ITALIAN CRADLE OF CHANCE CALCULUS

ŚLĄSKI
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Summary: Traditionally, the history of probability theory is dated for the year 1654, when a gambler de Méré proposed to Pascal two problems to solve. The problems were solved independently by B. Pascal and P. Fermat. One rather ignores that similar problems had been discussed and solved two hundred years earlier by Italian mathematicians. The article presents a sketch of the history of pre-Pascalian period of development of the doctrine of chances.

Keywords: history of probability theory, problem of points, problem of dice.

1. Introduction

Almost any traditional textbook on probability starts with a brief account of history. Traditionally this history is dated for the year 1654, when de Méré proposed to Pascal two problems to solve.

The first problem is known as a *dice problem*:
when one throws two dice, how many throws are needed to have a biger than even chance of getting two sixes at least once?

The second problem is known as *division problem (probleme des parties,* problem of points, Teilungs Problem):
how one shall equitably divide the prize of money in a tournament in a case when the series is interrupted before it is completed?

It is true that these two problems were solved in 1654, and this is well documented in the correspondence Fermat-Pascal. One may, however, be surprised why the foundation of calculation of chances was laid down so late, knowing that games of chances were as old as the humanity?

The reasonable answer is given by I. Hacking. The calculation use numbers. Greeks treated numerals more metaphysically, even mystically, than practically. In particular, they lacked a perspicuous notation for numerals.

ŚLĄSKI
PRZEGLĄD
STATYSTYCZNY
Nr 10 (16)

The proper notation and appropriate rules for calculations were developed by Hindu mathematicians between the first and fifth century. By the ninth century this system had been fully recognized and appreciated by Arabs in Islamic countries.

Persian mathematician of very long name: Abu Jáfar Muhammad ibn Musa al-Khwarizmi , wrote a treatise on Hindu arithmetic. The Arabic text is lost, however. The Latin translation with the title *Algoritmi de numero Indorum,* was the next one translated into English as *Al-khworizmi on the Hindu Art of Reckoning*. The word *algorithm*, so popular now, is a deformed name of that mathematician.

Due to Italian mathematicians this work, and as a consequence the entire Hindu-Arabic numeral system has been "imported" into West, and become an essential element of Western civilization.
The most significant contribution are presented below.

2. Gerbert d'Aurillac

Beyond any doubt, Gerbert d'Aurillac deserves the credit to introduce the Hindu-Arabic system of numerals to Western Europe.

The childhood of Gerbert varies in description, depending on whether the writers belong to church or not.

Certainly he was born about 945 apparently in Auvergune in the middle of France. Around 963 he entered the monastery of St. Gerard of Aurillac. In 967 this monastery was visited by the count Borrell II of Barcelona. He took a young Gerbert to Spain so he could acquire some knowledge of Arabic learning and study mathematics.

In the period from 972 to 982 Gerbert was appointed as the cathedral teacher in Rheims. In 983 he became a private teacher of the German emperor Otto III (983-1002). With a strong support of the emperor, Gerbert was elected a pope in 999 and took the name Sylvester II.

He was a prolific scholar, published scientific works, invented hydraulic-powered organ and wrote a dogmatic treatise (*De corpora at sanguine Domini*).

Due to these scientific interests as well as due to his efforts to root out a lot of corruption within the church, there were created lot of unfavorable stories about him which were spread over the long time.

ŚLĄSKI PRZEGLĄD STATYSTYCZNY
Nr 10 (16)

He was accused to be in contact with a devil. In 1648 his tomb was opened to see the absence of devil there.

Around 1000 A.D. he who wrote the treatise *Regula de Abaco Computi*, where he presented the rules for computing with abacus he had invented. This abacus is known as Gerbert's abacus.

3. Leonardo Pisano

The first significant step in the development of the modern algebraic calculi in the West was taken by Italian merchant Leonardo from Pisa, known as Fibonaccio. The name Fibonaccio was derived from the Italian expression "filio di Bonaccio" meaning literally a son of Bonaccio.

Leonardo was born in 1180 and died in 1240. In his book *Liber Abaci* published in 1202 he presented algorithms for performing arithmetic operation using Hindu system of numerals. The title of the book might suggest that this is the book on abacus as the instrument for making calculation. However, the word *abaci* in the title was probably used because then the word abacus was treated as a synonym of "calculations". Some other authors argue that the title was used for marketing reasons: term "abacus" was more attractive than algorithm.

The struggle between *abacists* and *algorists* ended not until the XVI century with the victory of the latter ones. The weapon for this battle was produced by Fibonaccio. One of the most remarkable contributions of this book was the introduction of the name of zero. Zero had been already discovered by Hindu and called *Śūnya* by them. In sanscrit Śūnya means nothing, empty. Arabs translated this word as *Sifr* which means empty. Fibomeci used a Latin word *zephirum*. After three hundred years this word become *zefiro*, and next in different languages it appears as *zero*. The given number was called by Fibonacci *numerus*, and the unknown as *res* (thing) or *radix* (root); the square of unknown was named *census*. This terminology was later adopted by other Italian writers.

