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**APPLICATION OF ADAPTIVE METHODS  
IN DEMOGRAPHIC VARIABLES FORECASTING**

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**WYKORZYSTANIE METOD ADAPTACYJNYCH  
DO PROGNOZOWANIA ZMIENNYCH  
DEMOGRAFICZNYCH**

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**Summary:** The research paper is focused on the assessment of the usefulness of adaptive methods in forecasting demographic variables. The goal of the paper is to conduct the retro and prospective analysis of selected demographic values in the sphere of changes in time, and also to indicate an efficient method for the forecasting of the studied values in subsequent periods. The time series for Poland for the period between 2000 and 2013 are the basis for the development of the forecast. Mean squared errors of ex post forecasts are used as forecast quality measures. The results of the study show that among the applied methods of forecasting, the method of creeping trend with harmonic weights is the most suitable as it gives the smallest forecast errors.

**Keywords:** forecasting, demographic variables, adaptive methods, forecast error.

**Streszczenie:** Niniejsze opracowanie poświęcone jest ocenie przydatności metod adaptacyjnych do prognozowania zmiennych demograficznych. Celem artykułu jest analiza retro i prospektywna w zakresie zmian w czasie wybranych wielkości demograficznych oraz wskazanie skutecznej metody prognozowania badanych wielkości na kolejne okresy. Podstawą sporządzenia prognozy były szeregi czasowe dla Polski z okresu 2000-2013. Jako miary jakości prognoz wykorzystano błędy średniokwadratowe prognoz wygasłych. Rezultaty badań wskazują, że spośród zastosowanych metod prognozowania za najlepszą, tzn. dającą najmniejsze błędy prognoz, należy uznać metodę trendu pełzającego z wagami harmonicznymi.

**Słowa kluczowe:** prognozowanie, zmienne demograficzne, metody adaptacyjne, błąd prognozy.

## 1. Introduction

This paper is focused on the assessment of the usefulness of adaptive methods in demographic processes forecasting. As is generally known, adaptive methods do not need a decision of a prognostic as for the form of the trend function; furthermore this function and its parameters can vary with the flow of time. The selection of these methods in forecasting allows for taking into constant consideration newly occurring information and thus, for the implementation of the idea of sequential proceeding<sup>1</sup>.

The goal of the paper is to conduct a retro and prospective analysis in the sphere of the changes of selected demographic values over time in Poland and to indicate an efficient forecasting method for the studied values in subsequent periods. The paper includes an empirical illustration of the forecasts formulated on the basis of the suggested methods and also a comparison of their results.

Three adaptive methods have been applied for the development of the forecasts. They are the method of creeping trend with harmonic weights, Holt's method and Brown's method. It is obvious that the very method of forecasting provides good results for some phenomena and for some others it does not. Thus it has been decided that each of the phenomena should be forecasted with the use of all the methods, and then the method that offers the most accurate forecast for a particular phenomena should be selected. Due to the fact that in the studied time series there occurs a systematic component in the form of trend and incidental fluctuations, trend models are also applied. Thereby it is possible to perform a comparative analysis of the results of the forecasts obtained with the use of four forecasting methods.

If the development trend and incidental fluctuations occur in time series, adaptive models, including Holt's model and the model of creeping trend as well as the aforementioned trend models, can be applied for forecasting. Selecting Brown's method may raise doubts. It is generally known that a simple model of exponential soothing is applied when a systematic component in the form of the stable level of the forecasted variable occurs in time series. This method is intentionally used in the research to show that in the type of series analysed in the study, this method is not applied due to serious projection errors. It ought to be remembered that when developing projections on the basis of this method, the forecaster assumes the principle of status quo, a passive attitude towards the projected phenomenon.

Errors of *ex post* forecasts are used for the assessment of forecast admissibility. Mean squared forecast error is calculated *ex post* in verification intervals. The presented forecast is considered admissible if the error is not higher than 5%.

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<sup>1</sup> A description of adaptation methods can be found in papers by: [Zeliaś, Pawelek, Wanat 2004; Radzikowska 2001; Snarska 2005].

