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MODELING AND PROJECTING LIFE EXPECTANCY. THE CASE OF THE EU COUNTRIES

MODELOWANIE I PROJEKCJA PRZECIĘTNEGO CZASU TRWANIA ŻYCIA NA PRZYKŁADZIE KRAJÓW UE

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Summary: In this article we investigate the latest developments on life expectancy modeling. We review some mortality projection stochastic models and their assumptions, and assess their impact on projections of future life expectancy for selected countries in the EU. More specifically, using the age- and sex-specific data of 20 countries, we compare the point projection accuracy and bias of six principal component methods for the projection of mortality rates and life expectancy. The six methods are variants and extensions of the Lee-Carter method. Based on one-step projection errors, the Renshaw and Haberman method provides the most accurate point projections of male mortality rates and the method is the least biased. The Quadratic CBD model with the cohort effects method performs the best for female mortality. While all methods rather underestimate variability in mortality rates and life expectancy, the Renshaw and Haberman method is the most accurate.

Keywords: life expectancy, mortality, Lee-Carter models family, stochastic models.

Streszczenie: W artykule poruszamy najważniejsze aspekty z zakresu modelowania przeciętnego trwania życia. Dokonujemy przeglądu wybranych stochastycznych modeli i ich założeń oraz ich wpływu na projekcje przeciętnego dalszego trwania życia dla wybranych krajów UE. Na podstawie danych pochodzących z 20 krajów, w podziale na płeć i wiek, porównujemy obciążenia i dokładność punktowej projekcji wskaźnika umieralności i przeciętnego trwania. Sześć analizowanych modeli należy do rodziny modeli Lee-Cartera. Z analizy wynika, że metoda Renshawa i Habermana zapewnia najbardziej dokładne punktowe projekcje wskaźników umieralności dla mężczyzn i najmniejsze obciążenia. Dla kobiet najmniejsze obciążenia i największą dokładność otrzymujemy w wyniku zastosowania metody QCBD.

Słowa kluczowe: przeciętne trwanie życia, umieralność, rodzina modeli Lee-Cartera, modele stochastyczne.

1. Introduction

With the accelerated aging of the population, life expectancy projection becomes very important, especially for the insurance industry and pension system. Reforms in the pension systems in Europe, which were necessary to ensure pensions remained sustainable, have made the link between pensions and changes in life expectancy more apparent. In general, monthly pension payments are based on remaining life expectancy when people retire. But whereas in some countries benefit levels are linked to life expectancy (Germany, Finland, and Portugal), in others the pension age is set to rise with increasing life expectancy (Denmark, the Netherlands), or the contribution period for pensions is set to be extended as people live longer (France) [OECD 2007]. The accurate modeling and projection of mortality rates and life expectancy are therefore of growing interest to researchers. Lots of projection methods are used, both between and within countries which produce different outcomes.

We review the different mortality projection models and their assumptions, and assess their impact on projections of life expectancy for selected countries in Europe. This study shows that comparing different variants and extensions does not automatically result in the identification of a single best method for all the considered countries.

This article is organized as follows: in Section 2, we briefly describe the life expectancy phenomenon and some facts about modeling life expectancy. Section 3 describes the six mortality projection methods that are included in our comparisons. In Section 4 we describe the data and we compare the point forecast accuracy of the methods. The evaluations include both mortality rates and life expectancy. Conclusions appear in the last section of the paper.

2. Life expectancy

The life expectancy at birth is the average number of years that a newborn baby could expect to live, if he or she were subject to the age-specific mortality rates of a given period.

2.1. Some facts about life expectancy in Europe

Economic development and the improvement in some environmental conditions, improved lifestyles and advances in healthcare and medicine have resulted in the continuous increase in life expectancy at birth throughout all Europe during the last century. This process has been going on for longer in Europe than in most other parts of the world, placing the EU-28 among the world leaders in life expectancy [Eurostat 2015]. Over the past 50 years, life expectancy at birth has increased by about 10 years for both men and women in the EU-28.

Life expectancy at birth in the EU-28 was estimated at 80.6 years in 2013, reaching 83.3 years for women and 77.8 years for men. During more than the decade

between 2002 (the first year for which data are available for all EU Member States) and 2013, life expectancy in the EU-28 increased by 2.9 years, from 77.7 to 80.6 years – the increase was 2.4 years for women and 3.3 years for men [Eurostat 2015].

While life expectancy has risen in all EU Member States, there are still major differences between and within countries. For men, the lowest life expectancy in 2013 was recorded in Lithuania (68.5 years) and the highest in Italy (80.3 years). For women, the range was narrower, from a low of 78.6 years in Bulgaria to a high of 86.1 years in Spain. Between 2003 and 2013, the rise in life expectancy at birth for men in the EU Member States ranged from a minimum of 2.1 years (in Lithuania) to a maximum of 6.4 years (in Estonia). For women, the increase ranged from 1.3 years (in Sweden) to 4.5 years (in Estonia) [Eurostat 2015].

