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## IS EDUCATION IN POLAND REALLY EFFECTIVE?

**Marek Biernacki, Katarzyna Czesak-Woytala**

**Abstract.** This paper discusses the results of the Programme for International Student Assessment and the results obtained by students of the Wrocław University of Economics. Special emphasis was placed on mathematics. Polish students obtained much better results in the latest PISA edition in maths, science and reading when compared to previous editions and to the average results calculated for the OECD countries. Despite this fact the scores of Polish students in the Creative Problem Solving test were much worse than the average for the OECD countries. It is worth noting that the Polish education system has been undergoing dynamic changes which are correlated with political changes and with the increase of social need for participating in decision-making in the education of the young generations. Unfortunately, the pressure to achieve good results in exams means that teachers have started to teach test solving instead of problem solving. This was also shown in the edition of the PISA exam. The authors of the article propose to change the structure of the didactic process, especially at universities, in a way that would make it possible to increase the number of exercises and laboratory hours.

**Keywords:** effectiveness of mathematics education, PISA – Programme for International Student Assessment.

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### 1. Introduction

In the latest PISA exam conducted in 2012, the results of Polish students improved significantly compared to previous editions of this exam. This is certainly an important achievement of both students and teachers.

In 2010 the salaries of Polish teachers increased by 25%. Despite this fact, the salaries are still lower than the average salaries calculated for all the OECD countries. Detailed data concerning teachers' salaries are presented in Table 1 and Table 2.

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Table 1. Average monthly teachers' salaries (PLN) and the number of vacancies in 2007 and 2010 by professional degree

	2007		2010	
	Salary (PLN)	Number of vacancies	Salary (PLN)	Number of vacancies
Trainee teacher	1 401	29 807	2 302	20 509
Contract teacher	1 925	96 810	2 791	108 264
Nominated teacher	2 676	227 658	3 472	165 928
Certified teacher	3 492	228 611	4 394	286 101

Source: Educational Information System: [www.cie.men.gov.pl/index.php/sio.html](http://www.cie.men.gov.pl/index.php/sio.html).

Table 2. Expenditure on education in selected countries

	A	B	C
USA	24 074	40 088	39
Canada	19 992	19 994	31
France	10 704	33 548	53
Germany	11 594	48 167	64
Italy	8 764	31 291	53
Czech Republic	6 774	37 925	59
Poland	4 589	10 263	15

Where:

A – average annual expenditure per student in dollars according to PPP; B – annual wage of a teacher working in a high school, with a 15-year work experience; C – wage per hour of didactic work of that teacher.

Source: own study based on the OECD data.

## 2. Results of Polish students in PISA

The transformation of the Polish education system is reflected in the results Polish students obtain in the Programme for International Student Assessment (PISA). Analysis of the results should help to increase schools' effectiveness and consequently ensure the optimal human capital growth which is the basis of a modern economy (Barro, Lee 2001; Zienkowski 2003).

In every edition of the PISA exam, the results of the Polish students are improving. There were 27 OECD countries which took part in the reading PISA exam in 2000, out of these countries Poland came 22<sup>nd</sup>. In 2009, Polish students were much better and came 13<sup>th</sup>. In the latest edition in 2012 the success of Polish students was spectacular. They took 5<sup>th</sup> place among all the 34 OECD countries. The detailed data are presented in Table 3.

Table 3. The position of Poland in the subsequent editions of the PISA exam in every area (among all the OECD countries which took part in this survey) when comparing mean scores

Year	Position of Poland in reading	Position of Poland in science	Position of Poland in maths
2000	22	-	-
2003	13	-	21
2006	8	19	21
2009	13	13	19
2012	5	5	8

Source: own study based on the OECD data.

When analysing these data it is worth comparing the results of Polish students with Finland which is the leader of almost every edition of PISA in almost every domain.