ŚLĄSKI PRZEGLĄD STATYSTYCZNY Nr 10 (16)

4. Luca Bartolomeo dal Borgo (Pacioli)

Luca was born in 1445 in Borgo Sansepolcro, not far away from Perugia. His father was Bartolomeo Pacioli. Luca died in 1517. We know little about his early life. As a young man he moved from Sansepolcro to Venice, where he went into service with the wealthy merchant Antonio Rompiasi. In 1477 Pacioli began life travel. Around 1496, Ludovico Sforza, duke of Milan, with a suggestion of Leonardo da Vinci invited Pacioli to come to Milan. Luca and Leonardo became close friends, so close that Leonardo made the illustration for the Pacioli's book *Divine proportione*. Historians wonder how many mathematicians can have so talented illustrators for their books.

One of the most famous mathematical books, *Summa de arithmetica, geometria, proportioni et proportionalitá*, covering about 600 pages, was written by Pacioli and published in Venezia in 1494. One of the chapters is devoted to games of chance, in which Pacioli stadied the *problem of points*, considered later by Fermat and Pascal. Borrowing from the excellent I. Hacking's book we quote the Pacioli's formulation of this problem [Hacking, 1975, p. 50]:

"A team plays a ball in such a way that a total 60 points is required to win the game, and each goal connote 10 points. The stake are 10 ducates. By some incident they cannot finish the game, and one side has 50 points and the other 20. One wants to know what share of the prize money belongs to either side".

Pacioli thought that the stake should be divided in the proportion 5:3 [Kendall 1956]. The error was noted in later writings by Tartaglia and Cardano.

Despite the erroneous solution, the book of Pacioli proved that problems of games of chance were considered by Italian mathematicians at least two hundred years earlier than for example by French.

Although the book *Summa* lacked the originality it provided a basis for the major progress in arithmetic and algebra, the branches of mathematics necessary for the development of probability theory, which took place in the XVI century. In the presentation of algebra, Pacioli followed the earlier works by Diofantos and Fibonacci. He continued the so-called rethoric algebra, where words are used rather than symbols. For algebraic symbols, which Pacioli called *caratteri algebraici*,

ŚLĄSKI PRZEGLĄD STATYSTYCZNY
Nr 10 (16)

there are used abreviations derived from appropriate words. Some examples are given below (see [Nikiforowskij 1979, p. 40]).

Table 1. Pacioli's algebraic symbols

Symbol	Derived from word	Meaning, in modern notation
co	*cosa*	x
ce	*censo*	x^2
cu	*cubo*	x^3
ce.ce	*censo de censo*	x^4
ce.cu	*censo de cubo*	x^6
p°r°	*primo relato*	x^5
p	*plus*	+
m	*minus*	−

Source: [Nikiforowskij 1979].

The main problem for algebraists was to find an unknown *thing* involved is some equation. The word “thing” in Italian is spelled *cosa*. For that reason unknown *x* was called *cosa* and algebraists were called *cosisti* [Stewart 2012].

5. Niccolo Fontana (Tartaglia)

Niccolo was born in Brescia in 1499 or 1500, as a son of an honest and respected mail rider Michele Fontana. Niccolo had one brother and two sisters. When he was six years old his father was murdered. The whole family lived in extreme poverty.

Six years latter another tragedy happened. In 1512 French army plundered Brescia. During one horrible slaughter, when 46 000 residents were killed and twelve year old Niccolo tried to hide with his family in the cathedral, a French soldier cut his jaw and probably also the month. This caused a serious speech impediment for his the whole life. Hence his nickname Tartaglia was derived which means stutterer or stammerer. In order to hide scars, he wore a long beard.

In the time when Tartaglia was living, mathematicians were very active in algebraic investigation. Almost one hundred years earlier, in

ŚLĄSKI
PRZEGLĄD
STATYSTYCZNY
Nr 10 (16)

his *Summa* Pacioli had stated that it was impossible to find the general method for solving cubic equations. Apparently the first person who solved the cubic equation was Scipione del Ferro about 1510. He died, however, before publishing his discovery, but he passed on the secret to his student Antonio Maria Fiore, who, with a strong desire to gain the reputation, challenged Tartaglia to the public dispute. It was organized in 1535. Tartaglia was able to solve all thirty problems proposed by Fiore, but Fiore was unable to solve any of the questions proposed by Tartaglia.

At that time G. Cardano was a lecturer of mathematics in Milan. Trying to find the general solution by himself he failed and asked Tartaglia for the solution. Tartaglia offered Cardono his discovery provided that Cardano swear an oath on the Holy Bible that he would never reveal the solution. It is amusing that Tartaglia offered his solution in the form of cryptic poem. The beginning of this poem reads [Cauli 2007/2008, p. 9]:

Quando che'l cubo con le cose appresso, { x^3+px}

Se agguaglia a qualche numero discreto, {= }

Trovan dui altri, differenti in esso… { $u-v=q$}

Dapoi ter rai, questo per consuento,

Che 'l loro produtto, sempre sia eguale {$uv=$ }

Al terzo cubo delle cose netto { $(p/3)$}

In parentheses { ,} there is given "algebraic" translation poem's verses.

