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**A MODERN TOOL FOR A MODERN STUDENT.  
VIDEO GAMES IN THE EXPLORATION  
AND LEARNING OF MATHEMATICS**

**Izabela Solarz**

**Abstract.** Solving equations is one of the central topics in mathematics curricula. Teachers would like to pay more attention to building formulas and formulating algebraic models because modern students dare to translate a problem situation into algebra, but very often their students are helpless because of the lack of practice. There is not much time for exercises. The problem is how to help students with acquiring algebraic skills, like transforming expressions and solving equations. There is a new medium in education that can be used for this – video games. Could they help? I designed a special learning arrangement using a special game that was built for learning algebra. The results suggest that This can be a really relevant didactic medium for teaching mathematics.

**Keywords:** algebraic skills, unintentional learning, game.

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### **1. Introduction**

What do we think about students these days? Are they really different than some years ago? I think they are the same in a way. They need a goal and some kind of activity that would help them to reach it. They are different because they live in a different world, they have different experiences, other things are interesting for them and something else is important, and this is the reason why our “old fashioned” teaching does not fit into students’ new possibilities. The best we can do – as teachers – is to enrich the good, old, tried and tested methods with something new, fresh and incentive.

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**Izabela Solarz**

Institute of Mathematics, Pedagogical University of Cracow, ul. Podchorążych 2, 30-084 Kraków, Poland.

E-mail: balt37@interia.pl

The question discussed in this paper is if a suitably designed computer game may encourage 12 year old children to enter the world of algebra, in particular – a game that concerns solving linear equations with one unknown. What algebraic obstacles do students overcome using the game? How do they become accustomed to symbols? How can they transfer the rules of the game to paper-and-pencil work? If we compare students who played the game with others – do they make fewer mistakes? What kind of mistakes are less frequent?

I designed and made an experiment. A group of 12 year old children were playing **DragonBoxAlgebra5+** (<http://dragonboxapp.com/>) before they got to know anything about solving equations. The students have already looked for solutions of  $x + b = c$  or  $ax = c$ , but only by guessing or finding the inverse operation. Starting the game, they did not know what it was about or that it was somehow connected with mathematics.

## 2. Rules of the game

The game starts with the presentation of the table, divided into two parts and different cards. There is one particular card between them – a blinking box. The main principle says: “In order to win you must isolate the box on one side”. Students follow the rules that say what move is needed to get rid of the useless cards: what to do if the cards are scattered, if they are stuck together, or if one is below another. One of the first rules says: “You can add the card from the deck”. From now on the student always gets this information with a leaping picture on the deck of the board. He/she cannot make the next move until he/she places the same card on the other side.

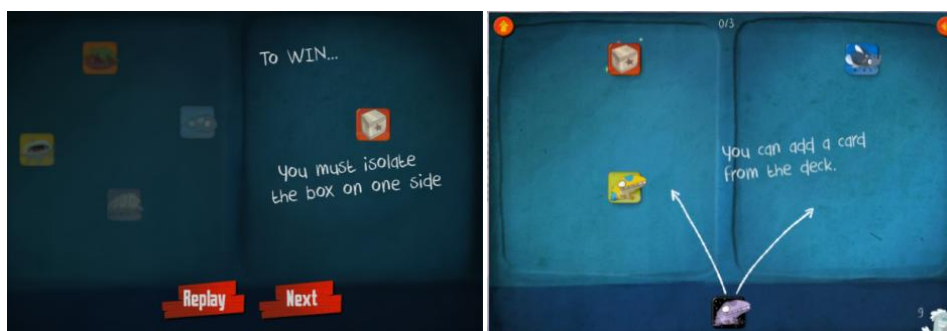


Fig. 1. Rules of the game

Source: own work.

The next principles come in slowly and they are used in several examples before anything new is introduced. Having solved an equation the student gets feedback. It covers three points and gives three stars as an award: the first one – for leaving the box alone, the second one – for the right number of moves, and the third one – for the right number of cards used. You can always move back. You can also solve your task again from the beginning. There is no timing in the game. Students have as much time as they need to finish.

