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**APPLICATION OF DIAGRAM METHODS  
AND HIERARCHICAL AGGLOMERATIVE  
PROCEDURES TO ASSESS THE RISK  
OF INVESTMENT FUNDS ON THE WARSAW STOCK  
EXCHANGE**

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**ZASTOSOWANIE METOD DIAGRAMOWYCH  
I HIERARCHICZNYCH PROCEDUR  
AGLOMERACYJNYCH W ANALIZIE RYZYKA  
FUNDUSZY INWESTYCYJNYCH NA GPW  
W WARSZAWIE**

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**Summary:** The issue of the article focuses on the need for the selection of investment funds of the Warsaw Stock Exchange with a certain similarity in terms of investment risk. For this purpose, the selected hierarchical agglomeration procedures and the diagrams methods that allowed creating a map of the funds, describing their diversity and diversity of the market, were applied. In addition, the compatibility of the obtained results were verified in terms of the measures of the investment risks designated by the fund managers.

**Keywords:** diagram methods, hierarchical agglomerative procedures, investment funds.

**Streszczenie:** Problematyka artykułu koncentruje się wokół potrzeby selekcji funduszy inwestycyjnych GPW w Warszawie o określonym podobieństwie pod względem ryzyka inwestycyjnego. W tym celu zastosowano wybrane hierarchiczne procedury aglomeracyjne i metody diagramowe, które umożliwiły stworzenie mapy funduszy opisującej ich zróżnicowanie i niejednorodność rynku. Ponadto zweryfikowano zgodność otrzymanych wyników z miernikami ryzyka inwestycyjnego wyznaczanymi przez zarządzających funduszami.

**Słowa kluczowe:** metody diagramowe, hierarchiczne procedury aglomeracyjne, fundusze inwestycyjne.

## 1. Introduction

The assessment of the portfolio management effectiveness of the investment funds is performed with methods which take into account the rate of return and the risk

indicators, but the most meaningful indicators combine both of these characteristics. This is understandable due to the reason that the fund managers themselves declare the level of risk by dividing the funds for stock, balanced, stable growth, bonds, money market, etc. This is the information based on which the individual investor selects the appropriate funds, takes a decision on the investment method and possible changes in investment strategy by customizing it to the market situation. Therefore, the proper assessment and classification of the funds, which would provide the investor with a major source of knowledge, are very important. This was the premise to investigate:

- the uniformity of the fund groups that are created according to the risk assessed by the managers,
- the application of the hierarchical agglomeration procedures and the diagrams methods based on the classical risk assessment indicators in order to create a map of the uniform groups of the funds,
- the comparison of the results of the assessment and classification of the investment funds managers.

In this thesis, the selected hierarchical agglomeration procedures and the diagrams methods that allowed to create a map of funds, describing their diversity and diversity of the market were applied. In order to group the funds we applied the indicators from the multi-factors hybrid models of market timing, to assess the risk and effectiveness of the investment funds. These models allow to verify the usefulness and investigate the financial instruments for the skills of the investment fund managers in Poland in terms of:

- the prediction of changes in prices of individual assets, i.e. selectivity of the securities,
- the prediction of changes in the market, that is global changes of the market factor (application of the techniques of market-timing).

## 2. Methodology

### 2.1. Hybrid models of market-timing

The idea of market-timing refers to the identification of market trends. A manager with this type of skills will adapt the composition of the managed fund to the market situation. Therefore, in order to test the skills of the portfolio manager in the field of the so-called ‘feeling the market’, the classic parametric models of market-timing are applied, in which there is a variable that represents the market. In practice, the rate of return from the market portfolio is the most common, which is a substitute for the relevant stock market index or the excess return on the market portfolio of risk-free return rate. The classic models of market-timing:

- the Treynor-Mazuy model (T-M) [1966]:

$$r_{i,t} = \alpha + \beta_1 \cdot r_{M,t} + \beta_2 \cdot r_{M,t}^2 + \varepsilon_{i,t}$$

and

- the Henriksson-Merton model (H-M) [1981]:

$$r_{i,t} = \alpha + \beta_1 \cdot r_{M,t} + \beta_2 \cdot \max[0; -r_{M,t}] + \varepsilon_{i,t}$$

where:  $r_{i,t}$  – vector of excess of the rate of return of the portfolio at the moment of  $t$  over the risk-free,  $r_{M,t}$  means an excess of the rate of return from the index of the market at the moment of  $t$  over the risk-free rate.