However, Cardano was unable to decipher the code and he asked again for the necessary clue. After receiving it, Cardano published the book *Ars magna sive de regulis algebraicis* in 1545, in which the Tartaglia's method was published, with the reference to Tartaglia as the author. Tartaglia, however, was very unsatisfied with it. He was even absolutely furious. The next year he published a book in which he described the whole story. In 1556 Tartaglia published a book *General Trattato*, where he presented the scheme now because of Kendall very unjustly known as Pascal's arithmetic triangle. In the same treatise

Tartaglia considered the problem of division, for the first time stated by L. Pacioli in his *Summa* in 1494.

The problem was following: two players *A* and *B*, playing a fair game agree to continue until one has won six rounds; but the match has to stop when *A* has won five and *B* has won three. How should the stake be divided?

Tartaglia observed that the solution given by Pacioli was wrong and gave his own solution. According to his rule, the total stake should be divided in the ratio 2:1 [Kendall 1956].

6. Girolamo Cardano

Girolamo was born on September 24, 1501 and died on September 20, 1576. He was a man of remarkable contrasts: a combination of genius and charlatan, an astrologer but at the same time an eternal student of philosophy, extremely ambitious but also dishonest and hot-tempered.

Even his birth was extremely unusual. He was born to the dismay of his parents, who were not married. His mother Chiara Micheri, who was a young widow with three children, made an attempt of the abort, but unsuccessfully, so Girlomo was born.

His father, Fazio, a friend of Leonardo da Vinci, was a learned lawyer and a notorious gambler. In 1524 he was killed during a game, apparently because of cheating. In 1531, thirty year old Girolamo married 15 year old Lucia Bandarini. They had three children: Chiara, Giambatista and Alolo. Giambatista was executed in jail because he had murdered his wife. Aldo was exiled from Bologna because he had stolen a large amount of cash and jewellery from his father's house. At the end of his life he was a professional "torturer" of the Inquisition.

Girolamo published 131 books. Before his death he burned himself 170 manuscripts. In one of his books, the book with the title *Liber de ludo Alea* written about 1526, but published after the death in 1576, one finds the important observation: if a die is honest, i.e. if we give equal weight to each side, can we calculate the chances [David 1955]. This book was translated into English and published in 1953.

ŚLĄSKI PRZEGLĄD STATYSTYCZNY
Nr 10 (16)

7. Galileo Galilei

Galileo was born in Pisa in 1564 and died near Florence in 1642. According to I Hacking, Galileo furnishes for the first time the most lucid solution to the combinatorial problems for the numeration possible outcomes in tossing three dice. This problem is considered in the work known with the title *Sopra le scoperte dei dadi* (Galileo's own title was *Considerazione sopra il Giuco dei Dadi*). It was written between 1613 and 1623. Following F.M. David we can only marvel today at the question Galileo was asked: why the total of 10 points is more advantageous than 9?

Galileo explains the problem by enumerating all possible outcomes in tossing three dice, which were presented in the form of the following table [David 1998, p. 94]:

10		9		8		7		6		5		4		3	
631	6	621	6	611	3	511	3	411	3	311	3	211	3	111	1
622	3	531	6	521	6	421	6	321	6	221	3				
541	6	522	3	431	6	331	3	222	1						
532	6	441	3	422	3	322	3								
442	3	432	6	332	3										
433	3	333	1												
	27		25		21		15		10		6		3		1

Source: [David 1998, p. 94].

Total number is equal to 108 = 27+25+21+15+10+6+3+1. Dice sides totalling 11, 12, … 18 amount also 108 possibilities. Since 9 can be found in 25 ways and 10 will show up in 27 ways out of all possible 216 throws, 10 is more advantageous than 9.

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ŚLĄSKI PRZEGLĄD STATYSTYCZNY
Nr 10 (16)

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WŁOSKA KOLEBKA RACHUNKU LOSÓW

Streszczenie: Jako początek historii rachunku prawdopodobieństwa podaje się zwykle rok 1654, kiedy to hazardzista de Mere zaproponował Pascalowi do rozwiązania dwa problemy. Były one rozwiązane w tym samym roku niezależnie od siebie przez Fermata i Pascala. Ignoruje się zaś fakt, że takie problemy dyskutowane i rozwiązywane były 200 lat wcześniej przez matematyków włoskich. W artykule przedstawiono szkic historii rozwoju nauki o losowości w tzw. okresie przedpascalowskim.

Słowa kluczowe: historia rachunku prawdopodobieństwa, problem podziału, problem kości do gry.