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REFLECTING INTERDEPENDENCIES BETWEEN RISK FACTORS IN CORPORATE RISK MODELING USING MONTE CARLO SIMULATION

ODZWIERCIEDLANIE WSPÓŁZALEŻNOŚCI POMIĘDZY CZYNNIKAMI RYZYKA W MODELOWANIU RYZYKA DZIAŁALNOŚCI GOSPODARCZEJ PRZEDSIĘBIORSTWA Z WYKORZYSTANIEM SYMULACJI MONTE CARLO

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Summary: Modern enterprises use various spreadsheet financial models to project their financial situation as well as to address potential entrepreneurial activity risk exposure. The most advanced solution is provided by the Monte Carlo approach that offers much broader possibilities in terms of entrepreneurial risk measurement than in the case of traditional methods. One of the most significant problems of the Monte Carlo approach is to identify, quantify and reflect interdependencies between variables that are risk factors in any risk analysis. The aim of this paper is to discuss possibilities to identify and quantify interdependencies in terms of historical data availability as well as to present a spreadsheet solution that would reflect interdependencies in risk simulation and which would be easy to implement. The solution presented is not the only one available, but it does not require too much effort to be implemented in any financial model developed in the form of a spreadsheet, especially by the individuals responsible for risk management in small and medium sized enterprises.

Keywords: Corporate finance, Risk, Monte Carlo.

Streszczenie: Współczesne przedsiębiorstwa stosują różnorodne modele finansowe sporządzone w arkuszu kalkulacyjnym służące do projekcji sytuacji finansowej przedsiębiorstwa, jak również do oceny ekspozycji przedsiębiorstwa na ryzyko działalności gospodarczej. Najbardziej zaawansowanym, dostępnym rozwiązaniem jest podejście Monte Carlo, które oferuje znacznie szersze możliwości w pomiarze ryzyka działalności gospodarczej niż rozwiązania tradycyjne. Jednym z najistotniejszych problemów związanych z podejściem Monte Carlo jest identyfikacja, kwantyfikacja i odzwierciedlenie współzależności pomiędzy czynnikami ryzyka. Celem artykułu jest omówienie możliwości identyfikacji i kwantyfikacji współzależności ze szczególnym uwzględnieniem problemu dostępności danych historycznych oraz zaprezentowanie rozwiązania pozwalającego na odzwierciedlenie współzależności w analizie ryzyka w arkuszu kalkulacyjnym, które byłoby łatwe do implementacji. Prezentowane w artykule rozwiązanie nie jest jedynym dostępnym, jednakże jest łatwe do zastosowania w dowolnym modelu finansowym sporządzonym w arkuszu kalkulacyjnym, szczególnie przez osoby odpowiedzialne za zarządzanie ryzykiem w małych i średnich przedsiębiorstwach.

Słowa kluczowe: finanse przedsiębiorstwa, ryzyko, Monte Carlo.

1. Introduction

The contemporary economic environment constantly affects entrepreneurial activity. External and internal influences are usually simultaneous, interdependent and non-linear. Modern enterprises use various spreadsheet financial models to project their financial situation as well as to address potential entrepreneurial activity risk exposure (e.g. prospective financial ratios analysis, market valuation, investment profitability assessment etc.). Choosing the right risk analysis method is an important issue. The most advanced solution is provided by the Monte Carlo approach that offers much broader possibilities in terms of entrepreneurial risk measurement than in the case of traditional methods. Most of all, Monte Carlo simulation ensures – close to reality – simultaneous, non-linear, interdependent changes of risk factors, also when historical data of risk factors is unavailable due to the easy integration of objective (made on the basis of historical data) and subjective (elicited from an expert opinion) assumptions. One of the most significant problems of the Monte Carlo approach is to identify, quantify and reflect interdependencies between variables that are risk factors in any risk analysis. The aim of this paper is to discuss possibilities to identify and quantify interdependencies in terms of historical data availability as well as to present a spreadsheet solution that would reflect interdependencies in risk simulation and which would also be easy to implement by individuals responsible for risk management in small and medium sized enterprises.

