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HOUSEHOLD RISK MANAGEMENT TECHNIQUES IN AN INTERTEMPORAL CONSUMPTION MODEL

Summary: The article focuses on implementing the Cash Flow at Risk method to assess the influence of different life insurance strategies on the financial situation of households. Three different types of life insurance are considered to depict different attitudes towards risk management by households. For the purpose of analysis a simplified intertemporal consumption model is assumed.

Keywords: Household finance, risk management, life insurance, Cash Flow at Risk.

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1. Introduction

A main goal of this paper is to depict how different risk management techniques in terms of life insurance influence long term financial situation of a household. In order to do that an intertemporal consumption model is assumed that includes some of the most important risk factors that household face.

There is a vast literature on optimal consumption choices, which mostly bases on findings on intertemporal consumption made by Fisher [1930]. Fisher relied on previous consumption theories formulated by Keynes and researches made by Kuznets. Further consumption models were developed independently by Friedman – permanent income hypothesis [1957], Ando and Modigliani – life cycle hypothesis [1957], or Yaari's – uncertain lifetime [1965]. These models were improved later on and expanded on other aspects of life cycle events such as unemployment risk, Markowitz's portfolio theory or risk aversion preference. Nevertheless increasing complexity of models and their analytical forms does not resolve the issue of two decision makers in a household. Usually the models for individuals as decision makers where discussed. Sometimes the models were simplified so that couple decision makers were treated as one abstractive decision maker. However, in general a household comprises of at least two persons. Therefore in this paper a consumption model including two persons is used.

Uncertainty of future events leads towards the necessity of stochastic approach implementation rather than deterministic in development of consumption models. Therefore the consumption model (presented in section 3) mainly focuses on taking into account the risk of premature death as well as longevity risk, the risk of future income and risk of rate of return on financial assets. In order to compare how different techniques of risk management influence the financial situation of a household a distribution of cumulated net cash flow is compared.

2. Household risk management

Statistical researches and mathematical models prove very useful in terms of examining the impact of future events on financial situation of households. Based on these researches and models, scientists have been equipped in tools that are used in risk measurement which is one of the most important elements in risk management process. Risk management process comprises of four steps:

1) risk identification,
2) risk measurement,
3) selection of risk management techniques,
4) monitoring and revision.

Risk identification is a key component of a robust framework. In case of absence of this step, a household is unable to effectively manage its key risks. Careful consideration needs to be given to the identification of risk factors. Due to abundance of both internal and external risk factors, the household has to focus on those that might have the most significant impact on household's financial situation. The following are pointed out in literature:

- length of life (premature death risk, longevity risk),
- market risk,
- interest rate risk,
- inflation risk,
- health risk,
- financial goals risk (in particular risk of not achieving goals),
- future income risk (in particular unemployment risk),
- durable goods risk (for instance, significant loss of value or damage of durable goods).

In terms of risk a household focuses mostly on the downside aspect of risk rather than the potential that the outcome might be better than expected. Therefore a decision maker above all is interested in achieving his/hers minimal level of consumption. Thus the household is interested in minimization of aspiration level (see for instance [Mezias 1988; Payne, Laughhunn, Crum 1981]). In simplest form the aspiration level is given by the following formula:

$$P(X \leq X_0) = P_a.$$

The aspiration level will be used in the assessment of risk management techniques analysed in this paper. Next to the aspiration level other risk measure focusing on downside risk might be used in the assessment of risk management techniques. In the author's opinion such measure should be based on the value at risk concept. However, households should be interested in monitoring their cash holding (including value of their investment portfolio) rather than pure portfolio. In order to achieve that we focus on cash-in-hand at the end of analysis' horizon which is determined by the death of the last living household member. The underlying parameter for analysis suggests that appropriate concept would be Cash Flow at Risk developed by RiskMetrics [1999]. Originally CFaR has been introduced as a risk measure used by companies. However, in the author's opinion CFaR concept might be applied in risk management for households as well. From analytical point of view it does not matter whether we analyze cash flow of a company or cash flow of a household. Nevertheless some amendments have to be introduced. Especially in terms of taking into account other types of risk beside market risk.