As people live longer, interest has shifted to the older generations. In 2013, once a man had reached the age of 65, he could, on average, expect to live between another 13.9 years (as in Latvia) and 19.3 years (as in France). The life expectancy of women at 65 was higher. In 2013 it ranged from 17.9 years in Bulgaria to 23.6 years in France).

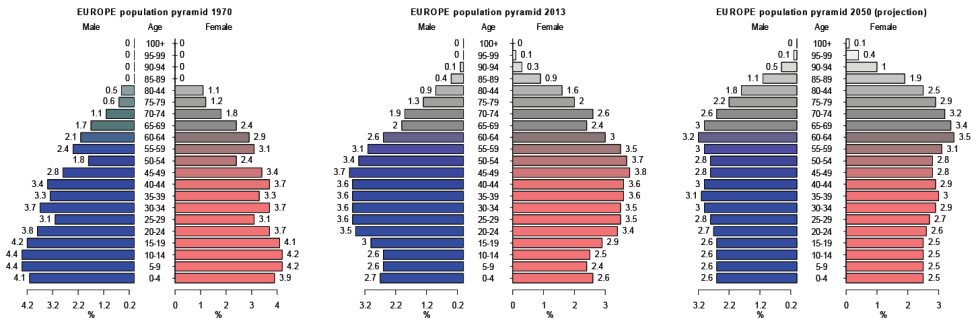

Fig. 1. Population pyramids for Europe: 1970, 2013 and 2050

Source: data from [http://data.worldbank.org] in R project.

Population pyramids (see Figure 1) of Europe show the distribution of the population by gender and by five-year age groups, in 1970, 2013 and a projection for 2050. The share of the population aged 65 and over is increasing in Europe as a whole. On the other hand, the share of the population aged less than 15 has decreased.

2.2. Modeling life expectancy

When looking at life expectancy rate, it is necessary to quantify the level of mortality rates at age x during calendar year t (denoted by $q(x,t)$) and their evolution in time. The close relationship between mortality and longevity modeling is particularly clear

when considering the survival probability. Mathematically, life expectancy appears to be the product of some correlated mortality rates as is underlined by the following expression for the survival probability until date $t + u$ of a person aged x at time t [Barrieu et al. 2012]:

$$S_t(x,T) = \prod_{i=0}^{T-1}\left[1 - q(x+i,t+i)\right].$$

As a consequence, the models described below can be used for both mortality and longevity.

As mortality projections have become increasingly important, numerous models for mortality modeling and projection have been developed (for reviews see [Pollard 1987; Tabeau 2001; Wong-Fupuy, Haberman 2004; Booth, Tickle 2008]). The various methods for mortality projection are divided in the literature into three approaches: extrapolation, explanation, and expectation (see [Booth, Tickle 2008]). The extrapolative approach makes use of the regularity typically found in age patterns and trends in time. The explanation approach makes use of structural or epidemiological models of mortality from certain causes of death for which the key exogenous variables are known and can be measured. The expectation approach is based on the subjective opinions of experts involving varying degrees of formality. It should be noted that some mortality projection methods include aspects of one or more approaches [Barrieu et al. 2012].

In the past most methods were relatively simple and were largely based on subjectivity [Pollard 1987]. Over time more sophisticated methods that make increasing use of standard statistical methods have been developed and applied [Booth, Tickle 2008]. The majority of these methods can be classified as extrapolative approaches, of which the Lee-Carter method is the benchmark stochastic mortality model. One of the strengths of the Lee-Carter method, and of extrapolation methods – in general, is their robustness in situations in which age-specific log mortality rates have linear trends [Booth et al. 2006]. However, some countries have less linear trends.

The Lee-Carter method has also been extended to include a cohort dimension [Renshaw, Haberman 2006] and other stochastic models have been used to include the cohort dimension in mortality projection [Cairns et al. 2011]. Other examples are projection methods using valuable medical knowledge and information on behavioural and environmental changes (e.g. smoking and/or obesity).

The advent of new methods has led to a variety of types of methods being used to produce projections within a single country [Wong-Fupuy, Haberman 2004], which have produced different projection outcomes. Most existing studies that have compared the outcomes of different methods have focused largely on variants within one model, such as the Lee-Carter model and its variants, extensions or generalizations. These include Booth et al. [2002; 2005; 2006], Li and Lee [2005],

Renshaw and Haberman [2006], Hyndman and Ullah [2007], Wang and Liu [2010], Shang, Booth, and Hyndman [2011]. Other studies (e.g. [Cairns et al. 2011]) have compared the Lee-Carter model (and its cohort extension) with other extrapolative statistical models, such as P-splines models and the statistical model CBD.