2. Identifying and quantifying interdependencies between risk factors

Every contemporary enterprise is exposed to numerous external and internal risk factors [Chapman 2006, p. 132] that affect entrepreneurial activity simultaneously, interdependently and non-linearly. The risk management process typically includes stages of risk identification, risk quantification and risk control [Zieliński 2010, p. 41]. In its first step, quantification should result in a list of risk factors and their probability distributions. Then risk variables distributions are quantified on the basis of risk factors distributions through a computer financial model. Probability distributions enable the assessment of expected values and volatilities of risk factors and risk variables as well as the identification of relevant interdependencies.

It has to be stated that both risk factor probability distributions and the interdependencies between them may be attributed in an objective way, a quasi-objective way or a subjective way, depending on historical data availability and adequacy. The essence of the objective and the quasi-objective way is that the risk factor probability distribution is obtained from historical data. The objective way assumes that historical probability distribution suits a particular analytical situation, whereas the quasi-objective way tends to modify the historical probability distribution by changing expected value or volatility range, while the essence of the subjective way is the use of special probability distribution types for risk factors with an expert opinion as a primary source of information [Hull 2015, p. 475; Vose 2008, p. 263, 393; Kaczmarzyk 2013, p. 25]

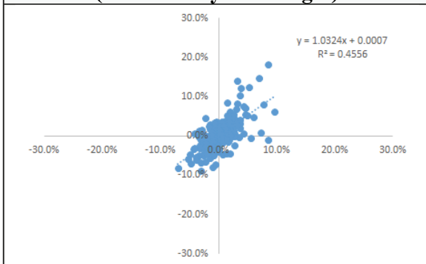
Some of the risk factors that are financial categories (e.g. prices, exchange rates or interest rates) have historical data that are easily available. This especially refers to financial categories that are constantly quoted on financial or commodity markets. However, there are many risk factors that belong to non-financial categories (e.g. having an operational or technological origin) with historical data not widely available or even unavailable. These categories are often individual issues of particular enterprises, therefore their historical data could only be available through the individual data collection process. Historical data collection is usually a reliable source of risk information, but generally it takes a lot of time. One should emphasize that historical data in many cases is also too expensive to obtain, no longer relevant or even sparse, thus consequently requiring an expert opinion "to fill in the holes" [Vose 2008, p. 393]. In terms of subjective risk factor quantification, an expert opinion can be a satisfying source of risk information due to special tools and techniques [Vose 2008, pp. 401–405; Kaczmarzyk 2013, pp. 23–34].

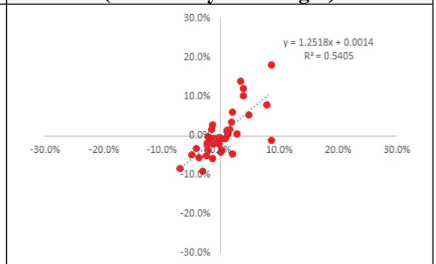
In the objective approach, interdependencies can be identified and quantified using various correlation coefficients: Pearson's linear correlation coefficient, Spearman's rank correlation coefficient or Kendall's Tau correlation coefficient [Jäckel 2002, pp. 42–45]. Every coefficient measures the direction and intensity of interdependency in a particular way. The question is whether historical data is relevant and adequate to the particular analytical situation. Market risk factors are an obvious example of such a phenomenon. Getting historical market risk data covering recession or recovery will result in quite different correlation intensities. Correlations between risk factors become higher in extreme market conditions [Hull 2015, p. 465]. Particular risk factors react more rapidly to changes of the other (higher sensitivity coefficient – β), also their changes are furthermore explained by the changes of the other risk factors (higher determination coefficient – R^2, squared linear correlation coefficient) (see Figure 1).