## 2. Research material – short description

The study includes eight demographic variables that characterise both the condition and structure of population of Poland by age, as well as the demographic processes that significantly influence population growth, and thereby they determine the process of its reproduction. The time series from the period of 2000 to 2013 are the basis for the forecast preparation. The values that were taken into consideration included the population as of 31 December, the proportion of people aged 0-4, 15-64 and 65 and older in total population, raw birth rate, fertility rate, infant mortality rate and standardised<sup>2</sup> total mortality rate.

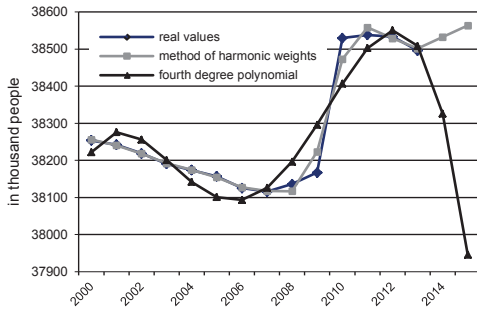
Analysing the population situation in Poland, changes of a multidirectional nature can be observed (Figure 1a). After the period of decline which occurred between 2000 and 2007, the population of Poland slightly increased, but as recently as in the last two years a decline in its number was observed again. As a result of favourable changes reported between 2008 and 2011, in the last year of the analysis, a population growth by 242 thousand people was observed in comparison to 2000. In a relative approach this gives a mean annual growth of 0.5%. With a population of 38,496 thousand in 2013, Poland was placed sixth in the European Union after Germany, France, Great Britain, Italy and Spain. The studies conducted by the author in the sphere of the demographic processes in the countries of Central and Eastern Europe (CEE) showed that in the period between 2000 and 2012, out of the fourteen states included in the analysis, only in Poland, Slovakia, Slovenia and the Czech Republic mean annual growth was observed. These favourable changes were significantly influenced by the surplus of births over deaths, which represents positive population growth.

In other countries of CEE a real decline in population could be observed. The largest, over 14%, was reported in the Baltic republics (Latvia, Lithuania), in Bulgaria (a decline of 10.5%) and in Ukraine (7.4%). Unfavourable changes in this sphere were caused both by negative population growth and the decrease in the number of people caused by migration.

According to the population projection developed by the UN (in mean option) in the perspective of 2050, the population potential of the studied group of states will decline from 303 million in 2012 to 254 million in 2050, which represents a decrease by over 16%. In the perspective of 40 years the largest real declines in population are forecasted for Bulgaria (30.7%), Moldova (30.2%) and in Ukraine (26%), while the smallest would be in Slovenia (1.6%) and in Slovakia (7.7%). In the middle of the 21st century Poland is expected to have a population of 34,079 thousand, that is 11% less than in 2012. Out of all the studied countries, population growth in 2050 in

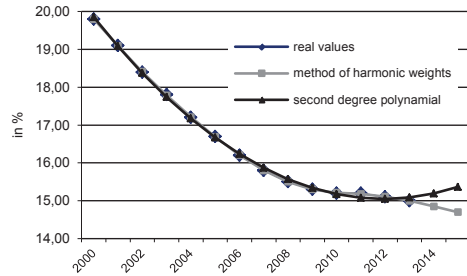
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<sup>2</sup> While calculating the standardised death rate, the model European population structure by age of 2013 was applied. Compare: <http://www.gro-scotland.gov.uk/files2/stats/age-standardised-death-rates-esp/age-standardised-death-ratesesp-background.pdf>.



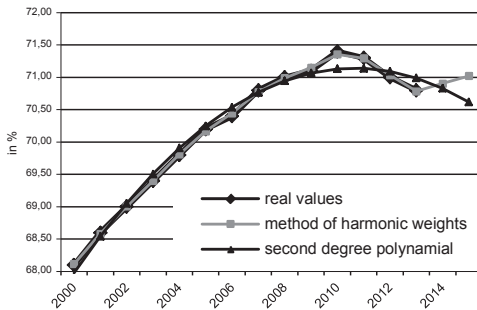
**Figure 1a.** Population of Poland between 2000 and 2015 (as of 31.12).

