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EQUIVALENCE SCALES FOR POLAND – NEW EVIDENCE USING COMPLETE DEMAND SYSTEMS APPROACH

Abstract: The concept of equivalence scales is considered as a measure of the welfare of different demographic types of household. The paper presents results of the estimation of equivalence scales using a complete demand system (CDS) approach. Within this approach, ESE/IB and GESE assumptions are considered. For empirical parametric specifications of CDS rank three quadratic almost ideal demand systems (QUAIDS) models are employed. The data used to estimate the QUAIDS is based on a pooled cross section of the 2005–2009 Household Expenditure Survey. The results of estimation for employees' households obtained from model with ESE/IB restrictions lead to equivalence scales similar to those of widely used "modified OECD" scales. It is also found that the equivalence scales decreased as the household's expenditures increased.

Keywords: equivalence scale, household, complete demand system, estimation.

1. Introduction

The equivalence scales are tools converting nominal incomes (or total expenditures) of heterogeneous households in comparable measures of welfare. They take into account differences in needs among household members and economies of scale within the household. The equivalence scale may be a simple *per capita* measure, but the needs of a household grow with each additional member not in a proportional way. Needs for housing space, electricity, etc. will not be four times as high for a household with four members as for a single person. For example, an income of, say, 4000 zlotys per head a month implies a different purchasing power for a household of two and four persons. In order to take into account the fact that, within any given household, economies of scale may operate with regard to the consumption of certain goods, more sophisticated approaches are proposed. Therefore, the following definition is usually adopted in economic literature: "the equivalence scale is a measure of the cost of living of a household of a given size and demographic composition, relative to the cost of living of a reference household, when both households attain the same level of utility or standard of living" [Lewbel, Pendakur 2008]. The reference household is usually a single adult or a childless couple.

Equivalence scales are a powerful instrument that allows interpersonal comparisons between individuals. They find application in social policy as well as poverty and inequality analysis, where households of different demographic composition have to be compared with respect to their well-being. Equivalence scales are also an important tool for assessing the effects of policy measures like tax reform, a change in social benefits on different household types. Another application is the evaluation of the cost of children [Schröder 2009].

It should be stressed that a satisfactory methodology for the calculation of equivalence scales has not yet been found [Szulc 2007]. Two different methodologies for setting equivalence scales can be distinguished: normative and empirical ones [Rusnak 2007]. The normative approach refers to the evaluation of household needs by experts. Some of the most commonly used normative scales include the OECD scales. The original OECD scale (also called 70/50 scale or "Oxford scale") was recommended in the 1980s for possible use in countries which had not established their own equivalence scale. This assigns a value of 1 to the first household member, of 0.7 to each additional adult and of 0.5 to each child. In the late 1990s the Statistical Office of the European Union (Eurostat) adopted the so-called "OECD-modified" equivalence scale. This scale (also called 50/30 scale) states[1] that the first adult should be assigned a value of one, subsequent adults are assigned a value of 0.5 and children 0.3.

The empirical methods of assessing equivalence scales can be split into two broad categories: subjective and objective ones. In the first category the computation is based on direct questions to individuals on levels of income corresponding with their opinion on standards of living. The second category, instead, encompasses all methods based on the observed behaviour of households. Methods based on consumption data belong to the most popular objective methods. Within this class, one can further distinguish between methods using appropriate proxy variables for household welfare and utility-based methods. In both cases, an econometric approach is required, which is based either on a single-equation analysis or on a multiple-equation one.

This study contributes to the limited literature on Polish equivalence scales research. It attempts to answer a question of whether the solutions proposed by Eurostat for Poland are appropriate in this area. The main goal of this work is the estimation of equivalence scales for Polish households for years 2005–2009. To achieve it, the methods using complete demand systems are used. A rank three quadratic almost ideal demand systems (QUAIDS) models are employed. To ensure the identification of equivalence scales ESE/IB and GESE approaches are applied.

[1] OECD-modified equivalence scale was first proposed by Hagenaars et al. [1994].

2. Theoretical framework

The study focuses on the utility-based approach. The basic hypothesis in this approach assumes that households choose a basket of goods that is preferred to all the other baskets they can afford. In other words, the households' choices follow the maximization of the utility function, subject to the budget constraint. Mathematically, the households are assumed to maximize household utility $U(\mathbf{q}, \mathbf{z})$ with respect to $\mathbf{q}$, subject to a budget constraint:

$$\max_{\mathbf{q}} U(\mathbf{q}, \mathbf{z}) \text{subject to budget constraint } \mathbf{p}'\mathbf{q} = x, \tag{1}$$

where: $\mathbf{q}$ is the vector of quantities of the different consumer goods,
$\mathbf{p}$ is the corresponding vector of prices,
$\mathbf{z}$ is the vector of demographic variables (i.e. household composition),
x is the total expenditures to consume $\mathbf{q}$.

