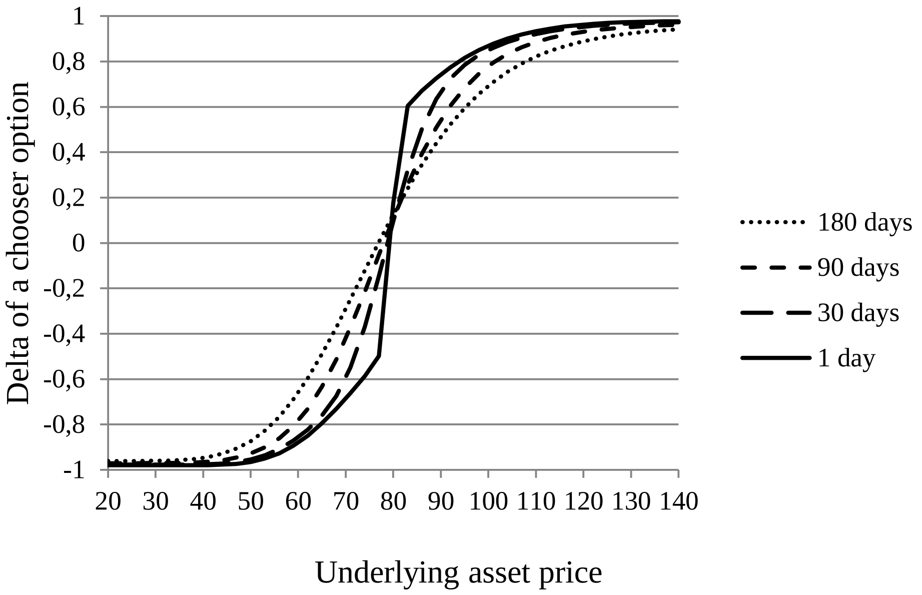

Figure 1 Variation of delta of chooser options with respect to the underlying asset price
Source: Own elaboration.

Also, in the case of $S << X$ or $S >> X$, the value of a chooser option is less sensitive to the changes in underlying asset price. That means, in case that the asset price is sufficiently different from the exercise price (higher or lower), the delta-neutrality of this portfolio is relatively stable. The value of parameter delta is stable, so that the investor does not need to adjust his portfolio. If the underlying asset price will change by a small amount, the value of parameter delta will not change very much, so that the hedging portfolio is still delta-neutral. This is valid for every time to choose, as the Figure 1 shows.

Figure 1 could be more interpreted as follows. In the case of $S >> X$ the value of delta parameter is close to one and for a small change of the underlying asset price it will still stays close to one for every time to choose. So the value of parameter delta is stable. In this case we can say that the holder of the chooser option will choose that his option will be call. This is usable especially in case that the time is close to choose time. Then the greater changes of the underlying asset price are improbable, so we could take into account that the chooser option will be call after choose time. And the same is true in case that $S << X$, for a close choose time the chooser option should be consider to be put option after choose time. In this case the value of parameter delta is close to minus one for every time close to choose time and it will stays similar also in case of small change in the underlying asset price.

For $S \approx X$ is $\ln(S/X) \approx 0$. If $t \to 0$ then $d_1^* \to 0$. In this case $N(d_1^*) \to 0{,}5$ and $\Delta_{chooser} \to e^{-qT}[N(d_1) - 0{,}5]$. This means that when the underlying asset price change by a small amount, the option price changes by about $\Delta_{chooser} \to e^{-qT}[N(d_1) - 0{,}5]$ % of that amount. That means, if the underlying asset price will change, but still will be close to exercise price, the value of parameter delta is instable. Every small change in underlying asset price will cause greater change in value of delta, especially in case that the choose time is approaching. Then the portfolio used for hedging must be adjusted more frequently, as we mentioned hereabove.

In the case of $S << X$ is $\ln(S/X) \to -\infty$ and $d_1, d_1^* \to -\infty$. Then $N(d_1), N(d_1^*) \to 0$. Now we have $\Delta_{chooser} \to -e^{-qT}$. Therefore when the underlying asset price increases by a small amount, then the option price decreases by a discounted value of this amount, and vice-versa. This result is similar to value of delta of a European put option, which chooser option holder will probably choose in this case. For the investor who wants to hedge his asset by using the chooser option that means the following. In case that he has the portfolio created and the price of the underlying asset is actually much under the exercise price, he does not need to reevaluate his portfolio in case that the underlying asset price will change by a small amount. This is because the value of delta is stable in this case and also it is probable, that owner of option will choose his option to be put and this will not change in case of small underlying asset price change. This is valid in all times to choose and also in case that the choose time is very close, for example one day.

