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DISCOUNTING UNDER IMPACT OF TEMPORAL RISK AVERSION – A CASE OF DISCRETE TIME

Summary: The main aim of this study is to describe temporal risk aversion impact on the present value. Here the case of discrete time is considered only. An initial problem with differential equation occurred as a result of these studies. Then the discounting function is given as the unique solution of this initial problem. The formal influence of risk aversion index on the discounting was pointed out. Among other things, there has been proved that for fixed current discount rate the condition of present value temporal monotonicity is equivalent to the condition that temporal risk aversion index is positive. Moreover, there has been shown that nominal discount rate increases with temporal risk aversion index. Obtained discounting model is applied to explain the behavioural paradox of conducting contradictory transactions under the same set of rational premises. All considerations have theoretical nature.

Keywords: Present value, discounting, discounted utility, risk aversion,

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1. Introduction

The current equivalent value of payments available in a fixed point in time is called the present value (PV for short) of this payment. In financial arithmetic any PV is used as discounting function for dynamic assessment of the money value. The starting point for the financial arithmetic development was the interest theory. Further development of the financial arithmetic theoretical foundations has resulted in the formulation axioms of the financial arithmetic theory [Peccati 1972]. Peccati has defined PV as an additive function of payment value. This theory has been extensively developed in recent years [Janssen, Manca, Volpe di Prignano 2009]. The Peccati's definition of PV is generalized in [Piasecki 2012], where PV is defined as a utility of the multicriterion comparison determined by the time preference [Mises 1962] and the wealth preference.

For any fixed value of payments its PV may be described as a discount factor which is time-dependent only. On the basis of the interest theory we meet only with

exponential discount factors. On the other hand, many different types of discounting have been determined during the research on the behavioural aspects of dynamic assessment of the money value. The hyperbolic discounting was introduced by Mazur [1987] who generalized some particular function applied in [Ainslie 1975]. In [Killeen 2009] we can find arithmetic discounting. This discounting describes bill discounting method specified by financial law. The hyperbolic discounting was generalized to hyperboloid one in [Loewenstein, Prelec 1992; (Myerson, Green 1995)]. An alternative way to generalize the hyperbolic discounting was proposed by Rachlin [2006]. Quasi-hyperbolic discounting is rarely used in psychological research, though it is used extensively by economists hoping to preserve as much of the exponential model as possible. In the discrete time version this discounting method was introduced in [Laibson 1997]. The quasi-hyperbolic discounting can also be used for the case of continuous time [Benhabib, Bisin, Schotter 2010]. Moreover, as an alternative to the percentage decrement for delayed payments Benhabib, Bisin and Schotter [2010] suggested using a fixed cost model to exponential discounting. Read [2001] presented his model as such exponential discounting that discount fraction occurs within the given time window. Arguing with the Weber-Fechner law of logarithmic psychophysical sensibility to stimuli [Masin, Zudini, Antonelli 2009], Roelofsma [1996] presented his model as exponential discounting where exponent is directly proportional to logarithm of time. Ebert and Prelec [2007] defined their Constant Sensitivity discounting factor as the Cobb-Douglas function. Constant Relative Decreasing Impatience discount factor is defined as the constant relative risk aversion function [Bleichrodt, Rohde, Wakker 2009]. In analogous way, Constant Absolute Decreasing Impatience discount factor is defined as constant absolute risk aversion function [Bleichrodt, Rohde, Wakker 2009]. In [Killeen 2009] discount factor is defined by means of the assumption that its marginal change follows the Stevens' power law [Stevens 1957, 1961]. Multithreaded results of studies on the discounting factors were competently discussed in [Doyle 2013]. Moreover, in [Piasecki 2015] discount factor is defined as such discounting under continuous capitalization which is determined by constant Arrow-Pratt's measure of absolute risk-aversion [Arrow 1971; Pratt 1964]. The last discount factor is different from Constant Decreasing Impatience discount factors introduced in [Bleichrodt, Rohde, Wakker 2009].

Any product of the discounted value of payment and the discount factor describes linear discounting. Using linear discounting we omit the impact of interaction of payment value and payment time. In general, this approach is contrary to the finance practice. Each Peccati's PV is a linear discounting. Therefore, the main goal of this paper is to show that for any payments set we can define PV satisfying the conditions:

- PV variability is fully justified by economic reasons,
- PV is not a linear discounting.