Figure 1 presents the percentage of Polish and Finnish students who did not reach the second level in the reading exam in subsequent editions of PISA. It is noticeable that the percentage of the weakest students in Poland has been constantly decreasing from 23% in 2000 to 10.6% in 2012. This means that the Polish education system is successful and efficient in such a way that it concentrates on the weakest students and provides them with the necessary skills to achieve better results in such exams. At the same time we can observe an increase in the percentage of the weakest students in Finland which equals over 11% in the last PISA edition and a decrease in the percentage of the best students from 18.5% in 2000 to 13.5% in 2012 (presented in Figure 2). We can also observe that the percentage of the best students in Poland is rather changeable and equals around 10%.

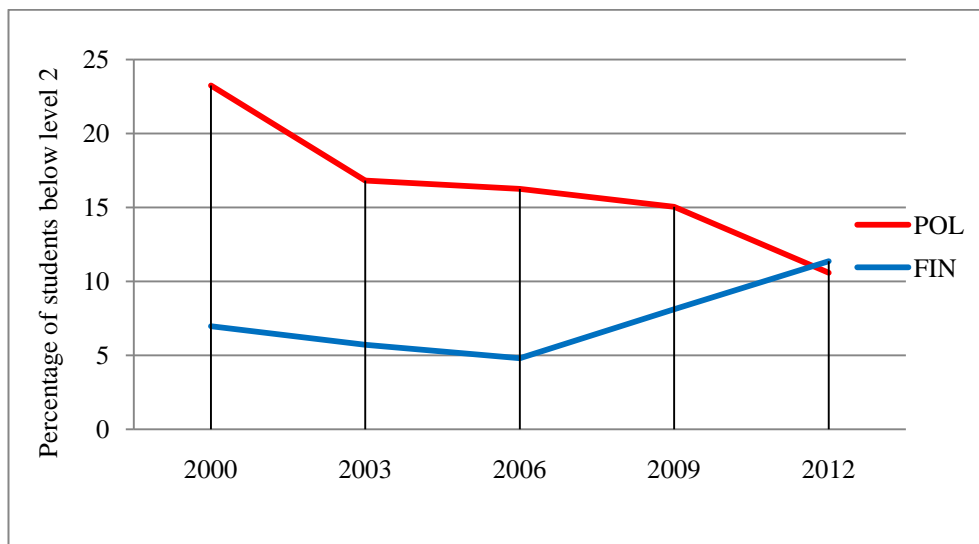


Fig 1. Percentage of Polish and Finnish students who reached at the most 1<sup>st</sup> level in reading in the PISA exam in 2000-2012

Source: own study based on the OECD data.

