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Introduction

One of the fastest growing areas in the economic sciences is broadly defined area of finance, with particular emphasis on the financial markets, financial institutions and risk management. Real world challenges stimulate the development of new theories and methods. A large part of the theoretical research concerns the analysis of the risk of not only economic entities, but also households.

The first Wrocław Conference in Finance WROFIN was held in Wrocław between 22nd and 24th of September 2015. The participants of the conference were the leading representatives of academia, practitioners at corporate finance, financial and insurance markets. The conference is a continuation of the two long-standing conferences: INVEST (Financial Investments and Insurance) and ZAFIN (Financial Management – Theory and Practice).

The Conference constitutes a vibrant forum for presenting scientific ideas and results of new research in the areas of investment theory, financial markets, banking, corporate finance, insurance and risk management. Much emphasis is put on practical issues within the fields of finance and insurance. The conference was organized by Finance Management Institute of the Wrocław University of Economics. Scientific Committee of the conference consisted of prof. Diarmuid Bradley, prof. dr hab. Jan Czekaj, prof. dr hab. Andrzej Gospodarowicz, prof. dr hab. Krzysztof Jajuga, prof. dr hab. Adam Kopiński, prof. dr. Hermann Locarek-Junge, prof. dr hab. Monika Marcinkowska, prof. dr hab. Paweł Miłobędzki, prof. dr hab. Jan Monkiewicz, prof. dr Lucjan T. Orłowski, prof. dr hab. Stanisław Owsiak, prof. dr hab. Wanda Ronka-Chmielowiec, prof. dr hab. Jerzy Różański, prof. dr hab. Andrzej Sławiński, dr hab. Tomasz Słoński, prof. Karsten Staehr, prof. dr hab. Jerzy Węcławski, prof. dr hab. Małgorzata Zaleska and prof. dr hab. Dariusz Zarzecki. The Committee on Financial Sciences of Polish Academy of Sciences held the patronage of content and the Rector of the University of Economics in Wroclaw, Prof. Andrzej Gospodarowicz, held the honorary patronage.

The conference was attended by about 120 persons representing the academic, financial and insurance sector, including several people from abroad. During the conference 45 papers on finance and insurance, all in English, were presented. There were also 26 posters.

This publication contains 27 articles. They are listed in alphabetical order. The editors of the book on behalf of the authors and themselves express their deep gratitude to the reviewers of articles – Professors: Jacek Batóg, Joanna Bruzda, Katarzyna Byrka-Kita, Jerzy Dzieża, Teresa Famulska, Piotr Fiszeder, Jerzy Gajdka, Marek Gruszczyński, Magdalena Jerzemowska, Jarosław Kubiak, Tadeusz Kufel, Jacek Li-

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Wanda Ronka-Chmielowiec, Krzysztof Jajuga

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EXTREME VALUE THEORY FOR DETECTING HEAVY TAILS OF LARGE CLAIMS

ROZPOZNAWANIE GRUBOŚCI OGONA ROZKŁADÓW WIELKICH ROSZCZEŃ Z UŻYCIEM TEORII WARTOŚCI EKSTREMALNYCH

DOI: 10.15611/pn.2016.428.22
JEL Classification: C13, C63, C88

Abstract: One of the most prominent facets, that arises in risk management in an insurance company, is a difficulty in detecting large claims' distribution properly. Therefore, a quite large variety of stochastic methods and models have been proposed to solve the mentioned difficulty. The main goal of the presented study is to discuss and to compare two alternative approaches to theoretical distribution estimation. The first approach consists in collective estimation of all parameters of theoretical distribution, while the second approach is based on two-step procedure, in which previous estimation of tail-heaviness parameter is followed by conditional estimation of the rest of parameters. The theoretic discussion is illustrated by simulation research and empirical data analysis, as well. As a result, one may state that the introduced two-step approach enables more flexible, and adequate detection of tail asymptotics without negative impact on a quality of the remaining parameters' estimates.