Our attention will be paid to the problem of finding such nonlinear discounting which is dependent on temporal risk aversion measure. In [Piasecki 2015] it was

shown that such PV exists for the case of continuous time. Therefore all considerations will be restricted here to the case of discrete time.

My paper is organized in the following way. In Section 2 I discuss two axiomatic definitions of PV , which are theoretical background base for further considerations. Section 3 is devoted to the problem of determining the impact of risk aversion on discount. The model of this impact is given as the unique solution of initial problem formulated for differential equation. In Section 4 the result obtained in this way is applied to explain the paradox of conducting contradictory transactions under the same set of rational premises.

The mathematical deduction is the main tool of presented considerations. Therefore this paper has theoretical nature. Nevertheless, obtained results may be applied in behavioural finance. An example of these applications is given in Section 4.

2. Axiomatic definitions of present value

Each available at a fixed time point payment can be described as a financial flow. Let be given the set of time moments $\{0\} \subset \Theta \subseteq \mathbb{R}_0^+$. In the particular case it may be the set of capitalization moments. Each financial flow is described by the pair $(t, C) \in \Phi = \Theta \times \mathbb{R}$, where $t \in \Theta$ represents flow moment and $C \in \mathbb{R}$ describes the nominal value of the flow. The set Φ is called the payments set. In addition, by the symbol $\Phi^+ = \Theta \times \mathbb{R}^+$ we denote the receivables set. Peccati [1972] has defined PV as any function $PV: \Phi \rightarrow \mathbb{R}$ satisfying the conditions:

$$\forall_{C \in \mathbb{R}}: PV(0, C) = C, \quad (1)$$

$$\forall_{(t_1, C), (t_2, C) \in \Phi^+}: t_1 > t_2 \implies PV(t_1, C) < PV(t_2, C), \quad (2)$$

$$\forall_{(t, C_1), (t, C_2) \in \Phi}: PV(t, C_1 + C_2) = PV(t, C_1) + PV(t, C_2). \quad (3)$$

There are many economically well-justified PV functions which meet the Peccati's definition. Each PV satisfying the conditions (1), (2), (3) can be expressed by means of the identity

$$PV(t, C) = C \cdot v(t), \quad (4)$$

where discounting factor $v: \Theta \rightarrow [0; 1]$ is decreasing function additionally fulfilling the condition

$$v(0) = 1. \quad (5)$$

On the other hand, each function PV in the form (4) satisfies the conditions (1), (2) and (3). All of this means that any Peccati's PV is a linear discounting and any linear discounting is a Peccati's PV .

However, such proceedings did not explain the phenomenon of growing the money value. This explanation was obtained by showing that any payment PV is identical with the utility of the financial flow representing this payment (Piasecki, 2012). Then the generalized PV is defined as any function $PV: \Phi \rightarrow \mathbb{R}$ satisfying the conditions (1), (2) and

$$\forall_{(t,C_1),(t,C_2) \in \Phi}: C_1 > C_2 \implies PV(t, C_1) > PV(t, C_2). \quad (6)$$

$$\forall_{(t,C) \in \Phi}: PV(t, -C) = -PV(t, C). \quad (7)$$

Fulfillment of condition (7) may be obtained via the assumption that the utility of any liability is non-positive.¹ The above PV definition is a generalization of the Peccati's definition. In [Piasecki 2015] we can find such generalized PV which does not fulfill the condition (3). The significance of generalization Peccati's PV to the generalized PV was showed in this way. Moreover, such approach fully explains the essence of the PV notion.

3. The temporal aversion risk impact on present value

The concept of time preference is inextricably linked with the notion of temporal aversion. Reinterpreted Saint Petersburg paradox is a theoretical background for the phenomenon of temporal aversion [Fishburn, Edwards 1997]. There is pointed out that probability of deferred payment realization decreases with maturity lengthening. Hence, the liquidity risk increases with the extending of payment term. The cost of this risk reduces the value of assessed financial flows. Thus, the increase in liquidity risk implies a decrease in the unit price of each deferred payment. This means that the discount value is implied by the temporal risk. In this situation, we cannot exclude that also the investor's risk aversion has an impact on the discount assessment.