2.3. Projection methods in Europe

The primary purpose of national projections is to provide an estimate of future population, mortality and life expectancy as a common framework for planning in a number of different fields. The projections are based on the assumptions judged to be the best that could be made at the time they were adopted.

The approaches currently used by statistical offices in Europe to project future mortality vary considerably. Extrapolation methods are used most frequently. These methods are either a direct linear extrapolation of the logarithm of the age-specific mortality rates (Austria, Belgium, France, Spain), or a variant of the Lee-Carter model (Denmark, Italy, Norway, Portugal, Sweden). Ireland, Luxembourg, Poland, and the UK use a more subjective target approach. For Poland and Luxembourg, information on trends in other countries is included directly in the projection. Portugal, France, Ireland, the Netherlands and the UK also include expert opinion in their mortality projections. Indirectly, through the knowledge of the experts, this could include trends in other countries and epidemiological information [Stoeldraijer et al. 2013].

In addition to the differences in the methods used, there are differences in the variants and the extensions employed. Denmark, Italy, Portugal, and Sweden use different variants of the original Lee-Carter method. Norway and Denmark extend the original method. Belgium and Spain extend the direct extrapolation method with a re-estimation after smoothing the age-specific parameter, but use a different period for the re-estimation. Belgium and France both make some adjustments for old-age mortality. Ireland and the UK make a similar assumption about the target value − a constant improvement rate after some year in the future. The UK includes a cohort approach for the convergence because of the apparent cohort effects. Moreover, the historical period used differs considerably by country. There is also variation in the length of the projection period, although this does not seem to correlate with the length of the historical period [Stoeldraijer et al. 2013].

It appears that the observed past trends determine which method and historical period is used. Life expectancy at birth in Western Europe has increased by six to ten years since 1970 [WHO Health Database]. All of the countries in Western Europe have experienced a rise in life expectancy, although at different rates and with periods of stagnation. Countries with a more linear trend (e.g. France) use extrapolation methods with an average historical period, while countries with more non-linear trends (e.g. Denmark, the Netherlands and Norway) use different approaches in order to take non-linearity into account. Denmark, which has a history of having a less linear trend among women in particular, uses an extrapolation method, but

with a short historical period. The Netherlands, which has non-linear trends among both men and women, uses epidemiological information in the projection. Norway, with a period of stagnation in the 1980s among men, uses a very long period but includes a quadratic age effect to account for the non-linearity.

Past life expectancy projections from official sources have generally underestimated the gains in life expectancy at birth. Commentators have argued that as a consequence, governments may have underestimated the potential budgetary impact of ageing populations. Underestimating life expectancy has a significant impact on the solvency of pensions (for probabilities of financial ruin of pensioners in Poland and countries of Central Europe see [Trzpiot, Majewska 2015a; 2015b].

3. Generalised Age-Period-Cohort stochastic mortality models (GAPC family)

In this section we describe some of the stochastic mortality models highlighting how they can be framed within the GAPC family. A GAPC stochastic mortality model is comprised of four components:

1. The random component: the numbers of deaths D_{xt} follow a Poisson or a Binomial distribution.

2. The systematic component: following Hunt and Blake [2014] the effects of age x, calendar year t and year-of-birth (cohort) $c = t - x$ are captured through a predictor η_{xt} given by:

$$\eta_{xt} = \alpha_x + \sum_{i=1}^{N} \beta_x^{(i)} \kappa_t^{(i)} + \beta_x^{(0)} \gamma_{t-x},$$

where

- the term α_x is a static age function capturing the general shape of mortality by age,
- $N \geq 0$ is an integer indicating the number of age-period terms describing the mortality trends, with each time index $\kappa_t^{(i)}$, $i = 1, \ldots, N$, contributing in specifying the mortality trend and $\beta_x^{(i)}$ modulating its effect across ages,
- the term γ_{t-x} accounts for the cohort effect with $\beta_x^{(0)}$ modulating its effect across ages.

In the GAPC family we assume that the period indexes $\kappa_t^{(i)}$, $i = 1, \ldots, N$, and the cohort index γ_{t-x} are stochastic processes. This is the key feature that allows the stochastic projection of GAPC models and thus the generation of probabilistic forecasts of future mortality rates.

3. The link function g associating the random component and the systematic component.

4. The set of parameter constraints: most stochastic mortality models are only identifiable up to a transformation and thus require parameter constraints to ensure unique parameter estimates.

Most stochastic mortality models proposed in the literature belong to the GAPC family.

