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DOES MATH EDUCATION IN POLAND TEACH CREATIVE THINKING?

Marek Biernacki

Abstract. This work is a continuation of the article (Biernacki, Czesak 2013), which examined the most recent results (2012) of the Program for International Student Assessment. It turned out that Polish students have significantly improved their results in general subjects, but have great difficulties with creative thinking (the results are below the median). Analysis of the results of examinations in mathematics for students of the first year of two departments confirmed that students have great difficulty with problem solving. The author proposes to encourage students through the use of exercises and labs to take more time to solve tasks that require creative thinking, especially those relating to real-world problems of physics and economics.

Keywords: effectiveness of mathematics education.

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1. Introduction

The reform of the Polish educational system in 1999 restored the 3-year middle school (“gimnazjum”) where students go after 6 years of studying in primary school. It also shortened language training from 4 years to 3 years. The reforms introduced examinations for external students in the final year of each stage of the career development of teachers, and introduced the concept of changing the educational subsidy base amount per student. National external exams were to help in the analysis of the level of education and improve the quality of learning in schools. Unfortunately, as was confirmed by the results of the latest test OECD PISA and especially the Creative Problem Solving Test from 2012, the most difficult work for teachers is to teach students creative thinking. Incidentally, according to the 2012 PISA research ranking “who reported being happy at school”, Polish 15-year-old

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students occupy 7th place from the bottom (only 65% are happy students). The question then becomes why so few good students (5th place in reading and science and 8th in mathematics) are happy in school? Exploring the mathematical and physical human nature should be a basic human desire at any age, especially in one's youth. According to the same study, Polish students have little incentive to learn and do not use the potential of the group (class), which are too large (Biernacki, Ejsmont 2011). This confirms the hypothesis that Polish students are trained to take tests, and this does not bring them satisfaction or happiness. We have been getting better craftsmen, but we also need artists who bring innovation in all areas of life, particularly in IT.

In 1996 the World Bank offered recommendations that were to help the education of countries of the former Eastern bloc and customize the process of training to the requirements of a market economy:

- keep the “old system” of general education;
- develop skills to solve difficult and new problems;
- develop an attitude of innovation;
- learn how to take responsibility for themselves.

These recommendations were to help significantly improve the quality of human capital in those countries and also in Poland.

In 2009 the Council of the European Union introduced the strategic objectives for European cooperation in education and training:

1. Implementation of the concept of lifelong learning and mobility.
2. Improving the quality and effectiveness of education and training.
3. The promotion of equality, social cohesion and active citizenship.
4. Enhancing creativity and innovation, including entrepreneurship, at all levels of education and training.

Unfortunately, the last three recommendations of the World Bank and the purpose of the Fourth Council of the European Union, have not been implemented well in Poland which is shown in the results of the Creative Problem Solving Test. Hence, Poland's high human potential measured by the number of years of schooling with a large percentage of adults having at least a medium-level education, and very good results in standardized tests do not translate into an increase in GDP per capita (Biernacki, Łyko 2010).

The work is a continuation of the analysis made in the article *Is the Education in Poland Really Effective?* (Biernacki, Czesak-Woytala 2013), in which it was found that Polish students obtained much better results in the last PISA edition in math, science and reading when compared to previous editions and the average results calculated for the OECD countries.

Despite this fact, the scores of Polish students in Creative Problem Solving tests are much worse than the average for the OECD countries. In order to verify the claim that Polish students are poor at solving problems that require creative thinking, a faculty at Wroclaw University proposed an exam in mathematics to use by NE and ZIF, two tasks that require students to think creatively. In the first group (NE) of 101 people there were 62 (61%) students who passed the matriculation examination in mathematics only at basic level. In the second group (ZIF) there were 208 students of which 75% (157) graduated in mathematics at extended level. On the basis of Programmatic (as decided by MEN), out of the subject matter of Mathematics scope the extended contents are presented below, where there is no basic items: strings and string boundaries, the concept of the derivative of its interpretation of geometrical and physical, and the use of derivatives to solve simple, practical problems.

2. Results of Polish students in PISA

In the latest PISA exam which was conducted in 2012, the results of Polish students improved significantly compared to previous editions of the exam. This is certainly an important achievement for both students and teachers.

In every edition of the PISA exam the results of the Polish students have continued to improve. There were 27 OECD countries which took part in the reading PISA exam in 2000 and, among these countries, Poland ranked 22nd. In 2009 Polish students were much better and were ranked thirteenth. In the last edition in 2012, the success of Polish students was spectacular. They took fifth place among all 34 OECD countries. The detailed data are presented in Table 1.