The objective way of risk quantification assumes the direct use of historically attained correlations. Choosing the right period of historical data is the most important issue to be solved. Historically attained correlation measures may be inadequate to the particular analytical situation. Therefore one should consider a quasi-objective

EUR/PLN versus USD/PLN exchange rates

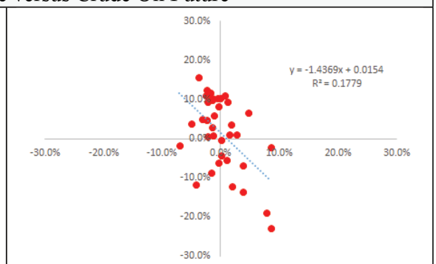
31/12/1998 – 31/10/2015 (203 monthly % changes)	**31/05/2007 – 31/07/2010 (38 monthly % changes)**





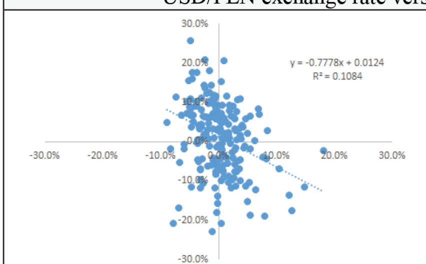
$\rho = 0.67$, $\beta = 1.0324$, $R^2 = 45{,}56\%$	$\rho = 0.74$, $\beta = 1.2518$, $R^2 = 54.05\%$

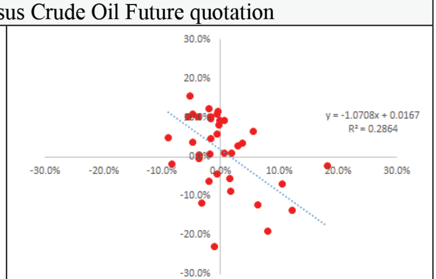
EUR/PLN exchange rate versus Crude Oil Future



$\rho = -0.19$, $\beta = -0.6728$, $R^2 = 3.47\%$	$\rho = -0.42$, $\beta = -1.43$, $R^2 = 17.79\%$

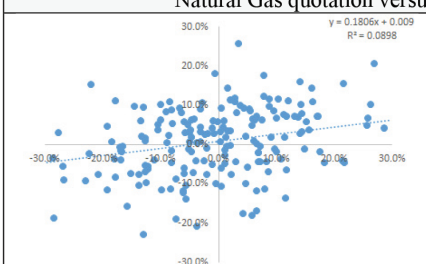
USD/PLN exchange rate versus Crude Oil Future quotation

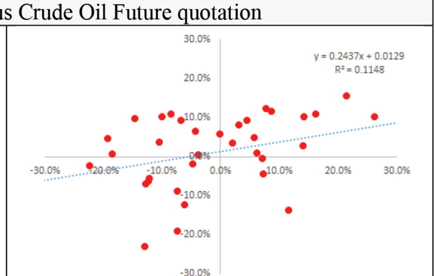




$\rho = -0.33$, $\beta = -0.7778$, $R^2 = 10.84\%$	$\rho = -0.54$, $\beta = -1.0708$, $R^2 = 28.64\%$

Natural Gas quotation versus Crude Oil Future quotation





$\rho = 0.30$, $\beta = 0.1806$, $R^2 = 8.98\%$	$\rho = 0.34$, $\beta = 0.2437$, $R^2 = 11.48\%$

Figure 1. Correlations between monthly changes of market risk factors

Source: own study on the basis of published market data of www.stooq.pl (accessed on 3/11/2015).

way of quantification and modify correlations if necessary (making them higher or lower depending on the forecasted economic conditions).

The subjective way of risk quantification is based on an expert's or experts' group opinion. Depending on experts' statistical experience, they can associate particular risk factors indicating correlation measures. Giving the coefficient of determination R^2 being squared linear correlation coefficient ρ seems to be the easiest way in the subjective approach to interdependencies quantification. An expert defines how much a risk factor change explains another risk factor change [Kaczmarzyk, Zieliński 2010, p. 178; Kaczmarzyk 2013, p. 29] (0-100%) as well as the direction of change (+ or –). An enterprise could introduce its own system of varying and describing correlation direction and its intensity (see Table 1).

Table 1. The coefficient of determination R^2 as a tool of subjective correlation quantification

Correlation intensity	R^2	ρ
High	From 80% to 100%	From +/–0.89 to +/–1
Medium high	From 60% to 80%	From +/–0.77 to +/–0.89
Medium	From 40 to 60%	From +/–0.63 to +/–0.77
Medium low	From 20 to 40%	From +/–0.45 to +/–0.63
Low	From 0% to 20%	From +/–0 to +/–0.45

Source: own study.

