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Contents

Introduction

One of the fastest growing areas in the economic sciences is broadly defined area of finance, with particular emphasis on the financial markets, financial institutions and risk management. Real world challenges stimulate the development of new theories and methods. A large part of the theoretical research concerns the analysis of the risk of not only economic entities, but also households.

The first Wrocław Conference in Finance WROFIN was held in Wrocław between 22nd and 24th of September 2015. The participants of the conference were the leading representatives of academia, practitioners at corporate finance, financial and insurance markets. The conference is a continuation of the two long-standing conferences: INVEST (Financial Investments and Insurance) and ZAFIN (Financial Management – Theory and Practice).

The Conference constitutes a vibrant forum for presenting scientific ideas and results of new research in the areas of investment theory, financial markets, banking, corporate finance, insurance and risk management. Much emphasis is put on practical issues within the fields of finance and insurance. The conference was organized by Finance Management Institute of the Wrocław University of Economics. Scientific Committee of the conference consisted of prof. Diarmuid Bradley, prof. dr hab. Jan Czekaj, prof. dr hab. Andrzej Gospodarowicz, prof. dr hab. Krzysztof Jajuga, prof. dr hab. Adam Kopiński, prof. dr. Hermann Locarek-Junge, prof. dr hab. Monika Marcinkowska, prof. dr hab. Paweł Miłobędzki, prof. dr hab. Jan Monkiewicz, prof. dr Lucjan T. Orłowski, prof. dr hab. Stanisław Owsiak, prof. dr hab. Wanda Ronka-Chmielowiec, prof. dr hab. Jerzy Różański, prof. dr hab. Andrzej Sławiński, dr hab. Tomasz Słoński, prof. Karsten Staehr, prof. dr hab. Jerzy Węcławski, prof. dr hab. Małgorzata Zaleska and prof. dr hab. Dariusz Zarzecki. The Committee on Financial Sciences of Polish Academy of Sciences held the patronage of content and the Rector of the University of Economics in Wroclaw, Prof. Andrzej Gospodarowicz, held the honorary patronage.

The conference was attended by about 120 persons representing the academic, financial and insurance sector, including several people from abroad. During the conference 45 papers on finance and insurance, all in English, were presented. There were also 26 posters.

This publication contains 27 articles. They are listed in alphabetical order. The editors of the book on behalf of the authors and themselves express their deep gratitude to the reviewers of articles – Professors: Jacek Batóg, Joanna Bruzda, Katarzyna Byrka-Kita, Jerzy Dzieża, Teresa Famulska, Piotr Fiszeder, Jerzy Gajdka, Marek Gruszczyński, Magdalena Jerzemowska, Jarosław Kubiak, Tadeusz Kufel, Jacek Lisowski, Sebastian Majewski, Agnieszka Majewska, Monika Marcinkowska, Paweł Miłobędzki, Paweł Niedziółka, Tomasz Panek, Mateusz Pipień, Izabela Pruchnicka-Grabias, Wiesława Przybylska-Kapuścińska, Jan Sobiech, Jadwiga Suchecka, Włodzimierz Szkutnik, Mirosław Szreder, Małgorzata Tarczyńska-Łuniewska, Waldemar Tarczyński, Tadeusz Trzaskalik, Tomasz Wiśniewski, Ryszard Węgrzyn, Anna Zamojska, Piotr Zielonka – for comments, which helped to give the publication a better shape.

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RISK PARITY PORTFOLIOS FOR SELECTED MEASURES OF INVESTMENT RISK

PORTFELE PARYTETU RYZYKA DLA WYBRANYCH MIAR RYZYKA INWESTYCYJNEGO

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Abstract: The risk is an important factor taken into account in the construction of each investment portfolios. Usually, portfolios are constructed in this way to minimize total risk of investment. Another approach is the selection of weights of individual stocks included in the portfolio so that the risk of the investment was equally distributed over all the components of portfolio. Such portfolios are called risk parity portfolios or equal risk contribution portfolios. In research carried out so far on the risk parity, the risk was measured only by the standard deviation. The main goal of this article is to introduce optimization models that will determine the risk parity portfolios for selected risk measures such as Gini's mean difference and mean absolute deviation. Also in the article the results of empirical research concerning the practical implementation of proposed models are presented.

Keywords: risk parity portfolio, equal risk contribution, mean absolute deviation, Gini's mean difference.