The game starts from replacing colour icons but slowly the pictures are replaced by cards with numbers and letters. Soon the “blinking box“ is transformed to the card with “x”. On one of the last levels the signs of arithmetic rules appear and the line dividing the board is replaced by the equals sign. The change from various cards to letters and the initiation of using mathematical signs is barely noticeable, it is only the new kind of pictures on which students have already made moves according to the well-known rules. However, the manipulation of symbols starts to look like solving equations.

### **3. Learning arrangement**

#### **3.1. Discovering**

I carried out the plan of special learning arrangement in a group of 12 year old children. There were 20 students in the class. At first the pupils were playing at school using the interactive white board, but very soon most of them bought the game and got it on their own tablets or smart phones. During the first three lessons they had the opportunity to play, discuss every example and look for the best strategy. They often moved back some steps, solving the task from the beginning several times. Some students cooperated, others worked without any help. Everybody wanted to take part in it and were really involved in the game. I acted rather like an observer – sometimes I helped students to understand the English commands. I did not interrupt nor make suggestions.

I listened to my students and watched what they discovered. How did they join the pictures with the numbers and the moves with the arithmetic rules? How did they notice that bright and dark icons were like the opposite numbers and what did their disappearing after being moved one upon the other mean? If the same pictures were situated one below the other you could get rid of them by dragging one to its twin. It looked for children like reducing a fraction. Some cards were stuck. It was sufficient to put one of

such cards below that pair to make a fraction. Then one could use a preceding rule and divide the numbers. The students were not surprised when the pictures started to gather in groups joined with the sign of addition, and in the place of line dividing the board the sign of an equation appeared. Before this change there was not anything meaningful in that the right side is equal to the left one. Students only linked it with the necessity of adding everything to both sides of the board. This rule is stressed all the time – you can do nothing if the move is not repeated.



Fig. 2. Playing on the board

Source: own work.

In more difficult examples, the students thought about the order of the moves. What was more effective – to start from putting a card below another one – then you had to do the same to all groups of cards; or maybe it is better to get rid of the useless cards from the side where the “ $x$ ” card was blinking? Sometimes they took away the pictures from the side without the box, but then the opposite cards appeared on the other side. They noticed that it is very important to see where the box is at the beginning to decide about the order of moves.

The students made a lot of mistakes and moved back many times. I can say that they learned by their mistakes. Sometimes they solved the problems together or they worked in pairs so there was an opportunity to argue a lot. I did not interfere. After getting feedback about the stars they knew what was wrong – if it was correct, if the order was the best and if there were no useless cards. The students did not want me to help – they preferred to look for their mistakes by themselves. The game gave the children a lot of pleasure. They enjoyed the mathematics lessons. They also played after the lessons, during the breaks. They discussed and compared their achievements.

### 3.2. Creativity – “paper-and-pencil work”

After such an amusing introduction by playing I said that it was time for transferring this to paper-and-pencil work. We started from the easiest equations. I encouraged the students to transfer the rules of the game to code all the needed operations in such a way that it would be legible and understandable. The students created the notation for adding cards from the deck, moving one on the other and dragging cards. Their ideas were very rich. We chose the best solutions.

The first problem was how to write down the addition of cards to both sides. There was no possibility to drag a picture with a finger. The students added the numbers, writing them on the border sides of the equation. They drew the arrows to code the addition and they wrote the results below them. Putting a number below another one they added the fraction line, or they stuck on the number to the number to get rid of a fraction. Sometimes it was clear in the writing that a student only placed numbers next to each other – like in the game – and after a while he/she put the multiplication sign. Every new operation was written down using a different color to mark it off. They used game language – “I drag on top, I place below...”.