3.1. Lee-Carter model under a Poisson setting

The Lee-Carter model, as implemented by Brouhns et al. [2002], assumes a Poisson distribution of the deaths using a log link function to target the force of mortality μ_{xt}. The predictor structure proposed by Lee and Carter [1992] assumes that there is a static age function, α_x, a unique non-parametric age-period term ($N = 1$), and no cohort effect. Thus, the predictor is given by:

$$\eta_{xt} = \alpha_x + \beta_x^{(1)}\kappa_t^{(1)}.$$

In order to project mortality, the time index $\kappa_t^{(1)}$ is modeled and projectioned using ARIMA processes. Typically, a random walk with drift has been shown to provide a reasonable fit. To ensure identifiability of the model, Lee and Carter [1992] suggest the following set of parameter constraints

$$\sum_x \beta_x^{(1)} = 1, \ \sum_t \kappa_t^{(1)} = 0.$$

3.2. Renshaw and Haberman model: Lee-Carter with cohort effects

Renshaw and Haberman [2006] generalise the Lee-Carter model by incorporating a cohort effect to obtain the predictor:

$$\eta_{xt} = \alpha_x + \beta_x^{(1)}\kappa_t^{(1)} + \beta_x^{(0)}\gamma_{t-x}.$$

Mortality projections for this model are derived using time series projection of the estimated $\kappa_t^{(i)}$ and γ_{t-x}, generated using univariate ARIMA processes under the assumption of independence between the period and the cohort effects. To estimate the model, Renshaw and Haberman [2006] assume a Poisson distribution of deaths (random component) and use a log link function targeting the force of mortality μ_{xt}.

Identifiability of the model can be ensured using the following set of parameter constraints:

$$\sum_x \beta_x^{(1)} = 1, \ \sum_t \kappa_t^{(1)} = 0, \ \sum_x \beta_x^{(0)} = 1, \ \sum_{c=t_1-x_k}^{t_n-x_1} \gamma_c = 0.$$

3.3. Age-Period-Cohort (APC) model

Another commonly used substructure of the Renshaw and Haberman model is the Age-Period-Cohort model, corresponding to $\beta_x^{(1)} = 1, \ \beta_x^{(0)} = 1$

$$\eta_{xt} = \alpha_x + \kappa_t^{(1)} - \gamma_{t-x}$$

which has a long-standing tradition in the fields of medicine and demography (see, e.g. [Clayton, Schifflers 1987; Hobcraft et al. 1982]), but has not been widely used in the actuarial literature until it was considered by Currie [2006].

We can ensure identifiability of the model by imposing the set of constraints:

$$\sum_t \kappa_t^{(1)} = 0, \quad \sum_{c=t_1-x_k}^{t_n-x_1} \gamma_c = 0, \quad \sum_{c=t_1-x_k}^{t_n-x_1} c\gamma_c = 0,$$

where the last two constraints imply that the cohort effect fluctuates around zero with no discernible linear trend.

3.4. Cairns, Blake and Dowd (CBD) model

Cairns et al. [2006] propose a predictor structure with two age-period terms ($N = 2$) with pre-specified age-modulating parameters $\beta_x^{(1)} = 1$ and $\beta_x^{(2)} = x - \bar{x}$, no static age function and no cohort effect. Thus, the predictor of the CBD model is given by:

$$\eta_{xt} = \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)},$$

where $\bar{x}$ is the average age in the data. Cairns et al. [2006] obtain mortality projections by projecting the period effects $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ using a bivariate random walk with drift.

The CBD model does not have identifiability issues and hence the set of parameter constraints is empty.

3.5. Quadratic CBD (QCBD) model with cohort effects

Cairns et al. [2009] extend the original CBD model by adding a cohort effect and a quadratic age effect to obtain the predictor:

$$\eta_{xt} = \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)} + ((x - \bar{x})^2 - \hat{\sigma}_x^2)\kappa_t^{(3)} + \gamma_{t-x},$$

where $\hat{\sigma}_x^2$ is the average value of $(x - \bar{x})^2$.

To identify the model Cairns et al. [2009] impose the set of constraints:

$$\sum_{c=t_1-x_k}^{t_n-x_1} \gamma_c = 0, \quad \sum_{c=t_1-x_k}^{t_n-x_1} c\gamma_c = 0, \quad \sum_{c=t_1-x_k}^{t_n-x_1} c^2\gamma_c = 0,$$

which ensure that the cohort effect fluctuates around zero and has no discernible linear or quadratic trend.

