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OPTIMIZATION OF DEPLOYMENT OF GOODS SUPPLY IN A HIGH STORAGE WAREHOUSE BY MEANS OF THE GREEDY ALGORITHM

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Abstract. A warehouse is one of the key areas in logistics. A large group of logistic problems is associated with the storage process. The optimization of logistic processes undoubtedly brings many benefits for businesses, both financial and organizational. This paper presents one of the most common problems which occurs in a specific type of warehouse – high storage warehouse. The problem is the location of the supply of goods in a high storage warehouse. Due to the large computational complexity of the problem, the paper proposes to use heuristic algorithms to solve the problem. The high storage warehouse is also a system that allows the comprehensive management of logistics operations in the warehouse by using information from a database and the corresponding data processing algorithms. The presented approach assumes the implementation of heuristic algorithms in the logistic warehouse management system and uses them in the high storage warehouse. This will allow the material management in the warehouse to become easier, faster and the decision-making to become more accurate.

Keywords: traveling salesman problem, high storage warehouse, greedy algorithm.

JEL Classification: C15.

1. Introduction

The optimization of the logistics processes and the easy access to complete information is essential in the management of each company. The storage of goods is one of the basic elements of logistics, and is related to the fact that the time of production of individual goods does not coincide with the time of their delivery and consumption. To overcome these differences, it is necessary to store the inventory in the respective warehouses. A high storage warehouse is a specific type of warehouse. It is a building of a great height and large storage area, designed to store goods.

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In order to support decisions in this type of warehouses, modules of ERP II systems are used, which support the problem management process in the warehouse. Warehouse management system (WMS) supports the decision making process in the warehouse and allows to manage the storage space by giving a bright and clear picture of use of the storage area, and the location of the stored goods. Furthermore, it provides information on the storage conditions of different goods, e.g. air temperature, humidity, sunlight, etc.

The aim of this study is to describe a particular type of warehouse, which is the high storage warehouse, and also to present a module of a system which manages the logistic processes inside the warehouse. The main objective of the article is to discuss and analyze one of the selected problems which occurs in the high storage warehouse and propose one of the optimization methods to solve it.

Due to the high computational complexity of the presented problem, it is necessary to propose a method that allows solving the problem in a satisfactory time, with a circulation of minimum cost. Such effects are achievable by using heuristic algorithms, because they minimize the number of solutions of the problem taken into consideration by rejecting variants that do not have a chance to succeed.

Heuristics algorithms are used when the solution is not obvious and requires a large amount of time, due to its computational complexity (sometimes it is well beyond cost-effectiveness to consider all of the solutions) (Cormen et al., 2004). Heuristics is an example of a technique that gives satisfactory solutions with an acceptable outlay of both cost and time, but without the guarantee of achieving an optimal solution. It often happens that the solutions are close to optimal. Due to the large computational complexity of the problem discussed in this paper, which is the traveling salesman problem, we can calculate the solution by using some heuristics, namely heuristic algorithms. Mathematical programming is important for both theoretical and practical aspects and provides the basis for many computational algorithms optimization problems (Sysło et al., 1993). Heuristic methods are used where one exact algorithm does not exist, and the classic models are not reaching satisfactory solutions and are not giving a guarantee of an optimal solution of this problem. Heuristic algorithms deserve special consideration when solving problems of a high computational complexity (where the exact algorithm fails), especially when solving NP-complete problems.

The article consists of five sections. The first one shows the definition and characteristics of a high storage warehouse. The second section contains a description of the selected optimization problem occurring in a high-bay warehouse. The third section focuses on a detailed description of the problem of the optimal placement of goods in the warehouse, taking into account the rate of rotation. The fourth section is devoted to the presentation of selected optimization methods available in the literature, which can be used to solve the chosen problem. At the end – in the conclusion section – we focus on the information gathered, draw conclusions and propose to continue further research and discussion.

2. High storage warehouse

2.1. Warehouse

A high storage warehouse is a building with a large storage area and great height, equipped with a multi-level shelving system for different purposes and different principles of operation, allowing the storage of goods for the purpose of manufacturing or logistic processes. The characteristic elements of the storage facilities are:

- fixed racks, shelving tunnel LIFO types;
- forklift trucks operated manually or automatically;
- lifts and cranes;
- transport lines with a crossover system;
- transport containers;
- pallets.

High bay warehouses are mechanized or automated warehouses. The amount of storage exceeds 7.2m and such warehouses are usually equipped with automated computer controlled manipulators. It should be noted that the costs of such storage are quite high and almost five times higher than the cost of the construction and the equipment of a standard warehouse (Gołembska (Ed.), 2001).

