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APPLICATION OF THREE-DIMENSIONAL COPULA FUNCTIONS IN THE ANALYSIS OF DEPENDENCE STRUCTURE BETWEEN EXCHANGE RATES

Summary: The multivariate analysis of financial data has gained a lot of attention. Investors are no longer interested in knowing only the dependence between two components of their portfolio but between all of them as it allows them to better understand andto assess the situation on the financial market. Copula functions seem to be a tool efficient enough to provide deep and understandable results regarding instruments dependence. This paper has a goal to analyze the relation between three currencies: USD, EUR and CZK against PLN. The literature research has shown that such analyses are performed but the use of elliptical copulas, namely normal and *t*-Student copula functions dominate. Another disadvantage is that instruments are grouped in pairs enabling a bivariate analysis. Multivariate approach can simplify calculations and lead to more reliable results. However, multivariate copula models are still under deep investigtion. Therefore another approach has been proposed: a decomposition of a joint multivariate distribution function into a product of marginal densities functions and a pair copula density function. In this paper, we will focus on Archimedean copula functions such as the Frank, Clayton and Gumbel families which constitute an introduction to multivariate analysis of financial underlying instruments.

Keywords: multivariate, copulas, exchange rates, vines, pair-copula.

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1. Introduction

Each company involved in international trade or each investor acting on foreign financial markets are interested in the dependence between various currencies. In the face of globalization processes, international exchange of goods, services and capital, products exportation and importation, entities do not longer monitor the behavior of the national currency but rather focus on relations between foreign currencies in order to be able to exhibit some common patterns. Given Polish currency market, knowing that the appreciation of euro carries the appreciation of ruble provides information about how import prices will be affected against Russia in case of appreciation of euro against Polish zloty.

Furthermore, interdependence relations between various exchange rates may have influence on the actions taken by central banks in order to fulfill their mission regarding currency interventions.

Exchange rates show asymmetric dependence. They are vulnerable to variations over time and hence are characterized by skewness and fat tails. For this reason, standard methods of measuring their relationship fail. Therefore, our field of interest is the copula theory. Copula functions allow for measuring non-linear dependence. Another advantage is that they capture dependence in tails and are free of distribution limitations like in case of multivariate normal or *t*-Student distributions as marginal functions do not need to belong to the same distribution family nor to have the tail of the same size or category. What is more, they are invariant under all strictly increasing transformations, which leads to the fact that the same copula can be applied either to exchange rates or toe exchange rates returns. For these reasons, bivariate copula functions have constituted an object of interest for many researchers (see [Boero, Silvapulle, Tursunalieva 2009; Dias, Embrechts 2007; Embrechts, Hofert 2013; Trivedi, Zimmer 2006; Genest, Favre 2007; Patton 2006]). Also Polish researchers have contributed to the study of copula functions with applications in insurances (see [Wanat 2011]), stock market analysis (see [Doman, Doman 2010; Pipień 2013]), risk analysis (see [Jajuga, Papla 2005]). However, it seems that copula functions of higher dimensions are still under investigation (see [Dias, Embrechts 2010; Bugienė, Šutienė 2011; Venter et al. 2007]). There are several attempts to build multivariate models; nevertheless, many researchers apply a pair-copula approach (see [Schirmacher, Schirmacher 2008; Aas et al. 2006; Bugienė, Šutienė, 2011; Doman 2010]) which seems to be quite an efficient method to measure dependence in higher dimensions.

The goal of this paper is to observe and determine the dependence within three exchange rates: US dollar, euro and Czech crown against Polish zloty by applying pair-copula approach and to show that copula functions are a more efficient method to model tail dependence in a multivariate space than other available tools. The article is organized in five sections, including introduction and conclusion.

Section 2 introduces basic notions and theorems related to copula functions. We will focus mainly on Sklar's theorem which is essential to properly apply copulas in the empirical study. In this section we will also discuss briefly main copula functions with their properties that will be further used in the experiment.

Section 3 is devoted to a pair-copula approach with introduction to regular vines theory and construction.