3.6. Plat model

Plat [2009] combines the CBD model with some features of the Lee-Carter model to produce a model that is suitable for all age ranges and captures the cohort effect. The proposed predictor structure assumes that there is a static age function, α_x, three age-period terms ($N = 3$) with pre-specified age-modulating parameters $\beta_x^{(1)} = 1$ and $\beta_x^{(2)} = x - \bar{x}$, $\beta_x^{(3)} = (\bar{x} - x)^+ = \max(0; \bar{x} - x)$ and a cohort effect with pre-specified age-modulating x parameters $\beta_x^{(0)} = 1$. The predictor is given by:

$$\eta_{xt} = \alpha_x + \kappa_t^{(1)} + (\bar{x} - x)\kappa_t^{(2)} + (\bar{x} - x)^+ \kappa_t^{(3)} + \gamma_{t-x}.$$

Plat [2009] targets the force of mortality μ_{xt} with the log link and estimates the parameters of the model by assuming a Poisson distribution of the deaths. The following set of parameter constraints can be imposed to ensure identifiability:

$$\sum_t \kappa_t^{(1)} = 0,\ \sum_t \kappa_t^{(2)} = 0,\ \sum_t \kappa_t^{(3)} = 0,\ \sum_{c=t_1-x_k}^{t_n-x_1} \gamma_c = 0,\ \sum_{c=t_1-x_k}^{t_n-x_1} c\gamma_c = 0,\ \sum_{c=t_1-x_k}^{t_n-x_1} c^2\gamma_c = 0.$$

Table 1. Model structures considered in this paper

Model	Predictor
LC	$\eta_{xt} = \alpha_x + \beta_x^{(1)}\kappa_t^{(1)}$
CBD	$\eta_{xt} = \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)}$
APC	$\eta_{xt} = \alpha_x + \kappa_t^{(1)} - \gamma_{t-x}$
RH	$\eta_{xt} = \alpha_x + \beta_x^{(1)}\kappa_t^{(1)} + \gamma_{t-x}$
QBCD	$\eta_{xt} = \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)} + ((x - \bar{x})^2 - \hat{\sigma}_x^2)\kappa_t^{(3)} + \gamma_{t-x}$
PLAT	$\eta_{xt} = \alpha_x + \kappa_t^{(1)} + (\bar{x} - x)\kappa_t^{(2)} + \gamma_{t-x}$

Source: own construction.

In the rest of this paper we will focus on the models summarized in Table 1. For the sake of comparability, in all cases we will use the logit function to link q_{xt} to the predictor structure η_{xt}.

4. Projection of mortality and life expectancy in the EU – empirical analysis

The data sets used in this paper were taken from the Human Mortality Database [2015]. Twenty countries were selected, and thus forty sex-specific populations

were obtained for all analyses. We divided each data set into a fitting period and a projection period. For projection purposes we used a rolling origin as follows: the projection period was initially set to be the last thirty years, ending in the year as seen in Table 2. Using the data in the fitting period, we computed one-step-ahead point projections, and determined the project errors by comparing the projections with the actual out-of-sample data. Then we increased the fitting period by one year, and computed one-step-ahead projections, and calculated the projection errors.

The twenty countries selected all have reliable data series in HDM database available after 1957 in order to maintain the full and consistent comparisons of the six methods. Age is in single years and we restrict the age range to 55-100 as the CBD model and the QBCD model have been particularly designed to fit older ages.

Table 2. Fitting period for each country

Country	Data set	Country	Data set
Austria	1947-2014	Lithuania	1945-2013
Belgium	1951-2012	Luxembourg	1951-2014
Bulgaria	1952-2010	Netherlands	1952-2012
Czech Republic	1950-2014	Poland	1958-2009
Denmark	1951-2011	Portugal	1951-2012
Finland	1954-2012	Slovakia	1958-2014
France	1951-2013	Slovenia	1951-2014
Germany	1938-2011	Spain	1950-2012
Ireland	1950-2013	Sweden	1951-2014
Latvia	1951-2013	United Kingdom	1957-2013

Source: own construction.

Due to the limitation of pages of this article in the tables we present only the results for males. However the descriptions also include the results for females.

4.1. Models fitting and goodness-of-fit analysis

Parameter estimates of GAPC stochastic mortality models are obtained by maximizing the model log-likelihood, which – in the case of a Poisson distribution of deaths – is given by

$$L(d_{xt}, \hat{d}_{xt}) = \sum_x \sum_t \omega_{xt} \left\{ d_{xt} \log \hat{d}_{xt} - \hat{d}_{xt} - \log d_{xt}! \right\},$$

where ω_{xt} are the weights taking the value 0 if a particular (x, t) data cell is omitted or 1 if the cell is included, and

$$\hat{d}_{xt} = E_{xt} g^{-1}\left(\alpha_x + \sum \beta_x^{(i)} \kappa_t^{(i)} + \beta_x^{(0)} \gamma_{t-x} \right)$$

is the expected number of deaths predicted by the model, with g^{-1} denoting the inverse of the link function g.

Additionally, since some models include cohort effects and in agreement with the usual practice (see e.g. [Cairns et al. 2009; Haberman, Renshaw 2011]), we exclude all cohorts that have fewer than three observations.

