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SINGLE–WORD SELF–REFERENTIAL SENTENCES

Ludomir M. Laudański

The man who knew his enemy's name
could, by means of it, acquire magic power over him

Bertrand Russell (1872-1970)

Abstract. The paper discusses the well-known logic paradox formulated by Grelling. It belongs to the class of paradoxes known as Russell's paradoxes. Analysing Grelling's paradox, the author arrives at the conclusion showing its apparent nature, in other words resolving the problem.

Keywords: Grelling's paradox, logic, string variables.

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1. Grelling's paradox

By a word[1] we understand an entry in a dictionary. To set a selective reservoir of English words we have adopted (Vlasova 1974) containing approximately 3500 words. As a reference we have also used (Dictionary 2003) supported by (Thesaurus 1986).

Grelling's paradox analysed here follows its description given by Jean van Heijenoort in (Encyclopedia 1972). We quote its essential part below:

A word is said to be "autological" if, and only if it applies to itself, that is, it satisfies the schema:

"W" is W (1.1)

otherwise it is called "heterological".

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[1] Here the idea belonging to Wittgenstein (1981), that a word is a family – proves to be extremely useful.

Based on such a definition, van Heijenoort explains the essence of the paradox, i.e. the word "heterological" is neither heterological nor autological. The paradox arises on applying the schema (1.1) to this word. In particular, while assuming that this word is autological, the application of the schema (1.1) leads to the statement:

"Heterological" is heterological (1.2)

but the statement (1.2) itself, taken literally, instructs us that this word is heterological. And, on the contrary, if we begin this consideration by stating that it is a heterological word, we would express it exactly as shown by (1.2). But this, in turn, via van Heijenoort's definition, says that the word "heterological" is autological.

It might be interesting to note that a reader of the references (Gardner 1982; Paulos 1985; Smullyan 1988), although they are almost entirely devoted to the paradoxes and related pleasures, would not find any mention of Grelling's paradox. From among the well-known, so to speak, contemporary magicians, only Douglas R. Hofstadter mentioned it in (Gardner 1982), although for an unprepared reader it would not be easy to follow what precisely he is talking about.

2. Grelling's words

Let us begin our consideration by applying such a reliable initial stage as is offered by a predicate sentence (Hodges 1981) written schematically as:

x is ****. (2.1)

Examples. The girl is a virgin. John is a liar. An orange is green. Today is Tuesday.

Letting the variable 'x' be a string variable, the above expression leads us to:

"x" is ****. (2.2)

And now to arrive at exactly the same schema (1.1) we have to replace the symbolic predicate **** by the variable 'x'. This finally gives us the string self-referential sentence:

"x" is x, (2.3)

which has to convey the essential features of (2.1). The details lost in such a way can be restored by proposing what follows.

It has to be acknowledged, by the way, that the schema (2.3) has been simplified by the absence of the definite and/or indefinite articles. More correctly we would write:

The "x" is a/an x or A/an "x" is x.

We hope that this is a forgivable simplification.

2.1. General Grelling's words

The first definition proposed instead of (2.3) is:

$$\forall x \exists y \ (\text{»“x” is y «} \text{ is a meaningful sentence}), \tag{2.4}$$

which says that there is such a word 'y' that every[2] sentence following the above pattern in which 'x' is replaced by a word must be a meaningful sentence or it should have a sense as stated by Wittgenstein (1981).

Let us now explain the meaning of the words denoted by 'y'; words which we would like to call the general Grelling's words. They state something about *every* string variable which is *always* true, or rather they are telling us something about the formal side of the string variable. Let us look at this example:

y = within. (2.5)

We can say now: whatever is placed within the quotation marks is within (the said quotation marks). Therefore the sentence:

"x" is within

is true for every 'x'. If so, it is also true for:

x = within

therefore the meaningful sentence:

"Within" is within (2.6)

leads us to the conclusion that the word 'within' is a Grelling's word. Moreover, while being discovered by the definition (2.4) it is called a general Grelling's word. Here are a few other examples:

bout, ambiguous, first, present, quotation, still, string, visible (2.7)

Moreover, some synonyms of these words will also be other Grelling's words.

[2] The domain of 'x' are all the words of a language.

Examples.

$$\text{within} \Rightarrow \text{between} \quad \text{ambiguous} \Rightarrow \text{vague} \tag{2.8}$$

In practice, while searching for such words we say to ourselves:

Every string variable is...

and try to finish the sentence by placing the successive word taken from the vocabulary.

2.2. Particular Grelling's words

The second definition is given by:

$$\exists R(\forall x(x \in R \rightarrow \text{»“x” is R«}\ \text{is a meaningful sentence})), \tag{2.9}$$

which says that there is such a set R that for each of its members, the above defined sentence becomes a meaningful sentence. For instance, let R stand for the set of basic words:

$$\text{R} = \text{the set of basic words} \tag{2.10}$$

then we read (2.9) like this: for every basic word 'x' the sentence »“x” is a basic word «is a meaningful sentence; therefore, taking x = basic, we arrive at the meaningful sentence:

“Basic” is basic. (2.11)

The word 'basic' is a basic English word. This leads us to the *particular* Grelling's word 'basic'. Other examples are given:

ball, correct, English, entry, name, noun,
orange, rose, third, short, simple, (2.12)
single, violet, word.

While searching for such words we can help ourselves by saying:

Something is...

and perusing successive words from the vocabulary in order to finish the sentence successfully.

3. Pseudo Grelling's words

The cases listed below are considered as apparent self-referential cases. Therefore they do not define NEW Grelling's words. Nevertheless they seem to be worth mentioning.