Streszczenie: Zazwyczaj portfele inwestycyjne konstruowane są tak, aby zminimalizować ryzyko. Innym podejściem jest taki dobór udziałów poszczególych instrumentów portfela, aby całowite ryzyko inwestycji było równo podzielone na poszczególne składniki. Tak skonstruowane portfele nazywamy portfelami parytetu ryzyka lub portfelami równego podziału ryzyka. W dotychczas prowadzonych badaniach parytet ryzyka definiowany był dla odchylenia standardowego. W ninejszym artykule przedstawione zostały modele wyboru portfeli parytetowych dla takich miar ryzyka, jak średnia różnica Giniego czy średnie odchylenie bezwzględne. Omówione zostały również wyniki analizy porównawczej dotyczącej zastosowania zaproponowanych modeli.

Słowa kluczowe: portfele parytetu ryzyka, równy podział ryzyka, średnie odchylenie bezwzględne, średnia różnica Giniego.

1. Introduction

Risk plays a crucial role in every investment. Usually investors want to reduce risk of investment and simultaneously they want to receive the profits on the fixed level. However, the financial markets are very unstable, what we could observe many times since the last economic crisis in 2007. Many investors maintain that the method to minimize the risk is not a good approach for such unstable markets. Accordingly, a better solution is investment in portfolios, whose every component has attributed the same part of total risk. This approach is called risk parity portfolios or equal risk contribution portfolios. The basic definition of the risk parity was formulated for the standard deviation as a measure of risk. However, in the portfolio analysis many other measures can be applied successfully.

In the first part of the article, the classical approach to construct the risk parity portfolios was described. In the next section, the risk parity conditions for the selected measure of risk were proposed. In these definitions, mean absolute deviation and Gini's mean difference will be used as a measure of risk. Last part is the presentation of results of short empirical research. The main goal of this research was the comparative analysis of application of the proposed methods to construct the risk parity portfolios for measures other than standard deviation.

2. Portfolios with equal risk contribution

In the recent years, all around the world the financial markets were characterized by high volatility. One of the main problems for many investors was how to create truly diversified portfolio. The first proposition on how to solve this problem was an optimization model of Markowitz [1952]. He worked out a mean-variance model, which had been criticised many times during the following years.

Alternative method was proposed by DeMiguel [DeMiguel et al. 2009]. He introduced equally weighted portfolios (naive portfolios). In the equally weighted portfolio all assets have the same weight. However, each component of portfolio has a different contribution in the total risk of the portfolio. The research conducted by De Miguel proved that the equally weighted portfolios provide diversification only in terms of capital while the investors want the portfolio diversified in the sense of risk.

Other approach to receive a diversified portfolio is to construct the risk parity portfolios. The idea of risk parity strategy is to identify weights of the portfolio in such a way that the total risk of the portfolio is equally divided on the individual its components. Using this approach, we can avoid the dominant role of one or few stocks in the portfolio [Qian 2005, 2006, 2011; Braga 2012]. A risk parity portfolio can be also understood as a trade-off between minimizing risk and maximizing nominal diversification [Maillard et al. 2010].

At first, it had been assumed that the weights of risk parity portfolio are proportional to the inverse of the standard deviations of individual stocks. This approach is called naive risk parity and it can be used only when all pairs of assets have the same coefficient of correlation. A more universal method to construct the risk parity portfolios was proposed by Maillard et al. [2010]. Their model allows to create a portfolio in which every component has the same contribution in the total risk, thus providing diversification in terms of risk [Bhensali et al. 2012]. Authors defined the risk parity portfolios using the total and marginal risk contribution.

Let us consider the portfolio consisting of N stocks. Portfolio risk measured by the classical standard deviation is equal to:

$$\sigma_p = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij}}$$

where: x_i – weight of the i-th stock in the portfolio, σ_{ij} – covariance between i-th and j-th stocks, $\sigma_{ii} = \sigma_i^2$ – variance of the i-th stock, σ_p – standard deviation of the portfolio.

To define parity portfolio, we need two measures. One of them is the marginal risk contribution for the i-th stock (MRC_i) described with the formula [Maillard et al. 2010; Chaves et al. 2011, 2012]:

$$MRC_i = \frac{\partial \sigma_p}{\partial x_i} = \frac{x_i \sigma_i^2 + \sum_{j \neq i}^{N} x_j \sigma_{ij}}{\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij}}} = \frac{\sum_{j=1}^{N} x_j \sigma_{ij}}{\sigma_p}$$

Marginal risk contribution determines the changes in the risk of the portfolio (measured by the standard deviation) caused by the infinitely small changes made on the weights of assets.