The solution of this optimization problem can be described by a complete demand system. Using indirect utility functions, demands can be interpreted as choices minimizing the expenditure required to achieve a certain utility level and so households' preferences can be represented by a household' cost function. Mathematically, the cost (expenditure) function $C(u, \mathbf{q}, \mathbf{z})$ equals the minimum expenditure required for a household with characteristics $\mathbf{z}$ to attain utility level u when facing prices $\mathbf{p}$. Equivalence scales relate the expenditures of a household with characteristics $\mathbf{z}$ to a household with a reference vector of characteristics $\mathbf{z}^0$. It is defined by:

$$S = \frac{C(u, \mathbf{p}, \mathbf{z})}{C(u, \mathbf{p}, \mathbf{z}^0)}. \tag{2}$$

Due to the fact that household utility and accordingly household cost cannot be observed directly, utility-based equivalence scales suffer from a fundamental identification problem that must be solved before any equivalence scales can be estimated. In practice, to solve the identification problem in the demand system approach, A. Lewbel [1989] and C. Blackorby and D. Donaldson [1993] considered the case where the equivalence scale function is independent of utility, which they call "independence of base" (IB) and "equivalence-scale exactness" (ESE), respectively. In this case there is a function m such that

$$C(u, \mathbf{p}, \mathbf{z}) = C(u, \mathbf{p}, \mathbf{z}^0) \cdot m(\mathbf{p}, \mathbf{z}). \tag{3}$$

Under the ESE/IB assumption, the expenditure function is separable in demographic characteristics and the utility level. The ESE/IB property assumes that household expenditure functions across families with different demographic compositions are proportional with respect to reference expenditure, hence equivalence scales are, *a priori*, independent of reference utility:

$$S(u,\mathbf{p},\mathbf{z}) = \frac{C(u,\mathbf{p},\mathbf{z})}{C(u,\mathbf{p},\mathbf{z}^0)} = m(\mathbf{p},\mathbf{z}). \tag{4}$$

Scales (4) are *a priori* independent of households' incomes or total expenditures. An approach to relax the exact equivalence scales was suggested by D. Donaldson and K. Pendakur [2004]. They introduced a property named "Generalized Equivalence Scale Exactness" (GESE) which implies a linear relationship between the logarithm of equivalence scales and the logarithm of reference incomes. D. Donaldson and K. Pendakur [2004] considered more general then (3) functional form of expenditure functions:

$$C\left(u,\mathbf{p},\mathbf{z}\right) = G(\mathbf{p},\mathbf{z})C\left(u,\mathbf{p},\mathbf{z}^0\right)^{K(\mathbf{p},\mathbf{z})} \quad , \tag{5}$$

where $G(\mathbf{p}, \mathbf{z}) > 0$ and K is a homogeneous function degree zero with respect to **p**.

Substituting (5) into the formula (2) and doing some manipulation yields equivalence scale SE as a function dependent on total expenditure (x), vector of prices **p** and vector of household characteristics **z**:

$$SE(x,\mathbf{p},\mathbf{z}) = G(\mathbf{p},\mathbf{z}) \cdot \left(\frac{x}{G(\mathbf{p},\mathbf{z})} \right)^{\frac{K(\mathbf{p},\mathbf{z})-1}{K(\mathbf{p},\mathbf{z})}} \tag{6}$$

Equivalence scale SE is increasing in x if $K(\mathbf{p}, \mathbf{z}) > 1$ and decreasing in x if $K(\mathbf{p}, \mathbf{z}) < 1$.

D. Donaldson and K. Pendakur [2004] showed that if GESE is a maintained hypothesis and the reference expenditure function is not PIGLOG (price independent generalized logarithmic[2]), the equivalence scale can be identified from demand behaviour. Since GESE nests ESE, the estimation of demand systems allows for an easy specification test for a more appropriate equivalence scale structure.

3. Econometric issues: Model QUAIDS

This paper presents results of the estimation of equivalence scales based on a complete demand system. Within this approach, ESE/IB and GESE assumptions are considered. For empirical parametric specifications of complete demand systems a rank three quadratic almost ideal demand systems (QUAIDS) models are employed. In such models expenditure share equations are quadratic function of the logarithm

[2] Expenditure function is PIGLOG if $\log C(u, \mathbf{p}, \mathbf{z}) = (1 - u)\log a(\mathbf{p}, \mathbf{z}) + u\log b(\mathbf{p}, \mathbf{z})$ for linear homogeneous with respect to **p** functions $a(\mathbf{p}, \mathbf{z})$ and $b(\mathbf{p}, \mathbf{z})$.