Interdependencies between risk factors prejudge the accuracy of the risk analysis. Not taking them into account may lead to the underestimation or overestimation of risk. As a result, an enterprise may be less or more secured in the context of possible economic environment changes.

3. Reflecting interdependencies between risk factors in the Monte Carlo approach to risk analysis

The Monte Carlo approach is widely accepted as a risk analysis method. The method involves quite basic mathematics required to perform the simulation of risk [Vose 2008, p. 45]. 'The thinking behind simulation is similar to the idea of carrying out multiple manual what-if scenarios' [Chapman 2006, p. 177]. In the Monte Carlo approach, scenarios are generated randomly on the basis of probability distribution assumptions assigned to the risk factors identified. Random scenarios are processed in a computer financial model (e.g. a projected financial statement model, an enterprise valuation model, an investment profitability model etc.). It is sometimes indicated that risk factors that are model input variables should be independent in the Monte Carlo approach [Krysiak 2008, p. 424]. Such an assumption is perceived as a disadvantage [Rogowski 2008, p. 292]. Taking the nature of the contemporary economic environment into account, one has to admit that there are not many situations in which risk factors could be independent.

Reflecting interdependencies in the Monte Carlo approach is actually possible. Of course spreadsheets are the natural environment of any financial model in the world of corporate finance. In spite of the possibility of random number generation with the desired probability distribution (the complexity of probability distributions catalogue available in spreadsheet is a separate problem to be discussed [Kaczmarzyk 2013, pp. 23-34]), spreadsheets themselves do not offer the assumption of interdependencies in any direct way. A spreadsheet user is able to conduct risk analysis by means of the Monte Carlo method in a blank spreadsheet, but risk factors should remain independent. The fastest and easiest solution to include interdependencies in any risk analysis is to upgrade spreadsheet standard capabilities by using high-end Monte Carlo analysis add-ons. A few high-end professional Monte Carlo analysis computer applications (mostly spreadsheet add-ons e.g. Palisade @Risk, Vose Software ModelRisk etc.) enable the assumptions in terms of risk factors interdependencies [Merna, Al-Thani 2008, p. 80].

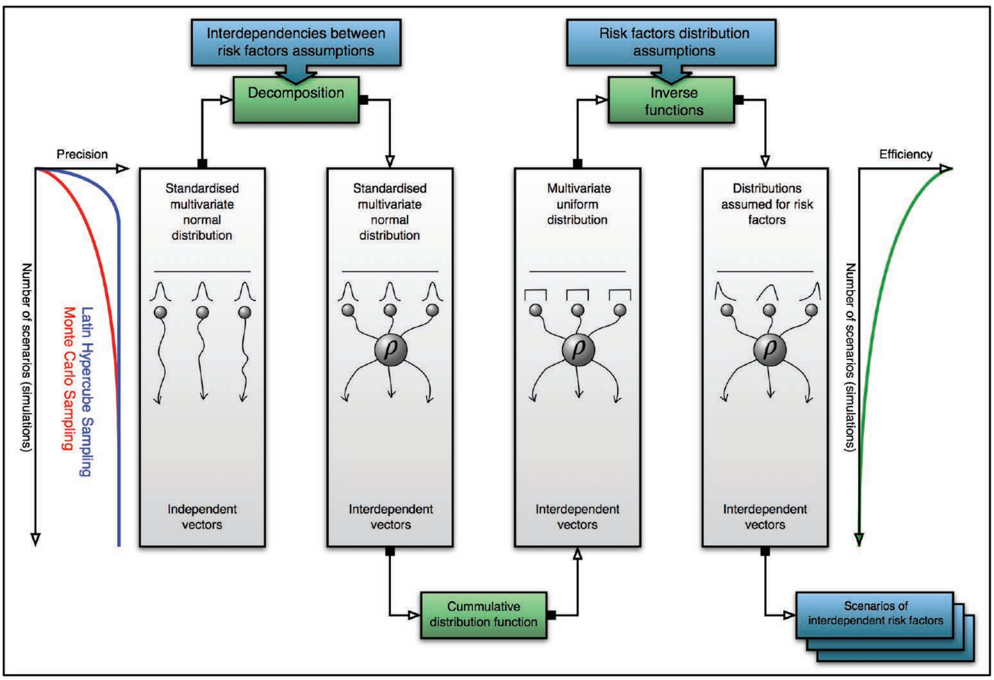


Figure 2. Generating interdependent risk factor scenarios

Source: own study.

