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Contents

Introduction

One of the fastest growing areas in the economic sciences is broadly defined area of finance, with particular emphasis on the financial markets, financial institutions and risk management. Real world challenges stimulate the development of new theories and methods. A large part of the theoretical research concerns the analysis of the risk of not only economic entities, but also households.

The first Wrocław Conference in Finance WROFIN was held in Wrocław between 22nd and 24th of September 2015. The participants of the conference were the leading representatives of academia, practitioners at corporate finance, financial and insurance markets. The conference is a continuation of the two long-standing conferences: INVEST (Financial Investments and Insurance) and ZAFIN (Financial Management – Theory and Practice).

The Conference constitutes a vibrant forum for presenting scientific ideas and results of new research in the areas of investment theory, financial markets, banking, corporate finance, insurance and risk management. Much emphasis is put on practical issues within the fields of finance and insurance. The conference was organized by Finance Management Institute of the Wrocław University of Economics. Scientific Committee of the conference consisted of prof. Diarmuid Bradley, prof. dr hab. Jan Czekaj, prof. dr hab. Andrzej Gospodarowicz, prof. dr hab. Krzysztof Jajuga, prof. dr hab. Adam Kopiński, prof. dr. Hermann Locarek-Junge, prof. dr hab. Monika Marcinkowska, prof. dr hab. Paweł Miłobędzki, prof. dr hab. Jan Monkiewicz, prof. dr Lucjan T. Orłowski, prof. dr hab. Stanisław Owsiak, prof. dr hab. Wanda Ronka-Chmielowiec, prof. dr hab. Jerzy Różański, prof. dr hab. Andrzej Sławiński, dr hab. Tomasz Słoński, prof. Karsten Staehr, prof. dr hab. Jerzy Węcławski, prof. dr hab. Małgorzata Zaleska and prof. dr hab. Dariusz Zarzecki. The Committee on Financial Sciences of Polish Academy of Sciences held the patronage of content and the Rector of the University of Economics in Wroclaw, Prof. Andrzej Gospodarowicz, held the honorary patronage.

The conference was attended by about 120 persons representing the academic, financial and insurance sector, including several people from abroad. During the conference 45 papers on finance and insurance, all in English, were presented. There were also 26 posters.

This publication contains 27 articles. They are listed in alphabetical order. The editors of the book on behalf of the authors and themselves express their deep gratitude to the reviewers of articles – Professors: Jacek Batóg, Joanna Bruzda, Katarzyna Byrka-Kita, Jerzy Dzieża, Teresa Famulska, Piotr Fiszeder, Jerzy Gajdka, Marek Gruszczyński, Magdalena Jerzemowska, Jarosław Kubiak, Tadeusz Kufel, Jacek Li-

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Wanda Ronka-Chmielowiec, Krzysztof Jajuga

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THE IMPACT OF DATA CENSORING ON ESTIMATION OF OPERATIONAL RISK BY LDA METHOD

WPŁYW CENZUROWANIA OBSERWACJI NA SZACOWANIE RYZYKA OPERACYJNEGO METODĄ LDA

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Abstract: The article raises the problem of determining the quantification of operational risk by LDA method for left-censored observations. In literature that concerns the modelling of operational risk, the term *censoring* can be found, but in practice it relates to the truncated observation. This article distinguishes truncated and censored data. Different properties of these processes are analysed by means of simulation, in the context of their impact on the size of operational risk estimates by the LDA method. In the world literature, the application of left-censoring process in operational risk has not been sufficiently analysed. Data censoring, which is a compromise between a full report of losses and collection of observation of only above mentioned level, consists in counting these observations that are below the accepted threshold. Such operations affect directly both the loss severity process and the process of loss occurrence frequency, which are components of the LDA model. What is very important in case of modelling is the uncertainty of the estimates in risk models.

Keywords: operational risk, censoring and truncation of severity distribution, Weibull distribution, LDA method.