Section 4 focuses fully on the numerical study. Analyzed data set consist of log-differenced returns of daily exchange rates: USD/PLN, CZK/PLN and EUR/PLN from January 2002 till June 2014. Its source is The National Bank of Poland database of daily average exchange rates.[1]

2. Bivariate copula functions

Copula functions were introduced for the first time by Sklar in 1959 in the article *Fonctions de repartition à* n *dimensions et leurs marges.* Its English version followed in 1973. Briefly speaking, copulas are functions that exhibit the relation between a joint multivariate distribution function and its marginal distribution functions. What is important, marginals can belong to various distribution functions families. It implies that copulas are a very powerful and intuitive method to measure dependence between various variables.

2.1. Definition and basic properties of a bivariate copula functions

In this section we will present the definition of a bivariate copula and the basic theorem in the copula theory, namely the Sklar's theorem. Extension to higher dimensions can be found in: [Nelsen 2006; Trivedi, Zimmer 2006; McNeil, Frey, Embrechts 2005].

Definition 2.1.1 [McNeil, Frey, Embrechts 2005]. *A two-dimensional copula is a distribution function on* $[0,1]^2$ *with standard uniform marginal distributions.*

A two-dimensional copula function C satisfies three conditions [Nelsen 2006]:

1. For every u, v in $[0,1]$,

$$C(u,0) = C(0,v) = 0.$$

2. For every u, v in $[0,1]$,

$$C(u,1) = u,$$
$$C(1,v) = v.$$

3. For every u_1, u_2, v_1, v_2 in $[0,1]$ such that $u_1 \leq u_2$ and $v_1 \leq v_2$,

$$C(u_2,v_2) - C(u_2,v_1) - C(u_1,v_2) + C(u_1,v_1) \geq 0.$$

The most important theorem in copula functions theory is Sklar's theorem stating that each distribution function can be represented by a junction of its marginal distributions functions and an appropriate copula function. Let us present this theorem in a bivariate case only.

[1] Available on http://www.nbp.pl/home.aspx?c=/ascx/archa.ascx (retrieved: 27.07.2014).

Theorem 2.1.1 [Nelsen 2006]. Let F be a joint distribution function with margins F_1 and F_2. Then there exists a copula $C\colon [0,1]^2 \to [0,1]$ such that for all $x_1, x_2 \in \mathbb{R}$,

$$F(x_1, x_2) = C(F_1(x_1), F_2(x_2); \theta).$$

Parameter θ is a dependence parameter which measures the dependence between marginals. In a bivariate case, it is assumed to be a scalar. Further in this paper it will be shown how to estimate the dependence parameter θ.

2.2. Archimedean copula functions

Archimedean copulas were recognized for the first time by Schweizer and Sklar in 1961 in their study of triangular norms used in fuzzy logic. Consider a function $\varphi\colon [0,1] \to [0,\infty)$ such that $\varphi(1) = 0$. Function φ is continuous, decreasing and convex. This function is called a generator. It is a strict generator when $\varphi(0) = \infty$. Archimedean copulas are constructed through a generator.

It is necessary to introduce the notion of pseudo-inverse.

Definition 2.2.1 [Cherubini, Luciano, Vecchiato 2004]. *The pseudo-inverse of a function* φ is defined as follows:

$$\varphi^{[-1]}(v) = \begin{cases} \varphi^{[-1]}(v) & 0 \le v \le \varphi(0) \\ 0 & \varphi(0) \le v \le \infty \end{cases}.$$

Definition 2.2.2 [Cherubini, Luciano, Vecchiato 2004]. Given a generator φ and its pseudo-inverse $\varphi^{[-1]}$, an Archimedean copula is defined by the formula:

$$C(u, v) = \varphi^{[-1]}\big(\varphi(u) + \varphi(v)\big).$$

Archimedean copula functions have been deeply studied and have found application in various financial areas such as modeling dependence between various financial assets (interest rates, exchange rates), pricing currency options, modeling portfolio credit risk, VaR calculation, systemic risk stress testing or survival functions construction.

Most commonly used Archimedean copula functions are a product copula and Clayton, Frank or Gumbel families. Each of these functions can be used to measure different types of a dependence structure between variables. We will present them in a brief manner.