Finally, if $S >> X$, then $\ln(S/X) \to \infty$ and $d_1, d_1^* \to \infty$. Then $N(d_1), N(d_1^*) \to 1$. Now we have $\Delta_{chooser} \to e^{-qT}$. Therefore if the underlying asset price increases by a small amount than the option price increases by a discounted value of that amount. This result is similar to value of delta of a European call option, which chooser option holder will choose in this case. The interpretation is similar as described hereabove. The investor owning the chooser option will probably choose that it will be call option. This will not change in case that the asset price will change by a small amount. The value of delta is stable and then the hedging portfolio does not need to be reevaluated. This is valid for every time to choose.

5. The gamma of a chooser option

The gamma is the rate of change of delta with respect to the price of the underlying asset. Therefore gamma can be calculated as a derivative of delta with respect to the price of the underlying asset. Then the gamma of a chooser options is

$$\Gamma_{chooser} = \frac{\partial^2 c_{chooser}}{\partial S^2} = \frac{\partial \Delta_{chooser}}{\partial S} = \frac{e^{-qT} N'(d_1)}{S\sigma\sqrt{T}} + \frac{e^{-qT} N'(d_1^*)}{S\sigma\sqrt{t}}, \tag{13}$$

where d_1 is defined in equation (3) and d_1^* in equation (9). Again, we can also use the value of gamma of a European call and put option

$$\Gamma_c(S, X, T) = \Gamma_p(S, X, T) = \frac{e^{-qT} N'(d_1)}{S\sigma\sqrt{T}}, \tag{14}$$

where d_1 is defined as in equation (3). Figure 2 shows typical variation of gamma of a chooser option with various times to choose time (180 days, 90 days, 30 days and 1 day). Time from choose time to maturity time is 180 days. Strike price of this chooser option is \$80, risk-free interest rate is 5%, volatility is 29% and dividend-yield is 4%.

In the case of $S << X$ or $S >> X$, a chooser options gamma is close to zero, as this figure shows. Situation is more complex for chooser option with $S \approx X$.

If there is $S << X$ or $S >> X$, therefore $\ln(S/X) \to \pm\infty$ and $d_1, d_1^* \to \pm\infty$. In both cases is $N'(d_1), N'(d_1^*) \to 0$ and thus $\Gamma_{chooser} \to 0$. That means the value of delta is insensitively to the small changes in the underlying asset price. As was mentioned hereabove, than the hedging

portfolio does not need to be reevaluated in case that the choose time is far. Figure 2 shows that the value of gamma parameter is close to zero so that the value of delta parameter is stable. For the investor who uses the chooser option to hedge the underlying asset the value of gamma means that the value of delta will not change very much in case of small changes in the asset price. The sensitivity of delta is increasing as the time to choose is decreasing.
For chooser options with $S \approx X$ it can be proved that $\Gamma_{chooser} \rightarrow +\infty$ is implied by $t \rightarrow 0$.

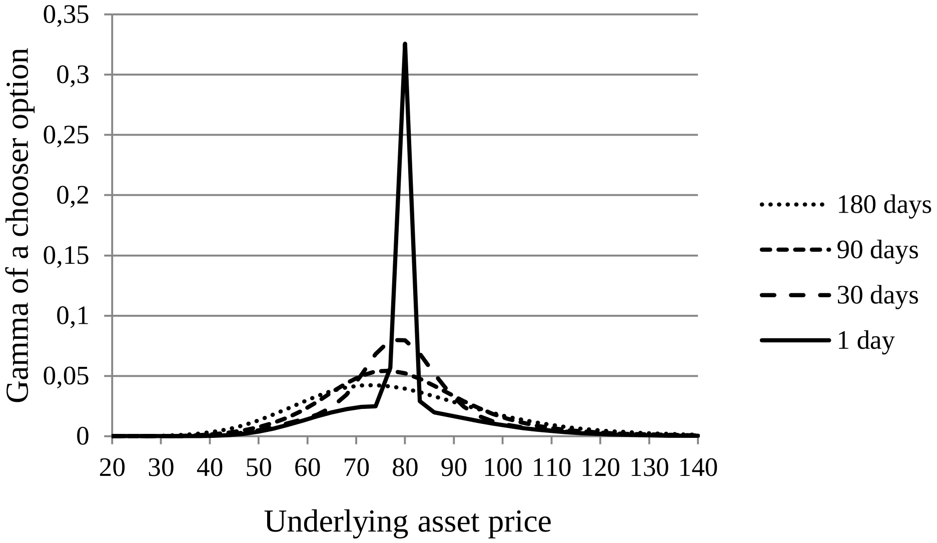

Figure 2 Variation of gamma of chooser options with respect to the underlying asset price
Source: Own elaboration.