Source: own case study.



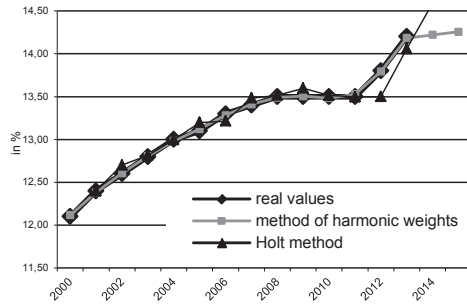
**Figure 1b.** Rate of people aged 0-14

Source: own case study.



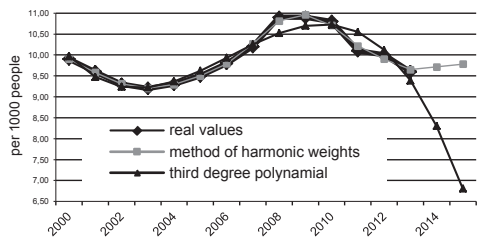
**Figure 1c.** Rate of people aged 15-64

Source: own case study.



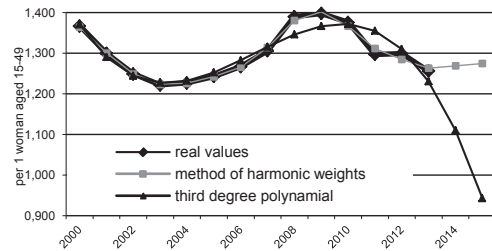
**Figure 1d.** Rate of people aged 65 and older

Source: own case study.



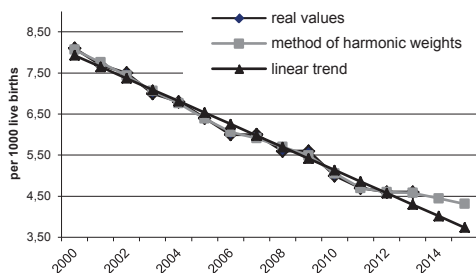
**Figure 1e.** Raw birth rate

Source: own case study.



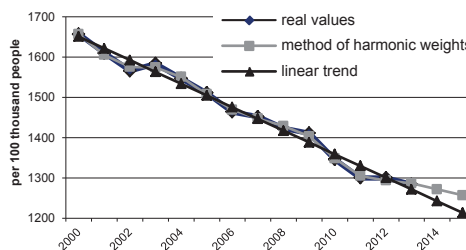
**Figure 1f.** Theoretical fertility rate

Source: own case study.



**Figure 1g.** Infant mortality rate

Source: own case study.



**Figure 1h.** Standardised total death rate

Source: own case study.

comparison with 2012 could only be predicted in the Czech Republic (6.8%) [Sojka 2013, p. 74].

In the studied years the number of people aged 0-14 was constantly decreasing; the share of this group in the total population declined from 19.8% in 2000 to 15% in 2013 (Figure 1b). Both the number as well as the share of children in the total population was constantly falling since the middle of the 1980s; at that time they constituted the quarter of the population and at the beginning of this century, it was less than 20%. On the other hand, a constant growth (by over 2%) of the rate of elderly people was observed. In 2013 it reached the level of 14.2% (Figure 1 d). The demographic perspectives for Poland (also for Slovakia) until 2050 are not optimistic in this respect either, because according to a UN forecast they are going to be the countries that will be getting older the fastest among the group of the states of Central and Eastern Europe (the largest increase in the rate of people aged 60 and older by 16.5% and the largest growth of the age median of 10-11 years).

Population ageing is accompanied, and is going to be accompanied in the nearest future, by one more unfavourable trend, that is a decrease in the share of potential labour resources in the total population (Figure 1c). At the turn of the 21st century, people born during the period of the second population boom (1980s) were replenishing the group of people fit to work. This is why still at the beginning of the decade the growth in the share of people aged 15 to 64 in the total population was observed. However, after the periods of constant growth up to 2010, a decline in this rate was observed. With respect to the determination coefficient, these changes are well described by the following second degree polynomial  $y_t = -0.0251t^2 + 0.6127t + 67.439$ ;  $R^2 = 0.9830$ .