CFaR represents the maximum shortfall of net cash generated, relative to a specified target, that could be experienced due to the impact of risk factors on a specified set of exposures, for a specified reporting period and confidence level.

$$P\,(CF \leq CF_0 - CFaR) = \alpha.$$

CFaR might be additionally useful in comparing different risk management techniques by analysing descriptive statistics of cash flow distribution which are generated in simulation process.

Once we have identified the risk factors and chose risk measures we can focus on selecting appropriate risk management technique. In general there are four following attitudes towards risk management:

1) avoidance,
2) minimization – either probability of occurrence or severity of loss,
3) risk transfer,
4) retention.

In Table 1 some risk factors and different risk management techniques have been presented. These techniques are divided into four above mentioned categories. Risk management techniques presented in the table are just examples of wide variety of possible actions that one can take in order to manage particular risk. The decision of which technique to choose is dependent on many aspects, like: age, risk aversion, wealth, etc. However in general a household should follow the "rule of thumb" which is presented as risk matrix in Figure 1.

The higher is the severity of loss as a result of materialization of particular risk factor, the higher the necessity to manage the risk. The higher is the probability, the higher the need to manage the risk. The techniques presented above have their limitations and conditions of usage. The best way to manage the risk is just avoid it.

Table 1. Risk factors management techniques

		Risk management techniques			
		Avoidance	Minimization	Risk transfer	Retention
Risk factors	Length of life: – Premature death – Longevity	– No possibilities – No possibilities	– Healthy lifestyle; cautiousness – Unhealthy lifestyle; lack of cautiousness	– Life insurance – Endowment	– Self-insurance – Self-insurance
	Market risk	Lack of investment	Diversification	Hedging	No action
	Interest rate risk	Lack of debt	Fixed rate debt; debt selection	Swaps (theoretically)	No action
	Inflation risk	No possibilities	Savings reinvestment	Investing in inflation indexed instruments	No action
	Health risk	No possibilities	Healthy lifestyle; prevention	Health insurance	Self-insurance
	Financial goals risk	Not to have financial goals	More feasible financial goals; dedicated investments	Dedicated investments	Increased savings
	Durable goods risk	Not to possess any high value durable goods	Purchase of less expensive goods; repairs; additional features	Property insurance; car insurance	Self-insurance

Source: own elaboration.

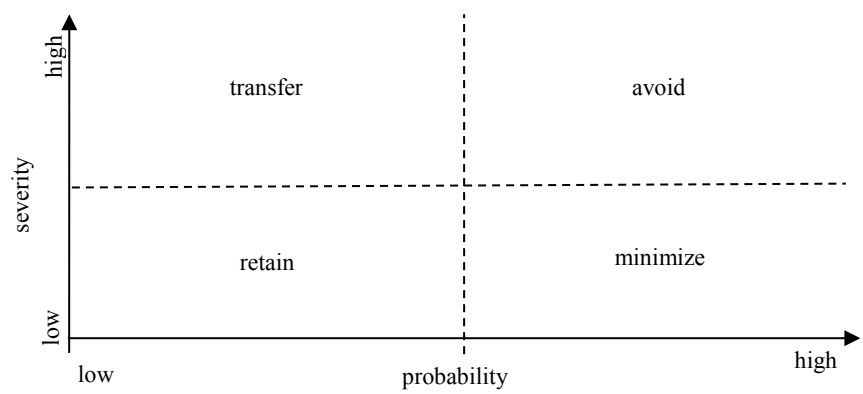

Figure 1. Risk matrix and risk management techniques

Source: based on [Baranoff , Brockett, Kahane 2010].