The first examples were very simple. They consisted of one operation only. The students have already solved such equations, but only by guessing or finding the inverse operation. I encouraged them to compare what they did before with the rules of DragonBox. I often asked the question: which way is easier for you in a definite task – doing operations on both sides or rather finding an inverse operation? The students decided what to do differently in different situations. I think they often tended to forget the methods that they learned earlier – trying to do everything in one way, without the reflection that it is possible to do it differently.

During such a transfer from the game to paper-and-pencil work it is very important to pay attention to the meaning of the expression “solution of the equation” and to verify its correctness. The students do not get the feedback yet. The only way to check the solution is to substitute and calculate.



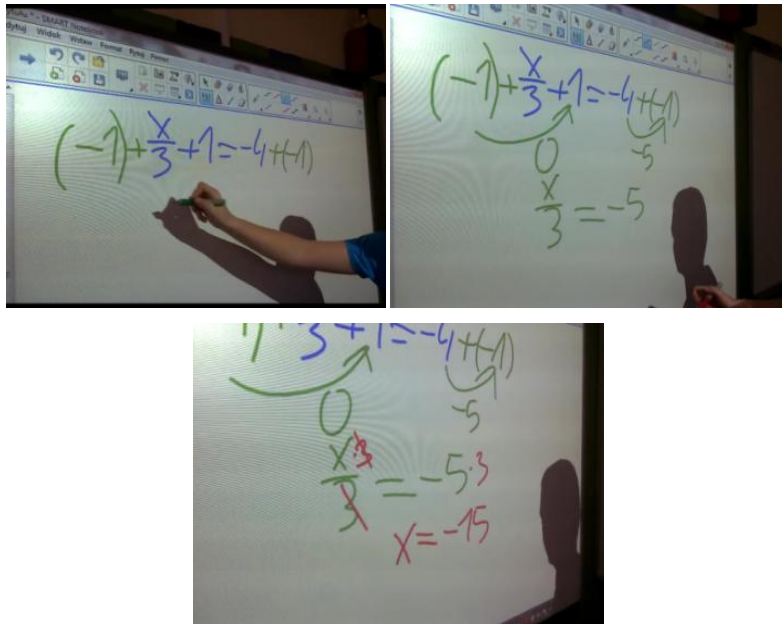


Fig. 3. Paper-and-pencil work

Source: own work.

The next problem was how to remember about repeating every operation on both sides. The game reminds the players about it from the first to the last level. The added card is leaping on the deck as long as the move is repeated on the other side to every group of cards. Now there is no reminder. The children agreed that using colored pencils helps them to remember about both sides. They often checked their solutions and compared them. I posted up the incorrect answers and they were looking for the mistakes. Very soon the students expected some kinds of errors so they were looking for special ones. Some students started to abandon writing single operations but more often this led to the wrong solution. Taking the notes of every transition gave them more chances that they would succeed.

Very slowly, students started to use the specific algebraic language in place of the language of the game, but they could always adduce to DragonBox; particularly in more difficult, doubtful situations, coming back to the game helped them to make the proper decision.

### 3.3. Creativity – “new types of equations”

The first part of DragonBox5+ is just an introduction to solving linear equations with one unknown. There are only equations with an unknown on one side of the equality sign.

However, it can give opportunity to conduct new research, to encourage students to transfer the rules to new equations, for example:  $2x + 5 = x + 6$ . The 12 year old students already knew how to add expressions like  $2x + 3x$ . In the game they got used to see  $(-x)$  like a dark version of  $x$ , so it was nothing new to add  $(-x)$  if it was necessary. The game also makes children familiar with leaving  $x$  on any side – it does not matter which one. The aim was the same: to isolate  $x$  on one side. The children asked their own questions too, for example: which side would be better? Very soon they decided that it was more safe to add  $(-x)$  to both sides than to add  $(-2x)$  because of the positive result on the left side.

It appeared that the new type of equation was not so hard, the students just applied their earlier experiences and it was like creating the next levels of the game.