A **product copula** is the simplest one and it serves to measure independence. It has the form:

$$C(u, v) = uv.$$

A product copula is a simplified version of a Farlie-Gumbel-Morgenstern copula which takes the form:

$$C(u,v) = uv[1+\theta(1-u)(1-v)], \quad \theta \in \langle -1,1 \rangle.$$

When $\theta = 0$ then we receive a product copula. The FGM copula is used to model marginal with a weak symmetric or negative dependence.

The Clayton copula is of the following form:

$$C(u,v;\theta) = \left(u^{-\theta} + v^{-\theta} - 1\right)^{-\frac{1}{\theta}}, \qquad \theta \in (0,\infty).$$

In the literature, it can be also recognized as Cook and Johnson function. This function is used to where variables exhibit strong lower/left tail dependence.

The **Gumbel copula**, on the other hand, is associated with strong upper tail/right dependence. It takes the form:

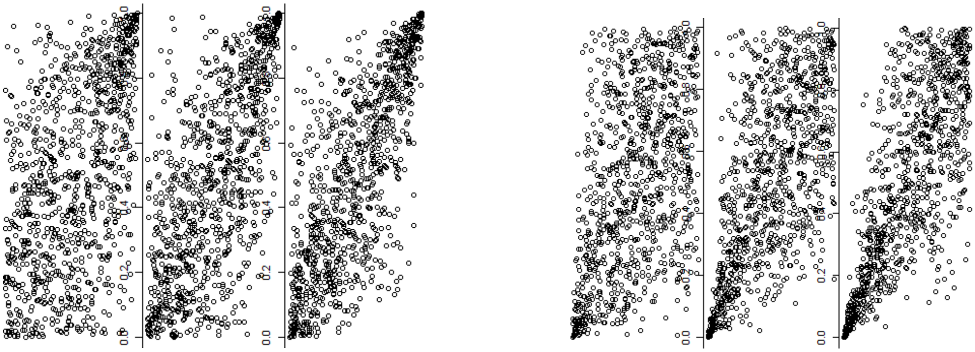
$$C(u,v;\theta) = \exp\left\{-\left[(-\ln u)^{\theta} + (-\ln v)^{\theta}\right]^{\frac{1}{\theta}}\right\}, \qquad \theta \in [1,\infty).$$

Finally, the **Frank copula** is defined by the formula:

$$C(u,v;\theta) = -\frac{1}{\theta}\log\left\{1 + \frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1}\right\}, \qquad \theta \in (-\infty,\infty).$$

The Frank copula is used whenever marginals exhibit symmetric dependence and it also allows negative dependence. It is usually applied to model a strong dependence, either a positive or a negative one.

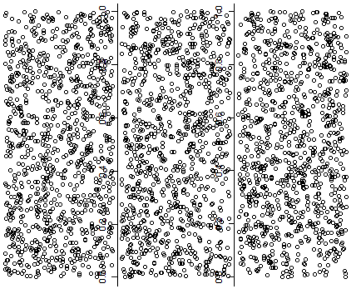
Figures 1 and 2 show scatterplots of 1000 random variables from Gumbel, Clayton, Frank and independence copula functions. The dependence parameters were set as $\theta_{G1} = 1.5, \theta_{G2} = 1.95$ and $\theta_{G3} = 2.4$ for Gumbel function, $\theta_{C1} = 1.1,\ \theta_{C2} = 1.9$ and $\theta_{C3} = 2.7$ for Clayton function, $\theta_{F1} = 0.3,\ \theta_{F2} = 0.4$ and $\theta_{F3} = 0.5$ and $\theta = 0$ for FGM functions respectively.



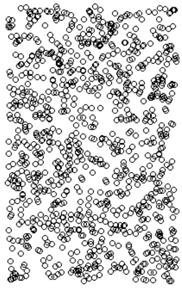
(a) (b)

Figure 1. Gumbel (a) and Clayton (b) copulas scatterplots

Source: own study.



(a)



(b)

Figure 2. Frank (a) and independence (b) copulas scatterplots

Source: own study.