Figure 2 depicts the fitted parameters of the RH model for Poland's male population, as an example.

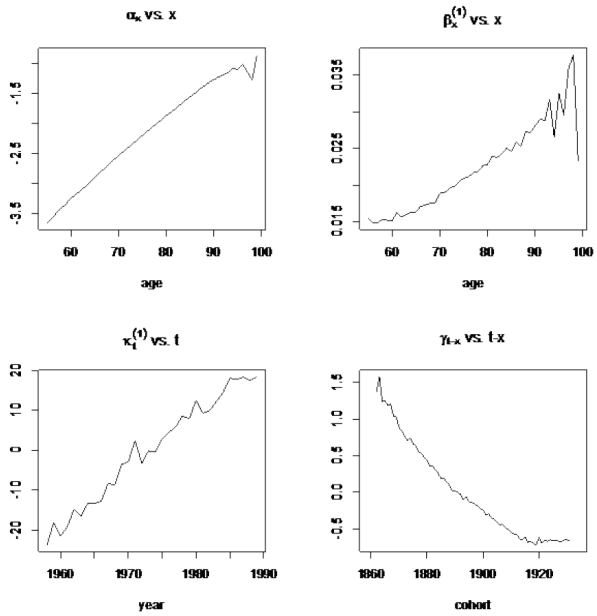

Fig. 2. Parameters for the RH model fitted to Poland's male population for ages 55-100 and the period 1958-1979

Source: own calculation in R.

The goodness-of-fit of mortality models is analyzed by inspecting the residuals of the fitted model. Regular patterns in the residuals indicate the inability of the model to describe all the features of the data appropriately. With a Poisson random component, it is appropriate to look at the scaled deviance residuals defined as

$$r_{xy} = \operatorname{sign}\left(d_{xt} - \hat{d}_{xt}\right)\sqrt{\frac{\operatorname{dev}(x,t)}{\hat{\varphi}}},$$