We assume that $\Theta = \{t_i: i \in \mathbb{N}_0\}$ where $t_0 = 0$ and for each $i \in \mathbb{N}$ we have

$$t_i - t_{i-1} = \Delta t > 0. \quad (8)$$

For fixed receivable value $C \in \mathbb{R}^+$ we define the sequence $\{v_i\}_{i \in \mathbb{N}_0}$ as follows

$$v_i = PV(t_i, C), \quad (9)$$

where $PV: \Phi \rightarrow \mathbb{R}$ fulfills conditions (1), (2), (6) and (7). Due to (2) the sequence $\{v_i\}_{i \in \mathbb{N}_0}$ should be decreasing. About considered PV we will additionally assume that current discount rate $\delta \in \mathbb{R}^+$ is independent of any receivable value $C \in \mathbb{R}^+$ which results in

¹ The notion of negative utility was discussed in the papers [Becker, Duesenberry, Okun 1960; Cooper, Garcia Peñaloza, Funk 2001; Rabin 1993].

$$\frac{C-v_1}{\Delta t \cdot C} = \delta. \quad (10)$$

From the viewpoint of financial practice the above condition is obvious because in this way current discount rate is defined by banking institutions.

We will characterize the investor's susceptibility to liquidity risk by temporal risk aversion index $\hat{\gamma}: \Phi^+ \rightarrow \mathbb{R}$ defined as the Arrow-Pratt measure of absolute risk-aversion [Arrow 1971; Pratt 1964] which is given here by the identity

$$\hat{\gamma}(t_i, C) = -\frac{\frac{v_i-2 \cdot v_{i-1}+v_{i-2}}{(\Delta t)^2}}{\frac{v_i-v_{i-1}}{\Delta t}} = -\frac{v_i-2 \cdot v_{i-1}+v_{i-2}}{\Delta t \cdot (v_i-v_{i-1})}. \quad (11)$$

In agreement with the First Gossen's Law [Begg, Fischer, Dornbusch 2005] the temporal risk aversion index is a non-increasing function of receivable value. We can suppose that the temporal risk aversion does not decrease with maturity lengthening. The formal image of this conjecture is the assumption that in general case the temporal risk aversion index is a non-decreasing function of payment term. Usually, we do not have information about the distribution of temporal risk aversion. Thus we assume that temporal risk aversion index is independent of the payment term. This means that this index is constant in time. Therefore we can replace the temporal risk aversion index (11) by the function $\gamma: \mathbb{R}^+ \rightarrow \mathbb{R}$ defined as follows

$$\gamma(C) = -\frac{v_i-2 \cdot v_{i-1}+v_{i-2}}{\Delta t \cdot (v_i-v_{i-1})}. \quad (12)$$

Immediately from above we obtain following differential equation

$$(1 + \Delta t \cdot \gamma(C)) \cdot v_i - (2 + \Delta t \cdot \gamma(C)) \cdot v_{i-1} + v_{i-2} = 0. \quad (13)$$

For the case $\gamma(C) = -(\Delta t)^{-1}$ the equation (13) reduces to the equation

$$v_{i-1} + v_{i-2} = 0. \quad (14)$$

Each solution $\{v_i\}_{i \in \mathbb{N}_0}$ of the problem (14) is a constant sequence, which is contrary to the condition (2).

For the case $\gamma(C) \neq -(\Delta t)^{-1}$ the equation (13) has the general solution $\{v_i\}_{i \in \mathbb{N}_0}$ defined by

$$v_i = A(C) \cdot (1 + \Delta t \cdot \gamma(C))^{-i} + B(C), \quad (15)$$

where $A: \mathbb{R}^+ \rightarrow \mathbb{R}$ and $B: \mathbb{R}^+ \rightarrow \mathbb{R}$ are some functions of payment value. Let us note that for the case $\gamma(C) < -(\Delta t)^{-1}$ sequence $\{v_i\}_{i \in \mathbb{N}_0}$ is not monotonic. Moreover, for the case $\gamma(C) = 0$ the sequence $\{v_i\}_{i \in \mathbb{N}_0}$ is constant. Last two facts are contrary to the condition (2). Thus we have

$$0 \neq \gamma(C) > -(\Delta t)^{-1}. \quad (16)$$

From the condition (1) we obtain

$$A(C) + B(C) = C. \quad (17)$$

This and the condition (10) imply that

$$A(C) = (1 + \Delta t \cdot \gamma(C)) \cdot \frac{\delta}{\gamma(C)} \cdot C, \quad (18)$$

$$B(C) = \left(1 - (1 + \Delta t \cdot \gamma(C)) \cdot \frac{\delta}{\gamma(C)}\right) \cdot C, \quad (19)$$