3. Pair-copula approach

The pair-copula approach is a very intuitive method to work on multivariate models [Aas et al. 2006]. It consists of decomposing a joint multivariate distribution function into pairs of bivariate blocks and of scaling them with a bivariate copula function. Multivariate copula modeling is still considered to be theoretically and computationally hard to implement.

In order to determine the number of possible pair-copula functions, regular vines will be implemented. This is a graphical method introduced by Bedford and Cooke [Bedford, Cooke 2002] and founded on the grounds of graphs theory and statistical inference which enables to decompose a multivariate model into bivariate copulas and a nested set of threes satisfying the proximity condition. In this paper we will focus on D-vines and canonical vines which in case of three dimensions happen to be of the same form. However, the decision about which vine should be used is taken by observing the nature of dependence structure. If it is possible to select the variable that enters in interaction with the others, a canonical vine is the best fit. Otherwise, if we are not able to determine such a variable, D-vine should be used.

Therefore, a joint density can be represented as a product of marginal density functions and a pair copula density function. The general formula to describe the density function in terms of a canonical vine is the following [Aas et al. 2006]:

$$f(x_1, \dots, x_n) =$$

$$= \prod_{k=1}^{n} f(x_k) \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{j,j+1|1,\dots,j-1}\left(F\left(x_j|x_1, \dots, x_{j-1}\right), F\left(x_{j+i}|x_1, \dots, x_{j-1}\right)\right).$$

Furthermore, the general formula to describe the density function in terms of a D- vine is the following:

$$f(x_1, \dots, x_n) =$$
$$= \prod_{k=1}^{n} f(x_k) \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{i,i+j|i+1,\dots,i+j-1} \left(F(x_i|x_{i+1}, \dots, x_{i+j-1}), F(x_{i+j}|x_{i+1}, \dots, x_{i+j-1})\right).$$

Let us consider a three-dimensional density function which can be expressed in terms of conditional densities:

$$f(x_1, x_2, x_3) = f_1(x_1) f_{2|1}(x_2|x_1) f_{3|1,2}(x_3|x_1, x_2).$$

By Sklar's theorem we arrive to the following formula:

$$f(x_1, x_2) = c_{12}\left(F_1(x_1), F_2(x_2)\right) \cdot f_1(x_1) f_2(x_2).$$

Moreover, by conditional density formula:

$$f_{2|1}(x_2|x_1) = \frac{f(x_1, x_2)}{f_1(x_1)} = \frac{c_{12}\left(F_1(x_1), F_2(x_2)\right) \cdot f_1(x_1) f_2(x_2)}{f_1(x_1)} =$$
$$= c_{12}\left(F_1(x_1), F_2(x_2)\right) \cdot f_2(x_2).$$

The same applies to the conditional density $f_{3|1,2}(x_3|x_1, x_2)$. Let it depend on the variable X_2.

$$f_{3|1,2}(x_3|x_1, x_2) = \frac{f(x_1, x_2, x_3)}{f_{1,2}(x_1, x_2)} = c_{13|2}\left(F_{1|2}(x_1|x_2), F_{3|2}(x_3|x_2)\right) f_{3|2}(x_3|x_2) =$$
$$= c_{13|2}\left(F_{1|2}(x_1|x_2), F_{3|2}(x_3|x_2)\right) c_{23}\left(F_2(x_2), F_3(x_3)\right) f_3(x_3).$$

Hence, the three-dimensional density function can be presented as follows:

$$f(x_1, x_2, x_3) = f_1(x_1) f_{2|1}(x_2|x_1) f_{3|1,2}(x_3|x_1, x_2) =$$
$$= f_1(x_1) f_2(x_2) c_{12}\left(F_1(x_1), F_2(x_2)\right) f_{3|1,2}(x_3|x_1, x_2) =$$
$$= f_1(x_1) f_2(x_2) f_3(x_3) c_{12}\left(F_1(x_1), F_2(x_2)\right) c_{23}\left(F_2(x_2), F_3(x_3)\right) \cdot$$
$$\cdot c_{13|2}\left(F_{1|2}(x_1|x_2), F_{3|2}(x_3|x_2)\right).$$