of total expenditure [Banks et al. 1997]. As in D. Donaldson's and K. Pendakur's study [2004] the specification of the QUAIDS model with demographic variable is used:

$$W_i(x,\mathbf{p},\mathbf{z}) = \alpha_i(\mathbf{z}) + \sum_{l=1}^{n} \gamma_{il} \log p_l + \log \beta_i(\mathbf{z}) \cdot \log\left(\frac{x}{a(\mathbf{p},\mathbf{z})}\right) + \frac{\lambda_i(\mathbf{z})}{b(\mathbf{p},\mathbf{z})}\left(\log \frac{x}{a(\mathbf{p},\mathbf{z})}\right)^2, \quad (7)$$

where: w_i – a household's fraction of expenditure on i-th group of goods, $i = 1, 2, ..., n$,

n – number of equations (one equation for each group of households' expenditures),

$$\log a(\mathbf{p},\mathbf{z}) = \alpha_0(\mathbf{z}) + \sum_{j=1}^{n} \alpha_j(\mathbf{z}) \log p_j + \frac{1}{2} \sum_{j=1}^{n} \sum_{l=1}^{n} \gamma_{jl} \log p_j \log p_l , \quad (8)$$

$\sum_{j=1}^{n} \alpha_j(\mathbf{z}) = 1$, $\sum_{l=1}^{n} \gamma_{jl} = 0$ for each $j = 1, 2, \ldots, n$, $\gamma_{jl} = \gamma_{lj}$ for each $j, l = 1, 2, \ldots, n$.

$$b(\mathbf{p},\mathbf{z}) = \frac{1}{1-\beta_0(\mathbf{z})} \prod_{j=1}^{n} p_j^{\beta_j(\mathbf{z})} \text{ wherein } \sum_{j=1}^{n} \beta_j(\mathbf{z}) = 0 \quad (9)$$

$$\lambda(\mathbf{p},\mathbf{z}) = \sum_{j=1}^{n} \lambda_j(\mathbf{z}) \log p_j \text{, wherein } \sum_{j=1}^{n} \lambda_j(\mathbf{z}) = 0 \quad (10)$$

Generally, vector **z** can involve many demographic variables, but in this study, in order to simplify the estimation, one demographic variable is included. This variable denoted by z[3] is referred to natural logarithm of household size, i.e. the number of people in the household. Functions αi(**z**), βi(**z**) and γi(**z**) are linear functions of logarithm of z:

$$\alpha_i(\mathbf{z}) = \alpha_i^0 + \alpha_i z,\; i = 0, 1, 2, ..., n,\; \sum_{i=1}^{n} \alpha_i^0 = 1,\; \sum_{i=1}^{n} \alpha_i = 0 \quad (11)$$

$$\beta_i(\mathbf{z}) = \beta_i^0 + \beta_i z,\; i = 1, 2, ..., n,\; \sum_{i=1}^{n} \beta_i^0 = 0,\; \sum_{i=1}^{n} \beta_i = 0 \quad (12)$$

$$\lambda_i(\mathbf{z}) = \lambda_i^0 + \lambda_i z,\; i = 1, 2, ..., n,\; \sum_{i=1}^{n} \lambda_i^0 = 0,\; \sum_{i=1}^{n} \lambda_i = 0, \quad (13)$$

where α*'s*, β*'s*, γ*'s*, and λ*'s* are parameters to be estimated.[4] The rest of the variables and acronyms are as defined in the text.

[3] In the study vector **z** is reduced to scalar z.

[4] The "intercept parameter" denoted by α_0^0 is assumed to be equal to the minimum level of reference household expenditure in the sample.

GESE assumption implies that in QUAIDS model holds:[5]

$$\begin{cases} \log a(\mathbf{p},\mathbf{z}) = \log a^0(\mathbf{p}) \cdot K(\mathbf{p},\mathbf{z}) + \log G(\mathbf{p},\mathbf{z}) \\ b(\mathbf{p},\mathbf{z}) = b^0(\mathbf{p}) K(\mathbf{p},\mathbf{z}) \\ \lambda(\mathbf{p},\mathbf{z}) = \lambda^0(\mathbf{p}) \end{cases} \tag{14}$$

where $a^0(\mathbf{p})$, $b^0(\mathbf{p})$, $\lambda^0(\mathbf{p})$ are the functions $a(\mathbf{p}, \mathbf{z})$ $b(\mathbf{p}, \mathbf{z})$ and $\lambda(\mathbf{p}, \mathbf{z})$, respectively, which do not depend on the demographic variable z, for example $\lambda^0(\mathbf{p}) = \sum_{j=1}^{n} \lambda_j^0 \log p_j$.

