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DEGRESSIVE PROPORTIONALITY – NOTES ON THE AMBIGUITY OF THE CONCEPT

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Abstract. This article discusses the problem of the definition of degressive proportionality. This term is used in many areas of economics but its meaning is not standardized. It is understood differently when related to tax issues compared to distribution of seats in the European Parliament. The differences are small but significant. The article provides and analyzes the different meanings of the concept of degressive proportionality and proposes to introduce two new concepts – concave degressive proportionality and convex degressive proportionality.

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1. Introduction

Degressive proportionality is a compromise between equality and proportionality. If the distribution of goods or receivables may not be proportionate or equal, degressive proportionality (DP) is a natural alternative. The term was widely popularized as the way of allocating seats in the European Parliament among the Member States of the European Union and has been included into the Treaty of Lisbon as the nature of the allocation. At the same time the term of degressive proportionality is also used in other economic issues related to, for example, with taxation. However, in this case, it is defined in a slightly different way. In addition, degressive proportionality in the form proposed by the EU legislation does not appear to be entirely consistent with the idea of the implementation for which it has been established. This is probably the consequence of excessive conciseness in clarifying the DP contained in the report (Lamassoure, Severin 2007). DP within

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the meaning of Community law covers a wider class of divisions than in fact is taken into account when allocating parliamentary seats. Although most of the proposed methods of the distribution of seats in the EP (parabolic method (Ramírez González 2007), shifted proportionality – fix + prop – (Pukelsheim 2010), or its specification – base + prop method contained in the Cambridge Compromise (Grimmett et al 2011)) is consistent with the understanding of the DP in other areas of economics, at the same time it recognizes as degressively proportional the whole range of divisions, which in these areas would not be considered degressively proportional. At this point a unification of terminology seems to be most desirable. Until that happens one should be aware of this diversity.

The issue of the formal definition of degressive proportionality is widely discussed in the paper (Dniestrzański and others 2013). Definitions proposed and discussed there, however, are based exclusively on the findings of the Lisbon Treaty and successive acts of the Community.

2. Degressive proportionality in the European Parliament

The basis of degressive proportionality, under European Union law, are two principles formulated in the report (Lamassoure, Severin 2007).

The principle of fair distribution – no State will have more seats than a larger Member State or a smaller amount of seats than a smaller Member State.

The principle of relative proportionality – the ratio of the population size to the number of seats is greater the larger the State, and smaller the smaller the State.

The principle of fair distribution in later considerations will be referred to as DP1, and the principle of relative proportionality as DP2. Additionally, the allocation of seats in the EP is subject to so-called boundary conditions fixing the minimum and maximum number of seats that can be allocated to a Member State and the total number of seats in the EP. The Lisbon Treaty establishes the conditions as follows:

B1. The minimum number of seats a Member State receives is $m = 6$,

B2. the maximum number of seats a Member State receives is $m = 96$,

B3. the total number of seats in Parliament is $H = 751$.

The importance of boundary conditions in divisions DP is discussed in an article (Łyko 2012) and in publications (Misztal 2012) and (Dniestrzański et al 2013).

Let $f(x)$ be the allocation function[1] of establishing the allocation of seats in the EP. The DP1 principle says that the function $f(x)$ is non-decreasing. The DP2 condition says, however, that the segment joining points $(0,0)$ and $\big(x, f(x)\big)$ must have a smaller inclination angle of abscissa the greater x is. The condition DP2 in the presented interpretation will henceforth be referred to as **angular condition**. How would one describe the DP1 and DP2 conditions assuming the differentiability of the $f(x)$ function? The DP1 and DP2 conditions require the function $f(x)$ to be non-decreasing and

$$\frac{f(x)}{x}$$

non-growing, which is equivalent to inequalities

$$f^{I}(x) \geq 0 \quad \text{and} \quad f^{I}(x) \leq \frac{f(x)}{x} \quad \text{respectively.}$$

Thus, degressive proportionality here means that the allocation function must satisfy the relation:

$$f^{I}(x) \in \left\langle 0, \frac{f(x)}{x} \right\rangle \tag{1}$$

The closer the derivative of the function $f(x)$ is to 1, the closer to proportional allocation the distribution becomes. In turn, the closer to 0, the distribution is closer to an equal division. The values $f^{I}(x)$ can be treated as an indicator of the strength allocation degression, where the approaching equal distribution means an increase in the strength of the degression of division. The equal division is an extreme example of degressive proportionality. In the case of restrictions imposed by boundary conditions, an equal division may, however, be unreachable. For example, conditions B1 and B2 make it impossible to achieve an equal distribution. Reflections on the strength of the divisions' degression can be found in Haman (2007).