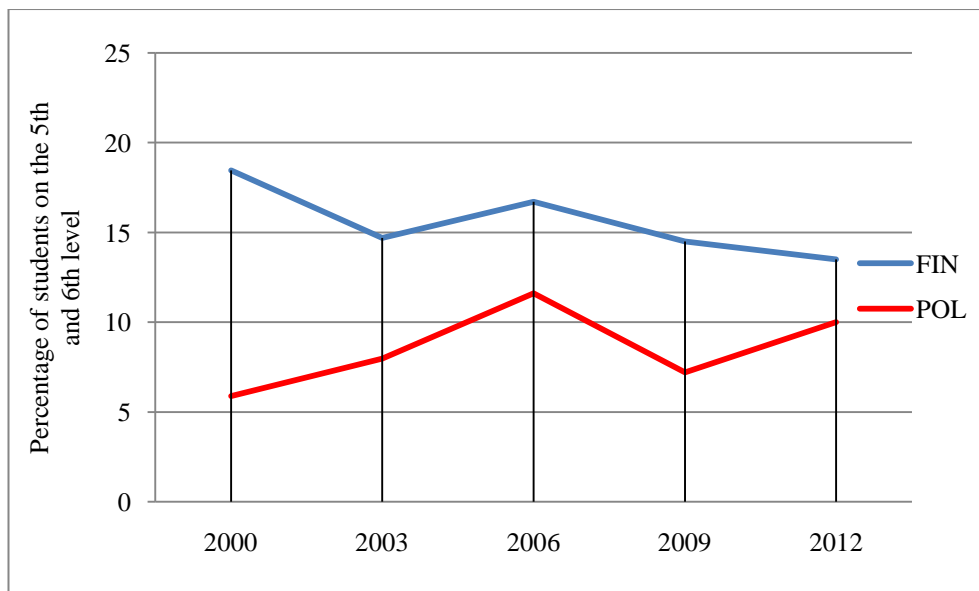


Fig 2. Percentage of Polish and Finnish students who reached 5<sup>th</sup> or 6<sup>th</sup> level in reading in the PISA exam in 2000-2012

Source: own study based on the OECD data.

In Table 3 we can see that Polish students are also becoming better and better in science. In 2006 they took part in the PISA exam in science for the first time and they came 19<sup>th</sup>. In the latest edition Poland took 5<sup>th</sup> place. The percentage of the weakest students in science decreased in Poland from 17% in 2006 to 9% in 2012, while the share of the weakest students in Finland increased from 4.1% to 7.7% in 2012. When analysing the tendency for the best students we can observe an opposite trend both in Poland and Finland.

The most interesting situation can be observed in the domain of mathematics. In 2003 Poland had 21<sup>st</sup> place among the 29 OECD countries. Then, in 2006 and 2009, Polish students achieved 21<sup>st</sup> and 19<sup>th</sup> position respectively among the 34 OECD countries. But in 2012 the high mean score of Polish students resulted in 8<sup>th</sup> position in the ranking of the OECD countries. It has to be mentioned that the mean score in Poland in 2012 equalled 518, which is much higher than the results of previous editions. For the first time, Polish students achieved a better average score than the average for the OECD countries (496).

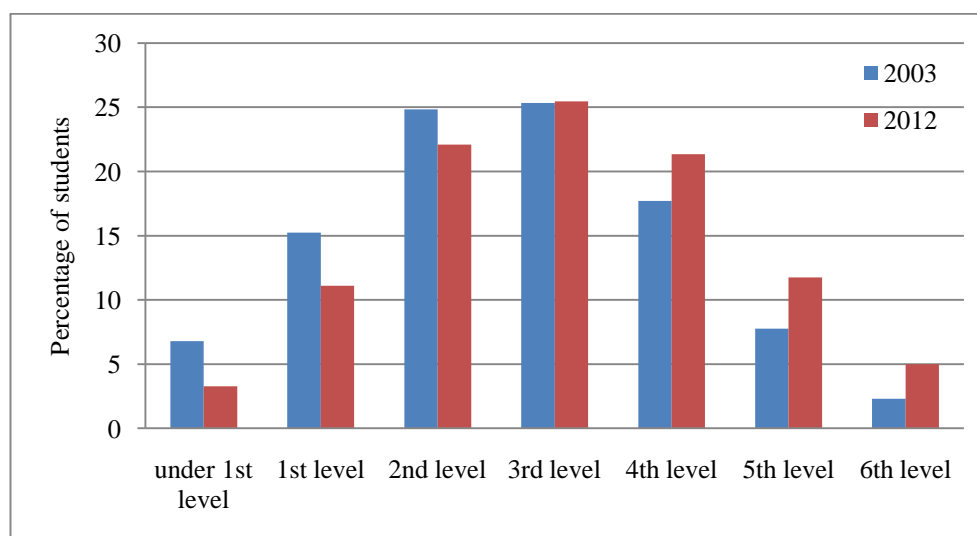


Fig 3. Distribution of the results of the Polish students in maths in 2003 and 2012 (on the ordinate axis is the percentage of students)

Source: own study based on the OECD data.

When comparing the distribution of the Polish students' results in maths in 2003 and 2012 (see Figure 3), we can observe its shifting to the right. This means that in 2012 the whole population of 15-year-old students



achieved better results than in the previous edition. This is a very good tendency and without further detailed investigation it could be treated as a great success of the Polish education system, students and teachers.