Thus, one can estimate $K(\mathbf{p},\mathbf{z}) = \dfrac{b(\mathbf{p},\mathbf{z})}{b^0(\mathbf{p})}$ and

$\log G(\mathbf{p},\mathbf{z}) = \log a(\mathbf{p},\mathbf{z}) - \log a^0(\mathbf{p},\mathbf{z}) \cdot K(\mathbf{p},\mathbf{z})$ then computing values of equivalence scale (6).

In the study three hypotheses presented in Table 1 are concerned. Their verification enables to compare specifications of nested models by assessing the significance of restrictions to an extended model.

Table 1. Tested hypotheses in QUAIDS model

Null hypothesis	Alternative hypothesis	Rejection of the null hypothesis means:
$\lambda(\mathbf{p}, \mathbf{z}) = \lambda^0(\mathbf{p})$	$\lambda(\mathbf{p}, \mathbf{z})$ depends on $\mathbf{z}$	GESE approach is rejected
$b(\mathbf{p}, \mathbf{z}) = b^0(\mathbf{p})$	$b(\mathbf{p}, \mathbf{z})$ depends on $\mathbf{z}$	ESE approach is rejected in favour of GESE

Source: own elaboration on the base [Donaldson, Pendakur 2004].

Adding an error term to the right-hand side of equations (7) produces an estimable demand system:

$$\begin{aligned} W_1 &= \alpha_1(\mathbf{z}) + \sum_{l=1}^{n} \gamma_{1l} \log p_l + \log \beta_1(\mathbf{z}) \cdot \log\left(\frac{x}{a(\mathbf{p},\mathbf{z})}\right) + \frac{\lambda_1(\mathbf{z})}{b(\mathbf{p},\mathbf{z})}\left(\log \frac{x}{a(\mathbf{p},\mathbf{z})}\right)^2 + \varepsilon_1 \\ W_2 &= \alpha_2(\mathbf{z}) + \sum_{l=1}^{n} \gamma_{2l} \log p_l + \log \beta_2(\mathbf{z}) \cdot \log\left(\frac{x}{a(\mathbf{p},\mathbf{z})}\right) + \frac{\lambda_2(\mathbf{z})}{b(\mathbf{p},\mathbf{z})}\left(\log \frac{x}{a(\mathbf{p},\mathbf{z})}\right)^2 + \varepsilon_2 \\ &\vdots \\ W_n &= \alpha_n(\mathbf{z}) + \sum_{l=1}^{n} \gamma_{nl} \log p_l + \log \beta_n(\mathbf{z}) \cdot \log\left(\frac{x}{a(\mathbf{p},\mathbf{z})}\right) + \frac{\lambda_n(\mathbf{z})}{b(\mathbf{p},\mathbf{z})}\left(\log \frac{x}{a(\mathbf{p},\mathbf{z})}\right)^2 + \varepsilon_n \end{aligned} \tag{15}$$

[5] Detailed derivation of the formulas (14) can be found in [Donaldson, Pendakur 2004; Dudek 2011].

Since the expenditure shares W_1, W_2, …, W_n sum to one, then the variance-covariance matrix is singular. There are a number of ways of dealing with this, the usual procedure being to drop one of the share equations [Edgerton et al. 1996]. After the estimation of the remaining share equations,[6] the parameters of the omitted equation are obtained via the restrictions (8)–(13).

The estimation of demand systems based on data from individual households is often difficult because of the problem of "zero expenditures" on individual commodities. Since not all the households spent their income on certain expenditure items, numerous zero observations exist in the data. In this case the so-called censored sample problem occurs resulting in the dependent variable having a value of zero for some observations. Three main reasons for zero-expenditure on a good can be identified: 1) infrequency of purchase because the period of the survey is too short, 2) households' preference, 3) households do not purchase the good at the current prices and income levels. Unfortunately, it is not possible to identify which of these reasons are responsible for each of the reported zero expenditures from the data. While estimating the model, it is necessary to consider the aspect of censored data because if one includes the zero observations in the econometric estimation without special treatment, the estimation process would lead to biased and inconsistent estimators. To deal with censored data, two-step procedure proposed in the study [Shonkwiler, Yen 1999] is usually applied. However, in this approach, heteroscedasticity of error terms is introduced into the model by construction and in consequence Shonkwiler and Yen's procedure generates inefficient estimates. Therefore, to avoid the problems mentioned above, one can apply aggregation over commodities. It usually helps to reduce or eliminate the issue of "zero expenditure."

4. The data

Equivalence scales derived from the QUAIDS demand system are estimated using Polish microdata. The data are pooled cross-sections from the 2005–2009 Household Expenditure Survey. They were collected by the Central Statistical Office of Poland (GUS). The choice of the initial year was caused by a change of the classification made by Central Statistical Office in 2005. Since that year households in GUS's survey have been classified according to five basic socio-economic groups of the population. There are: employees' household, farmers' households, households of the self-employed, households of retirees and pensioners, households living on unearned sources. In the study employees' households are analysed. That choice is caused by relative homogeneity of the sample on the one hand, and demographic variability on the other.