[1] The number of seats falling to a Member State must of course be a natural number. At this point we do not deal with the issue of rounding. This is a separate problem that has been solved to some extent by the resolution of the European Parliament (Gualtieri, Trzaskowski 2013). which states that, among other things, the division must comply with the principle of DP before rounding. Therefore in practice distribution can be carried out taking into consideration only this principle at the level of quotas.

3. Degression concave and convex

The essence of degressive proportionality is the division that assigns the weaker more than the proportional division would have offered, and the stronger less than on the basis of proportional allocation. For example, the allocation of seats in the EP by the base + prop method makes Member States with a population of less than the EU average gain or not lose, whereas Member States with a population larger than the EU average will lose or not gain additional seats. In the case of other methods which comply with the DP, the boundary dividing the contenders into the ones gaining and losing in relation to the proportional division may be positioned differently.

Two further examples will show what the problem is with the definition of degressive proportionality.

Example 1. Let there be a sequence $P = (p_1, p_2, p_3) = (1, 2, 3)$. What values can a sequence take (with elements that are positive integers) $M = (m_1, m_2, m_3)$, with the boundary condition to $m_1 + m_2 + m_3 = 10$ for the division to be called degressively proportional? You will notice that there are only two sequences meeting the DP1 and DP2 conditions: $M_1 = (2, 4, 4)$ and $M_2 = (3, 3, 4)$ (Figure 1).

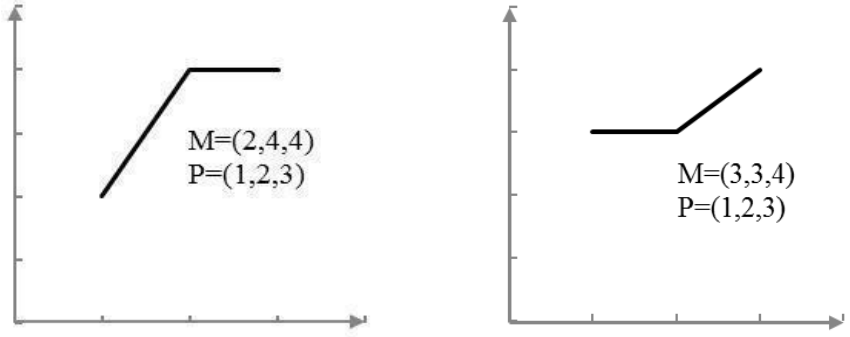

Fig. 1. Degressively proportional distributions

Source: own work.

Both distributions are degressively proportional, or using the equivalent terminology (Florek 2011), both sequences $M_1 = (2, 4, 4)$ and $M_2 = (3, 3, 4)$ are degressively proportional to sequence $P = (1, 2, 3)$. What is the fundamental difference between these distributions? The broken line connecting points (p_i, m_i) is in the case of distribution of $M_1 = (2, 4, 4)$ concave and for

$M_2 = (3, 3, 4)$ convex. At the same time the distribution $M_1 = (2, 4, 4)$ is more favorable for medium contenders[2] as opposed to the $M_2 = (3, 3, 4)$ distribution which prefers the weaker contenders. The question of the degree of preference of different categories of contenders depending on the degree of concavity or convexity is an important issue that requires accurate theoretical analysis. Let us look at another example.

Example 2. Figures 2-5 are examples of functions representing the four different classes of allocation functions. The f_1, f_2, and f_4 functions are suitable for the construction of degressively proportional distributions within the limitations of Community law. In the case of the f_1 function, f_2 and the f_4 angle condition is met. Only the f_3 function cannot be used to model DP. For example, if f_3 as given by the formula $f_3(x) = x^2$ we would have $f(1)/1 < f(2)/2$, which is contrary to the condition of DP2. But one can also see that in this case the angle condition is not met. In the case of function f_4 the angle condition is met, but in contrast to function f_1 and f_2 the function is convex. The distribution is based on the allocation function f_4 which results in that the greater the state, the greater the increase in the number of seats with a given increase of population. In the case of twice differentiable function f_4 this is of course equivalent to inequality $f_4^{II} \geq 0$.