Figure 4 and Figure 5 present the share of the Polish and Finnish students who reached at the most 1<sup>st</sup> level and at least 5<sup>th</sup> level in maths respectively. From these graphs we can learn that in the 2003, 2006 and 2009 editions the percentages of the best and the weakest students in Poland were rather stable. Only the latest edition of the PISA exam resulted in the rapid growth of the share of the best students and a corresponding significant decrease of the share of the weakest students.

At the same time we can observe the opposite tendency in Finland. The results of Finnish students were similar in 2003 and 2006. Then, in 2009, we can notice a slight decrease of the share of the best students and a corresponding increase in the percentage of the weakest students. From these data one can conclude that maybe the 8<sup>th</sup> position of Polish students in the latest PISA edition was not the result of the efficient education system in Poland, but of the weaker performance of other OECD countries.

This issue certainly requires further investigation.

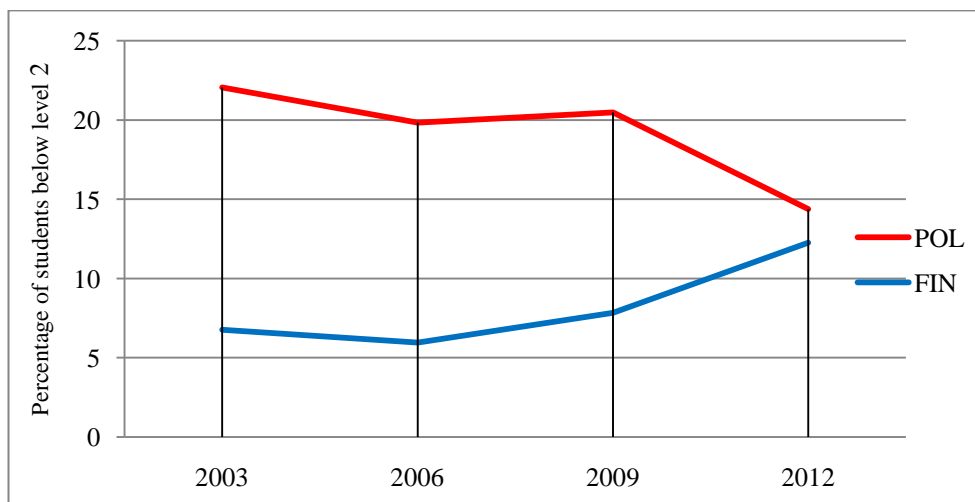


Fig 4. Percentage of Polish and Finnish students who reached at the most 1<sup>st</sup> level in maths in the PISA exam in 2000-2012

Source: own study based on the OECD data.