Keywords: extreme value index, heavy tails, large claims, estimation.

Streszczenie: Właściwe rozpoznanie rozkładów wysokości wielkich roszczeń jest jednym z bardziej kluczowych problemów pojawiających się w toku zarządzania ryzykiem w przedsiębiorstwach ubezpieczeniowych. Wychodząc naprzeciw rozwiązaniu tego problemu, proponuje się wiele różnych metod i modeli stochastycznych. Głównym celem artykułu jest porównanie dwóch alternatywnych podejść do estymacji rozkładu teoretycznego na podstawie danych empirycznych. Pierwsze z nich polega na jednoczesnej estymacji wszystkich parametrów rozkładu teoretycznego. Natomiast drugie podejście polega na dwuetapowej estymacji: najpierw estymowany jest parametr grubości ogona, a następnie warunkowo pozostałe parametry rozkładu. Ujęte w opracowaniu rozważania zobrazowane zostały badaniami symulacyjnymi i analizą danych empirycznych. W efekcie stwierdzono, że zaproponowane, dwuetapowe podejście umożliwia bardziej elastyczne i właściwsze rozpoznawanie asymptotyki ogona rozkładu bez pogorszenia jakości estymacji pozostałych parametrów.

Słowa kluczowe: indeks ekstremalny, grube ogony, wielkie roszczenia, estymacja.

1. Introduction

Extreme values occur in many types of data, and they usually cause difficulties in a proper detection of the underlying law that constitutes a fundamental benchmark for a decision-maker. This is the case of insurance data as well, especially with respect to large claims' amounts and portfolio risk management (see: [Bühlmann 2005]).

In view of the above, we discuss and compere two alternative approaches to theoretical distribution estimation (introduced in Section 2), which is the goal of the paper. As a result, we indicate some advantages of usage of the extreme value theory.

The initial motivation of the paper was an attempt to continue previous research (see: [Stachura, Wodecka 2012]) concerning parallel ideas arising from Value-at-Risk estimation problem, and then to reproduce, and to progress, the methods being discussed therein.

2. Theoretical background

From now on, let $X_1, X_2, \ldots$ be *independent and identically distributed* (i.i.d.) random variables with a common *cumulative distribution function* (cdf) F. For any fixed $n \in \mathbb{N}_+$, the order statistics of a sample $X_1, \ldots, X_n$ are denoted by $X_{1:n} \leq \cdots \leq X_{n:n}$.

The main theorem of the ***extreme value theory*** (EVT) states that if there exist constants $a_n > 0$, b_n for $n \in \mathbb{N}_+$, and some non-degenerate distribution function *G,* such that for all $x \in \mathbb{R}$ holds:

$$\lim_{n \to \infty} \mathbb{P}\left(\frac{X_{n:n} - b_n}{a_n} \leq x\right) = G(x),$$

then there exists a constant $\gamma \in \mathbb{R}$, such that the limit distribution *G* has the form:

$$G(x) = G_\gamma(x) = \begin{cases} \exp(-(1+\gamma x)^{-1/\gamma}) & 1 + \gamma x > 0 \quad \gamma \neq 0 \\ \exp(-e^{-x}) & x \in \mathbb{R} \quad \gamma = 0 \end{cases}.$$

The parameter γ is called the *extreme value index* (EVI), and it impacts the right tail asymptotics of the common cdf F (see, e.g.: [de Haan, Ferreira 2006]).