Streszczenie: Artykuł porusza problem kwantyfikacji ryzyka operacyjnego metodą LDA dla obserwacji lewostronnie cenzurowanych. W literaturze dotyczącej modelowania ryzyka operacyjnego, można znaleźć określenie procesu rejestrowania danych jako „cenzurowanie", ale w praktyce najczęściej w tych przypadkach jest to proces bardziej rygorystyczny polegający na ucinaniu obserwacji. Artykuł rozróżnia ucinanie oraz cenzurowanie informacji jako dwa różne procesy. Różne właściwości tych procesów są analizowane za pomocą symulacji, w kontekście ich wpływu na wielkość szacunków ryzyka operacyjnego metodą LDA. W literaturze światowej zastosowanie lewostronnego cenzurowania w procesach dotyczących ryzyka operacyjnego nie zostało dostatecznie przeanalizowane. Cenzurowanie danych, które jest kompromisem pomiędzy pełnym raportowaniem strat a ich rejestrowaniem powyżej określonych progów, polega na zliczeniu tych obserwacji, które są poniżej określonego poziomu.

Takie działania mają bezpośredni wpływ zarówno na proces dotkliwości strat oraz na proces częstości ich występowania, które są składowymi modelu LDA. Jest to istotne w przypadku badania niepewności szacowania ryzyka oraz wiarygodności i stabilności wyników.

Słowa kluczowe: ryzyko operacyjne, cenzurowanie, ucinanie, LDA, symulacje.

1. Introduction

The issue raised in the article refers to the method of estimating operational risk in banks. The method of quantification of operational risk, which is the Loss Distribution Approach (LDA), analysed in this article, belongs to the Advanced Measurement Approach (AMA). Under this method, the quantification of risk is based on the aggregated distribution of losses as a quantile at the relevant level. In practice, determination at actual analytical form of distribution in LDA is not always possible. Therefore, various forms of approximation are widely used in practice.

The article focuses on the issues related to the estimation of relevant risk value at the level of LDA model only. A formal construction of LDA is identical as in the studies of Szkutnik [2012a] or [2012b]. Determination of aggregated distribution of loss is based on Monte Carlo method, which, together with its thorough description, is included in works of Szkutnik and Basiaga [2013] or Shevchenko [2011].

The main point of interest of this article is the estimation of influence of data (referring to bank losses) truncation and censoring processes upon the value of estimated parameters of continuous probability distributions for loss severity area. Observation truncation is commonly considered in modelling of operational risk, however, there are no significant studies of the process that apply the concept of observation censoring which is presented in the article. The goal of the article is to show the scale of potential differences in the estimation of risk value for the only one process modelled by LDA method. The study focuses on the area of loss severity and total results of the presented concepts.

2. Influence of data truncation and censoring processes in risk quantification

2.1. Modelling of events severity

A long history of the use of censored data and a wide spectrum of problems which arise in reporting processes results in the fact that there is no single way of dividing censored data.

There are four basic ways of classification of censored data. This definition duplicates the term *censoring* that initially was used to define the overall processes connected with information limitation. This definition also includes a detailed differentiation between a total information loss in case of observation truncation and its counting in case of censoring. The article uses the convention which differentiates these two processes. According to Millard [2013, p. 175] censored data can be divided as follows:

- truncated or censored data,
- left-censored data, right-censored data, two-side-censored data,
- single or multiple (progressive) censored data,
- censoring type I or type II.

In reference to operational risk, the simplest censoring process can be left-single type I censoring. This means that full reporting and estimation of data occurs only above the defined "c" level. Below this level the information concerning operational loss will be calculated without complete information (i.e., with no information regarding financial value of loss). Moreover, single censoring means that "c" level is constant for all observations. Type I censoring requires the knowledge of "c" level, then the number of uncensored observations is a random variable (in case of type II censoring the number of uncensored observations is known and that is why this type of censoring cannot be used in operational risk).

In widening the concept of censoring for operational risk it is good to mention that a constant level of reporting may be a limiting factor in many analytical areas. In such situations, floating levels of data censoring may be considered in particular operational processes that generate losses.

From a practical point of view, it seems justified to know the process of data censoring in reference to LDA method, particularly in consideration of potential errors related to the selection of modelling method and evaluation of influence of such a process on the value of estimated risk.