However, the majority of risk factors that influence a households' financial situation cannot be avoided. For instance the income risk or the length of life risk cannot be managed in such a manner. Secondly in case of high severance or loss the risk should be transferred. But here one can face some limitations. Some of the risks cannot be

transferred or there is very high cost of transferring them. Furthermore the insurer might decline the insurance coverage or the coverage will be limited to some specific situations, thus only some part of the risk will be transferred. The third way of handling the risk is mitigation which can be aimed at minimizing the probability of an event or minimizing the severity of an event. Usually the action undertaken by individuals tends to handle the risk in both ways. The last approach toward the risk is retention. This technique should be used when the risk of an event is rather low and the severity is also insignificant.

As one of the most important risk factors that household face is associated with length of life. A two-person household faces both premature death risk and longevity risk (see [Feldman, Pietrzyk, Rokita 2015]). Therefore we examine the influence of different types of life insurance on cash flow distribution for a theoretical household.

3. Consumption model

A majority of literature devoted to consumption optimization assume that a household should be perceived as a single subject and therefore limit the consumption models to consumption of individual. Surprisingly not as many consumption models divide consumption among members of a household, which is in fact a crucial assumption, especially when we take into account that the death of one household member significantly influences the overall consumption. Usually households comprise of two or more members, of which there are two (husband and wife) that are decision makers (see [Feldman, Pietrzyk, Rokita 2014]). Thus a consumption model used for analysis has to include a division of consumption.

Let us assume that that consumption C in period t equals:

$$C_t = FC_t + VC_t^1 + VC_t^2,$$

where: C_t – overall consumption in period t; FC_t – fixed consumption (independent on number of household members); VC_t^1 – variable consumption of per-son 1 (strictly associated with person 1); VC_t^2 – variable consumption of person 2 (strictly associated with person 2).

Consumption expressed in the formula above includes any day-to-day expenses and expenses related to accomplishment of financial goals (i.e. mortgage instalments, car loan instalments, vacation, etc.).

The household's main goal is to sustain a minimum satisfactory standard of living over the whole lifespan. Based on Friedman's permanent income hypothesis and Ando-Modigliani's life cycle consumption hypothesis, we conclude that:

$$\begin{aligned} &C_t = c\left[wI_t + (1-w)I_{t-1}\right] \\ &0 < c < 1 \\ &0 < w < 1 \end{aligned},$$

where: c – marginal propensity to consume; w – weight; I_t – total household's income in period t.

Having also in mind that the public pension systems are not expected to maintain the pension income on the level of income generated during working years we assume that consumption will be constant in retirement period T for a household:

$$C_{t+1} = C_t \quad \text{for } t > T .$$

Note that period T might be different for different household members, thus one's variable consumption remains constant after entering retirement age. Additionally it is assumed that the household's consumption has to be higher than base level (C_0) for the whole lifespan.

The household has also a budget constraint given by:

$$C_t \le I_t - \mathrm{Ins}_t + S ,$$

$$S = \sum_{i=0}^{t-1} S_i ,$$

where: Ins_t – insurance premiums paid in the period t; S – cumulated surplus (savings accrued over time).

A cumulated surplus reflects a cash-in-hand at the end of every annual period. Thus we are going to identify S with CF from Cash Flow at Risk concept.

The savings from each period can be allocated into two asset classes. The first is risk free investment which brings real rate of return $r_f = 0$, and the other is risky investment the rate of return of which is given by normal distribution $R \sim N(\mu,\sigma)$. Investments are used to finance future consumption needs, especially when pension income significantly drops in retirement period. Additionally investments might be treated as a generalization of financing multiple goals with different investments (see [Pietrzyk, Rokita 2014]).

For the convenience the risk premiums are calculated assuming that there are no additional costs associated with insurance and that the insurer is a not for profit organization. Thus annual insurance premiums are given by:

$$\mathrm{Ins}_t = FVq_x \qquad \text{for annual term insurance}$$

$$Ins_t = \frac{FV\ {}_T q_x}{T - t_0} \quad \text{for term insurance to period } T,$$

$$\mathrm{Ins}_t = \frac{FV}{e_{t_0}} \quad \text{for whole life insurance (premium paid to } e_{t0}),$$

where: FV – face value of insurance; ${}_tq_x$ – probability that a person of age x will die before period t; t_0 – year of buying a policy; e_{t0} – expected length of life in period t_0.