Classical estimators of EVI are based on upper order statistics. Among wide variety of such estimators, the most popular are the Pickands', and the Hill's ones (see: [Gomes et al. 2008]), given respectively by formulas:

$$\hat{\gamma}_P^k = \log_2 \frac{X_{n-k:n} - X_{n-2k:n}}{X_{n-2k:n} - X_{n-4k:n}}, \qquad \hat{\gamma}_H^k = \frac{1}{k}\sum_{i=0}^{k-1} \ln X_{n-i:n} - \ln X_{n-k:n}.$$

An alternative, proposed in Berred [1995], is based on the k-th record idea, that had been defined in Dziubdziela and Kopociński [1976]. So for a fixed $k \in \mathbb{N}_+$, the k-th record times $\{T_n^{(k)}\}$, and the k-th record values $\{R_n^{(k)}\}$ are defined as:

$$T_1^{(k)} = k, \quad T_n^{(k)} = \min\{\, j : j > T_{n-1}^{(k)},\ X_j > X_{T_{n-1}^{(k)} - k + 1 \,:\, T_{n-1}^{(k)}} \} \quad \text{for } n \geq 2$$

$$R_n^{(k)} = X_{T_n^{(k)}-k+1\,:\,T_n^{(k)}}$$

In other words, a sequence of k-th record values $R_1^{(k)} < R_2^{(k)} < R_3^{(k)} < \cdots$ is constructed by eliminating repetitions in the non-decreasing sequence of k-th order statistics $X_{1:k} \le X_{2:k+1} \le X_{3:k+2} \le \cdots$.

The original Berred's estimator based on the k-th record values is of the form:

$$\hat{\gamma}_B^k = \ln \frac{R_{N(k,n)}^{(k)} - R_{N(k,n)-k}^{(k)}}{R_{N(k,n)-k}^{(k)} - R_{N(k,n)-2k}^{(k)}},$$

where $N(k,n)$ denotes the number of k-th records values in a sample of size n.

The Pickands', and the Berred's estimators are convenient for any real γ[1], while the Hill's one is improper for $\gamma \le 0$. Moreover, the Berred's estimator value depends on sample order, which allows re-sampling, since i.i.d. property is assumed.

In the presented paper, we consider **two approaches** to estimating theoretical distribution that serves as a model law for large claims from a homogeneous portfolio of policies. Both of the approaches rely on *a priori* assumed model law, and *a priori* assumed estimation method as well[2]. In the sequel, we focus our attention on the generalised Pareto distribution (GPD), with parameters $\gamma \in \mathbb{R}$, $\sigma > 0$, given as:

$$F_{\gamma,\sigma}(x) = \begin{cases} 1 - \left(1 + \frac{\gamma}{\sigma}x\right)^{-1/\gamma} & x \in (0,\infty) \quad \gamma > 0 \\ 1 - e^{-x/\sigma} & x \in (0,\infty) \quad \gamma = 0, \\ 1 - \left(1 + \frac{\gamma}{\sigma}x\right)^{-1/\gamma} & x \in (0, -\frac{\sigma}{\gamma}) \quad \gamma < 0 \end{cases}$$

since the GPD family covers all types of the right tail asymptotics[3]. However, the GPD is not the only reasonable choice. Additionally, we restrict just to the *maximum likelihood* (ML) method.

The first of the mentioned approaches – from now on referred to as (A1) – consists in straightforward collective estimation of all parameters of the chosen theoretical distribution, which, in our case, implies simultaneous ML-estimation of γ, and σ. At the same time, the other approach – referred to as (A2) – is based on two-step procedure, in which initial estimation of tail-heaviness parameter is performed with use of extreme value methodology, and then it is followed by conditional estimation of the rest of parameters. In our case, estimations of γ is provided by means of EVT, and then σ is estimated with use of ML method, conditionally on a known γ estimate.

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[1] These estimators are invariant under any linear transformation (with a positive slope) of data, which is fully concordant with the linear transformation appearing in the main EVT theorem.

[2] Additionally, one may assume different model laws, and different estimation methods for each of the approaches, but such a situation precludes comparing thus obtained results directly.

[3] Normalised maxima taken from a sequence, whose common law is given by $F_{\gamma,\sigma}$, converge to a limit that is distributed according to G_γ with the same value of γ. Thus the same symbol.