2.2. Construction of the likelihood function for censored and truncated observations for the estimation of model parameters

The presented construction of likelihood function assumes that the level value below, whose data will be subject to the process of truncation or censoring, is independent from the value of a random variable. The overall form of credibility function in the discussed case is as follows [Klein, Moeschberger 2003, p.74]:

$$L = \Pi_{i \in D} f(x_i) \cdot \Pi_{i \in R} S(C_r) \cdot \Pi_{i \in L} \big(1 - S(C_l)\big) \cdot \Pi_{i \in I} \left[S(L_i) - S(R_i)\right] \qquad (1)$$

The relation given in formula (1) shows a general problem which can be considered for various forms of observation truncation or censoring. The relevant values are: $f(x)$-observed loss value; $S(C_r)$: right-censored observation, $1 - S(C_l)$: left-censored observation, $[S(L_i) - S(R_i)]$: interval-censored observation. In case of data truncation, the values are: $f(x)/S(Y_L)$: left-truncated observations, $f(x)/[1 - S(Y_R)]$: right-truncated observations.

3. Construction of a simulation study

The simulation study adopts arbitrarily some values of parameters. The value of VaR is defined at 0,999 level and it is determined on the basis of aggregated loss distribution. It has been adopted that the number of iterations in Monte Carlo method (which is used to determine aggregated loss distribution) equals 10^6 .

Hypothetical period covered by the study equals 4 years. Poisson distribution with $\lambda = 50$ [Klugman et al. 2008, p. 102] has been adopted as a process of loss occurrence in a yearly scheme. Loss severity is consistent with Weibull distribution (with parameters in accordance with appendix A of Klugman study [Klugman et al. 2008, p. 675]), with variable values of parameters (they are given in a further part of the study) and it is marked as $Wbl(\cdot)$ (where τ means shape parameter and θ means scale parameter).

Weibull distribution was used in the issues concerning operation risk in such studies as Chernobai [Chernobai et al. 2007]. This distribution belongs to the family of transformed gamma distributions with $\mathbb{R}^+$ domain. Therefore, it is applied to describe the operational loss registered on the positive axis. Weibull distribution has a known property connected with the risk function h(·). Logarithm function h(·) is linear with slope parameter τ–1. It is known that the function, for Weibull distribution equals $h(x) = \tau / \theta^{\tau} \cdot x^{\tau-1}$, is a dynamic characteristic of the distribution which allows for classification in terms of shape of the distribution tail. In case of increasing (decreasing) h(·) for Weibull distribution parameter $\tau > 1(\tau < 1)$, the considered distribution will be thin-tailed (fat-tailed). Relevant parameter values were adopted for Weibull distribution, $\theta = 4$ and $\tau \in \{0,5; 0, 75; \ldots; 2\}$ (variable value τ is to reflect a potential influence of changes in distribution tail form on the value of estimated risk at various variants of information "censoring").

3.1. Simulation of severity process

In order to model loss severity distributions, three variants of data, dependent on a hypothetical process of data censoring or truncation, were adopted:

- Complete – complete scope of data is registered,
- Truncated – only observations above the known "c" level, are registered,
- Censored – where censoring "c" level is the same as for truncation scenario, data from below the level is counted.

It is also adopted that, depending on data availability, there are four methods for estimation of parameters of a relevant continuous severity process. It should be emphasized that the main goal here is to show the influence of the way of reporting depending on the selected estimation variant. Estimation variants, with reference to the nature of available data, are as follows:

- Basic variant. Complete data is available. Standard method of estimation (MLE).

- Truncated variant. Truncated data is available and it is considered in the process of estimation. Estimated data is truncated (variant of method given in formula (1)).
- Naive variant. Truncated data is available, but it is not considered in the process of estimation. Complete domain of data i.e. $\mathbb{R}^+$, not its improper subset, is estimated.
- Censored variant. Censored data is available, i.e.it is extended against truncated data by additional information concerning the frequency of loss occurrence below "c" level. The method of estimation (the variant of method defined by formula (1)) takes into account the fact of censoring in the process of optimization.

For each out of seven possible variants of $Wbl(\cdot)$ (constant θ and variable τ) distribution an identical scheme of simulation study was adopted and repeated 10^4 times (iterations).

A single iteration in a simulation study for selected distribution variant $Wbl(\cdot)$ assumes the availability of hypothetical random sample of 200 observations (four years, fifty observations per each year). On such a basis, three data variants have been constructed (basic variant – with no changes, truncated variant – information below "c" is lost, censored variant – information below 'c' level is counted). With reference to the variants which depend on 'c' level, the number of analysed cases will increase by the number of various reporting levels.