3.1. Etymological self-reference

If we recall the so called Ockham's razor with its claim:

» Plurality is not to be assumed without necessity «

then we would refute from the scope of Grelling's words all compound words. Here are some examples:

autological, blackboard, greenhouse, heterological,
multisyllabic, necktie, something, passport, steamboat, (3.1)
waterproof, windmill.

3.2. Indirect speech references

The verbal forms of words are required to be infinitives. Therefore such examples as below are refuted as well:

defined, employed, expressed,
given, learned, recognised, (3.2)
spoken, typed, written.

3.3. Homonyms

They seem to have something in common with the pseudo-referential cases.

Examples.

Mary is merry. Band is bend. (3.3)

do not need any special comments. At least they *sound* like Grelling's words.

3.4. Nomen-omen references

According to the story there was a bald person whose name incidentally was 'Bald'. Therefore the sentence which acknowledges that fact, i.e.:

"Bald" is bald (3.4)

falls into the schema (1.1) and **is** a meaningful sentence. Nevertheless we also propose to refute such a motivation and do not consider any word received entirely in this way as a Grelling's word.

3.5. Subjective self-references

Appropriate examples belong mainly to the general schema (2.4), here we have such words as:

amazing, boring, fascinating, funny, silly. (3.5)

But there are some examples which can be placed in the schema (2.9) like:

cruel, hard, sharp, soft. (3.6)

Is the word 'soft' really a soft word? There is no objective definition of how to judge whether a word is soft or not. Therefore we refute all the above words from the possible scope of Grelling's words as well.

3.6. Strange self-references

To say what we understand by the strange self-reference, let us look at the following examples:

"B l a c k" is black. "B I G" is big.
"$_{\text{small}}$" is small. (3.7)
"S t R a n G e" is strange.

If the sentences (3.7) possess some meaning, this is entirely due to some special reasons lying beyond the scope which would be assigned to a string variable. Therefore we do not consider that in such a way we may find new Grelling's words anyway.

4. Conclusions

4.1. Computability

Is it possible to write a computer program which would perform a search for Grelling's words?

It seems rational to believe that the answer to this question is positive. Although this matter has been left beyond the scope of our consideration.

4.2. Completeness

Are the general Grelling's words and the particular Grelling's words two mutually exclusive sets?

The suggested answer is 'yes'. The appropriate reasoning may go as follows. Intuitively each string variable is composed out of the two components: the string itself, and the appropriate variable placed between the

strings. We have here two definitions (2.4) and (2.9) bearing in mind that they would catch both these aspects accordingly. But we do not claim that the enclosed list of the pseudo-Grelling's words has been completed, although their importance does not look too impressive.

4.3. Undecidability

Is it possible to rule out Grelling's paradox by applying such a consideration as is presented here?

Let us approach the case step by step.

It seems reasonable that the procedure described here can be applied to the definitive record of the English words as presented by (Dictionary 1985). If so, we would be able[3] to complete a full list of Grelling's words. We would not see any room for a paradox in such circumstances. Therefore we infer that the possibility to capture – if at all – Grelling's paradox remains *beyond* the vocabulary.

Is it possible to step somewhat ahead in this direction?

Let us say that we employ a neologism – a newly born word which is not in (Dictionary 1985) – for instance, a snaeg, to denote every Grelling's word. This means that now any Grelling's word can be called 'a snaeg' due to the identity:

$$\forall x \text{ (`x' is a Grelling's word } \Leftrightarrow \text{ `x' is a snaeg)} \qquad (4.1)$$

We would be tempted now to ask whether this neologism – a word which does posses a single meaning described by (4.1) is a Grelling's word or not? To give the answer we should be able to judge whether the sentence:

$$\text{The `snaeg' is a snaeg} \qquad (4.2)$$

is a meaningful sentence. To avoid a possible trap we have to acknowledge that we arrive at an undecidable situation. If a word possesses a single meaning we have to say that there is no possibility to assign a meaning to such sentences as (4.2) – therefore we have no facilities to judge whether such a word is a Grelling's word or not. In this way we shall avoid applying the reasoning which states arbitrarily that a word either is a Grelling's word or not. This is an opening which leads directly to Grelling's paradox with respect to neologisms built in the way suggested by Grelling. With the proposed measures we have ruled out Grelling's paradox definitively.

[3] At least in principle.

To be convinced that Grelling's paradox is no more threatening to our considerations, let us employ another neologism by the identity:

$$\forall x \text{ (`x' is a non-Grelling's word } \Leftrightarrow \text{ `x' is snaerl)} \qquad (4.3)$$

In order to examine whether it is or not a Grelling's word we have to say whether the sentence:

$$\text{The `snaerl' is a snearl} \qquad (4.4)$$

is a meaningful sentence or not. But also this time, the only meaning which the word 'snaerl' possesses stems from (4.3) therefore also this time there is no chance to resolve the question. There is also no chance to arrive at the sentences built á lá Grelling:

$$\text{The snaerl is a snaeg} \qquad \text{The snaeg is a snaerl} \qquad (4.5)$$

being indispensably associated with Grelling's paradox.

We do not know precisely what Ludwig Wittgenstein (1981) had in his mind when he wrote the sentence quoted below, nevertheless, we are deeply impressed with its matchless applicability to the case under discussion. This is what he said:

> *It is impossible to represent in language anything that 'contradicts logic' as it is in geometry to represent by its coordinates a figure that contradicts the laws of space, or to give the co-ordinate of a point that does not exist.*

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