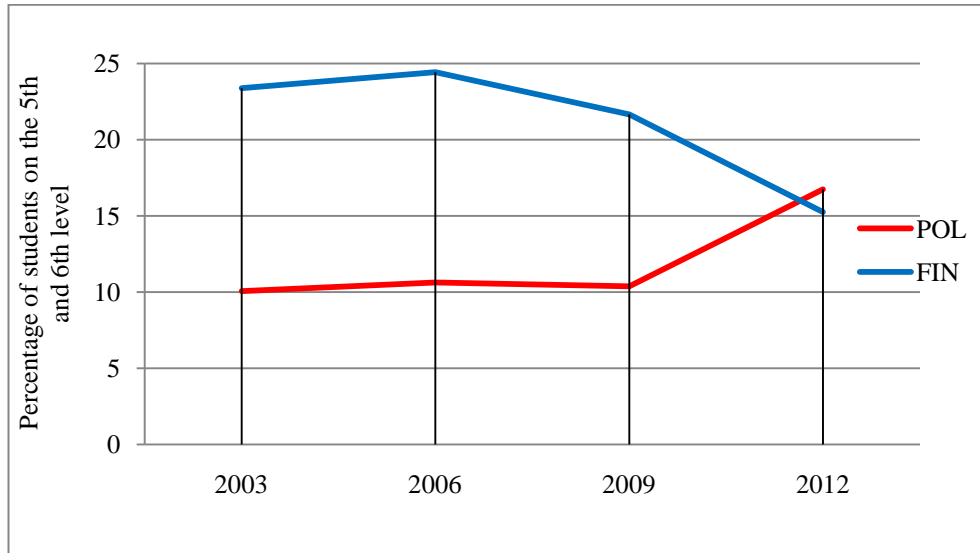


Fig 5. Percentage of Polish and Finnish students who reached 5<sup>th</sup> or 6<sup>th</sup> level in maths in the PISA exam in 2000-2012

Source: own study based on the OECD data.

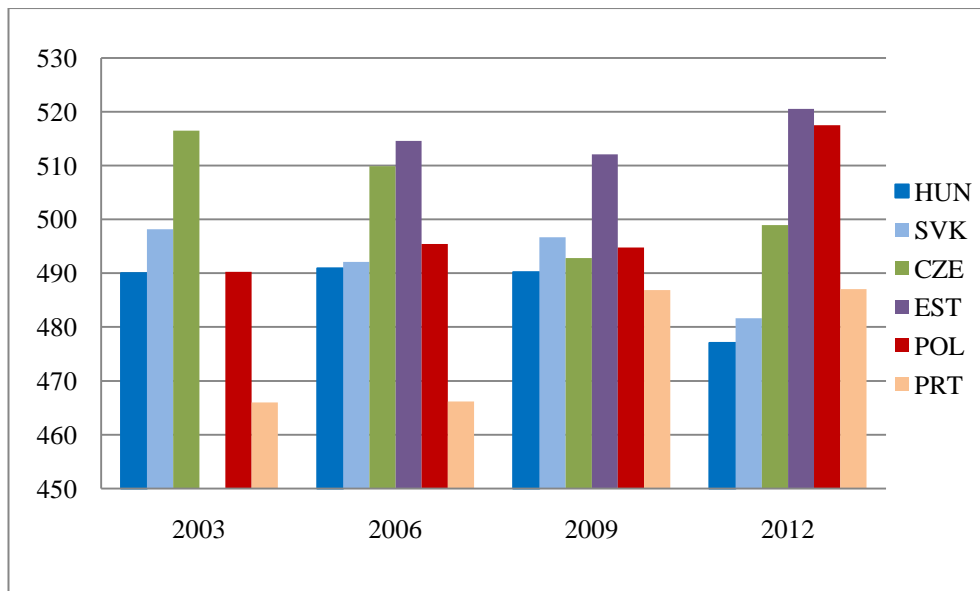


Fig. 6. Mean results in maths for Poland and similar countries in every edition of PISA

Source: own study based on the OECD data.

In the next section we decided to compare the results of Polish students with students from countries which are similar to Poland. In order to identify similar countries, the k-means analysis based on GDP per capita and cumulative expenditure per student between 6 and 15 years old was implemented. We took into account the OECD countries which are members of the European Union. Such an approach allowed us to define a set of countries similar to Poland: Hungary, the Slovak Republic, the Czech Republic, Estonia and Portugal.

Figure 6 presents the mean scores in maths for these selected countries. It is noticeable that up to 2009 the results of Polish students were very close to the results of students from Hungary and the Slovak Republic. However, in 2012 the results of the Polish students rapidly increased and were at a level similar to Estonia. A detailed analysis of the share of the weakest and the best students confirms these theses. In almost every edition, Portuguese students achieved the weakest results among the countries selected. Portugal also had the highest percentage of the weakest students (who did not even reach the 2<sup>nd</sup> level) and the lowest percentage of the best students (who reached at the least 5<sup>th</sup> level). But this situation improved in 2009 and 2012. The opposite situation can be observed in the Czech Republic which achieved very good results in 2003 and 2006 (mean value at the level of around 510). These were about 18% of best students. But in 2009 and 2012 their results worsened and were even below the average score for all the OECD countries.