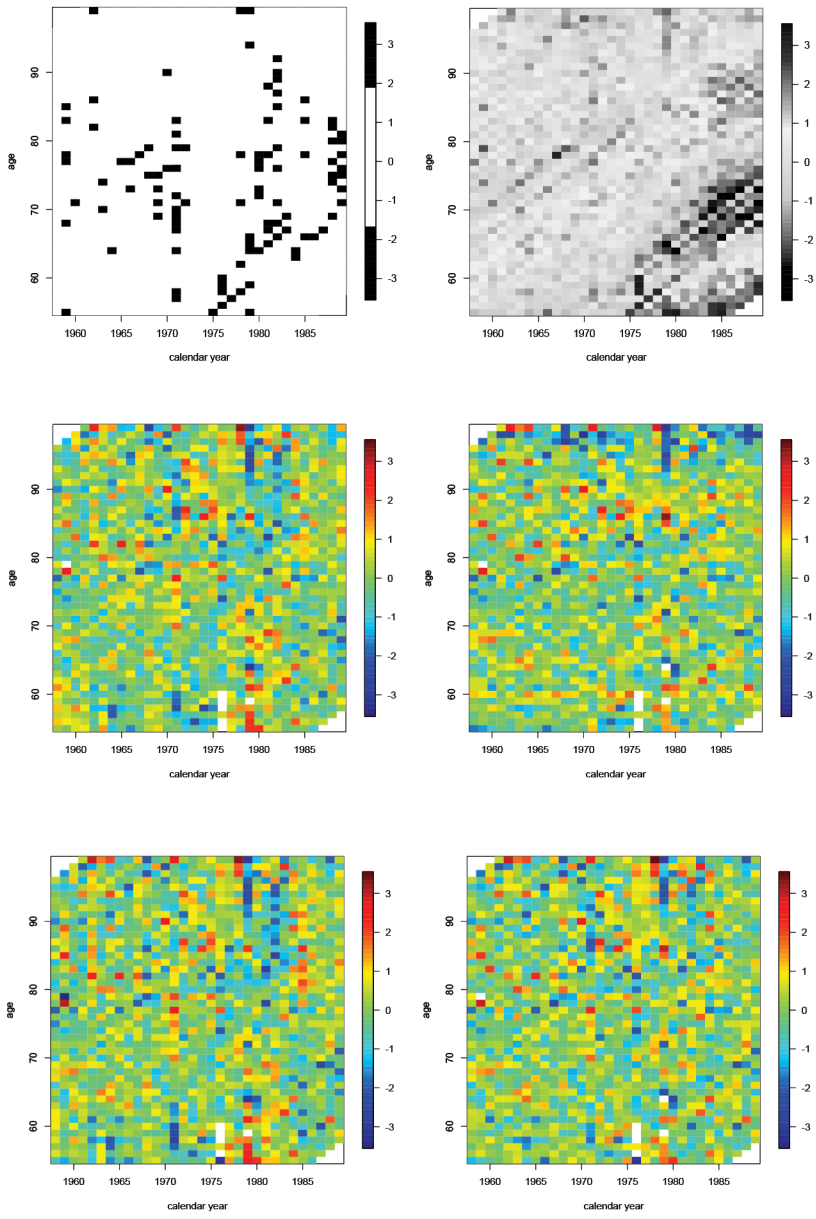

Fig. 3. Heat-maps of deviance residuals for different model fitted to the Polish male population for ages 55-100 and the period 1959-1979; first row (from left): LC and CBD, second row: RH and QBCD, third row: APC and PLAT

Source: calculations in StMoMo R package.

where $\hat{\varphi} = \frac{D(d_{xt}, \hat{d}_{xt})}{K - \nu}$, $K = \sum_x \sum_t \omega_{xt}$ is the number of observations in the data and ν is the effective number of parameters in the model, $\text{dev}(x,t) = 2 d_{xt} \log\left(\frac{d_{xt}}{\hat{d}_{xt}}\right) - \left(d_{xt} - \hat{d}_{xt}\right)$ and $D(d_{xt}, \hat{d}_{xt}) = \sum_x \sum_t \omega_{xt} \text{dev}(x,t)$.

Figure 3 presents heat-maps of the deviance residuals for the six models fitted to the Polish male mortality experience. We see that models LC and CBD display strong residual patterns while the residuals of models RH, QBCD, APC and PLAT look reasonably random. The LC and CBD models, which do not incorporate a cohort effect, show very marked diagonals patterns indicating the inability of these models to capture the well-known cohort effect observed in the Polish population [Willets 2004].

To rule out the possibility that the better fit observed in a model is the result of over-parametrisation and to compare the relative performance of several models, it has become common in the mortality literature to use information criteria which modify the maximum likelihood criterion by penalising models with more parameters. Two of these criteria are the Akaike Information Criteria (AIC) and the Bayesian Information Criteria (BIC), defined as AIC = 2ν−2L and BIC = νlogK−2L, respectively, with a lower value of AIC and BIC being preferable.

Table 3 presents the BIC values for the six models fitted to the male data of the EU countries. We note that this criterion leads to almost the same ranking of models with RH, PLAT, and QBCD being the best performing models. Overall, the Renshaw-Haberman extension of the Lee-Carter model and the QBCD model have been identified as good candidates for modeling mortality in the chosen population.

Table 3. Number of parameters and BIC values for different model fitted to the countries male population for ages 55-100

	LC	CBD	RH	QBCD	APC	PLAT
1	2	3	4	5	6	7
# parameters	120	64	189	163	144	174
Austria	19 845	18 563	18 256	17 137	17 473	17 376
Belgium	20 897	21 636	18 536	16 473	18 856	17 584
Bulgaria	18 897	20 748	16 689	16 515	17 576	16 415
Czech Republic	20 897	20 757	17 326	16 314	18 537	16 472
Denmark	21 564	17 848	16 689	16 885	16 691	16 954
Finland	21 645	20 748	16 689	16 515	16 576	17 145
France	20 897	20 746	17 845	16 721	17 576	17 253
Germany	20 897	18 456	16 689	16 503	18 576	19 635

1	2	3	4	5	6	7
Ireland	20 876	20 753	17 841	16 511	16 576	18 346
Latvia	21 635	20 748	16 689	16 502	19 576	19 456
Lithuania	21 564	17 848	16 689	16 085	16 671	16 354
Luxembourg	20 635	20 472	16 897	16 523	18 575	17 211
Netherlands	20 896	20 763	17 834	16 721	16 580	17 263
Poland	20 873	18 853	16 675	16 513	19 542	19 685
Portugal	20 986	20 851	17 888	16 516	16 573	18 853
Slovakia	21 605	20 730	16 692	16 518	18 646	19 635
Slovenia	21 651	17 673	16 685	17 985	16 678	16 435
Spain	18 643	18 583	18 261	17 957	17 524	15 256
Sweden	20 907	21 566	18 843	16 853	18 645	17 634
United Kingdom	19 807	20 758	16 689	16 734	16 673	17 613

Source: own calculations.

4.2. Projection of log mortality rates

The QBCD, PLAT, and RH methods tend to perform better than the LC classical method, and these methods perform best in the male and female data. The RH method also performs at least as well as any other method in twelve of the twenty populations. For both male and female rates, all RH methods overestimate mortality consistently for all countries. Among these methods, the QBCD method performs best for male rates. Among the LC methods there is less consistency. The RH method performs best overall, though the QBCD method has the lowest simple average for male rates. The LC method underestimates female rates for all fourteen countries and male rates for eleven of the twenty countries.