Considering the shaping of the birth rate, which is an important element of population growth, it ought to be stated that in the studied period, changes in this parameter had a varied nature (Figure 1e). After the periods of decline (between 2000 and 2003) there occurred a period of intensified increase in births, but only until 2009 after which, starting from 2010, a decline in the number of births per 1000

inhabitants was observed again. Similar tendencies are evident in the case of the theoretical fertility rate which, as is generally known, since 1980 has not guaranteed the simple replacement of generations. Currently this rate is in Poland at the level of 1.26, which means that there are 126 children born by 100 women of childbearing age (Figure 1f).

Infant mortality has been constantly declining since the 1990s when our country started to reach the level of other states of Western Europe. The mortality rate of infants younger than 1 year old fell to 4.6 per 1000 live births, while in 2000 it was over 8 per million (Figure 1g). Although the level of infant mortality in Poland has been gradually decreasing and the pace of decrease is higher than the mean for the states of the European Union, the rate in Poland is still higher than the mean rate in the European Union (in 2013 – 3.8 per 1000 live births).

Between 2000 and 2013 a constant decline in total mortality was also observed (Figure 1h). The death rate standardised by a European model of age structure decreased in total from 1657 to 1284 deaths per 100 thousand people, which represents a fall by one fifth.

### 3. Presentation and assessment of forecast results

Tables 1 and 2 show the calculated forecasts of previously described demographic values with forecast errors for two subsequent periods of 2014 and 2015. On the other hand, Figures 1a – 1h show the real values of the analysed volumes with the values of ex post forecasts. However, it should be mentioned that every time the forecasting method that provided the lowest forecast errors was applied.

**Table 1.** Forecasts of the number and structure of people by age and assessment of admissibility of predictions

Forecasting method	Period	Forecast	Mean squared error ex post	Relative forecast error in %
1	2	3	4	5
Total population				
Method of harmonic weights	2014	38,532.40	22.647	0.060
	2015	38,563.45		0.060
Holt's method ( $\alpha = 0.95$ ; $\beta = 0.1$ )	2014	38,517.37	111.157	0.290
	2015	38,535.59		0.290
Brown's method ( $\alpha = 0.9$ )	2014	38,499.74	100.884	0.262
	2015	38,576.35		0.262
4 degree polynomial trend	2014	38,325.99	57.389	0.150
	2015	37,945.24		0.151

1	2	3	4	5
Rate of people aged 0-14				
Method of harmonic weights	2014	14.85	0.0087	0.059
	2015	14.70		0.059
Holt's method ( $\alpha = 0.95$ ; $\beta = 0.95$ )	2014	14.90	0.0854	0.573
	2015	14.80		0.577
Brown's method ( $\alpha = 0.9$ )	2014	15.01	0.4658	3.103
	2015	14.39		3.238
2 degree polynomial trend	2014	15.19	0.058	0.382
	2015	15.36		0.378
Rate of people aged 15-64				
Method of harmonic weights	2014	70.90	0.0275	0.039
	2015	71.02		0.039
Holt's method ( $\alpha = 0.8$ ; $\beta = 0.9$ )	2014	70.53	0.1709	0.242
	2015	70.27		0.243
Brown's method ( $\alpha = 0.9$ )	2014	70.82	0.3391	0.479
	2015	71.48		0.474
2 degree polynomial trend	2014	70.83	0.1236	0.174
	2015	70.62		0.175
Rate of people aged 65 and older				
Method of harmonic weights	2014	14.22	0.0154	0.108
	2015	14.26		0.108
Holt's method ( $\alpha = 0.95$ ; $\beta = 0.95$ )	2014	14.59	0.1139	0.781
	2015	14.99		0.760
Brown's method ( $\alpha = 0.9$ )	2014	14.16	0.2078	1.468
	2015	14.03		1.481
Linear trend	2014	14.17	0.1461	1.031
	2015	14.30		1.021

Source: own case study.