In order to calculate the CFaR the Monte Carlo simulation method will be used. For that purpose a probability distributions of particular risk factor has to be known or at least approximated. For the analysis the following risk factor are going to be analysed:

- length of life,
- return from risky investment,
- income.

Probability distribution of mortality has been widely examined. There are five classic mathematical models used to describe the mortality rates: de Moivre's law; exponential model; Gompertz's law, Makeham's law, Weibull's law. These models may be used for the analytical solutions in finding the optimal consumption. However, for the purposes of this paper mortality rates given by Central Statistical Office of Poland were used. In terms of return from risky investment and income process the traditional Geometric Brownian Motion was used, thus:

$$\frac{dX_t}{X_t} = \mu_X dt + \sigma_X dB_t,$$

$$dI_t = \mu_I I_t dt + \sigma_I I_t dB_t,$$

where: X_t – price of risky instrument in period t; μ; σ – distribution parameters; B – classic Geometric Brownian Motion.

4. Risk management techniques assessment

As it was stated earlier, the Monte Carlo simulation method has been used to determine distribution of cash-in-hand at the end of the analysis horizon. In Table 2 a set of parameters used in simulation is presented.

For the purposes of the analysis 10.000 simulation sets have been generated. Based on those sets an amount of cash-in-hand at the end of analysis period has been calculated.

Base model assumes that a household does not buy any life insurance therefore a self-insurance technique of risk management is adopted. On Figure 2 a histogram of outcomes is presented.

In order to assess the risk management techniques, three different strategies have been analysed. These strategies focused on buying three different types of life insurance:

- annual life insurance bought each year with new premium,

Table 2. Parameters and variable

Parameter and variable	Person 1	Person 2
Age	30	28
Retirement age	67	67
Replacement rate	40%	35%
Income in period t_0	36 000	30 000
Income distribution parameters: – μ – σ	2.22% 1.61%	
Fixed consumption in period t_0	23 400	
Variable consumption in period t_0	18 000	18 000
Marginal propensity to consume	0.90	
Weight	0.80	
Risky investment distribution parameters: – μ – σ	3.55% 16.58%	
Face value of life insurance	180 000	150 000
Savings division among: – risk-free instrument – risky instrument	25% 75%	

Source: own elaboration.

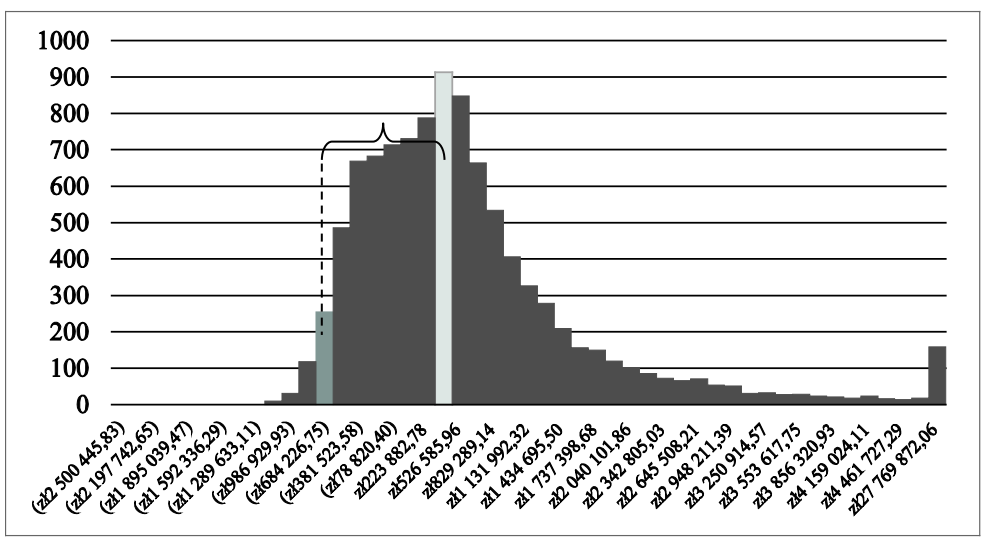

Figure 2. Cash flow histogram (without insurance)

Source: own elaboration.