Table 1. Position of Poland in the subsequent editions of PISA exam in every area (among all OECD countries which took part in this survey) when comparing mean scores

Year	Position of Poland in reading	Position of Poland in science	Position of Poland in maths
2000	22	-	-
2003	13	-	21
2006	8	19	21
2009	13	13	19
2012	5	5	8

Source: own study based on the OECD data (Biernacki, Czesak-Woytala 2013).

This success has been strongly publicized in the Polish media and by the Polish government. However, no one mentioned another aspect of the PISA survey: the Creative Problem Solving Test. “The problem-solving assessment in PISA 2012 focuses on students’ general reasoning skills, their ability to regulate problem-solving processes, and their willingness to do so, by confronting students with problems that do not require expert knowledge to solve”. The next step of our analysis concentrates on the results of this domain and their relationship with the results in mathematics.

There were 40 countries which took part in the Creative Problem Solving test. Taking into account the mean score of Polish students of 480, they only ranked 27th. This means that their result was much worse than the average calculated for all OECD countries. Over 25% of Polish students did not achieve even the second level and less than seven percent passed the fifth level. For all 40 countries, the Pearson correlation coefficient between the percentage of the weakest students in mathematics and the percentage of the weakest students in problem solving equals 0.91. An analogous correlation for the percentage of the best students equals 0.82. Both correlations are statistically significant at the 5% significance level. This means that the results in mathematics are strongly correlated with the results in the problem solving test.

What does it look like for countries which are similar to Poland?

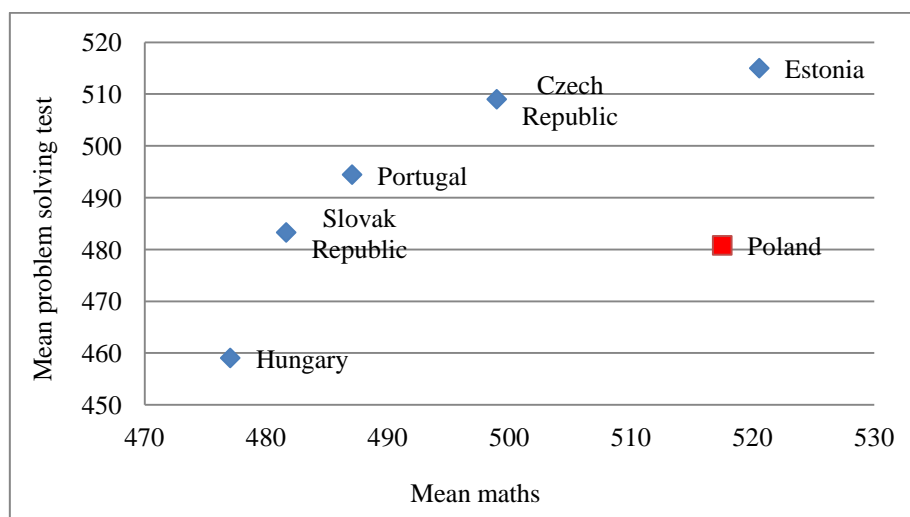


Fig. 1. Relationship between mean results in maths and mean results in the problem solving test

Source: own study based on the OECD data (Biernacki, Czesak-Woytala 2013).

One can easily see that the relationship for Poland is different than the relationships created for Estonia, Hungary, Portugal, the Slovak Republic and the Czech Republic. Despite the fact that Polish students achieved relatively good results in mathematics, they performed rather poorly in the problem solving test.

3. How do students perform in math at the Wrocław University of Economics?

The verification capacity of Polish students to solve problem tasks were this year's (2014) exams in mathematics for the first year students of the Faculties and ZIF in the Wrocław University of Economics.

In the first group (NE) of 101 people there were 62 (61%) students who passed the matriculation examination in mathematics only at basic level. In the second group (ZIF), there were 208 students of which 75% (157) graduated in mathematics at extended level.

The NE Department, despite the fact that most students was just after graduation from high school, spent at the primary Mathematics a total of only 45 hours. Only some elements of the courses: Analysis 1, Analysis 2, Algebra 1 and Algebra 2 are taught during one of their study courses (30 hours of lectures and 15 hours of exercises). In the final exam this year, an assessment of at least good was received by 23 persons (22.8 %). Evaluation of the maximum of sufficient was received by 57 people, including an assessment of inadequate or were failed miserably in the first period by 52 people (51.5%) Compared with last year, the percentage of people with at least a good increased by 4.8 percentage points, and those with insufficient diminished by 1.5 percentage point. Five people with 101 (5%) got "task problem" No 2.

During the exam in mathematics in 2014, the NE students had to face following tasks:

- For the linear transformations given:

$$f(x, y, z) = (-7x + 4y - 2z, 3x - y + z, 4x - 2y + z)$$

and

$$g(x, y, z) = (x - 2y + z, -2x + 4y - 3z, x - y + z)$$

find $f \circ g^{-1}(x, y, z)$.