This formula can be still simplified if we assume the conditional independence of variables X_1 and X_3 given X_2 which leads to the fact that:

$$c_{13|2}\left(F_{1|2}(x_1|x_2), F_{3|2}(x_3|x_2)\right) = 1.$$

Under this assumption, the joint density function can be finally represented by:

$$f(x_1, x_2, x_3) = f_1(x_1) f_2(x_2) f_3(x_3) c_{12}\left(F_1(x_1), F_2(x_2)\right) c_{23}\left(F_2(x_2), F_3(x_3)\right).$$

The above decomposition is called a regular vine. As it was already mentioned, in case of three variables, a D-vine is equal to a canonical vine.

Its graphical form is presented in Figure 3.

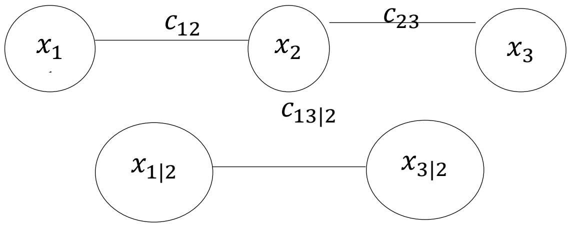


Figure 3. A D-vine with 3 variables and two trees

Source: own study.

A more detailed analysis about regular vines decomposition can be found in [Bedford, Cooke 2002].

4. Application: foreign exchange rates

The set of data consists of daily observations of three currencies: US dollar (USD), euro (EUR) and Czech Crown (CZK) against Polish zloty (PLN). The sample covers the period of 2nd January 2002 till 30th June 2014 (3159 observations). Data were collected from the National Bank of Poland database of average daily exchange rates. For estimation purposes we have converted observed data into log-differenced returns. Moreover, in order to detect any patterns, it is crucial to remove marginal distributions and to transform the analyzed set of data through its empirical distribution [Genest, Favre 2007].

4.1. Data analysis

Time plots of three analyzed currencies (Figure 4) exhibit already some dependence over time between euro and US dollar as a significant depreciation is noticed during a recent financial crisis. One can also observe that Czech crown seems to behave in a different way, although, starting 2009, there is a high depreciation against polish zloty, similarly to US dollar and euro.

Table 1 contains basic statistics for analyzed time series. One can observe that kurtosis in three cases manifests high values which is the property of leptokurtic distributions. Regarding skewness values, they are positive which means that distributions are slightly right-tailed. Also Jarque-Bera statistics confirms that marginal distributions are not symmetric and far from being normal.

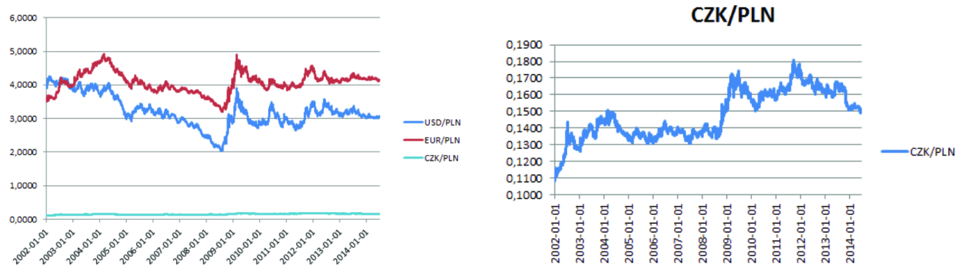


Figure 4. Exchange rate time series

Source: own study.

Table 1. Sample descriptive statistics

	USD/PLN	EUR/PLN	CZK/PLN
Mean	–0.000085	0.000053	0.000099
Median	–0.000627	–0.000172	0
Standard deviation	0.009607	0.006413	0.005976
Kurtosis	4.628397	5.753758	5.007672
Skewness	0.350914	0.218365	0.179627
Jarque-Bera statistic	2873.23	4365.98	3304.77
Number of observations	3159	3159	3159

Source: own study.

The above is confirmed by QQ plots form (Figure 5). The left panel presents USD/PLN, the middle one – EUR/PLN and the right panel – CZK/PLN.