Interdependencies between risk factors can be reflected in the Monte Carlo approach using spreadsheets by employing another solution. The most effective solution is to reproduce interdependencies through the decomposition of random numbers having a standardised normal distribution. There are two general decompositions used in practice: the eigenvector method and Cholesky's method [Korn, Korn, Kroisandt 2010, p. 44].

Cholesky's approach is easy to implement in any spreadsheet by inserting a self-prepared function giving matrix M, which multiplied by M^T gives Σ being the correlation matrix [Wilmott 2006, pp. 1275-1276]. The correlation matrix consists of linear correlation coefficients. The procedure of generating random, interdependent scenarios of risk factors in a spreadsheet (Figure 2) involves the following:

1. Generation of a random standardised multivariate normal distribution with independent column vectors using random number generator included in the spreadsheet.
2. Decomposition of the random standardised multivariate normal distribution with independent column vectors into multivariate distribution with interdependent column vectors on the basis of interdependencies assumptions by multiplying the multivariate distribution by the transposed matrix M^T.
3. Transformation of the random standardised multivariate normal distribution with interdependent column vectors into multivariate uniform distribution using the cumulative distribution function of standard normal distribution (available in any spreadsheet).
4. Transformation of each interdependent column vector having uniform distribution into the vector with the desired type of distribution on the basis of its parameter assumptions[1].

The initially assumed and finally attained linear correlation coefficients are affected by distribution transformation, whereas the attained rank correlation coefficients are preserved[2]. The important condition of decomposition is that the correlation matrix should be positive-definite [Korn, Korn, Kroisandt 2010, p. 44]. In the case of economic environment actual nature can be numerous analytical situations when Cholesky's decomposition simply will not work due to the assumed correlation matrix. A questionable solution is to modify correlations (or abstain from the decomposition). Such a transformation available in some high-end risk analysis applications is only theoretically useful. Changing correlations will affect the risk analysis causing the under or overestimation of risk. It has to be emphasized that including modified correlations might be better than not including them at all.

The additional negative effect of the decomposition is distribution deformation especially visible with the lower number of scenarios involved in the Monte Carlo simulation. The solution is to simply use a higher number of scenarios (Figure 3).

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[1] Step 1 and 2 – compare [Wilmott 2006, pp. 1275-1276]. Step 3 and 4 make use of the inverse method – compare [Korn, Korn, Kroisandt 2010, p. 31; Vose 2008, p. 57; Gentle 2003, p. 102]. For the overall procedure – compare [Cherubini, Luciano, Vecchiato 2004, p. 181].

[2] Spearman's rank correlation coefficient or Kendall's tau correlation coefficient are invariant to variable transformations [Jäckel 2002, pp. 43-45].