The values of quantiles of relevant levels (marked c(q)) have been adopted as 'c' point, i.e., the level above which there is a complete registration of data. The values of quantiles represent the points of left-censoring of data and they are to present a proportional hypothetical scale of information loss in each case. The values of quantiles will differ depending on the variant of $Wbl(\cdot)$ distribution parameters (seven variants). Basing on the defined levels of quantiles makes it possible to compare the results in view of percentage of lost information. It has been adopted that values $q \in (0; 0.05; 0.10; ...; 0.5)$ will be eleven-element-vector, where the first element will mean lack of information loss and the following elements will mean, respectively, 5%; 10%; ...; 50% loss of information.

Simulation study that concerns the influence of reporting in comparison with the method of estimation of the selected probability distribution can be analysed in various dimensions. Graphs in R^3scheme are presented in order to create a collective presentation of at least parts of results. The OXY plane will always represent a grid created from vector of value of shape parameter τ (representing the influence of change in distribution tail nature) and vector of level of considered quantiles (influence of information loss) for various measures presented on a vertical axis OZ.

The process of data reporting of the above defined level and the character of basic distribution, from which the data are originally taken, influences all the parameters of considered probability models. Therefore, 6 possible sets are presented (three pairs of parameters).

3.2. The impact of loss reporting method upon Weibull distribution parameters

At first, the analysis concerns the degree of proportional deviation of estimator values (these are two parameters for distribution *Wbl*(·) obtained from the estimation of truncated, naive and censored variants with reference to the values of estimated parameters of the basic variant (estimated for complete data). Such a combination is presented in Figures 1-3 (with indices (a) and (b), which indicate, respectively, changes of τ and changes of θ for pairs: basic/truncated variant, basic/naive variant, basic/censored variant.

Vertical axis OZ, visible in Figures 1-3, indicates the value of deviations expressed in percentage as an error MAPE [Cieślak 2005, p. 51]. The estimator of basic distribution is always the current value while the estimator of one out of three remaining estimation variants is the compared value.

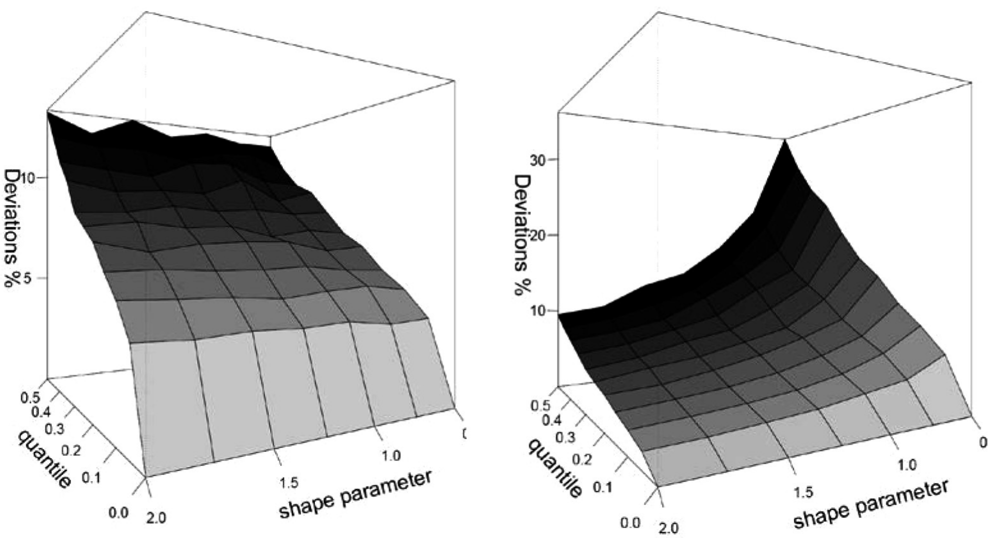

Figure 1. (a)-left;1(b)-right. Proportional differences in values of parameters τ and θ. Comparison of basic and truncated variants

Source: Author's own study.

The perspective of projections R^3 in Figures 1-3 shows the configuration where left axis of plane OXY concerns the percentage level 'q', which represents the influence of loss of least observation proportion. Influence of censoring or truncation of observation, can be seen as moving away from the beginning for q = 0 to the value of maximum rejection accepted at level of 50%. The right axis of plane OXY concerns the value of shape parameter of distribution *Wbl*(·), which represents the transition from a thin-tailed distribution to a thick-tailed distribution together with the heading of value τ from 2→0,5. The vertical axis OZ indicates the values of MAPE errors.