The second measure used to define risk parity is the total risk contribution (TRC_i). The total risk contribution is calculated as a product of the allocation of the given stock in the portfolio and the marginal risk contribution of this stock. Formally, the total risk contribution is defined in the following way:

$$TRC_i = x_i \frac{\partial \sigma_p}{\partial x_i} = x_i \frac{x_i \sigma_i^2 + \sum_{j \neq i}^{N} x_j \sigma_{ij}}{\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij}}} = \frac{\sum_{j=1}^{N} x_i x_j \sigma_{ij}}{\sigma_p}$$

By using the TRC_i we can divide the total risk of the portfolio onto all the individual components. The marginal risk contribution can be used to determine the weights for portfolio with the minimal variance. The necessary condition is that the MRC_i measures for all components should be equal. While the necessary condition for equal risk contribution portfolio is that the measures of the total risk contribution for all components should be the same:

$$x_i \frac{\partial \sigma_p}{\partial x_i} = x_j \frac{\partial \sigma_p}{\partial x_j} \quad \text{for} \quad i, j = 1, 2, \ldots, N$$

Currently, a few methods to construct risk parity portfolio were presented in the literature. An example of the algorithm of selection of this type of portfolios has been described among others by Chaves et al. [2011, 2012]. These authors presented iterative methods in which the linear approximation of the system of equations solved by the Newton's method was used. In the other approach the diversification distribution [Meucci 2009] and the principal portfolios analysis [Lohreet et al. 2012] were applied.

However, the method most often used to construct the risk parity portfolios is solving the following optimization model proposed by Maillard et al. [2010]:

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\left(x_i \frac{\partial \sigma_p}{\partial x_i} - x_j \frac{\partial \sigma_p}{\partial x_j}\right)^2 \to min$$

$$\sum_{i=1}^{N} x_i = 1$$

$$0 \le x_i \le 1 \ \text{for} \ i = 1, 2, \ldots, N$$

The main assumption in the construction of the risk parity portfolios is that the necessary condition (for equal risk contribution) should be satisfied. The objective function in the above optimization model satisfies this condition. To solve this model, we should use the algorithm of the sequential quadratic programming (we can use e.g. MATLAB software). According to the definition, the risk parity portfolio includes all *N* stocks which are taken into account in the analysis. In other words, using the above method of construction for portfolios with equal risk contribution, we received the portfolio of *N* components, each of them with non-zero weight. The lower volatility of the given asset and the lower correlation with the other assets, the higher weight of this asset we received.

Model proposed by Maillard et al. was compared with the minimum variance model and with maximum diversification model. Examples of such research were presented among others in Clark et al. [2013], Braga [2012], Chaves et al. [2011].

3. Risk parity for selected measures of investment risk

Generally, for every measure of risk which is linear – homogeneity in the risk, and additive, we can define the risk parity. One of such measures is the Gini's mean difference (GMD). The Gini's mean difference is defined as an expected value of the absolute differences between possible rates of return. In the portfolio analysis context, the Gini's mean difference for the i-th stock is defined as: [Yitzhaki 1982; Shalit, Yitzhaki 2005]:

$$\Gamma_i = 2cov\big(R_i, F_i(R_i)\big)$$

where R_i denotes rate of return of the i-th stocks in portfolio $R_i = [r_{i,1}, r_{i,2}, ..., r_{i,T}]$, $r_{i,t}$ – the rate of return of the i-th stock in the t-th period, (for $i = 1, 2, \ldots, N$), $F_i(R_i)$ – cumulative distribution function of rates of return of the i-th stock.

Let's introduce the following notations: Γ_p – Gini's mean difference for portfolio, R_p – portfolio rate of return, $F_p(R_p)$ – cumulative distribution function of rates of return of portfolio. The Gini's mean difference for portfolio can be calculated in the following way [Yitzhaki 1982; Shalit, Yitzhaki 2005]:

$$\Gamma_p = 2\sum_{i=1}^{N} x_i cov\left(R_i, F_p(R_p)\right)$$

For the Gini's mean difference, the necessary condition for the risk parity is:

$$x_i cov\left(R_i, F_p(R_p)\right) = x_j cov\left(R_j, F_p(R_p)\right) \text{ for all } i, j = 1, 2, ..., N$$

In the optimization model for construction of the risk parity portfolio in the Gini's mean difference sense ($\mathrm{RP_{GMD}}$), we should use this following objective function:

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\left(x_i cov\left(R_i, F_p(R_p)\right) - x_j cov\left(R_j, F_p(R_p)\right)\right)^2 \to min$$