- Determine the dimensions of a cuboid with the largest volume among cuboids, the sum of the lengths of the edges is equal to 12.
- Find the monotonicity ranges, extremes, features, local bands, bulge, and points of inflection of the function: $f \circ g^{-1}(x, y, z)$.
- Using the differentials and row calculate the approximate value of the expression: $\ln(\sqrt{1,03} + \sqrt[3]{0,98} - 1)$.
- Examine the linear independence of vectors: $\{[1, 1, 2, 3], [-1, 1, 3, 2], [-1, 5, 4, 3], [1, 0, -1, 3]\}$.
- Calculate an integral double normal area D :

$$\iint_D (x^2 - xy) dx dy,$$

where: $D: y = \sqrt{x} \wedge y = x^2$.

The scope of material during the annual course of mathematics for students of the Faculty of ZIF was Analysis 1, Analysis 2, Algebra 1 and Algebra 2. This material was intended for 105 hours (60 lectures + 45 exercise classes). Of 208 students, the score of at least a good plus was received by 25 students (12.5%) or at least a percentage of the students solved the quest “problem” (No. 1 and no. 2). The students who rated more than adequate (less than 60% of the possible points) were 80, which is 38.5%.

Students of the Faculty of ZIF in the exam had the following tasks:

- Linear transformation S with R^3 in R^3 is the submission of two linear transformations T that is in symmetry with respect to the plane of the OXY and the transformation of L :

$$L(x, y, z) = (3x - 2y, 2x - 3y, 4z).$$

Find the proper subspaces of invariant S .

- Designate the dimensions of a cuboid with the largest volume among cuboids, the sum of the lengths of the edges is equal to 12.
- Appoint a reversion to transform the linear

$$f(x, y, z, t) = (x + y, x + y, z + t - z + t).$$

- Calculate the gradient vector function $f(x, y) = (x^2 + y^2)^{0.5}$ at the point $P(3, 4)$ and then nominate and draw a contour passing through the point and vector gradient.

- Find the local extreme of the function

$$\iint_D \sin(x+y) dx dy,$$

where:

$$D: y=0 \quad \text{and} \quad y=x \quad \text{and} \quad x+y=\pi.$$

For the functions of two variables, most students were able to find the extremes of functions which confirmed the results of task 5. Therefore, it is worth noting that in task 2, which was solved by the smallest number of students, the greatest difficulty was the finding and signing depending on the volume of the cuboid to the length of the edge. Just as in task 1, as far as most students were able to find the proper subspaces of linear transformations invariant and knew how to save in the form of a linear transformation matrix, then the difficulty appeared in finding and saving the image to a vector in \mathbb{R}^3 formal symmetry with respect to the plane of the OXY.

4. Conclusions

The percentage of Polish students who best solved tasks in the Creative Problem Solving Test was 7%. Because our students are two years older than the student examinees in PISA (15-year-olds in 2012), I hypothesize that, in Poland, the percentage of students who think creatively diminishes with time. It is worth noting that in the tasks similar to task 2, ten years ago students of these faculties in the exams and homework resolved them without a problem.

Hence, the natural question is how, “on the occasion” of the teaching of mathematics covering such a wide range of material in a limited number of hours, to teach students creative thinking. One of the proposals is placing on the practice problem lists unusual and multi-stage tasks. I tried during lectures to put economic problems to the University students related to the application of mathematical tools in economics, but it turned out that due to their “little knowledge of economics” this is time consuming and the interest is negligible (!). Another proposal is to use mathematical programs starting with Excel through Matlab to Mathematica. You can give students real problems to solve, for example, physics or economics, in which they should solve the problem in mathematical form, and then using the programs, look for solutions. Issues for students with the solutions in Matlab can be found for example in the book (Łyko, Maciuk 2013), or on the website: www.mathworks.com/classroom.

References

- Biernacki M., Czesak-Woytala K. (2013). *Is the education in Poland really effective?* Didactics of Mathematics. No 10, pp. 5-18.
- Biernacki M., Ejsmont W. (2011). *Efektywność kształcenia we wrocławskich liceach*. Acta Universitatis Lodziensis. Folia Oeconomica. No 253.
- Biernacki M., Łyko J.(2010). *Economic Growth, Social Wealth and Investing In Human Capital*. Social Research. No 4, pp. 5-11.
- Łyko J., Maciuk A. (2013). *Algebra liniowa dla ekonomistów. Ćwiczenia z programem Matlab*. Wydawnictwo uniwersytetu Ekonomicznego we Wrocławiu.
- OECD. <http://www.oecd.org/pisa/keyfindings/pisa-2012-results-overview.pdf>
- World Bank (1996). *World Bank – World Development Report: From Plan to Market*. Oxford University Press. New York.
- www.mathworks.com/classroom.