[6] The models are estimated as a non-linear system of seemingly unrelated regression equations.

Taking into account composite commodity theorem[7] households' expenditures on consumer goods and services are aggregated to five broad groups:

1) food and non-alcoholic beverages,
2) clothing and footwear,
3) housing and energy, household equipment, furniture and running costs, restaurants and hotels,
4) recreation and culture, education, transport, communications,
5) alcoholic beverages, tobacco and narcotics, health and other expenditures.

The above classification is similar to that presented in [Vernizzi et al. 2003]. Due to the fact that about 20% households did not spend money on clothing and footwear, this item is included into the fifth group of expenditures. As a result of such an attachment the amount of households with "zero expenditures" is very small (does not exceed 1% sample size) in all four goods groups and, in consequence, the problem of censored data is passed over.

5. Results and discussion

Parameters of QUAIDS model are estimated by iterative feasible generalised non-linear least squares (IFGNLS) which is equivalent to the maximum likelihood (ML) [Poi 2008]. Fourth budget-share equation is dropped to prevent the singularity of the error covariance matrix. Iterative non-linear seemingly unrelated regression procedures implemented in Stata v.11 statistical package are used. Visual inspection as well as statistical tests show evidence of heteroscedasticity. For this reason robust, asymptotically consistent standard errors are calculated.

Table A1 (see Appendix) presents results of the estimation of QUAIDS model under the ESE/IB assumption.[8] As can be seen in this table, the demand model fits the available data reasonably well. Most of the coefficients on the variables are statistically different from zero at a 5% significance level. Based on estimates of the parameters values of the equivalence scale are calculated. As pointed out in the papers [Buhmann et al. 1988; Kot 2000] all equivalence scales might be approximated as $Scale \approx (N)^e$ where N is household's size and e is the scale elasticity parameter. Such scales are called Buhmann's scales. Since the logarithm of the equivalence scale in QUAIDS models with ESE/IB property and only one demographic variable $z = \log N$ can be expressed in the form [Dudek 2011]:

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[7] Composite commodity theorem asserts that if a group of prices move in parallel, the corresponding group of commodities can be treated as a single good [Deaton, Muellbauer 1999, pp. 120–122].

[8] The results are obtained on the basis of data which refer to households with maximum of seven people whose logarithm equivalent expenditure (by a modified scale) ranged within the limits [Q_1 – 1.5*IQR; Q_3 + 1.5*IQR], where Q_1 and Q_3 – first and third quartile, *IQR – interquartile range*. Other observations, representing approximately 1.5% of the sample, are considered outliers and are excluded from the analysis. Final *sample size* is 87 642 households.

$\log S = \left(\alpha_0 + \sum_{j=1}^{n} \alpha_j \log p_j \right) \log N$, then scale elasticity is linear combination of alphas parameters: $e = \alpha_0 + \sum_{j=1}^{n} \alpha_j \log p_j$. Table 2 shows the estimated equivalence scale elasticity obtained under the ESE/IB assumption.

Table 2. Scale elasticity parameters – results for the model with the ESE/IB assumption

Years	Estimated scale elasticity parameter (e)	Lower bound of 95% confidence interval	Upper bound of 95% confidence interval
2005–2009	0.6465	0.6373	0.6556
2005	0.5852	0.5759	0.5944
2006	0.6089	0.5996	0.6181
2007	0.6445	0.6354	0.6536
2008	0.6835	0.6745	0.6926
2009	0.6997	0.6906	0.7087

Source: own elaboration using Stata v.11 statistical package.

The results of the estimation presented in Table 2, obtained from models with ESE/IB restrictions lead to equivalence scales presenting moderate economies of scale. It can be seen that values of scale elasticity are not constant in each year. The average estimate *amounting to* 0.6465 is relatively similar to those in the widely used "OECD-modified" scales. It is found that for the analysed data the original *OECD scale corresponds* to a value of elasticity e=0.7796 and the modified scale OECD – 0.6160, i.e. Scale original OECD ≈ N0.7796 and Scale modified OECD ≈ N0.6160 with respectively (0.7793; 0.7798) and (0,6156; 0.6163) 95% confidence intervals. It means that scale 70/50 considerably overestimates empirical equivalence scales for employees' households in 2005–2009.

Table 3 shows estimated equivalence scales. The presented results refer to average values for the period 2005–2009.