Recently in the portfolio analysis, the mean absolute deviation (MAD) is often used instead the standard deviation. Mean absolute deviation indicates the average deviation of the rates of return from the expected rate of return. The mean absolute deviation for the i-th stock is defined here in the following way:

$$MAD_i = \sum_{i=1}^{T} p_i\left|r_{i,t} - r_i\right|$$

Using the scenario approach, the mean absolute deviation for the portfolio is calculated according to this formula [Konno, Yamazaki 1991]:

$$MAD_p = \frac{1}{T}\sum_{t=1}^{T}\left|\sum_{i=1}^{N}\left(r_{i,t} - r_i\right)x_i\right|$$

In the above formula r_i denotes the mean rate of return of the i-th stock. The necessary condition for the risk parity for mean absolute deviation is defined as:

$$\frac{1}{T}x_i\sum_{t=1}^{T}\left|r_{i,t}-r_i\right|=\frac{1}{T}x_j\sum_{t=1}^{T}\left|r_{j,t}-r_j\right| \text{ for all } i,j=1,2,\dots,N$$

To construct the risk parity portfolio in the sense of mean absolute deviation ($\mathrm{RP}_{\mathrm{MAD}}$) the following objective function should be used:

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\left(x_i\sum_{t=1}^{T}\left|r_{i,t}-r_i\right|-x_j\sum_{t=1}^{T}\left|r_{j,t}-r_j\right|\right)^2\rightarrow min$$

Similarly as in the classical approach, in both optimization models $\mathrm{RP}_{\mathrm{MAD}}$ and $\mathrm{RP}_{\mathrm{GMD}}$ we have constrains concerning the weights of the portfolio. Both models are examples of the quadratic programming, so we can solve them using e.g. the MS Excel.

4. Empirical analysis of risk parity portfolios for different measures of risk

The main goal of the research presented below was a comparative analysis of the risk parity portfolios constructed for alternative measures of risk such as the mean absolute deviation ($\mathrm{RP}_{\mathrm{MAD}}$), Gini's mean difference ($\mathrm{RP}_{\mathrm{GMD}}$) and standard deviation ($\mathrm{RP}_{\mathrm{SD}}$). Portfolios were constructed for the group of selected stocks quoted on the Warsaw Stock Exchange. Analysis concerned 20 stocks that were included in the WIG20 index in the second quarter of 2015. Portfolios were constructed on the base of the daily rates of return from the period of 01.07.2010 – 30.06.2015. Portfolios were constructed for the different number of components (from 2 to 20). Below only the results for portfolios consisting of 5, 10 and 15 stocks are presented. For the rest of the portfolios, the results were similar. Three groups of portfolios, depending on the criteria of stocks selection were considered. The stocks were selected according to the rate of return (I group), the standard deviation (II group) and the correlation coefficient (III group). For the first and the second group, the results were similar.

For the received weights of all parity portfolios, a few characteristics of portfolios were computed, such as the value of risk (variance) and expected rate of return. The optimization model proposed by Maillard et al. [2010] allows to calculate the weights of portfolio in such a way that the difference between the parts of the risk attributed to the individual stocks is minimal. It means that by using this model we received the division of risk of portfolio only approximately equal. Because of that, the level of the inequality of risk contribution was also compared. The level of the inequality was measured with Gini's coefficient.

Table 1. Characteristics for portfolios with components selected according to the rates of return

Number of stocks	Risk			Rate of return			Gini's coefficient		
	rP_{MAD}	RP_{GMD}	RP_{SD}	RP_{MAD}	RP_{GMD}	RP_{SD}	RP_{MAD}	RP_{GMD}	RP_{SD}
5	0.00014	0.00014	0.00015	1.0006	1.0006	1.0006	1E-08	1E-05	0.0712
10	0.00013	0.00013	0.00013	1.0004	1.0004	1.0004	2E-06	0.00357	0.0822
15	0.00012	0.00012	0.00012	1.0003	1.0003	1.0003	4E-06	0.0071	0.10095

Source: Author's own study.

All risk parity portfolios, for the fixed number of stocks, were characterized with the similar level of risk as well as the expected rate of return. In the case when the stocks were selected on the base of the rate of return, for all measures of risk we received the portfolios almost of the same variance and with very similar value of the rate of return. However, comparing portfolios of 15 components selected on the basis of variance we can notice, that parity portfolios in the sense of Gini's mean difference were a little less risky than the corresponding parity portfolios for the other risk measure.