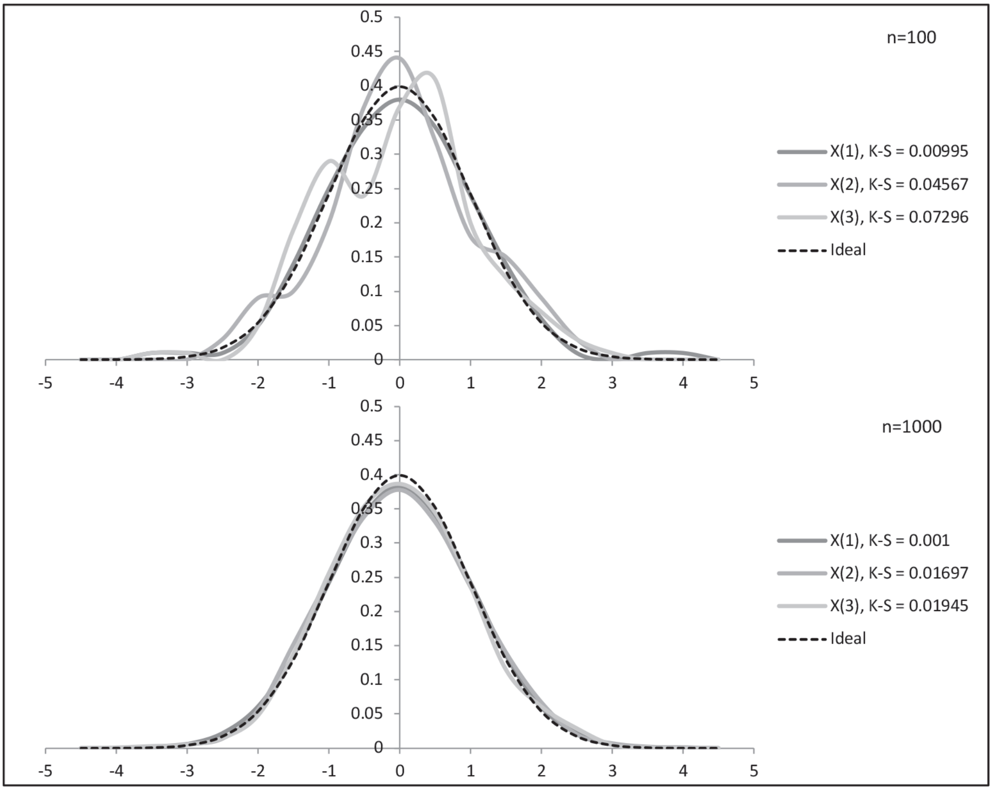

Figure 3. Cholesky's decomposition and the deformation of vectors of a standardised multivariate normal distribution measured with Kolmogorov-Smirnoff (K-S) statistics for $n = 100$ and $n = 1000$ simulations

Source: own study.

The generation process of a random standardised multivariate normal distribution with independent vectors can be based on the Monte Carlo sampling or Latin Hypercube sampling. Generally, Latin hypercube reproduces a distribution with better quality involving a lower number of scenarios when compared to the Monte Carlo sampling. 'Monte Carlo sampling (…) will over- and undersample from various parts of the distribution and cannot be relied upon to replicate the input distribution's shape unless a very large number of iterations are performed' [Vose 2008, p. 59]. 'The major difference between the two approaches is that the Latin hypercube has a memory of the process while the Monte Carlo simulation does not' [Cruz 2002, p. 217]. The fewer the scenarios the faster the risk analysis (fewer scenarios to be processed in the computer financial model) (Figure 2). Unfortunately, Latin Hypercube sampling is not directly available in spreadsheets, while Monte

Carlo sampling is. Nevertheless, the Latin hypercube can be implemented into every spreadsheet using a simple procedure [Cruz 2002, p. 217; Vose 2008, pp. 59-60].

The procedure of generating random, interdependent scenarios in the Monte Carlo approach necessitates assumptions in terms of risk factors distributions types and the interdependencies between them. The accuracy of assumptions should be considered to be a primary analytical problem, whereas the distribution deformation and sampling method as a secondary one.

4. Conclusion

In terms of interdependencies' identification and quantification, the availability of historical data prejudges only on an objective, a quasi-objective or a subjective way of measuring correlation between risk factors. In the objective and quasi-objective approach, getting historical data is not the only important issue to be faced. It is also essential to choose the right period of time, reflecting the projected future situation of an enterprise at its best. Having historical data to hand is generally a reliable source of risk information, but the individual responsible for any risk analysis should address data adequacy and eventually modify empirically found correlations. When historical data is unavailable, inadequate, too hard or too expensive to get, a subjective way of defining correlation can be the definite solution.

Reflecting interdependencies in any risk analysis that employs the Monte Carlo method is quite easy to implement thanks to the decomposition of random numbers. Cholesky's decomposition is a useful solution that can easily be implemented in a spreadsheet. Potential distribution deformation in the presented procedure can be avoided by using a sufficiently large number of scenarios.

Reflecting interdependencies properly is required in order to get the desired level of accuracy when it comes to the risk analysis process. Risk analysis should be involved whenever an enterprise is going to make decisions in terms of its operating, investment or financial activity. Therefore enterprise financial models that are based on the Monte Carlo approach should have a tool that would reflect the interdependencies implemented. The solution presented is not the only one available but it does not require too much effort to be implemented in any financial model developed in the form of a spreadsheet, therefore it is dedicated primarily to the individuals responsible for risk management in small and medium sized enterprises.

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