Table 3. Equivalence scale – results for the model with the ESE/IB restriction

Household's size (N)	Estimate of equivalence scale (S)	Lower bound of 95% confidence interval	Upper bound of 95% confidence interval
1	1	1	1
2	1.5653	1.5554	1.5752
3	2.0345	2.0141	2.0548
4	2.4503	2.4193	2.4812
5	2.8305	2.7890	2.8721
6	3.1846	3.1325	3.2367
7	3.5183	3.4558	3.3508
8	3.8355	3.7627	3.9083

Source: own elaboration using Stata v.11 statistical package.

Table 3 compares different equivalent scales for all considered demographic types of households. For example for three-persons household estimated value of equivalence scale equals 2.0345. It means that such a household requires on average about two times higher income than a reference single-person household to experience the same level of utility.

The next step is the analysis of levels of "social minimum" and the "subsistence minimum" specified (calculated) by the Institute of Labour and Social Studies (IPISS). The social minimum indicates a particular threshold needed for a household to lead a decent life and is based on the cost of a "basket of goods" considered necessary for this. The social minimum describes the indispensable level of consumption determining social participation and social integration, which demands satisfying not only basic needs but also certain other needs beyond them. This is reflected in the contents of the basket of goods and services indicated as "basic" for satisfying needs on a social minimum level. The contents of the basket allow for participation in social life: work, children's education, family life and socialising, participation in culture – all these on a modest level. The subsistence minimum is based on a more restricted "basket of goods" considered necessary for survival, i.e., to sustain one's vital functions and psychophysical capabilities. It takes into account those needs that cannot be postponed and if the consumption level is below this minimum it leads to biological emaciation. There is no special attention devoted to social contacts which were so important in social minimum. According to that, there are no needs relating to transport, communication and participation in culture, etc. the Institute of Labour and Social Studies estimated average yearly levels of the social minimum and the subsistence minimum for employees' household with various demographic composition. In this study these values are used to the calculation of adequate equivalence scales. It is estimated, that for the period 2005–2009:

$\text{Scale}_{\text{min\_subsistence}} = N^{0.8898}$ with 95% confidence interval: [0.8748; 0.9049],

$\text{Scale}_{\text{min\_social}} = N^{0.8232}$, with 95% confidence interval: [0.8143; 0.8322].

It can therefore be concluded that researchers from the Institute of Labour and Social Studies applied greater values of equivalence scales for poorer households.

In order to provide *more insight* into dependence of equivalence scales on households' financial situation QUAIDS model with GESE restriction is considered. Table A2 (see Appendix) shows detailed results of the estimation. *On the base of estimates* received for that model values of the equivalence scale are calculated and presented in Table 4.

As can be seen from Table 4, equivalence scale decreases as the household's expenditures increase. For example, *value of scale* for a household containing *four people equals 2.52 if its* total expenditure is at the level of the first quartile, 2.43 in the case of the second quartile (median), and *2.35 for the* third quartile of the total expenditure. It indicates that poor households require more compensation than rich households to keep the same level of utility when more household members are added. Due to the fact that poorer households in Poland consist of an average of more people than richer households, then the application of the improper equivalence scale distort more the image of poverty than wealth.

Table 4. Equivalence scale – results for the model with the GESE restriction

Household's size (N)	Estimate of equivalence scale (S)		
	For first quartile of x	For second quartile of x	For third quartile of x
1	1	1	1
2	1.5487	1.5223	1.4955
3	2.0466	1.9915	1.9362
4	2.5199	2.4346	2.3497
5	2.9788	2.8619	2.7463
6	3.4281	3.2787	3.1315
7	3.8709	3.6880	3.5084
8	4.3090	4.0917	3.8792

Source: own elaboration using Stata v.11 statistical package. Households' total expenditure is denoted by x.

According to the best knowledge of the author of this article, results of the estimation of QUAIDS models with the GESE assumption were reported only in two papers: [Donaldson, Pendakur 2004] and [Balli, Tiezzi 2011]. Empirical estimations of these models using Canadian and Italian data, respectively, give the result of equivalence scales declining with expenditure. Other applications giving similar results using different approach were made by C. Koulovatianos, C. Schröder and U. Schmidt for Germany and France [2005a], and for Cyprus [2005b]. In these studies equivalence scales were derived directly from people's assessment concerning the relationship between income, household type and economic well-being. It should also be mentioned that in A. Majumder and M. Chakrabarty's paper [2010], where equivalence scales declining with expenditure for India were estimated through the Engel's curves analysis. As pointed out by F. Balli, and S. Tiezzi [2011] and J. Faik [2012] variable equivalence scales causes distinctive increases in expenditure (or income) inequality compared with constant equivalence scales (i.e. expenditure (or income) independent).

The last stage of this study is verification of the hypotheses listed in Table 1. In order to do it likelihood ratio tests are conducted. Such tests compare specifications of nested models by assessing the significance of restrictions to an extended model. Table 5 contains the results of estimation for the considered models.