### **3. PISA 2012 results in Creative Problem Solving**

We have already discussed in detail the results of Polish 15-year-old students in reading, science and maths. It has been mentioned that Poland achieved very good scores in the last PISA edition in almost every domain. This success has been heavily publicized in the Polish media and by the Polish government. But no one mentioned another aspect of the PISA survey, which is the Creative Problem Solving test. “The problem-solving assessment in PISA 2012 focuses on students’ general reasoning skills, their ability to regulate problem-solving processes, and their willingness to do so, by confronting students with problems that do not require expert knowledge to solve”<sup>1</sup>. That is why the next step of our analysis concentrates on the

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<sup>1</sup> OECD (2014). PISA 2012: Results Creative Problem Solving. Students’ skills in tackling real-life problems. Volume V, page 28.

results in this domain and their relationship with the results in maths. There were 40 countries which took part in the Creative Problem Solving test. Taking into account the mean score of Polish students (which equals 480), they achieved 27<sup>th</sup> position, which means that their result was much worse than the average calculated for all the OECD countries. Over 25% of Polish students did not achieve even the 2<sup>nd</sup> level and less than 7% achieved at least the 5<sup>th</sup> level. For all 40 countries the Pearson correlation coefficient between the percentage of the weakest students in maths and the percentage of the weakest students in problem solving equals 0.91. An analogous correlation for the percentage of the best students equals 0.82. Both correlations are statistically significant at the 5% significance level. This means that the results in mathematics are strongly correlated with the results in the problem solving test.

How does it look for countries which are similar to Poland?

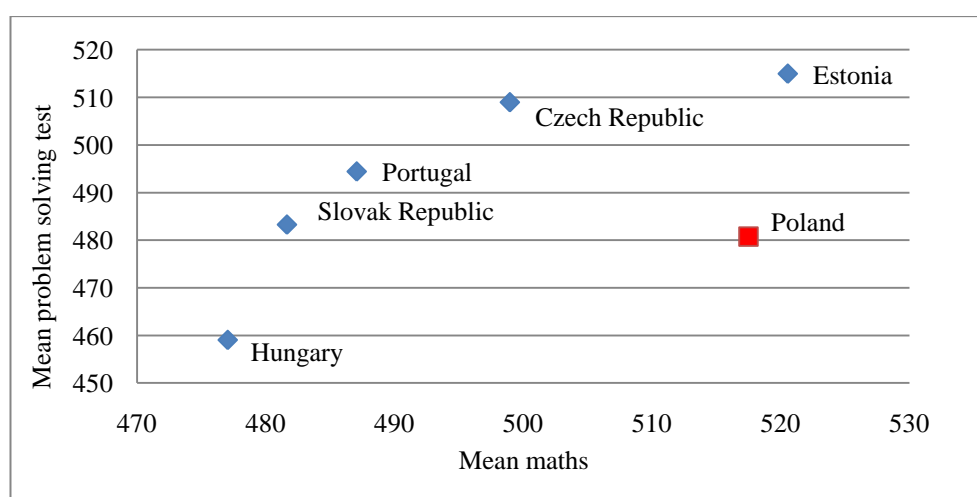


Fig. 7. Relationship between the mean results in maths and the mean results in the problem solving test

Source: own study based on the OECD data.

Figures 7-9 present the relationships between the results in maths and the problem solving tests as mean scores and percentages of the weakest and the best students respectively. One can notice that the relationship for Poland is different than the relationships created for Estonia, Hungary, Portugal, the Slovak Republic and the Czech Republic.