Table 2. Characteristics for portfolios with components selected according to the risk

Number of stocks	Risk			Rate of return			Gini's coefficient		
	RP_{MAD}	RP_{GMD}	RP_{SD}	RP_{MAD}	RP_{GMD}	RP_{SD}	RP_{MAD}	RP_{GMD}	RP_{SD}
5	0.00011	0.00011	0.00011	1.00019	1.00019	1.0002	4E-09	0.0039	0.0004
10	0.00011	0.00010	0.00011	1.00010	1.00010	1.0001	5E-07	0.0036	0.0599
15	0.00010	0.00003	0.00010	1.00010	1.00010	1.0001	2E-06	0.0048	0.0932

Source: Author's own study.

Comparing the portfolios according to the equality of the risk contribution, it can be noticed that the mean absolute deviation was the best criterion to construct the risk parity. The lowest values of Gini's coefficient were obtained for RP_{MAD} portfolios. The highest values of Gini's coefficient were obtained for portfolios with equal standard deviation contribution. The values of Gini's coefficient for these portfolios were on the level from 2% to even 10%. Values of coefficient of the inequality for portfolios with the equal contribution of Gini's mean difference were lower than 2%.

Additionally, all constructed portfolios were compared according to the future profits. For this purpose, for the data from the successive days of July 2015, the values of portfolios were calculated. Then the coefficient of the value of portfolio on the given day, relative to the value of the portfolio on the day when the portfolio was constructed (30.06.2015) was calculated. In this way it was possible to establish whether the given portfolio was profitable or not. The results were presented in Tables 3-4.

Table 3. Future profits of portfolios with components selected according to the rates of return

Data	5 components			10 components			15 components		
	RP_{MAD}	RP_{GMD}	RP_{SD}	RP_{MAD}	RP_{GMD}	RP_{SD}	RP_{MAD}	RP_{GMD}	RP_{SD}
02.07.15	1.0083	1.0084	1.0083	1.0042	1.0047	1.0059	1.0036	1.0046	1.0058
07.07.15	1.0168	1.0170	1.0168	1.0106	1.0114	1.0132	1.0096	1.0111	1.0128
14.07.15	1.0194	1.0196	1.0194	1.0124	1.0134	1.0153	1.0104	1.0124	1.0142
21.07.17	1.0178	1.0180	1.0178	1.0019	1.0037	1.0074	0.9989	1.0025	1.0059
23.07.15	1.0404	1.0410	1.0438	1.0238	1.0259	1.0305	1.0205	1.0247	1.0290

Source: Author's own study.

Table 4. Future profits of portfolios with components selected according to the risk

Data	5 components			10 components			15 components		
	RP_{MAD}	RP_{GMD}	RP_{SD}	RP_{MAD}	RP_{GMD}	RP_{SD}	RP_{MAD}	RP_{GMD}	RP_{SD}
02.07.15	0.9841	0.9840	0.9846	0.9886	0.9882	0.9902	0.9886	0.9881	0.9902
07.07.15	0.9828	0.9826	0.9834	0.9883	0.9881	0.9905	0.9872	0.9871	0.9895
14.07.15	0.9873	0.9872	0.9878	0.9888	0.9891	0.9906	0.9829	0.9834	0.9848
21.07.17	0.9547	0.9544	0.9560	0.9540	0.9536	0.9586	0.9431	0.9430	0.9484
23.07.15	0.9588	0.9585	0.9600	0.9594	0.9588	0.9636	0.9537	0.9531	0.9586

Source: Author's own study.

It should be noticed that all portfolios, whose components were selected according to risk, had lower value in July than in the moment of construction. If components were selected according to the rates of return or according to the correlation coefficient, for almost all portfolios higher profits were obtained. The highest profits were obtained for portfolios with the equal variance contribution whereas the lowest profits obtained for portfolios with equal mean absolute deviation contribution.

In addition, the obtained parity portfolios were compared with the portfolios constructed according to the classical model of Markowitz (without assumption about the rate of return of the portfolio). Markowitz portfolios were characterized by a significantly higher degree of unequal risk contribution (Gini's coefficient equal 17%-20%) and lower future profits than the corresponding parity portfolios. The analysed models to construct the risk parity portfolios were also applied to different groups of indices (WIG20, mWIG40, sWIG80, WIG). The results for all groups were similar.

5. Conclusion

Usually the risk parity is analysed only when the risk is measured by the standard deviation. In this article, the risk parity was formulated for such measures of risk as mean absolute deviation and Gini's mean difference. Conducted research proved that all three measures can be used alternatively to calculate the risk parity. In every case, we received portfolios with the similar value of risk and expected rate of return. Also, the future value of the portfolio was similar, no matter which measure was applied to express the risk parity. The biggest differences were obtained when the level of inequality of risk contribution was compared. Contribution of risk at the level closest to the equal was obtained for risk parity defined for the mean absolute deviation.

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