2.2. High storage warehouse management system

The high storage warehouse management system (WMS) is a system module used in high-tech warehouses. This module is useful especially in high bay warehouses, where the internal structure and the storage processes are complicated. This module allows reflecting the hierarchical structure of the warehouse for any number of levels of hierarchy. The inventory management module is a computer-controlled system which supports warehouse management processes, which include:

- support for supply,
- support for order picking,
- expedition,
- inventory,
- reorganization of the warehouse.

The basic elements of the system are:

- tools for warehouse management,
- tools for records storage operations,
- tools for marking goods,
- monitoring tools.

This module provides the optimum flow of products and information in the warehouse compatible with the mission and strategy of a company. A high storage warehouse is a type of logistic system with the advantage of storage elements, which provides the management of a series of technical and organizational activities and also participates in the phase of location and supply.

All warehouse operations can be performed using mobile terminals with bar code scanners, with forklift trucks, by using terminals or desktop computers. Personnel management functions allow splitting the work between the operators and controlling their work with full service access rights. The system enables the faster completion of goods in stock than in standard warehouses (not managed with computer) and provides a more effective deployment of goods. Personnel management in WMS is also becoming more efficient through a clear division of responsibilities, full control of time and the state of implementation of tasks by the employee. The system is able to reflect the actual organization of storage space.

For each of the elements of the warehouse structure one can assign individual storage parameters, such as dimensions of products (land use), temperature of storage (special conditions for storage of goods) and temporary lock slot.

3. Description of the optimization problem

The optimization problem raised in this article is the optimal placement of goods in the warehouse according to the inventory turnover ratio. Inventory turnover ratio is used to measure the inventory management efficiency of a business. In general, a higher value indicates better performance and lower value means inefficiency in controlling inventory levels. A lower inventory turnover ratio may be an indication of overstocking, which may pose the risk of obsolescence and increased inventory holding costs. However, a very high turnover may result in the loss of sales due to inventory shortage. Inventory turnover is different for different industries. Businesses which trade in perishable goods have a higher turnover in comparison to those dealing in durables. A comparison would be fair only if made between businesses of the same industry.

A function analysis of rotation of the goods provides information on trade in goods in a given period of time and enables one to know the market trends and needs in constantly changing conditions. Inventory turnover ratio allows evaluating the effectiveness of inventory management by the company (Zaleska, 2005). Analysis of the indicator allows one to easily manage the process of purchasing goods and minimizing inventory in stock.

From the rate of rotation of the goods analysis, one can place orders directly without having to go into the complex analysis of historical orders. Based on a rotation pointer, the corresponding goods can precisely determine the demand for the commodity in a given point of time.

The aim of the project is minimization the discharge time of goods, taking into account the inventory turnover ratio. In its implementation may indicate limitation:

- the number of empty locations in the warehouse has to be at least equal to the number of goods for unloading,
- storage conditions comply with the conditions required,
- each location can be visited only once,
- the end point of the previous location of the transition can be the starting point for a new location.

4. Problem definition

The problem definition is the optimal placement of goods in the warehouse according to the inventory turnover ratio.

Stage 1: The first stage is an analysis of the inventory turnover ratio. This analysis shows how many days in a calendar year the company stores particular goods. On average, the value of the indicator should be between 37 and 52 days. If there is a large diversity of inventory in the warehouse, inventory turnover ratios should be used for calculating groups of stock.

Assumptions:

– the number of batches of goods in delivery is well-known,

– each item has its inventory turnover ratio,

– goods on the order list are serialized in descending order according to the inventory turnover ratio:

$$Rd = \frac{s * m}{o}, \quad (1)$$

where:

Rd – warehouse inventory turnover ratio;

s – inventory level per day X;

m – number of days elapsed from the beginning of the year;

o – turnover at the end of the month, obtained from the beginning of the year.

Generally, a high inventory ratio means that a company is efficiently managing and selling its inventory. The faster the inventory sells, the less funds a company has tied up. Companies have to be careful if they have a high inventory turnover as they are subject to stockouts.

If a company has a low inventory turnover ratio, this indicates that inventory has been stocked for a long time, because the stocks are rarely ordered. Such a situation certainly contributes to reducing company's profits. Obviously, there are reasons for which a company may be holding a lot of inventory – there may be goods that have a long shelf life and a long waiting period (from the manufacturer)

The determination of the inventory turnover ratio is the first of the five stages of deployment solution to the problem of the supply of goods in stock. According to this indicator, at a later stage (in descending order) goods will be delivered by the warehouseman onto the shelf. In this way, the goods with a high rate of rotation will be stored nearer to the unloading zone.

Stage 2: In the second stage of the solution to the problem, we should determine the number of *p* courses that the warehouse worker will have to take in order to deliver the goods onto the shelves. Because of the limited capacity of the forklift, whose purpose is the distribution of goods inside the warehouse, it is possible to determine the quantity of goods to be loaded for a single forklift course.