3. Simulation research

In order to compare the both estimation approaches, simulation research is executed as follows[4]. **Firstly**, for a fixed pair of GPD parameters γ and σ – taken from arbitrarily chosen ranges $\gamma \in \{-2, -0.5, 0.5, 2, 5, 10\}$, $\sigma \in \{1, 10\}$ – a pseudorandom i.i.d. sample of size $n = 200$ is generated, and then:

a. with respect to (A1), ML-estimators $\hat{\gamma}_{\mathrm{ML}}$, $\hat{\sigma}_{\mathrm{ML}}$ are calculated,

b. due to (A2), for all $k \in \{1, 2, \ldots, \left[\frac{n}{4}\right]-1\}$, EVT-estimators $\hat{\gamma}_P^k$, $\hat{\gamma}_H^k$, $\hat{\gamma}_B^k$ are evaluated, and respectively on the basis of them ML-estimators $\hat{\sigma}_P^k$, $\hat{\sigma}_H^k$, $\hat{\sigma}_B^k$ are evaluated as well,

c. for every of 100 randomly taken permutations of the sample, estimators $\hat{\gamma}_B^k$, $\hat{\sigma}_B^k$ are calculated (likewise in b.), and then $\hat{\gamma}_{BB}^k$, $\hat{\sigma}_{BB}^k$ are assumed to be the medians of $\hat{\gamma}_B^k$, $\hat{\sigma}_B^k$ over all permutations.

Secondly, medians (over all k's) of $\hat{\gamma}_P^k$, $\hat{\gamma}_H^k$, $\hat{\gamma}_B^k$, $\hat{\gamma}_{BB}^k$, and of $\hat{\sigma}_P^k$, $\hat{\sigma}_H^k$, $\hat{\sigma}_B^k$, $\hat{\sigma}_{BB}^k$ are evaluated, and assumed to be the actual estimates of γ and σ. **Finally**, both previous steps are replicated 300 times independently, so that accuracy of the estimators may be revealed by minimum value, median, and maximum value (over all replications), which is presented in Tables 1 and 2[5].

Table 1. Estimators accuracy for $\sigma = 1$

		$\gamma = -2$						$\gamma = -0.5$					
		$\hat{\gamma}$			$\hat{\sigma}$			$\hat{\gamma}$			$\hat{\sigma}$		
		Min.	Med.	Max.	Min.	Med.	Max.	Min.	Med.	Max.	Min.	Med.	Max.
(A2)	*B*	–4.034	–1.981	–1.040	0.520	0.991	2.017	–1.450	–0.457	0.084	0.601	0.941	2,618
	BB	–2.398	–1.845	–1.258	0.629	0.923	1.199	–0.749	–0.410	0.009	0.629	0.889	1,395
	P	–2.726	–2.025	–1.544	0.772	1.013	1.363	–0.964	–0.498	–0.067	0.645	0.994	1,872
	H	0.002	0.011	0.024	0.304	0.332	0.365	0.094	0.170	0.255	0.500	0.619	0,707
(A1)		–2,001	–2.000	–2.000	1.000	1.000	1.000	–0.691	–0.523	–0.377	0.797	1.025	1.253

		$\gamma = 0.5$						$\gamma = 2$					
		$\hat{\gamma}$			$\hat{\sigma}$			$\hat{\gamma}$			$\hat{\sigma}$		
		Min.	Med.	Max.	Min.	Med.	Max.	Min.	Med.	Max.	Min.	Med.	Max.
(A2)	*B*	–0.348	0.545	1.390	0.596	0.967	7.418	1.157	2.049	3.912	0.570	1.010	1,829
	BB	–0.037	0.541	1.278	0.575	0.974	2.469	1.283	1.963	2.847	0.639	1.027	1,769
	P	0.072	0.514	1.270	0.555	0.985	1.596	1.271	1.997	3.040	0.593	1.016	1,831
	H	0.408	0.683	1.036	0.594	0.899	1.190	1.204	1.999	2.859	0.582	1.018	1,786
(A1)		0,000	0.494	0.869	0.636	1.006	1.512	1.362	1.982	2.636	0.624	1.008	1.789

Source: own study.