Degressively proportional distribution based on a concave allocation function is proposed to be called **concave degressively proportional distribution** or **concave degression** and based on convex functions – **convex degressively proportional distribution** or **of convex degression.** In the case of distribution $M_1 = (2, 4, 4)$ of Example 1 and distributions based on functions f_1 and f_2 in Example 2 we are dealing with examples of concave degression. In his article, Dniestrzański (2011), demonstrated that if the non-decreasing function of the allocation satisfies the $f(0) > 0$ condition, the concavity of the function f is a sufficient condition of degressive proportionality. If the function f is convex, the situation is more subtle. Take for example the $f(x) = x^2 + a$ function with $a > 0$. It is easy to show that for $x \geq 0$ this function satisfies DP1, DP2 and the condition is met by $x \in \langle 0, \sqrt{a} \rangle$ only. This case is easily generalized to the class of $f(x) = x^n + a$ functions, $a > 0$ DP2 condition is then satisfied only for

[2] In this example, there is one "average" pretender. It is easy to see, however, that this is in a general case if the broken line connecting points would be an allocation function for more contenders.

$$x \in \left\langle 0, \left(\frac{a}{n-1} \right)^{\frac{1}{n}} \right\rangle .$$

Thus, in cases when the allocation function is convex, condition $f(0) > 0$ is not sufficient for degressive proportionality.

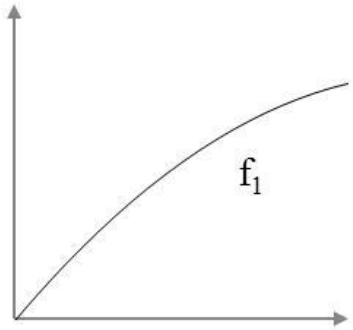

Fig. 2. The allocation of the DP
For example
$f_1(x) = \sqrt{x}$

Source: own elaboration.

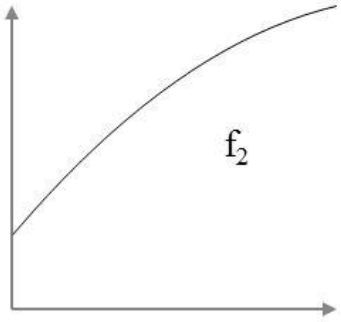

Fig 3. The allocation of the DP
For example
$f_2(x) = \sqrt{x} + a\,,\, a > 0\,,\, x \in \langle 0,1 \rangle$

Source: own elaboration.

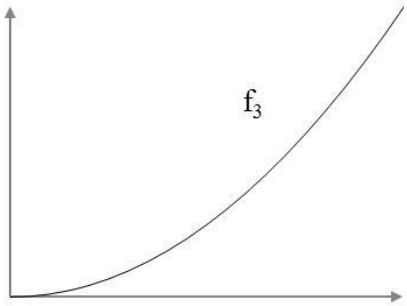

Fig. 4. The allocation of the DP
For example
$f_3(x) = x^2$

Source: own elaboration.

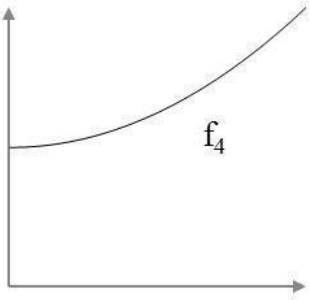

Fig. 5. The allocation of the DP
For example
$f_4(x) = x^2 + a,\ a > 0\,,\ x \in \left\langle 0, \sqrt{a} \right\rangle$

Source: own elaboration.

Differentiation between concave and convex degression is essential for economic modeling. In the case of the distribution of seats in the EP, concave degression means that the larger the state, the smaller the increase in the number of seats with the increase of population. This seems to be in line with the idea of degressive proportionality. All the methods of distribution mentioned in the introduction (shifted proportionality, fix + prop and base + prop), and other methods known from literature give (before rounding) concave degressively proportional distributions. Concavity allocation seems to be an (unwritten) rule associated with degressive proportionality. In some studies on this issue it is even considered that degressive proportionality implies the concavity of the allocation function. However, as can be seen from the above analyses these are not equivalent conditions. Keeping strictly to the definition of degressive proportionality of EU legislation, divisions based on certain classes of convex functions are fully eligible to be taken into account in the construction of the composition of the EP.

What is the problem of distinguishing concave and convex degression in taxes? Degressive tax is mentioned mostly in a situation of decreasing tax rates on fixed levels of income. This is the direct equivalent of concave degression. One can meet a broader definition of degressive tax saying that it is a kind of taxation, in which the average tax rate falls as income rises. It is then equivalent to DP1 and DP2 and with appropriate future interest rates it is possible to achieve convex degression. However, in all cases known to me, if this kind of tax is applied it has the character of concave degression.

4. Summary

The introduction of the concepts of convex and concave degression as special instances of DP will introduce greater precision in the used methods of distribution. What is now meant by the term degressive proportionality is the practice of a concave degression. In special cases in order to use DP distribution it has to have (at least locally) a convex nature. Such a situation may arise in the case of indivisible goods with contenders that are not greatly diversified – for example, the allocation of seats in the EP among Member States. However, this is a consequence of boundary conditions and not a form of deliberate action.

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