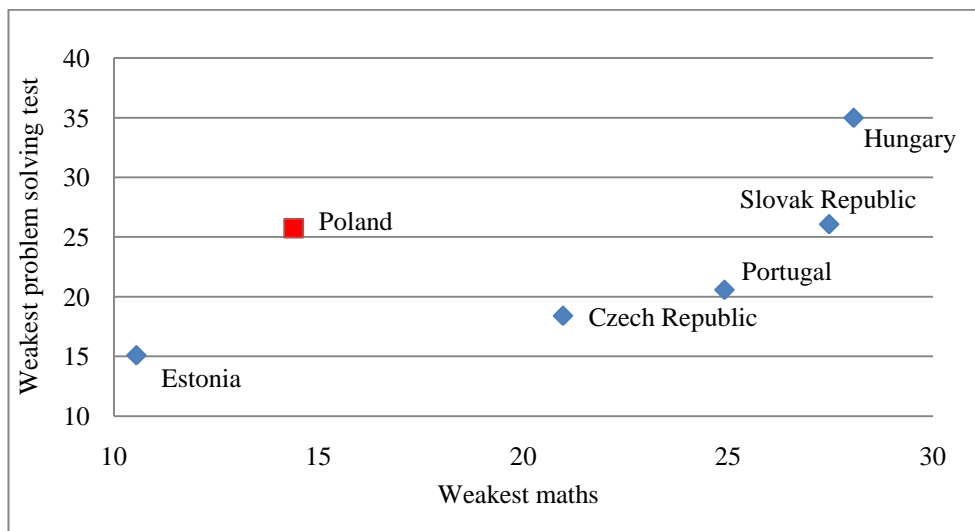


Fig. 8. The relationship between the percentage of the weakest students in maths (at the most 1<sup>st</sup> level) and the percentage of the weakest students in the problem solving test (at the most 1<sup>st</sup> level)

Source: own study based on the OECD data.

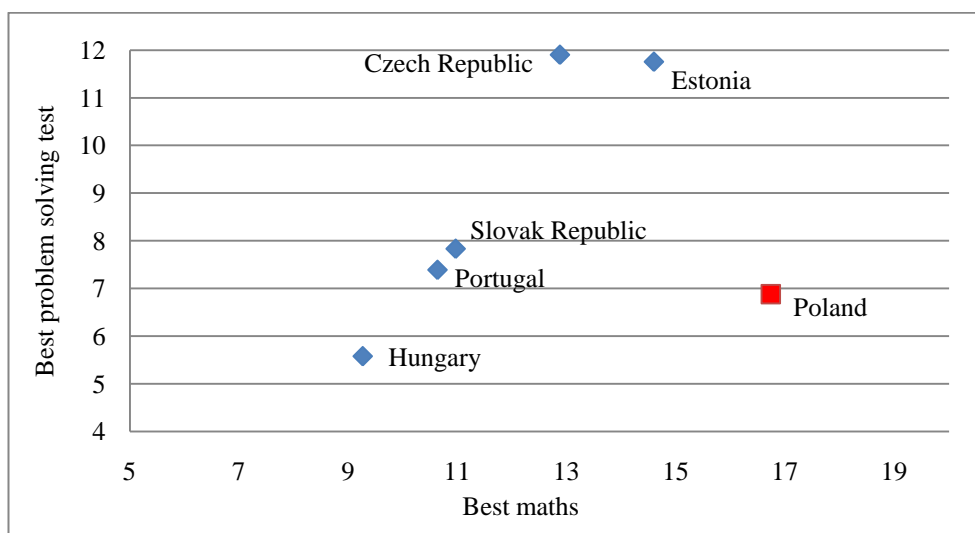


Fig. 9. The relationship between the percentage of the best students in maths (at the least 5<sup>th</sup> level) and the percentage of the best students in the problem solving test (at the least 5<sup>th</sup> level)

Source: own study based on the OECD data.

Table 4. Pearson correlation coefficient between the results in maths and the problem solving test

	Mean scores	Percentage of the weakest students	Percentage of the best students
Poland and similar countries	0.59	0.63	0.37
Similar countries	0.87	0.79	0.95

Source: own study based on the OECD data.