Table 5. Values of logarithm of likelihood for estimated models

Model	Logarithm of likelihood	Degrees of freedom
QUAIDS with ESE/IB restriction	224 347.9	19
QUAIDS with GESE restriction	225 276.6	23
QUAIDS without any restrictions	225 521.0	26

Source: own elaboration using Stata v.11 statistical package.

The large sample distribution of *LR* is chi-squared, with degrees of freedom equal to the number of restrictions imposed. It can be seen that, *on the one hand*, ESE/IB restriction is rejected in favour of GESE[9] (because $LR > \chi 2$ (4; 0.05), where LR= 1857.4 and $\chi 2(4; 0.05) = 9.4877$, but on the other hand GESE restriction is also rejected (due to the fact that $LR > \chi 2$ (3; 0.05), where $LR = 488.8$ and $\chi 2(3; 0.05) = 7.8147$). Similar result was obtained by D. Donaldson and K. Pendakur [2004]. It can be commented that dependence of equivalence scales on households' expenditures is more complex than those permitted by GESE.

6. Concluding remarks

The main objective of the study is the quantification of equivalence scales through household cost functions built on estimates of consumer demand systems. Such approach suffers from the fundamental identification problem – equivalence scale cannot be derived from demand data alone without further assumptions.[10] The most well-known identification assumption is the "independence of base" or the "equivalence scale exactness" assumption. It assumes constant equivalence scales which do not depend on the reference level of utility, and which, in consequence, are independent of income. More general functional form for identification was suggested by D. Donaldson and K. Pendakur [2004] allowing equivalence scales to be expenditure-dependent. The study uses both approaches. In an empirical comparison, ESE/IB is rejected in favour of GESE. Although more elaborate equivalence scales exist, the Buhmann et al. equivalence scale is chosen in the empirical part of the paper for simplicity and due to the fact that they approximate all scales well.

The study is important due to the scarcity of published papers on empirical equivalence scales analysis for Poland. It finds that equivalence scales are not independent of households' total expenditures. In consequence, ignoring that result understates poverty rates and expenditure (or income) inequality. It should be stressed that Donaldson and Pendakur's approach assuming that the logarithm of the equivalence scale is a linear function of the logarithm of total expenditure is rejected by Polish employees' households. It means that equivalence scales depend on expenditures, but it is not known what the nature of this relationship is. For this reason, further theoretical and empirical evidence is needed to explore the equivalence scales' expenditure dependence.

[9] It is also found that QUAIDS does not reduce to AIDS model (i.e. preference is not PIGLOG) what provides identification of scales obtained under GESE restriction.

[10] R. Blundell and A. Lewbel [1991] showed that demand equations for goods alone provide no information about equivalence scales at any one point in time (i.e., at any given price regime), but demand equations completely determine the way equivalence scales change over time in response to price changes.

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SKALE EKWIWALENTNOŚCI DLA POLSKI – NOWE OSZACOWANIA UZYSKANE NA PODSTAWIE KOMPLETNYCH MODELI POPYTU

Streszczenie: Skale ekwiwalentności są ważnym narzędziem we wszystkich analizach z zakresu porównywania zamożności gospodarstw domowych. Informują, ile razy więcej musi wydać gospodarstwo o danym składzie demograficznym, aby osiągnąć poziom satysfakcji gospodarstwa odniesienia. Oszacowania skal ekwiwalentności dokonano na podstawie wyników estymacji parametrów nieliniowych modeli typu QUAIDS. W celu umożliwienia ich identyfikowalności na preferencje gospodarstw domowych nałożono warunki ESE/IB oraz GESE. Wykorzystano dane z gospodarstw domowych pracowników biorących udział w badaniach budżetów GUS w latach 2005–2009. Skala ekwiwalentności oszacowana przy założeniu ESE/IB była zbliżona do zmodyfikowanej skali OECD. Ponadto stwierdzono, że skale malały wraz ze wzrostem całkowitych wydatków gospodarstw domowych. Uboższe gospodarstwa domowe w Polsce są przeciętnie bardziej liczne niż gospodarstwa znajdujące się w lepszej sytuacji finansowej, dlatego przyjęcie niewłaściwej skali bardziej zniekształca obraz ubóstwa niż dostatku.

Słowa kluczowe: skala ekwiwalentności, gospodarstwo domowe, kompletny model popytu, estymacja.

Appendix

Tables A1 and A2 report the results of estimation of demand systems basing on the sample of 87642 employees' households. Monthly price indices (with January 2005 as the base month) are used.