- long term life insurance bought at fixed annual premium for the whole period,
- whole life insurance at fixed premium paid until the year of expected life length.

The annual renewable term life insurance is usually designed for a person who has to secure his/her financially in short term. The death benefit remains unchanged throughout the term, but the premium increases yearly and for longer periods it may become very expensive. Therefore if the decision makers are more concerned of the premature death risk in medium and long term they probably should choose long-term life insurance or whole life insurance. Long-term life insurance with fixed premium (indexation is voluntary) has higher premiums than annual renewable insurance in earlier stage of insurance coverage, whereas in longer term the premium is significantly smaller. The increase in earlier years of policy allows for smoothening the cost of insurance coverage for the whole fixed period. The main flaw of that type of life insurance is fixed period and usually lack of cash value of the policy. The fixed period may be a main issue when the policy expires and a decision maker still needs insurance coverage. That may result in extremely high premiums for new insurance or even a decision maker would not be able to insure himself due to too much risk. In order to prevent these situations one can buy whole life insurance. The insurance term is infinite, which means that it suites best a person who is mostly concerned for the very long time horizon. Furthermore this type of policy includes cash value which can be withdrawn once the policy is cancelled by the policy owner.

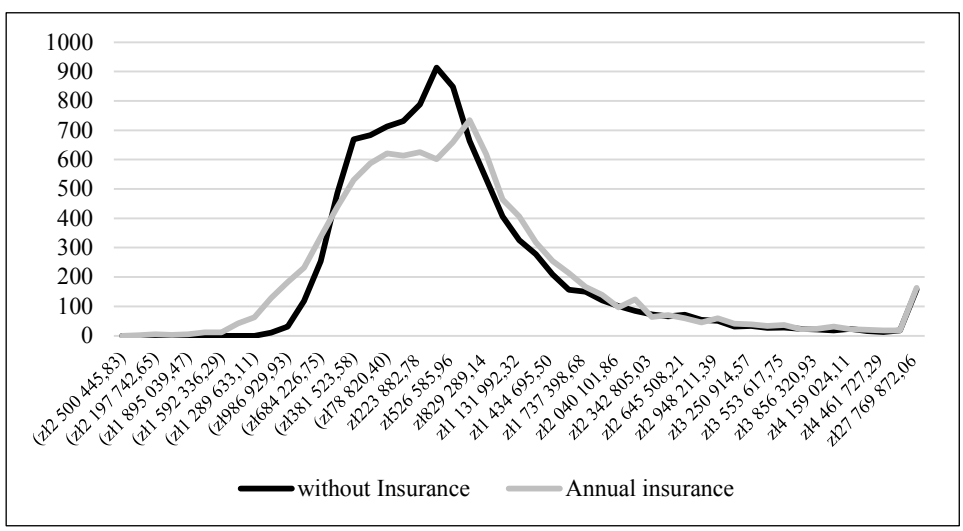

Figure 3. Cash flow distribution comparison (without insurance vs. annual insurance)

Source: own elaboration.