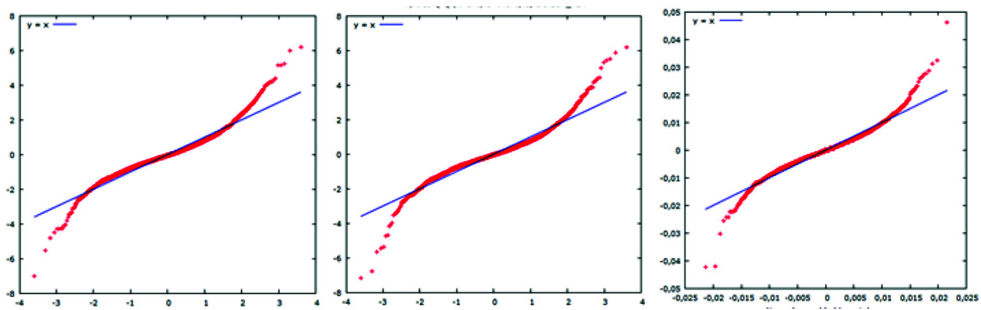


Figure 5. Q-Q plots for analyzed currencies

Source: own study.

4.2. Copula estimation

Let $X_1 = USD/PLN, X_2 = EUR/PLN$ and $X_3 = CZK/PLN$. Then, by formulas derived in Section 3, the joint distribution can be written in terms of a canonical vine:

$$f(x_1, x_2, x_3) = f_1(x_1)f_2(x_2)f_3(x_3)c_{12}\big(F_1(x_1), F_2(x_2)\big)c_{23}\big(F_2(x_2), F_3(x_3)\big)\cdot\cdot$$
$$c_{13|2}\left(F_{1|2}(x_1|x_2), F_{3|2}(x_3|x_2)\right).$$

This vine decomposition will allow depicting the following pairs to be constructed: {USD/PLN, EUR/PLN}, {EUR/PLN, CZK/PLN} and {USD/EUR, EUR/CZK} given PLN. The goal is to define copula functions which model the best the dependence structure between these pairs. Pair-wise correlation takes the values shown in Table 2 Values in bold will be used for further calculations.

Table 2. Pair-wise linear correlation

	USD/PLN	EUR/PLN	CZK/PLN	USD/EUR	EUR/CZK
USD/PLN	1				
EUR/PLN	**0.745278**	1			
CZK/PLN	0.551489	**0.778951**	1		
USD/EUR	0.748853	0.116244	0.047284	1	
EUR/CZK	0.358652	0.424665	-0.23692	**0.112163**	1

Source: own study.

In order to choose the appropriate copula function to model the dependence within constructed blocks of exchange rates, a non-parametric approach will be applied. There are studies showing that the chi-plot is a very powerful method of depicting dependence structure [Fisher, Switzer 1985]. The chi-plot has a key property of the invariance under all strictly increasing transformations of marginals [Boero, Silvapulle, Tursunalieva 2011]. Several authors have applied this method to detect the relation between several assets while applying copula theory (see, for example, [Schirmacher, Schirmacher 2008; Boero, Silvapulle, Tursunalieva 2011]).

Figures 6 and 7 present the chi-plots and scatterplots of analyzed three blocks of data: {USD/PLN, EUR/PLN}, {EUR/PLN, CZK/PLN} and {USD/EUR, EUR/CZK} respectively.

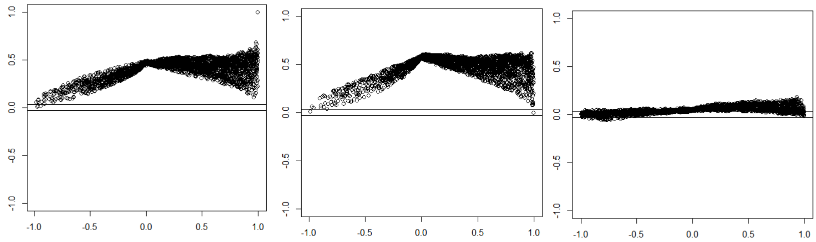


Figure 6. The chi-plots for analyzed pairs of exchange rates

Source: own study.