The corresponding correlation coefficients calculated first for Poland and similar countries and then only for similar countries are presented in Table 4. In every case the correlations calculated only for our competitors are much higher. We can notice that despite the fact that Polish students achieved relatively good results in maths, they performed rather poorly in the problem solving test (see Figure 7). Furthermore, although there were only 14% of the weakest students in maths in Poland, over 25% of Polish students did not cope with the easiest tasks from the problem solving test. Moreover, the high percentage of the best students in maths (almost 17%) does not entail a high percentage of the best students in the problem solving test. This means that Polish students have already learned how to write tests and achieve good results, but they still cannot apply their knowledge to solve real problems (Projekt EKOLOGIA 2010).

The World Bank recommended in the World Bank report (1996) that in order to support education in the countries of the former Eastern Bloc and to adjust their education processes to the requirements of the market economy, one of the necessary changes is to develop students' skills to solve difficult and new problems. The conclusion presented above proved that the Polish education system did not fulfil this recommendation.

#### **4. How do students perform in maths at the Wroclaw University of Economics?**

A good way to verify one's ability to learn and solve problems is the exam in maths for students of the 1<sup>st</sup> year of study. Only some elements of Analysis 1, Analysis 2, Algebra 1 and Algebra 2 are taught during one semester of study (30 hours of lectures and 15 hours of exercises). Such an approach requires from students a lot of self-reliance. Unfortunately they often do not fulfil this requirement. The absence rate on the lectures which are not obligatory equals about 50%.

During the exam in maths in 2013 the students had to face following tasks:

1. Calculate the limit:  $\lim_{n \rightarrow \infty} \left( \frac{2n+3}{2n-5} \right)^{7n}$ .

2. Draw a graph of the function:  $f(x) = x^2 \cdot e^{-2x}$ .

3. Calculate the field of the area which is bounded by the graphs of the following functions:

$$f(x) = x^2$$

and

$$f(x) = 2x + 3.$$

4. For the linear transformations given:

$$f(x, y, z) = (x + y, y + z, x + z)$$

and

$$g(x, y, z) = (x + y + 2z, -x + z, -2x - y).$$

Find  $f \circ g^{-1}(x, y, z)$ .

5. Solve the system of equations:

$$\begin{cases} x + y + t = 1 \\ x + 2y + z - 2t = -2 \\ -x + 2z + t = 3 \\ x + 3y + 3t = 2 \end{cases}$$

6. Find the local extremum of the function:

$$f(x, y) = 2x^3 + 2x^2 - 2xy - y^2 + 1.$$

74 out of the 124 first year students failed (60%). There were 22 (18%) students who received at least a good appraisal. Compared to the years 2009 and 2012 (see: Biernacki 2009; Biernacki, Czesak 2012), we can observe an increase in the percentage of students who received an unsatisfactory appraisal. In 2009 there were 50% of students and in 2012 there were 53% of students who received an unsatisfactory appraisal. Up to now, the differences in the percentages of students with an unsatisfactory appraisal

between subsequent years of the exam are statistically insignificant, but these differences are getting greater. However, we can also notice an increase in the best students (students who achieved a good or a very good appraisal) when compared with the results from the 2012 exam ( $p$ -value = 0.06).

## 5. Conclusions

Both the better results in the final exam in 2013 and the better results in the PISA exam in 2012 can be a positive effect of education after the enhancement of teachers' salaries in 2010.

Unfortunately, the results of the final exam in maths confirm our conclusions concerning the Creative Problem Solving test. Tasks which require finding some algorithm or combining several solutions simultaneously are the most problematic for students. Students do not have problems with calculating an inverse matrix, but solving an unmarked system of equations with the use of the Gauss method (elementary transformations method) causes serious problems.

This is why it is necessary for the Polish education system to undergo changes based on our conclusions concerning the Creative Problem Solving test. In primary, lower and upper secondary schools this means implementing changes in the way of teaching skills. At a higher education level the emphasis should be put on practical classes (exercises and laboratories). This would entail significant changes in the quantity and structure of teaching hours, because in almost every specialisation in the Wroclaw University of Economics the number of lectures in maths is at least equal to the number of exercises (financial criteria).

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