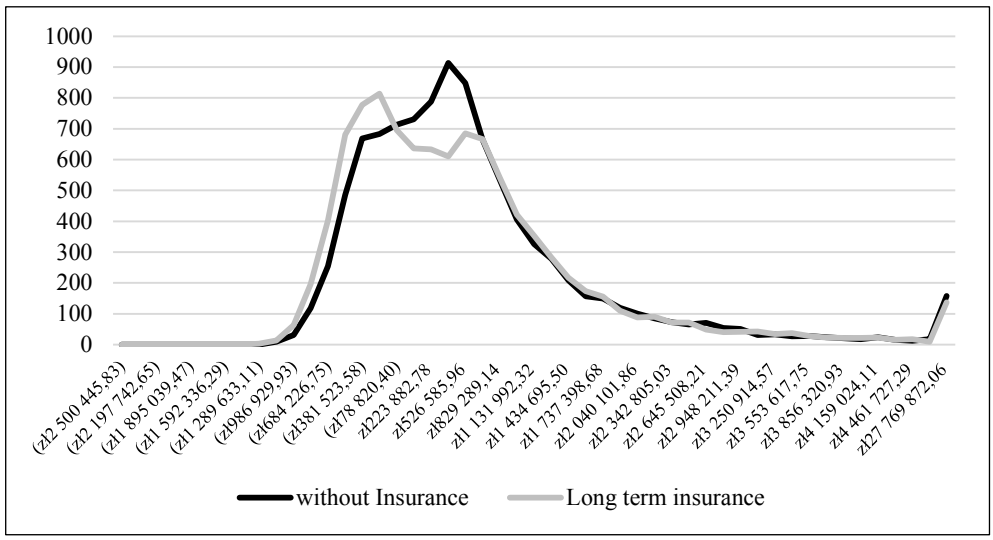

Figure 4. Cash flow distribution comparison (without insurance vs. long term insurance)

Source: own elaboration.

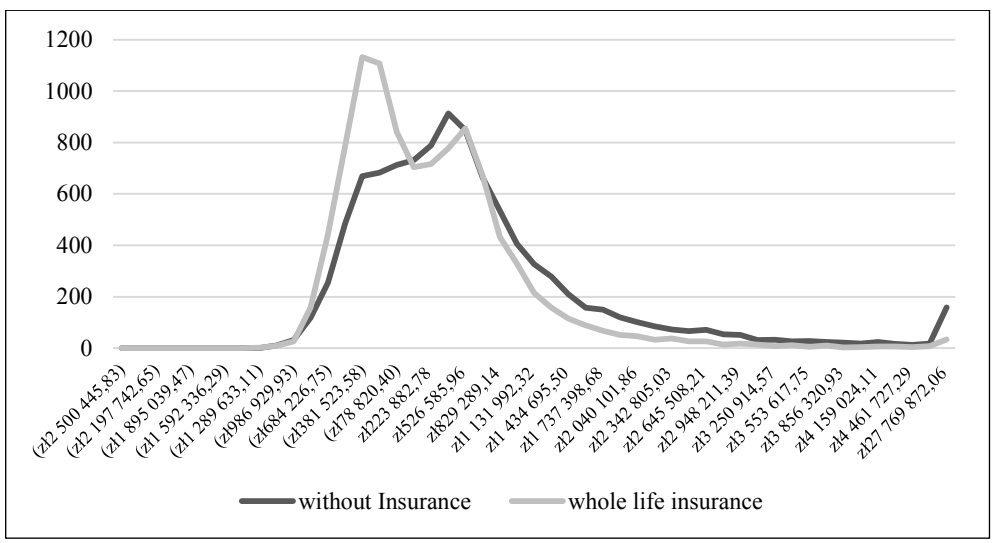

Figure 5. Cash flow distribution comparison (without insurance vs whole life insurance)

Source: own elaboration.

The abovementioned three types of insurance reflect different attitude towards the risk of decision makers from the point of view of their concern period (early age,

middle age, old age). Therefore this type of policies has been taken into account for the purpose of the analysis.

A comparison of cash-in-hand distributions between base strategy and the analysed ones is presented on Figures 3–5.

The descriptive statistics of the cash-in-hand distribution are compared in Table 3.

The CFaR and an aspiration level for $CF = 0$ are presented in Table 4.