Table A1. Results of estimation of QUAIDS model with ESE/IB assumption

Parameters	Estimates	Standard error	95% confidence interval	
			Lower bound	Upper bound
α_1^0	−2.1286	0.4467	−3.0041	−1.2532
α_2^0	1.4650	0.2270	1.0202	1.9099
α_3^0	1.8067	0.2136	1.3880	2.2254
α_4^0	−0.1431*	0.1787	−0.4935	0.2072
β_1^0	−0.2368	0.0033	−0.2434	−0.2303
β_2^0	0.0406	0.0049	0.0310	0.0503
β_3^0	0.0621	0.0045	0.0532	0.0710
β_4^0	0.1341	0.0036	0.1270	0.1411
γ_{11}	0.4524	0.0735	0.3083	0.5964
γ_{12}	−0.3211	0.0374	−0.3944	−0.2478
γ_{13}	−0.1851	0.0330	−0.2498	−0.1203
γ_{14}	0.0538	0.0273	0.0003	0.1073
γ_{22}	0.2266	0.0245	0.1786	0.2747
γ_{23}	0.0781	0.0152	0.0483	0.1079
γ_{24}	0.0163*	0.0137	−0.0106	0.0433
γ_{33}	0.0631	0.0213	0.0213	0.1049
γ_{34}	0.0439	0.0159	0.0127	0.0751
γ_{44}	−0.1140	0.0161	0.0824	0.1456
λ_1^0	0.0202	0.0008	0.0185	0.0218
λ_2^0	−0.0045	0.0015	−0.0074	−0.0017
λ_3^0	0.0073	0.0014	0.0046	0.0100
λ_4^0	−0.0229	0.0010	−0.0250	−0.02089
α_1	0.6619	0.0037	0.6546	0.6692
α_2	0.2026	0.0041	0.1945	0.2106
α_3	0.1026	0.0036	0.0956	0.1096
α_4	−0.9671	0.0033	−0.9736	−0.9606
α_0	0.5827	0.0047	0.5734	0.5919
Goodness of fit				
Uncentred R^2 for each equation: 0.9225, 0.7872, 0.7920, 0.7688, LL = 224347.9				

* denotes not significant parameter at the 0.05 significance level, LL is logarithm of likelihood.

Source: author's own computations in STATA statistical package.

Table A2. Results of estimation of QUAIDS model with GESE assumption

Parameters	Estimates	Standard error	95% confidence interval	
			Lower bound	Upper bound
α_1^0	–2.4061	0.3901	–3.1706	–1.6416
α_2^0	1.2859	0.1977	0.8983	1.6734
α_3^0	1.9763	0.2069	1.5707	2.3818
α_4^0	0.1366*	0.1564	–0.1699	0.4432
β_1^0	–0.2237	0.0045	–0.2325	–0.2149
β_2^0	0.0735	0.0039	0.0658	0.0812
β_3^0	0.0351	0.0034	0.0284	0.0418
β_4^0	0.1151	0.0039	0.1074	0.1227
γ_{11}	0.4994	0.0675	0.3671	0.6317
γ_{12}	–0.2886	0.0343	–0.3558	–0.2213
γ_{13}	–0.2170	0.0335	–0.2825	–0.1514
γ_{14}	0.0074	0.0247	–0.0409	0.0558
γ_{22}	0.1973	0.0226	0.1530	0.2416
γ_{23}	0.0666	0.0149	0.0374	0.0958
γ_{24}	0.0246*	0.0129	–0.0007	0.0498
γ_{33}	0.0899	0.0230	0.0448	0.1349
γ_{34}	0.0598	0.0153	0.0297	0.0899
γ_{44}	–0.0918	0.0151	0.0622	0.1213
λ_1^0	0.0179	0.0011	0.0157	0.0200
λ_2^0	–0.0095	0.0011	–0.0116	–0.0073
λ_3^0	0.0107	0.0010	0.0088	0.0125
λ_4^0	–0.0191	0.0011	–0.0213	–0.0169
α_1	0.6385	0.0047	0.6293	0.6476
α_2	0.2075	0.0034	0.2009	0.2141
α_3	0.1097	0.0033	0.1032	0.1161
α_4	–0.9556	0.0035	–0.9624	–0.9488
α_0	0.6606	0.0127	0.6357	0.6855
β_1	–0.0066	0.0024	–0.0113	–0.0019
β_2	–0.0215	0.0010	–0.0235	–0.0195
β_3	0.0176	0.0009	0.0159	0.0193
β_4	0.0105	0.0019	0.0067	0.0143
β_0	–0.0955	0.0454	–0.1845	–0.0065
Goodness of fit				
Uncentred R^2 for each equation: 0.9226, 0.7912, 0.7941, 0.7690, LL = 225276.6				

* denotes not significant parameter at the 0.05 significance level, LL is logarithm of likelihood.

Source: author's own computations in STATA statistical package.