4.3. Projection of life expectancy

The corresponding MFEs for one-step-ahead point forecasts of life expectancy are shown in Tables 4 and 5. In general, the average underestimation in mortality rates does not necessarily translate into the overestimation in life expectancy and vice versa, because of the implicit weights applied to errors by age. However there is a clear association between differences in the age patterns in forecast errors and differences in the size and sign of forecast errors in life expectancy. For both sexes, all RH and QBCD methods and the PLAT (in particular) method tend to underestimate life expectancy, both on average and almost consistently across countries, while the LC method overestimates life expectancy on average and for most countries. Based on the simple average the RH method is superior for male life expectancy, while for female life expectancy the QBCD method is superior according to the simple average.

Table 4. Point forecast accuracy of male life expectancy by method and country, as measured by the MAFE for one-step-ahead forecasts

Country	LC	CBD	RH	QBCD	APC	PLAT
Austria	0.616	1.836	0.693	0.282	0.371	0.330
Belgium	0.313	0.513	0.322	0.286	0.194	1.326
Bulgaria	0.484	0.150	0.263	0.190	0.223	0.395
Czech Republic	0.288	0.157	0.381	0.380	0.574	0.223
Denmark	0.420	0.297	0.339	0.560	0.554	0.230
Finland	0.936	1.837	0.486	0.176	0.235	0.491
France	0.198	0.633	0.274	0.255	0.587	1.861
Germany	0.580	0.188	0.282	0.545	0.327	0.475
Ireland	0.412	0.390	0.921	2.298	0.285	0.128
Latvia	0.172	0.291	0.522	0.188	1.032	0.210
Lithuania	0.832	0.854	1.032	0.854	0.915	1.512
Luxembourg	2.081	1.636	1.921	1.526	0.608	0.254
Netherlands	1.411	0.921	0.201	0.244	0.606	0.229
Poland	0.937	0.310	0.286	0.348	0.534	0.769
Portugal	0.190	0.575	0.327	0.235	0.333	0.284
Slovakia	0.244	0.606	0.725	0.895	0.212	0.559
Slovenia	0.336	0.329	0.413	0.434	0.259	1.303
Spain	1.728	0.446	0.204	0.285	0.448	0.357
Sweden	0.354	0.657	1.554	0.586	0.179	0.266
United Kingdom	0.237	0.359	0.474	0.417	0.637	0.468
Average	**0,591**	**0,580**	**0,535**	**0,531**	**0,594**	**0,552**

Source: own calculations.

Table 5. MFEs for one-step-ahead point forecasts of male life expectancy by method and country

Country	LC	CBD	RH	QBCD	APC	PLAT
1	2	3	4	5	6	7
Austria	1.836	−0.570	−0.179	1.326	0.216	0.036
Belgium	1.837	−0.914	−0.571	1.861	−0.921	2.298
Bulgaria	1.405	−0.047	0.310	−0.213	0.364	0.094
Czech Republic	1.033	−0.263	0.357	1.472	−1.257	0.728
Denmark	1.264	−0.381	0.291	−0.311	−0.468	0.921
Finland	0.613	−0.445	0.427	0.196	0.655	0.269
France	0.124	0.256	0.493	0.228	0.161	0.187
Germany	0.265	0.394	0.253	0.276	0.518	0.497

1	2	3	4	5	6	7
Ireland	0.136	0.491	0.633	0.244	0.540	0.248
Latvia	0.283	0.916	0.314	0.188	0.212	0.259
Lithuania	0.223	0.309	0.382	0.361	0.412	0.323
Luxembourg	0.288	0.456	0.247	0.349	0.381	0.374
Netherlands	0.124	0.210	0.110	0.225	0.367	0.291
Poland	0.257	1.468	0.219	0.190	0.223	0.062
Portugal	0.210	0.315	0.305	0.213	0.573	0.108
Slovakia	0.078	0.187	0.598	0.136	0.578	0.095
Slovenia	0.171	0.687	0.235	0.484	0.120	0.251
Spain	0.151	0.113	0.569	0.077	0.166	0.282
Sweden	0.109	0.079	0.918	0.589	0.835	0.913
United Kingdom	0.167	0.123	0.769	0.121	0.555	0.895
Average	0,541	0,459	0,410	0,413	0,461	0,450

Source: own calculations.

5. Conclusions

The above comparative analysis of mortality forecasting methods is the most comprehensive to date. It constitutes an evaluation of point projection for log mortality rates and life expectancy based on ten principal component methods and twenty populations. The methods include the LC method and five LC variants, itself an extension of the LC method. Based on the simple average we found that the RH method is superior for male life expectancy, while for female life expectancy the QBCD method is superior according to the simple average.

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