Table 3. Descriptive statistics of cash-in-hand distributions

	Without insurance	Annual insurance	Long term insurance	Whole life insurance
Average	564 437.60 zł	566 158.09 zł	492 175.49 zł	169 850.96 zł
Standard error	13 447.11 zł	14 616.74 zł	13 546.56 zł	8 892.40 zł
Median	309 156.82 zł	361 185.30 zł	243 747.45 zł	27 550.72 zł
St. deviation	1 344 710.88 zł	1 461 674.43 zł	1 354 655.95 zł	889 240.43 zł
Kurtosis	43.41	47.31	54.08	74.33
Skewness	4.72	4.57	4.95	5.40
Range	24 438 777.96 zł	30 134 077.37 zł	29 103 049.02 zł	19 977 965.82 zł
Minimum	–1 272 780.78 zł	–2 500 445.83 zł	–1 333 176.96 zł	–1 320 730.28 zł
Maximum	23 165 997.19 zł	27 633 631.54 zł	27 769 872.06 zł	18 657 235.54 zł
Sum	5 644 375 998.44	5 661 580 939.19	4 921 754 932.65	1 698 509 575.48
Numerator	10 000	10 000	10 000	10 000

Source: own elaboration.

Table 4. CFaR and aspiration level for different techniques

	Without insurance	Annual insurance	Long term insurance	Whole life insurance
CFaR at 5%	1 027 780.39 zł	1 327 230.07 zł	1 113 780.16 zł	1 091 858.16 zł
Aspiration level $CF = 0$	33.33%	35.11%	39.76%	48.74%

Source: own elaboration.

5. Conclusions

The cash-in-hand distributions suggest that self-insurance is performed significantly better than any other strategy including life insurance. Not only the CFaR but also the aspiration level confirms that conclusion. That in turn suggests that self-insurance through investments is better solution for households. That conclusion holds as long as the death of a household member does not occur in short period. Thus buying a life insurance for short period (i.e. 5 to 10 years) might improve the results and therefore decrease the risk of a household.

Furthermore among the strategies that included the insurance the whole life insurance has been significantly outperformed by every other strategy. Surely that conclusion may be a result of too general consumption model assumption. Another conclusion that arises from comparison of strategies is that the long-term life insurance did decrease the CFaR but the probability that the household will not achieve its aspiration level was higher.

Nevertheless the author finds the results very surprising, especially in terms of the reasonability of buying a life insurance. Entering a life insurance should result in more leptokurtic cash flow distribution at the end of the household's lifetime than the distribution of base scenario. Those results surely need further and more thorough analysis before they will be finally confirmed.

The Cash Flow at Risk along with aspiration level might be useful tools for measurement of household's financial plan risk. Furthermore those measures may facilitate the risk factors decomposition of households' financial plans.

In terms of future research on assessment of risk management techniques an additional risk factor will be included into model. That in turn requires statistical analysis of distributions of those risk factors. Further extension of the model as well as cross-sectional analysis will be conducted in order to further conclusion. Also analysis of the whole trajectory of cash flow distribution will be taken into account.

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ZARZĄDZANIE RYZYKIEM W GOSPODARSTWACH DOMOWYCH Z WYKORZYSTANIEM MIĘDZYOKRESOWEGO MODELU KONSUMPCJI

Streszczenie: Autor skupia się na wykorzystaniu metody opartej na *Cash Flow at Risk* do oceny wpływu różnych rozwiązań w zakresie ubezpieczeń życiowych na sytuację finansową gospodarstw domowych. W modelu wykorzystane zostały trzy rodzaje ubezpieczeń życiowych odpowiadające różnemu podejściu do ryzyka utraty życia. Ponadto analiza została przeprowadzona na podstawie skonstruowanego uproszczonego międzyokresowego modelu konsumpcji.

Słowa kluczowe: finanse gospodarstw domowych, zarządzanie ryzykiem, ubezpieczenia na życie, *Cash Flow at Risk*.