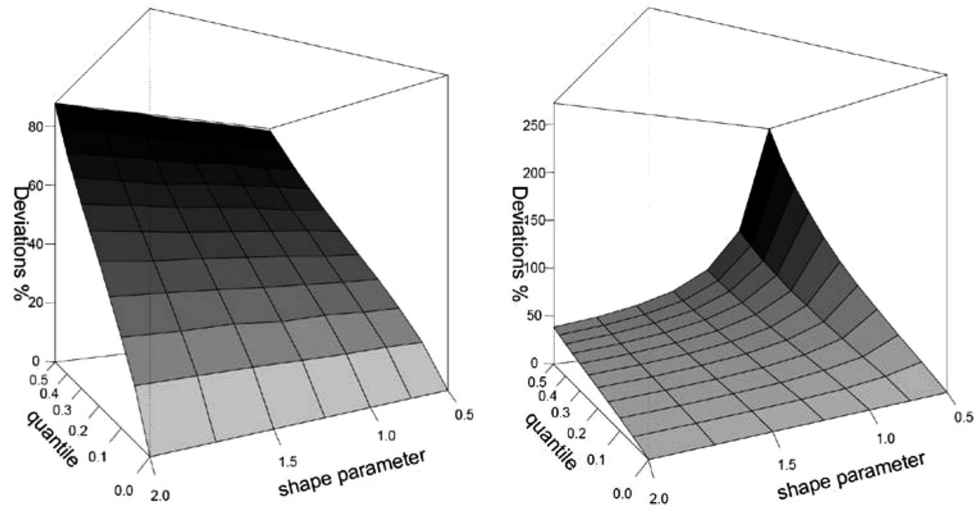

Figure 2. (a)-left; 2(b)-right. Proportional differences in values of parameters τ and θ. Comparison of basic and naive variants

Source: Author's own study.

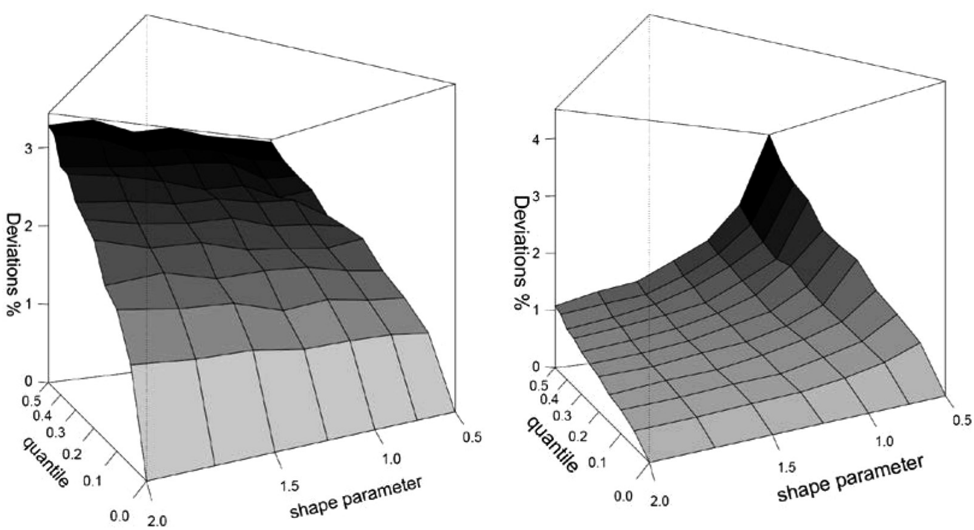

Figure 3. (a)-left; 3(b)-right. Proportional differences in values of parameters τ and θ. Comparison of basic and censored variants

Source: Author's own study.

In the process of comparing the obtained results, one can observe similar shapes of planes among various variants of estimation in reference with the basic variant within the area of the same parameters. For the results that concern the changes of estimators of parameter one can see a lack of sensitivity with regard to distribution tail (lack of significant interactions along the axis of plane OXY). As it comes to the influence of reporting level (expressed as a value of quantile ‘q’) one can observe almost a linear dependence between the particular steps for left axis of plane OXY against the percentage values from axis OZ.