In this way it is shown that the initial value problem (1), (10) and (13) has the unique solution

$$v_i = (1 + \Delta t \cdot \gamma(C)) \cdot \frac{\delta}{\gamma(C)} \cdot C \cdot (1 + \Delta t \cdot \gamma(C))^{-i} + \left(1 - (1 + \Delta t \cdot \gamma(C)) \cdot \frac{\delta}{\gamma(C)}\right) \cdot C. \quad (20)$$

It is very easy to check that the sequence $\{v_i\}_{i \in \mathbb{N}_0}$ is decreasing iff the temporal risk aversion index fulfills the condition

$$\gamma(C) > 0. \quad (21)$$

The foregoing inequality is consistent with the First Gossen's Law. In this way we prove that for fixed current discount rate the condition of PV temporal monotonicity is equivalent to the condition that temporal risk aversion index is positive.

In this situation, according to (9), for given temporal risk aversion index $\gamma: \mathbb{R}^+ \rightarrow \mathbb{R}$ the function $PV(\cdot | \gamma): \Phi^+ \rightarrow \mathbb{R}$ is determined by the identity

$$\begin{aligned} PV(t_i, C | \gamma) &= \\ &= C \cdot \left[1 - (1 + \Delta t \cdot \gamma(C)) \cdot \frac{\delta}{\gamma(C)} \cdot \left(1 - (1 + \Delta t \cdot \gamma(C))^{-i}\right)\right]. \end{aligned} \quad (22)$$

Let us note that due to (7) we can extend the domain of PV to the payments set Φ . Then the function $PV(\cdot | \gamma): \Phi \rightarrow \mathbb{R}$ is defined as follows

$$PV(t_i, C | \gamma) = C \cdot \left[1 - (1 + \Delta t \cdot \gamma(|C|)) \cdot \frac{\delta}{\gamma(|C|)} \cdot \left(1 - (1 + \Delta t \cdot \gamma(|C|))^{-i}\right)\right]. \quad (23)$$

This function can be used to discount the value of future financial flow $(t_i, C) \in \Phi^+$. Then the discount rate is equal to

$$\begin{aligned} D(t_i, C | \gamma) &= \frac{c - c \cdot \left[1 - (1 + \Delta t \cdot \gamma(C)) \cdot \frac{\delta}{\gamma(C)} \cdot \left(1 - (1 + \Delta t \cdot \gamma(C))^{-i}\right)\right]}{c} = \\ &= (1 + \Delta t \cdot \gamma(C)) \cdot \frac{\delta}{\gamma(C)} \cdot \left(1 - (1 + \Delta t \cdot \gamma(C))^{-i}\right). \end{aligned} \quad (24)$$

In an elementary way, we can prove that the discount rate determined above is an increasing function of the value $\gamma(C)$ of temporal risk aversion index. On the other hand, any temporal risk aversion index is non-decreasing function of receivable value. It means that discount rate is independent of receivable value iff the temporal risk aversion index is constant function of receivable value. In other case, defined by (23) PV describes nonlinear discounting.

4. Diversified risk aversion as a behavioural premise of market equilibrium

In situations where temporal risk aversion is an individual characteristic each investor, the above PV model can be used to build formal models of behavioural finance. In this model, objective fundamental factors are represented by the value δ of current discount rate.

Let us take into account any security \mathcal{Y} which is the object of trading on a fixed financial market. About financial market will assume that it is highly effective. In this case, for any future term $t > 0$ all market participants anticipate the same future value $C > 0$ of security. This instrument is represented by the financial flow (t, C) . The investor identifies the *PV* of this financial flow, as the financial equilibrium price of the security \mathcal{Y} .