Constraints:

– the weight and dimensions of goods are known,

– the forklift capacity is known,

– the goods are loaded/unloaded in accordance with the principle of LIFO.

On the second stage of the solution to the problem, the number of p courses which the warehouse worker will have to take to deliver the goods should be fixed.

$\{Rd_1, Rd_2, Rd_3,$	$Rd_4, Rd_5, Rd_6, Rd_7,$	$Rd_8, Rd_9, Rd_{10}, Rd_{11},$	$Rd_{12}, \ldots, Rd_m\}$
p_1	p_2	p_3	p_n

$$\min \{Rd_1, Rd_2, \ldots, Rd_m\}. \tag{2}$$

According to formula (2), we will load the forklift up with goods in such a way that at the bottom there will be items with the lowest inventory turnover ratio. The distribution of the refills of goods should begin with the course that contains the item with the highest rate of rotation of goods.

Stage 3: The third step is to determine which element will be minimized to get the best solution. The traveling salesman problem can be minimized, following the distance criterion, and expecting the shortest route. We can also use the time criterion, then it will be the fastest route. There is also a criterion of cost minimizing, where we expect the cheapest route.

Problem: How to plan the location of goods in the warehouse in such a way that the warehouseman starting from the start point (START), and returning to it, takes the shortest possible route?

Goal: minimization of:

- distance (in units of distance): the shortest route;
- time (in units of time): the fastest route;
- the real cost (money): the cheapest route.

Stage 4: With respect to the traveling salesman problem (TSP), traveling salesperson is a warehouse worker, who asks the following question: given a list of goods for distribution and the distances between each pair of locations in the warehouse, what is the shortest possible route that visits each location only once and returns to the unloading zone?

A linear programming model for solving the traveling salesman problem involves minimizing the objective function, which is the sum of all the Hamilton cycles in the graph.[1]

[1] Salesman route is a cycle which is passing through every vertex graph exactly once.

The objective function:

$$Z(X) = \sum_i \sum_j c_{ij} \, x_{ij} -> \min \tag{3}$$

$$CX -> \min \tag{4}$$

$$c_{11} x_{11} + c_{12} x_{12} + \ldots + c_{23} x_{23} -> \min \tag{5}$$

$$c_{ij} \geq 0, \; n \geq k, \tag{6}$$

where:

$x_{ij} = 1$ (do follow the path from i to j);
$x_{ij} = 0$ (do not follow the path from i to j);
n – number of empty locations;
k – number of goods to be located;
c_{ij} – route cost from location i to j.

when $k = n$, then permutations Number of permutations = $(n)!$ for $n = 5$, $(5)! = 120$	when $n \geq k$, then variations without repetition $V_n^k = \frac{n!}{(n-k)!}$ $n = 10$, $k = 5$ $V_{10}^5 = \frac{10!}{(10-5)!} = 30{,}240$

Fig. 1. Number of permutations/variations without repetition

Source: own elaboration.

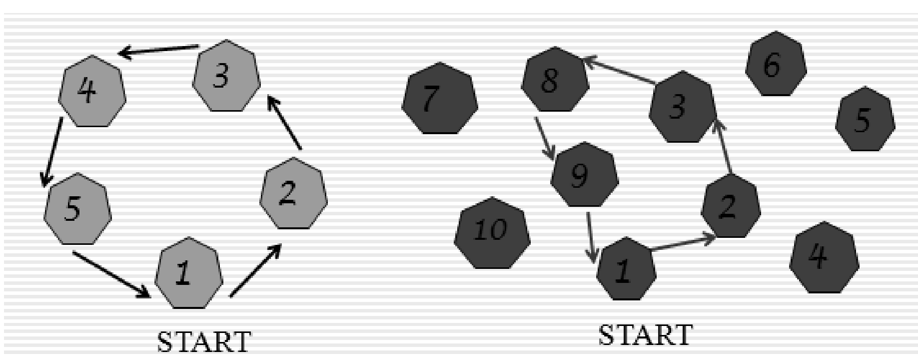

Fig. 2. Permutations and variations without repetitions
for the traveling salesman problem

Source: own elaboration.

5. Proposed solution of the problem

Brute Force Algorithms (called “brute force”) rely on the successive checking of all the possible combinations in search of a solution (Wróblewski, 2009) and determine all the Hamiltonian cycles. Such a technique allows finding the optimal solution, but only after searching through all the possible variants. This method is cheap and effective, but for more complex cases it is time-consuming and costly. In the case of the problem raised in this article – empty locations in the warehouse, of which there may be thousands – it is necessary to use methods that in no time and with low inputs give comparatively satisfactory results.

5.1. Greedy algorithm

“A game like chess can be won only by thinking ahead: a player who is focused entirely on immediate advantage is easy to defeat” (Vazirani, Dasgupta, Papadimitriou, 2006).