The analysis of proportional changes of parameter 0 looks similar, i.e., the planes in parts (b) in Figures 1-3 show some similarities. Diversity is visible similarly to parameter τ in the range of OZ axis. In this case, in contrast to the results for parameter τ, here the main differences can be expected at the biggest loss of information (q close to 50%), but in connection with a fat-tail-nature of the severity distribution.

A rapidly increasing plane in the back part of the presented perspectives together with the moving away of the value at the plane OXY from the point ($\tau = 0$, $\theta = 2$) reflects the nature of the phenomenon. Similarly, consideration of data censoring causes maximum deviations of a few percent in the most extreme cases and few-dozen percent deviations in case of truncated variant. In case of naive variant, the differences are more than twofold.

Such an example makes it possible to demonstrate to what extent the selection of reporting level, in connection with the change of modelling of complete range of data, may influence various practical variants. Therefore, it should be emphasized which possibilities lie in the change of concept of modelling based on the censored variant.

With reference to operational risk, the reporting level does not constitute a technological barrier. It is possible to register all the losses. Many times the losses are registered in bank accounts’ systems, however, they are not always available in operational loss data base or they are omitted in the process of modelling. The most important practical aspect of reporting below the defined level has economic dimension, e.g., the employee’s commitment in the process of loss reporting.

In order to avoid needless procedures that limit the scale of reporting, one can propose, for instance, collective reporting with reference to events below the defined ‘c’ level in clearly defined time units. i.e., weekly or monthly time schemes. In such a case it is possible to avoid the necessity of complete loss registration, together with the assignment of its owner (department or organisation unit), while the aggregate that calculates loss occurrences will allow a further processing of such information, both in terms of severity distribution estimations and also consideration of such information in the process of modelling of loss frequency.

Moreover, the introduction of censoring as a possible variant in estimation of severity distribution parameters can be used in stress tests, the realization of which partly results from the records included in the BCBS directive [BIS 2010, 2011, 2013].

3.3. The impact of loss reporting method upon the value of risk estimated by LDA method

The results of the first phase of the study, which concern the impact of the reporting method upon the estimators of Weibull distribution for various variants of estimation gives, in a sense, a direction of further studies. Proportional deviations for parameters show that the estimations derived from naive or basic variants can be used as input parameters for final LDA models. Such an operation will make it possible to emphasize the significant differences between the two variants that represent different status of knowledge and quality of modelling. The selection of basic variant is motivated by slight deviations of parameter values against censored estimation. The naive variant constitutes some kind of border point of reference and each case that increases given information, i.e., consideration of truncated or censored estimation, will be located between this variant and basic variant. In determination of VaR values from LDA model it has been adopted that the values that describe the loss frequency given originally as Poisson distribution will be respectively decreased for these LDA models which refer to the naive estimation with value of ‘q’ quantile levels. This is a correct action, when one assumes that a homogeneous Poisson

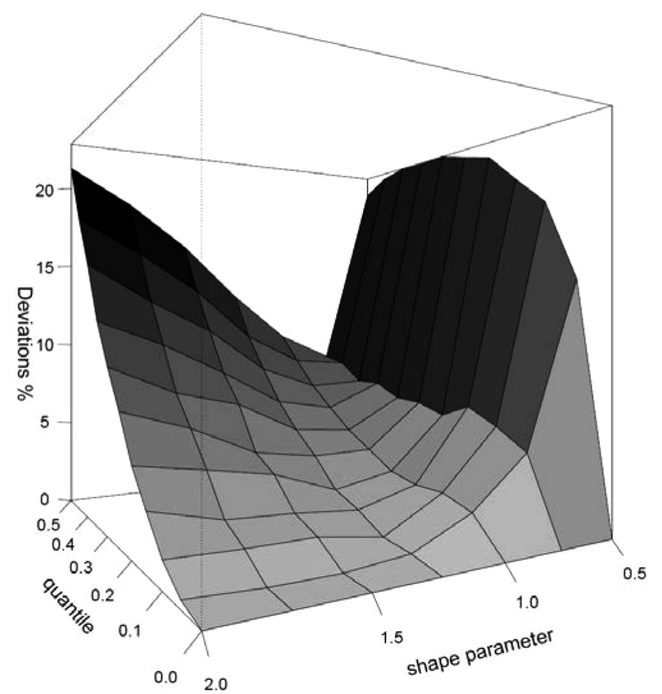

Figure 4. Proportional deviations as positive values

Source: Author’s own study.

process will be the process of frequency (but it is not true for all the cases). This means that if a defined result concerns the case of ‘q’ quantile level, then the LDA method assumes that the original frequency E(N) = λ will be replaced by the value $(1-q)\lambda$. Such an action results from the nature of Poisson distribution, particularly, from theorem concerning addition and Rajkow theorem (compare: Fisz [1969, p. 103, 157] and Shevchenko [2011, p. 181]).