Let us now consider a pair of investors $(\mathcal{P}_1, \mathcal{P}_2)$ who differ among themselves by temporal risk aversion only. The investor \mathcal{P}_i 's risk aversion is characterized by the value $\gamma_i(C)$ of temporal risk aversion index. Suppose now that the investor \mathcal{P}_1 is characterized by higher risk aversion than the investor \mathcal{P}_2 . Then we have the inequality.

$$\gamma_1(C) > \gamma_2(C), \quad (25)$$

which finally results in

$$C_{0,1} = PV(t, C | \gamma_1(C)) < PV(t, C | \gamma_2(C)) = C_{0,2}. \quad (26)$$

Here we see that partially subjectively estimated equilibrium price decreases with increasing risk aversion. Both investors observe the same value \check{C} of market price. If this price satisfies the condition

$$C_{0,1} < \check{C} < C_{0,2}, \quad (27)$$

then the investor \mathcal{P}_1 intends to sell the security \mathcal{Y} . At the same time the investor \mathcal{P}_2 plans to buy this security. The investor \mathcal{P}_1 offers supply which meets the demand declared by the investor \mathcal{P}_2 .

The inequality (27) and its interpretation explain the paradox of conducting contradictory transactions under the same set of rational premises. In a formal way it has been shown here that premise to explanation of this paradox may be differences

in individual investors' aversion to risk. In addition, it is worth noting here that the demand declared by investors with less risk aversion is always offset by the supply offered by the investor with higher risk aversion. This conclusion suggests that on the highly efficient financial markets, capital is concentrated in the hands of investors characterized by high susceptibility to risk. This conclusion is a specific commentary on the global financial crisis observed at this time, because many financial commentators discern the widespread use of high-risk financial instruments as one of the reasons for the crisis.

5. Conclusions

For the variable temporal risk aversion index the function $PV: \Phi \rightarrow \mathbb{R}$ determined by (23) is not linear discounting. Variability of defined above PV depends only on temporal risk aversion index variability and current discount rate variability. Each of these characteristics is an element of economic environment description. In this situation we can say that variability of proposed here PV is fully justified by economic reasons. All this shows that the main goal of this research work is realized.

There is shown that defined above PV is nonlinear discounting iff used temporal risk aversion index is variable. Moreover, we proved that for fixed current discount rate the condition of PV temporal monotonicity is equivalent to the condition that temporal risk aversion index is positive. Both of these conclusions can be useful in further studies devoted to non-linear discounting.

The temporal risk aversion index describes the behavioural aspects of financial management. The current discount rate is an image of the capital appreciation process. The capital appreciation process depends on the objective fundamental properties of the financial market, and it may depend on various behavioural factors. It means that we can determine PV using on the interaction of fundamental and behavioural factors.

Financial arithmetic should be treated as a subjective extension of the interest theory, which is based on objective premises. In this paper, it was shown that this extension is important.

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DYSKONTOWANIE POD WPLYWEM AWERSJI DO RYZYKA TERMINU – PRZYPADEK CZASU DYSKRETNEGO

Streszczenie: Głównym celem tego artykułu jest opisanie wpływu awersji do ryzyka terminu na wartość bieżącą. Rozpatrywany jest tutaj tylko przypadek czasu dyskretnego. Wynikiem tych rozważań jest pewne zagadnienie początkowe z równaniem różnicowym. Wtedy funkcja dyskontująca została wyznaczona jako jedyne rozwiązanie tego zagadnienia początkowego. Formalny wpływ czynnika awersji do ryzyka terminu na dyskontowanie został wskazany. Między innymi zostało tutaj dowiedzione, że dla ustalonej aktualnej stopy dyskonta, warunek malejącego trendu wartości bieżącej jest równoważny temu, że czynnik awersji do ryzyka terminu jest dodatni. Pokazano ponadto, że nominalna stopa dyskonta rośnie wraz ze wzrostem indeksu awersji do ryzyka terminu. Uzyskane wyniki zostały zastosowane do wyjaśnienia behawioralnego paradoksu utrzymywania się przeciwstawnych transakcji dokonywanych pod wpływem tych samych racjonalnych przesłanek. Wszystkie rozważania mają charakter teoretyczny.

Słowa kluczowe: wartość bieżąca, dyskontowanie, użyteczność zdyskontowana, ryzyko do awersji.