[4] The simulation research, and additionally all the calculations and plots presented hereunder, are accomplished in R environment.

[5] Cases of $\gamma = 5$, and $\gamma = 10$ are omitted, as the discrepancies among estimates, within a fixed pair of parameters γ and σ, are similar for any $\gamma > 0$.

Table 2. Estimators accuracy for $\sigma = 100$

		$\gamma = -2$						$\gamma = -0.5$					
		$\hat{\gamma}$			$\hat{\sigma}$			$\hat{\gamma}$			$\hat{\sigma}$		
		Min.	Med.	Max.	Min.	Med.	Max.	Min.	Med.	Max.	Min.	Med.	Max.
(A2)	*B*	–3.587	–1.965	–1.124	56.2	98.25	179.4	–1.103	–0.424	0.061	60.63	90.51	211,4
	BB	–2.462	–1.848	–1.263	63.15	92.39	123.1	–0.701	–0.404	0.074	58.91	87.87	130,4
	P	–2.784	–2.038	–1.444	72.19	101.9	139.2	–0.895	–0.494	–0.058	63.36	97.55	168,7
	H	0.003	0.010	0.026	29.84	33.34	37.10	0.104	0.164	0.283	50.68	61.72	72,44
(A1)		–2,001	–2.000	–2.000	99.92	100.0	100.0	–0.744	–0.519	–0.357	79.78	101.2	128.0
		$\gamma = 0.5$						$\gamma = 2$					
		$\hat{\gamma}$			$\hat{\sigma}$			$\hat{\gamma}$			$\hat{\sigma}$		
		Min.	Med.	Max.	Min.	Med.	Max.	Min.	Med.	Max.	Min.	Med.	Max.
(A2)	*B*	–0.117	0.530	1.300	66.89	99.15	209.6	1.309	2.079	4.152	60.94	97.61	178,8
	BB	0.130	0.543	0.926	69.89	98.35	150.1	1.155	1.959	2.727	63.18	100.8	167,2
	P	0.024	0.504	0.966	70.33	101.0	179.0	1.132	2.021	2.924	64.36	98.77	163,5
	H	0.356	0.684	0.950	68.59	89.93	133.1	1.171	1.979	3.536	59.48	99.80	173,5
(A1)		0,234	0.498	0.857	75.06	100.8	151.7	1.331	1.977	2.699	66.69	99.78	152.1

Source: own study.

Table 3. Percentages of non-rejection of null hypothesis for all repetitions for $\sigma = 1$

	$\gamma = -2$		$\gamma = -0.5$		$\gamma = 0.5$		$\gamma = 2$	
$\sigma = 1$	Independence tests							
	runs	Bartels	runs	Bartels	runs	Bartels	runs	Bartels
	0.953	0.957	0.947	0.947	0.933	0.960	0.940	0.937
	Goodness of fit tests							
	–	+	–	+	–	+	–	+
	0.957	0.000	0.947	0.007	0.007	0.950	0.000	0.960
$\sigma = 100$	Independence tests							
	runs	Bartels	runs	Bartels	runs	Bartels	runs	Bartels
	0.927	0.930	0.920	0.963	0.927	0.943	0.940	0.970
	Goodness of fit tests							
	–	+	–	+	–	+	–	+
	0.960	0.000	0.927	0.007	0.010	0.937	0.000	0.927

Source: own study.

Moreover, independence tests (runs test, Bartels' test), and a bootstrap goodness of fit tests (for negative '–', and positive '+' value of γ) are performed[6], which shows appropriate characteristics of the simulated data. Details are gathered in Table 3[7].

[6] We use runs.test(lawstat), bartels.test(lawstat), gpd.test(gPdtest) functions of the R environment.

[7] See footnote 5.

The simulation research carried out reveals that:

- in the context of EVT, Pickand's, and especially resampled Berred's estimators emerge to be preferable, since they are featured by the highest accuracy[8].
- in the context of the both approaches comparison, it appears that all the provided estimates have similar quality for $\gamma > 0$ which is the main case of interest.