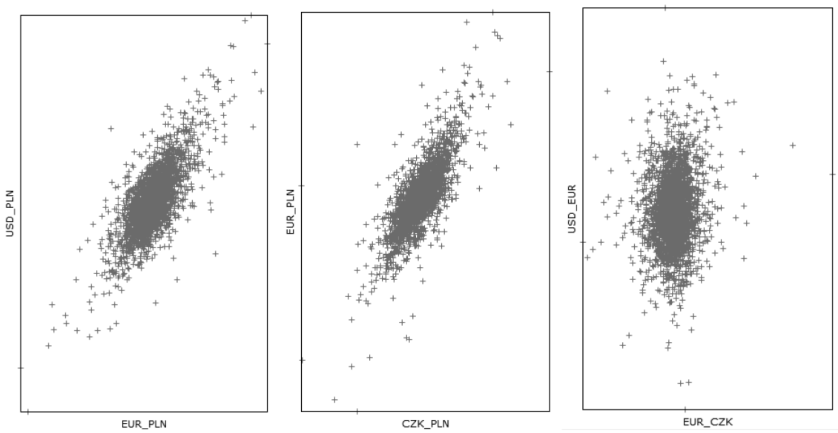


Figure 7. The scatterplots for analyzed pairs of exchange rates

Source: own study.

By theory, independent random variables are plotted around zero. The positive dependence is manifested by deviations on the right side of the chi-plot. The larger it is, the larger the peak of the plot is. Moreover, this peak corresponds to Kendall's tau value.

The question is now about the appropriate choice of a copula function to model dependence structure between analyzed data. The shape of two first plots is similar. Both chi-plots and scatterplots are proper to Gumbel copula functions family. The highest points are located near 0.5 for the pair {USD/PLN, EUR/PLN} and near 0.6 for the pair {EUR/PLN, CZK/PLN}. Regarding the last one, {USD/EUR, EUR/CZK}, points are scattered around zero and fall within bands which implies that there is a very weak positive dependence between these pairs of variables; hence a hypothesis about data independence can be assumed and the joint distribution function can be reduced to:

$$f(x_1,x_2,x_3)=f_1(x_1)f_2(x_2)f_3(x_3)c_{12}\big(F_1(x_1),F_2(x_2)\big)c_{23}\big(F_2(x_2),F_3(x_3)\big)$$

as, in case of independence between analyzed data the below condition holds:

$$c_{13|2}\left(F_{1|2}(x_1|x_2),F_{3|2}(x_3|x_2)\right)=1.$$

In order to estimate the dependence parameters of mentioned Archimedean copula functions the method of moments using rank correlation was used. The reason is that there exists a correspondence between the dependence parameter θ and Kendall's tau or Spearman's rho as these coefficients can be expressed in terms of copula functions [Trivedi, Zimmer 2006]. However, we will use only Kendall's tau as integrals calculations lead to the comprehensive results grouped in Table 3.

$$\tau_K=4\int_0^1\int_0^1 C(u,v)dC(u,v)-1.$$

Table 3. Correspondence between dependence parameter and Kendall's tau

Copula function	Kendall's tau
Gumbel	$1-\frac{1}{\theta}$
Clayton	$\frac{\theta}{\theta+2}$
Frank	$1+\frac{4[D_1(\theta)-1]}{\theta}$, $D_1(\theta)=\frac{1}{\theta}\int_0^\theta\frac{t}{e^t-1}dt$

Source: [Cherubini, Luciano, Vecchiato 2004, p. 126].

By applying these formulas, we have obtained the results presented in Table 4.

Table 4. Kendall's tau values for analyzed time series and dependence parameter values for given copula functions

	Kendall's tau	Dependence parameter		
		Clayton	Gumbel	Frank
USD/PLN & EUR/PLN	0.48794858	1.905858	**1.952929**	0.4316069
EUR/PLN & CZK/PLN	0.57765739	2.735492	**2.367746**	0.4996035
USD/EUR & EUR/CZK	0.06273680	–	–	–

Source: own study.