## 4. Result of the experiment

A year later I tested a group of pupils from the point of view of solving linear equations with one unknown. This was a group of 100 students from five different classes, taught by three different teachers. There were 20 pupils among them that had played DragonBoxAlgebra5+ and then continued with DragonBoxAlgebra12+. The others were taught in the traditional way – using the balance metaphor.

The students had to solve a few equations – with the unknown on one side only or with the unknown on both sides. There were also different coefficients – integer and fractional. Some of the equations needed only a single operation and in others the students had to transform the expressions.

Testing the students had two main goals – the first one was to answer the question: “what kind of mistakes did the students make?” The second one was to compare the students using the game with others. Did they make fewer mistakes? Which mistakes were less frequent?

## 5. Comparison

- Group E – (experimental group – students using the game),
- Group T – (traditional group – students who did not play the game).

Table 1. Results of the comparison

		Group E	Group T
1	Operations on both sides	8%	19%
2	Mistakes like: $4x - x = 4$	0%	5%
3	Incomprehension of the algebraic structure	0%	6%
4	Students did not start on a task	2%	7%
5	Troubles with the sign of equation	0%	4%
6	Creating “quasi methods”	2%	5%

Source: own study.

The experimental students less frequently forgot about both sides of the equations. They did not mistake the unknown with numbers. The misconception of the algebraic structure was also less frequent. Group T did not start on the task more often and had more troubles with the use of the equation sign. They also created their own – wrong – “quasi-methods” more often.

### 5.1. How could playing the game influence the results?

In my opinion playing such a game can encourage children to solve equations because:

- From the beginning the students get used to distinguish between the “blinking box” and the other cards, so then they do not mistake the unknown with numbers.
- It is impossible to make any move if you do not repeat the operation on both sides of the board – the main rule in balance metaphor is also the main one in the game.
- The children solve the same problem many times looking for the best solutions so they get to know the algebraic structure of expressions, considering the order of the moves.
  - The box can be situated on each side – it does not matter which one.
  - They discover the rules and create the notation by themselves so they remember it all better.
- The students are not afraid of symbols and operations, they use them, try them. They are not afraid of failure so they want to start on every task.

- When the students solve equations, they can call up the rules of the game at any moment to remember which move was necessary in such a situation.

## 6. Conclusions. Plans for future research

A well-built computer game used as a tool for discovering mathematics is obviously a relevant didactic medium. It encourages students to learn. It is also very close to students' interests as most of them spend many hours playing different games. I think it is worth taking advantage of this situation. Firstly – the game shows something new, secondly – the game provides incentives for the player to keep practicing. Solving equations seems to be boring and sometimes never-ending for many pupils, but as we know it is essential to be able to use algebra in the future.

I also agree with Keith Devlin who said that games cannot be the only way that students should learn mathematics. They can be a powerful supplement to other forms of teaching, because they are ideally suited for learning basic mathematics. But they cannot replace a good textbook, cannot replace the teacher first of all (Devlin 2011).

There are other good sources of mediums that I use to teach algebra – for example:

- special kinds of blocks:  
<http://www.mathedpage.org/manipulatives/alhs/alhs-0.pdf>,
- or well prepared computer applications:  
<http://www.fisme.science.uu.nl/wisweb/en/welcome.html>.

If children have more varied models to learn it is easier for them to cross the following algebraic thresholds.

The results of testing showed me some problems with the understanding of the concepts of a “solution” or a “set of solutions”. Sometimes students have no idea how to finish solving the equation  $2x = 5x$  or  $0 = 3x$  or  $2x = 2x + 1$ . They have problems with answering the question: what numbers are the solutions? And their answers suggest a lot of misconceptions. I am looking for the reason for such trouble. I think it is a good idea to use special blocks for it. I mean Lab Gear – special manipulatively designed to model algebra concepts (<http://www.mathedpage.org/manipulatives/alhs/alhs-0.pdf>). In the future I am also going to investigate its use for solving more equations with two variables, for example, and for inequalities.

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