4. Empirical research

In our empirical research, we aim to apply both approaches to real data, and to detect heavy tails of large claims from several insurance portfolios. In order to do so we consider the sequence of total charges (feature examined in the sequel) from Group Medical Insurance Large Claims Database provided by the Society of Actuaries (SOA)[9]. Three samples, properly chosen from the database, are constituted for the year 1992 in the way to make them homogeneous by the following characteristics:

- Sample 1 – sex: female, age: 30-60, diagnosis: malignant neoplasm of breast,
- Sample 2 – sex: male, age: 50-75, diagnosis: malignant neoplasm of prostate,
- Sample 3 – sex: both, age: 18-75, diagnosis: malignant neoplasm of brain.

For each sample, we select such a threshold (denoted by u) that claims higher than it forms an i.i.d. sequence[10] (of length denoted by n), which yields:

- Sample 1 – $u =$ \$70,000.00, $n = 202$,
- Sample 2 – $u =$ \$35,000.00, $n = 379$,
- Sample 3 – $u =$ \$55,000.00, $n = 207$.

Then we evaluate all the estimators of γ, and σ with respect to both approaches[11].

The left panels of Figures 1-3 present samples, and relevant thresholds (dashed horizontal lines), while the right panels of these figures represent sequences of Pickand's (black dotted line), Hill's (black dashed line), Berred's (grey solid line), resampled Berred's (black solid line) estimators for all k's going on horizontal axes.

Eventual EVI estimates, defined as medians of relevant sequences, serve to evaluate estimates of parameter σ. Simultaneously, for each sample, we estimate both parameters using (A1). All the obtain estimates are gathered in Table 4.

Casting an eye over the derived results shows that the considered approaches may assess differently, not only intensity of right tail heaviness, but also a type of its asymptotics, which obviously implies discrepancy among estimates of σ.

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[8] It is so even for $\gamma > 0$, where Hill's estimator exhibits properly.

[9] This database consists of about 171,000 claims covering costs of medical treatment that exceed \$25,000.00 disaggregated by types of charges gathered with respect to diagnosis, age, sex, employee status, insurance plan, to mention but a few.

[10] We perform the same independence and goodness of fit tests as in the simulation research.

[11] In such circumstances, the parameters γ, and σ refer to truncated distribution of claims, which is an analogue of Peak Over Thresholds method used for VaR estimation.

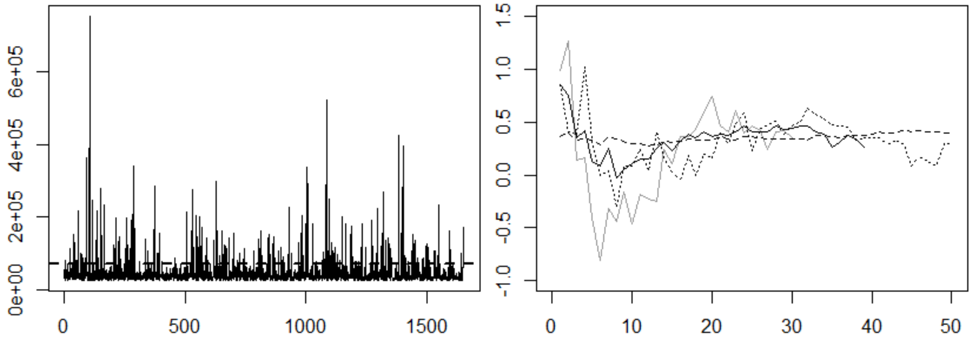

Figure 1. Sample 1 and estimates' sequences

Source: own study.