The research showed that the joint distribution function is of the form

$$f(x_1,x_2,x_3) = f_1(x_1)f_2(x_2)f_3(x_3)c_{12}\big(F_1(x_1),F_2(x_2)\big)c_{23}\big(F_2(x_2),F_3(x_3)\big).$$

where c_{12} is the Gumbel function with parameter $\theta = 1.95$ for the pair {USD/PLN, EUR/PLN} and c_{23} is the Gumbel function with parameter $\theta = 2.37$ applied to the pair {EUR/PLN, CZK/PLN}. Regarding a conditional copula $c_{13|2}$ joining the pair {USD/EUR, EUR/CZK}, independence between analyzed data was assumed. Therefore structure of a regular vine is as in Figure 8.

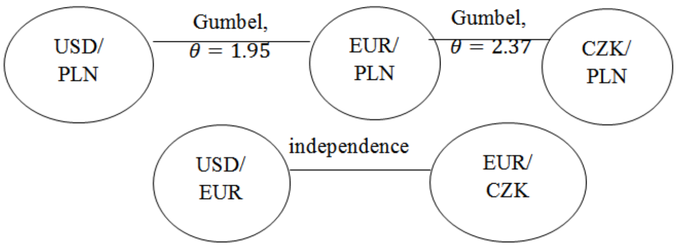


Figure 8. A canonical vine to model dependence between USD, EUR and CZK

Source: own study.

5. Conclusions

The paper has presented the pair-copula construction as a method to examine the nature of dependence between various variables. It was shown that copula functions are more efficient as they allow modeling tail dependence. The statistical inference has shown that analyzed currencies exhibit upper tail dependence, therefore the Gumbel function should be used to model their dependence structure.

The research has been conducted in a very general setting. There are some researches that measure dependence structures between three currencies with regards to economic situation, such as introduction of euro or recent financial crisis. A more detailed study can be found in [Azam; Boero, Silvapulle, Tursunalieva 2011].

However, the pair-copula approach permits only to measure this dependence using bivariate copula functions. Therefore, there is a large field for development of the use of three dimensional copula functions for such cases.

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ZASTOSOWANIE TRÓJWYMIAROWYCH FUNKCJI COPULA W ANALIZIE ZALEŻNOŚCI MIĘDZY KURSAMI WALUTOWYMI

Streszczenie: Wiedza dotycząca zależności między dwoma elementami portfela wydaje się niewystarczająca do podejmowania decyzji finansowych. Stąd inwestorzy przejawiają coraz większe zainteresowanie analizą zależności między wieloma składnikami swoich portfeli, co pozwala na lepsze zrozumienie oraz ocenienie sytuacji na rynku. Funkcje copula wydają sie być odpowiednim narzędziem do przeprowadzenia wnikliwej i zrozumiałej analizy dotyczącej zależności między wieloma instrumentami finansowymi. Celem niniejszego artykułu jest zbadanie relacji między trzema kursami walutowymi: USD/PLN, EUR/PLN oraz CZK/PLN. Badanie literaturowe pokazało, że takie analizy są przeprowadzane ale przy użyciu eliptycznych funkcji copula, w szczególności funkcji normalnej oraz t-studenta. Wadą tego podejścia jest grupowanie elementów w pary, co pozwala na dwuwymiarową analizę. Zatem, aby zbadać zależność między n instrumentami, tworzy się $\binom{n}{2}$ par. Podejście wielowymiarowe pozwala na uproszczenie obliczeń oraz prowadzi do bardziej wiarygodnych rezultatów. Jednak, należy zwrócić uwagę, że wielowymiarowe funkcje copula są obecnie przedmiotem wielu badań i można zaproponować inne rozwiązane, mianowicie rozbicie wielowymiarowej funkcji rokładu prawdopodobieństwa na iloczyn brzegowych funkcji gęstości oraz funkcji pair-copula. W tym artykule skupiono się na archimedejskich funkcjach copula, takich jak funkcje Franka, Claytona i Gumbela, które stanowią wprowadzenie do wielowymiarowej analizy zależności między instrumentami finansowymi.

Słowa kluczowe: wielowymiarowy, copula, pair-copula, kursy walutowe.