5.1 Making a portfolio gamma-neutral

A position in the underlying asset has zero gamma. The only way a trader can change the gamma of his portfolio is by taking a position in a traded option. Suppose that a delta-neutral portfolio has gamma equal to Γ and a traded option has a gamma equal to Γ_O. For chooser option is given by (13). If the number of traded option added to the portfolio is w_O, the gamma of the portfolio is $w_O\Gamma_O + \Gamma$. Hence, the position in the traded option necessary to make the portfolio gamma-neutral is $-\Gamma/\Gamma_O$. Of course, including the traded option is liable to change the delta of the portfolio, so the position in the underlying asset then has to be changed to maintain delta-neutrality. Note that the portfolio is only gamma-neutral instantaneously. As time passes, gamma-neutrality can be maintained only if the traded option is adjusted so that it is always equal to $-\Gamma/\Gamma_O$. Making a portfolio gamma-neutral can be regarded as a first correction for the fact that the position in the underlying asset cannot be changed continuously when delta-hedging is used. (Hull, 2012)

6. Chooser option hedging

In the case of the simple chooser hedging is straightforward. Since the simple chooser decomposes exactly into a portfolio of a call and put option then, if the required strike and maturity are available in the market, these can be used to perfectly hedge the chooser. Often, the precise strike and maturity are not available. In the case of the complex chooser hedging can be used.

The following figure shows the error of parameter delta. This error was calculated as a difference of two values. The first value is the change in the chooser option price (7) with respect to the unit change in underlying asset price. And the second is the value of delta (10) which is the estimate of this change.

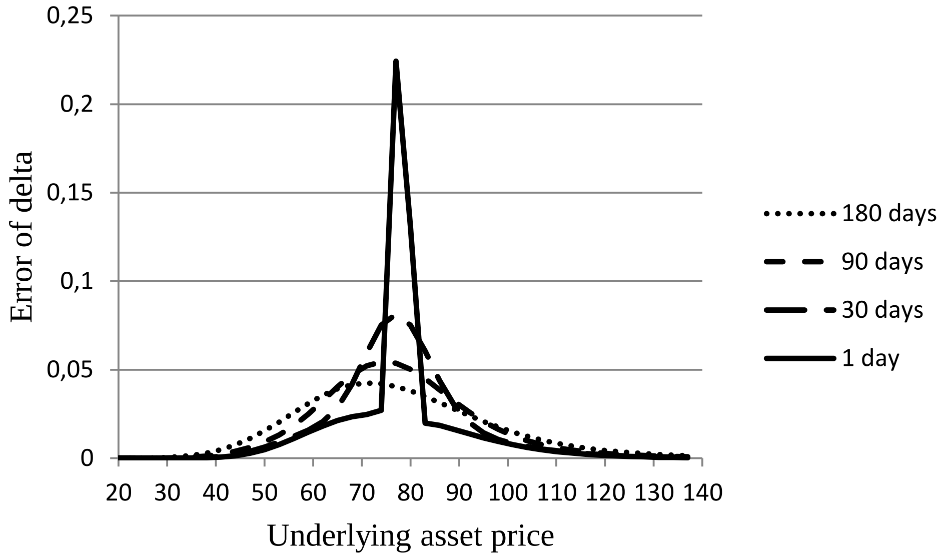

Figure 3 Error of parameter delta of chooser option with respect to the underlying asset price
Source: Own elaboration.

7. Conclusion

Previous analysis shows that the sensitivity of a chooser option price to changes in the underlying asset price, especially for options with underlying asset price close to strike price, is evident. Furthermore, it is clear that the sensitivity increases as the time to maturity decreases.

Gamma of a chooser option with the underlying asset price far from the strike price is almost zero. Therefore, a static delta-hedging is sufficient in this case. It is sufficient to create a delta-neutral portfolio and to review of this portfolio only in the case of major changes in underlying asset price. Whenever the underlying asset price gets closer to the strike price of the option, the parameter gamma can take on large values, especially for time close to the choice time. In this case, it is necessary to use dynamic delta-hedging and review the portfolio more often, which may be related with high transaction costs.

For larger changes in underlying asset price especially for option with $S \approx X$ the reliability of parameter delta is limited. In this case the gamma and Taylor expansion properties must be used. Expected change in an option price can be obtained as the sum of

$$\Delta + \frac{1}{2}\Gamma. \tag{15}$$

This method is significantly more precise. This may appear especially for portfolio with very high value and for big changes in underlying asset price. The modification of delta for the futures options by Ďurica and Švábová was suggested. (Ďurica, Švábová, 2013)

Of course it is necessary to check the impact of other parameters on the chooser option price, for example, the impact of changing time, risk-free interest rate, etc.

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