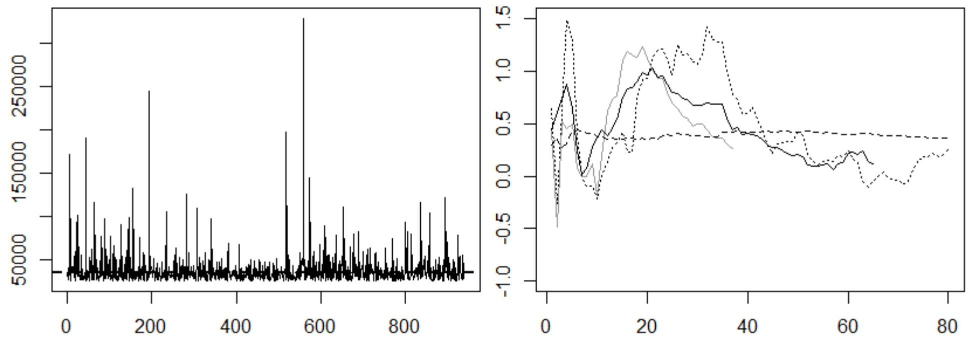

Figure 2. Sample 2 and estimates' sequences

Source: own study.

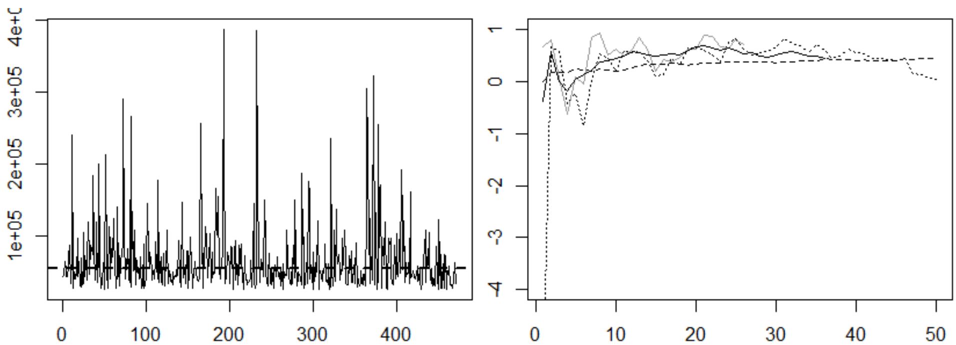

Figure 3. Sample 3 and estimates' sequences

Source: own study.

Table 4. Parameters' estimates

		sample 1		sample 2		sample 3	
		$\hat{\gamma}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\sigma}$
CML	*B*	0.3948	43,236.35	0.6240	8,026.65	0.6208	27,403.66
	BB	0.3560	44,360.26	0.6651	7,867.10	0.5612	28,163.92
	P	0.3941	43,254.43	0.7262	7,648.50	0.5750	27,981.03
	H	0.3359	44,982.08	0.3885	9,226.14	0.3582	31,540.35
ML		–0.3672	271,381.70	0.4055	9,117.55	–0.4077	148,241.91

Source: own study.

Table 5. Concordance with GPD, *p*-values of goodness of fit tests

sample 1		sample 2		sample 3	
–	+	–	+	–	+
0.000	0.108	0.000	0.569	0.000	0.077

Source: own study.

Moreover, negative ML-estimates of γ seem to be quite surprising, as they contradict tests' evidence (see Table 5), and common-sense visual evaluation of samples' plots. A probable reason is that the actual underlying law may not be the GPD one, and that fact biases our estimates. Similar question was raised by Gilli and Këllezi [2006], and Stachura and Wodecka [2012] with respect to financial time series data.

5. Summary

The presentation of the two approaches, and their empirical application clearly give evidence of some special features of the approach based on the EVT. Among these features the most important, which seems to be an advantage, is that one may be able to assess type and intensity of a tail heaviness appropriately, avoiding the impact of unsuitably chosen model distribution. In this context, if GPD is assumed to be the underlying law, the first step in the (A1) may be regarded as a specific test for the sing of parameter γ.

Concluding generally, it should be also noted that it is much more convenient to verify asymptotics of distribution tails at first – as long as the extreme values are concerned – and afterwards to model this distribution with a given theoretical approximate, rather than to find the approximate straightforwardly.

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