**Table 2.** Forecasts of selected demographic rates and assessment of admissibility of predictions

Forecasting methods	Period	Forecast	Mean squared error ex post	Relative forecast error in %
Raw birth rate				
Method of harmonic weights	2014	9.72	0.0608	0.620
	2015	9.78		0.610
Holt's method ( $\alpha = 0.9$ ; $\beta = 0.95$ )	2014	9.28	0.3347	3.560
	2015	8.94		3.980
Brown's method ( $\alpha = 0.9$ )	2014	9.64	0.3615	3.750
	2015	10.04		3.603
3 degree polynomial trend	2014	8.30	0.1916	2.308
	2015	6.81		2.815
Theoretical fertility rate				
Method of harmonic weights	2014	1.269	0.0073	0.575
	2015	1.275		0.572
Holt's method ( $\alpha = 0.95$ ; $\beta = 0.95$ )	2014	1.221	0.0445	3.480
	2015	1.185		3.586
Brown's method ( $\alpha = 0.9$ )	2014	1.260	0.0451	3.581
	2015	1.289		3.500
3 degree polynomial trend	2014	1.110	0.0234	2.111
	2015	0.943		2.486
Infant mortality rate				
Method of harmonic weights	2014	4.45	0.0600	1.348
	2015	4.31		1.392
Holt's method ( $\alpha = 0.2$ ; $\beta = 0.9$ )	2014	4.20	0.1847	4.398
	2015	3.99		4.629
Brown's method ( $\alpha = 0.9$ )	2014	4.60	0.3394	7.377
	2015	4.08		8.325
Linear trend	2014	4.02	0.1505	3.746
	2015	3.74		4.026
Standardised death rate				
Method of harmonic weights	2014	1271.72	5.7496	0.452
	2015	1256.76		0.457
Holt's method ( $\alpha = 0.9$ ; $\beta = 0.2$ )	2014	1254.91	26.4866	2.111
	2015	1227.63		2.158
Brown's method ( $\alpha = 0.9$ )	2014	1285.65	36.982	2.877
	2015	1245.68		2.969
Linear trend	2014	1242.25	15.6807	1.263
	2015	1213.08		1.293

Source: own case study.



The analysis of data from Tables 1 and 2 allows for the statement that the method of creeping trend with harmonic weights is the best to forecast these values. In all cases the forecast errors are small (they were not higher than 1.5%), which means that the calculated forecasts are admissible. The curves of the values of ex post forecasts presented in Figures 1a-1h totally coincide with real values. Therefore the model of creeping trend with harmonic weights can be an efficient tool for forecasting the studied demographic processes. This model is quite flexible and has an adjustment capability both in the case of irregular changes in the direction and the speed of trend and when the course of the phenomena is not stable enough.

It results from the data of the Central Statistical Office that the number of births ceased to grow for the first time in 2010. Between January and June 2014, 186.9 thousand children were born, that is 5.5 thousand more than in the same period of the previous year. The growth may not be impressive but we can expect that this positive trend will be maintained. Currently people in their thirties make the decision about having children. They realise that they cannot wait any longer to be parents. At present, the generation of the second population boom of 1979 to 1984 when there were between 690 thousand to 700 thousand children per year, are in their thirties. In 2014 the situation was even better than in 2013 when in twelve months Polish women gave birth to only 370 thousand children. This favourable trend could be influenced by the introduction of the one year long paternal leave. It might have encouraged some groups of people to make the decision about being a parent. It can also perform such a function in the future. Therefore the forecasts of growth in birth rate and fertility rate with the use of the method of harmonic weight may prove successful.

Observing the shaping of mortality rates (Figure 1g, 1h) it can be noticed that their values have been constantly decreasing since 2000 and according to formulated forecasts this tendency shall remain the same in subsequent years.

Regardless of the forecasting method, in the next few years the share of people aged 65 and older in the total population will be growing, which means that the process of population ageing will be progressing. This is also proved by the results of the latest population projection for 2014-2050 developed by the Central Statistical Office<sup>3</sup>. According to the projection, the share of elderly people in the total population will grow from 15.3% in 2014 to 18.8% in 2020, reaching the level of 32.7 percentage points in the middle of the 21<sup>st</sup> century. This means that every third person in Poland will be aged 65+.