The most important collective results are presented in Figure 4. Similarly to Figures 1-3, the description of projections is identical as it comes to OXY plane. In Figure 4 it can be observed that the process of censoring determines the precedent in definition of risk value. Figure 4 shows a proportional decrease as positive values (upward deviations from OXY plane). Some theoretical barrier for shape parameter can be observed, $\tau = 1$. With such a parameter, Weibull distribution will particularly be an exponential distribution, which separates the border of thick-tail and fat-tail natures. From this different dynamics of changes (separated against values) follows:

4. Conclusion

The performed simulation study had a double nature. The first nature was directly associated with the verification of scale of variability of Weibull distribution parameters, with the use of various modelling concepts in the process of its parameters estimation. The study was motivated by practical reasons related to the process of reporting and loss registration with reference to losses occurring in banks. The considered approaches, in the discussed case, make it possible to clearly state that the process of data truncation has a significant impact on the process of parameter estimation for continuous probability distributions. The application of naive approach, which is often used in banking practice, does not allow to estimate the scale of phenomenon in a credible way.

Moreover, in case of truncation process the actual proportion of lost information will have a significant impact on the values of estimated parameters. In case of actual availability of a single series of loss values, on the basis of which parameters are estimated, one can expect significant discrepancies in case of high levels of data truncation, as referred to the actual process that in fact will be unknown (the process of data truncation misses the information concerning actual proportion of rejected information).

Parts of these disadvantages can be omitted in the approach that assumes that the amount of lost detailed information is known (information is aggregated as observations below the defined ‘c’ level). In practice, it is connected with the change of concept of data registration and transition from the truncation process to the information censoring process. Of course, a complete registration of data would be an ideal solution because it would allow a total freedom in terms of data processing. However, it could be related to an over-expanded system of loss reporting. In such a context, the process of censoring allows, to a significant extent, the maintenance of original information concerning distribution parameters, of course within reasonable limits of censoring.

It should be emphasized that the knowledge of original process that generates data will not be available in practice and the only possibility is to select a proper distribution by means of relevant statistical tests. This fact constitutes an additional hindrance in the process of constructing models that concern loss severity, however, it has not been considered in this article.

With reference to the second phase of the study, its main aim was to verify the extent to which the process of data truncation influences the value of VaR (0,999), with the assumption of constant parameters for severity processes. On the basis of presented information one can conclude that the process of data truncation significantly disturbs the results, even by several percent, particularly with reference to thick-tail-distributions. In the considered case, these are Weibull distributions with parameter $\tau < 1$. For thin-tail-distributions such influence has no such rapid course, which constitutes, a lesser threat of incorrect evaluation and underestimation if risk value.

Data censoring has an impact both on the processes of loss frequency and loss severity. The approach based on data truncation is dangerous for fat-tail-distributions, as the influence of lowering the frequency against the actual number of occurrences has the most destructive influence on the value of the estimated risk. In connection with the incorrect way of estimation of severity distributions by means of the naive method, the scale of single losses will also be significantly disturbed. The process of severity distribution estimation that assumes data truncation can significantly improve the maladjustment of the area of operational loss severity that results from limited information. However, in this case the area of frequency will also be underestimated, as it comes to the so-called high-frequency/low-severity events, i.e., the events with a high frequency and low severity. The process of censoring, as shown by the simulation study, can, to a high extent, improve the credibility of estimation of parameters within the scope of loss severity modelling. What is most important, the approach based on data censoring in such a case, i.e., with reference to operational risk modelling by means of LDA model, gives almost complete knowledge of the events frequency.

The proposed approach concerning the change of way of operational loss registration in banks must be extended in terms of some analytical solutions. Also, the aspect of adjusting truncated and censored distributions to actual data is highly important in case of testing and evaluation of correct adjustment of considered theoretical distributions that present some hypothetical process related to loss generation.

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