A greedy algorithm is an example of an uncomplicated algorithm that suggests phased approaches to solving complex computational problems. The heuristics is based on making every stage of locally optimal choices with the hope of finding the global optimum (Cormen et al., 2004).

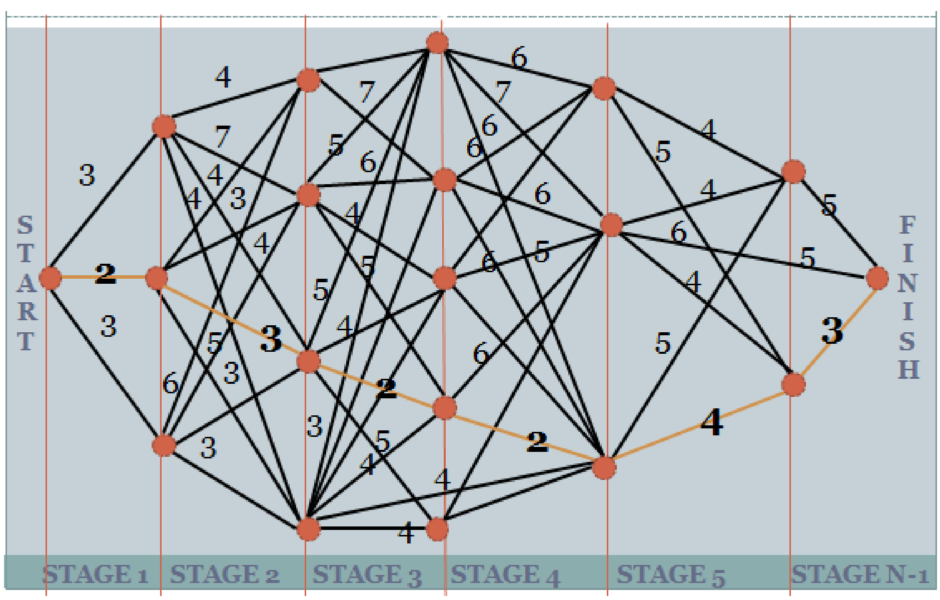

Fig. 3. Greedy algorithm

Source: own elaboration.

A greedy algorithm is an example of heuristics which uses the nearest neighbor algorithm strategy (Michalewicz, 1999), which involves the gradual joining of the next closest location until all vertices are attached (see Fig. 3). The greedy algorithm makes a decision locally optimal and makes the best choice at that moment. The disadvantage of the algorithm is the low quality of solution, which does not guarantee finding the optimal solution (Kaufmann, Fustier, Drevet, 1975).

In the example of using the greedy strategy to solve the traveling salesman problem, the algorithm can be defined as follows: after each stage of the visit to an empty location in the warehouse, leave goods there, and then go to the next nearest unoccupied location.

The course of the algorithm is shown in Fig. 3. The unloading ramp represents the starting point, where the traveling salesman starts on a forklift truck. Each of the nodes reflects an empty location in the warehouse, which can be filled up with goods. The solution is built piece by piece, always choosing the next piece that offers the most obvious immediate benefit (the nearest neighborhood).

6. Conclusions

The article proposes to use the heuristic algorithm as an attempt to optimize the shortest path which the warehouse worker takes during the supply of goods in stock. Storage processes are among the major elements of logistics, and also one of the many spaces where the use of optimization methods is necessary. The optimization of logistic processes undoubtedly raises many benefits for businesses, both financial and organizational.

The problem presented in this paper is one of the main aspects of daily work in high-rise warehouses. The rotation of goods in such a type of warehouse is huge, due to the scale and assortment diversity, which does not have a fixed location on the shelves. In this case, using methods which consider all the possible options is pointless, because of the time-consuming and disproportionate cost that can significantly exceed the profitability of the project. With the presented approach, by using heuristic algorithms and implementing them in warehouse management systems, satisfactory results can be achieved with an acceptable effort.

In order to improve the quality of solutions, it is possible to extend and modify the classical algorithm. This is possible to achieve by taking into account the actual data on the supply, transport options and the actual size of the warehouse. The increase of the efficiency of the algorithms can also be achieved by repeatedly running them or processing a parallel algorithm, starting with a random location. In addition, sometimes the modification of classical algorithms can be caused by the necessity to constrain the considered space of solutions (because of performance reasons) (Shukla, Deshpande, Naughton, 1998).

Heuristics does not guarantee finding the optimal solution, but allows determining correctly the best performance in the desired and given time, thereby not excluding achievement of an optimum solution. Further considerations on the problems of optimization in high storage warehousing could be focused on a selection of other heuristic algorithms, such as the use of a climbing algorithm (Michalewicz, 1999), which extends the concept of the greedy algorithm.

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