Population ageing is an irreversible process. It has a global character and results mostly from the decrease in the number of children born per woman and the increase in average life expectancy. This process is also affected by the population explosions and population declines of previous periods and migrations abroad [Frątczak 2002;

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<sup>3</sup> *Prognoza ludności na lata 2014-2050*, [in:] <http://stat.gov.pl/obszary-tematyczne/ludnosc/prognoza-ludnosci/prognoza-ludnosci-na-lata-2014-2050-opracowana-2014-r-1,5.html> (access: 17.08.2015).

Kurek 2008]. This remarkable increase in the number of elderly people brings a lot of challenges to a broadly considered social assistance. In gerontological studies, attention is drawn to the growing demand on assistance services associated with limited mobility and self-service capabilities that are growing, together with the progressing population ageing. As a result of the increase in the ratio of people aged 80+ it can be expected that in the near future the number of elderly people with disabilities will increase. They form one-person households and therefore will need daily support in the future [Sojka 2013a; Kowaleski, Gajdzińska 2012].

According to the forecast prepared with the use of adaptive methods, in the next two years population growth can also be expected (Figure 1a). However, this results from Central Statistical Office data which show that 2014 was the next, third successive year, in which a decline in population by around 17 thousand people was reported. At the end of 2014 the population of Poland was 38,478.6 thousand. Comparing this value with the obtained projections, it ought to be stated that the smallest absolute error of *ex post* prediction was obtained while Brown's method was applied (-21.14 thousand), and then, the relative error was only 0.05%. In the case of other methods, relative errors of *ex post* projections were also observed on a very low level which allows for stating that determined projections are admissible.

Unfortunately the population projection developed by the Central Statistical Office for 2014, according to which the population was predicted to reach the level of 38 461.8 thousand also did not come true, even though the absolute error of *ex post* projection was slightly smaller and amounted to 16.8 thousand people.

#### 4. Conclusions

Adaptive methods can be an efficient tool for demographic variables data forecasting. Among the applied methods, the method of creeping trend with harmonic weights ought to be considered the best as it gives the smallest forecast errors. Developed population projections show that the population potential in Poland will slightly increase between 2014 and 2015. Annual population growths will be respectively 36.4 thousand people in 2014, and will be 31.05 thousand in 2015 compared with the previous year. However, considering the actual data for 2014 and the population projection of the Central Statistical Office for 2015, according to which the population of Poland will further decline and will reach the level of 38,419 thousand, it ought to be stated that the population projections determined with adaptive methods will not prove to be true. On the other hand, the projections of growth in the birth rate and the rate of children born per woman will prove to be true. In 2014 the birth rate in Poland amounted to 9.7‰ and was by 0.1 per mill point higher in comparison with the previous year, whereas in the same year there were 129 children born per 100 women of reproductive age (aged 15-49) against 126 in 2013.

Quantitative relations between the major age groups of population (0-14, 15-64 and 65+) lets us determine relations between generations existing in society

and assess the level of demographic senility. They also constitute the basis for the calculations of various rates describing the population ageing process.

In the forthcoming years, the process of population ageing, reflected in the decline in the ratio of children in the total population and the increase in the rate of elderly people, will be growing. However, these changes could be insignificant considering the fact that in 2013 the respective rates of the structure were 15% and 14.2%.

A gradual decline in population aged 15-64 and a decrease in the share of this age group in the total population will be the implication of the decline in the number of births observed for several years, as well as the ageing of people who were born during the post war population explosion. They will successively join the group of people aged 65+. This results from the aforementioned population projection by the Central Statistical Office for 2014-2050 that the rate of people aged 15-64 will decrease within the period of the projection horizon from 69.8% to 55.2%.

In the sphere of infant mortality and total mortality the declining tendency observed since 2000 shall remain favourable in the nearest future. Therefore in 2014, according to the forecast, there will be 4.45 deaths of children up to one year old per 1000 live births against 4.6 deaths in 2013. On the other hand, the standardised total death rate would decrease in 2014 by 12 deaths per 100 thousand people as compared with the previous year.

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