In both models the skills of applying the techniques of market-timing (short-term market trends) is proved by the parameter  $\beta_2$ , the value of which is the correction of any pessimistic expectations of the fund manager as to the future shape of the market rate. If it accepts values greater than zero, the portfolio manager correctly predicts the movements of the market, and the value of this coefficient demonstrates the extent of this skill. If the coefficient is close to zero, the investor does not have the market prediction skills of the market. Indeed, the negative value of the estimated coefficient of the parameter indicates the negative influence of the technique of market-timing on the value of the portfolio.

Based on the research performed in many countries (including in the United States: [Henriksson 1984; Kao, Cheng, Chan 1998; Bollen, Busse 2001; in the United Kingdom Fletcher 1995, Portugal Romacho, Cortez 2006, in Poland Olbryś 2009; 2010a-c; Homa, Mościbrodzka 2015], the models (T-M) and (H-M) were modified due to the fact that their incorrect indications were observed in terms of the diversity of the actual return rates. Among others, the thesis of Bhanadari [1988] and Fama and French [1992] show that the balance sheet indicators such as: the book value/the market value and size of the company influence on the expected value of the return rate from the stock portfolio. In another article in 1993, Fama and French presented the three-factor stock pricing model of the shares price balance. As explanatory variables in the basic model, they proposed:

- an excess of the market return rate over the risk-free return rate,
- *SMB* (small minus big) – a factor created mainly based on the market value (so-called size factor);
- *HML* (high minus low) – a factor created mainly based on the value of the indicator  $BV/MV$  (the so-called book-to-market factor).

Taking into account the proposed new correction factors for the previous classic capital pricing model, the models of market-timing were extended with additional factors (the so called Fama-French factors) based on which the managers take the allocation decisions. Thus, the hybrid models of market-timing were obtained ( $T - M - F$ ) and ( $H - M - F$ ) following forms [Olbryś 2010a]:

$$r_{i,t} = \alpha + \beta_1 \cdot r_{M,t} + \beta_{SMB} \cdot r_{SMB,t} + \beta_{HML} \cdot r_{HML,t} + \beta_2 \cdot r_{M,t}^2 + \varepsilon_{i,t}$$

$$r_{i,t} = \alpha + \beta_1 \cdot r_{M,t} + \beta_{SMB} \cdot r_{SMB,t} + \beta_{HML} \cdot r_{HML,t} + \beta_2 \cdot \max[0, -r_{M,t}] + \varepsilon_{i,t},$$

where  $r_{SMB,t}$  excess of the return rate of the portfolio that imitate the *SMB* over the risk-free return rate at the moment of  $t$ ,  $r_{HML,t}$  excess of the return rate of the portfolio that imitate the *HML* over the risk-free return rate at the moment of  $t$ .

The coefficients  $\beta_{SMB}$  and  $\beta_{HML}$  are the measures of the possibility of the return rate from the investment to change of the return rate from the portfolios that imitate *SMB* and respectively. Therefore, their loads represent the additional risk bonus related to the investments in the company of low capitalisation and the high value of the book value to the market value ratio of the company.

## 2.2. Selected diagram methods – the Czekanowski method

The classification of the investment funds of the shares was performed applying the Czekanowski diagram due to the level of risk and their effectiveness. The method is the oldest method of taxonomy, published for the first time in 1909 by the renowned Polish anthropologist – Jan Czekanowski. Now it is also used in other fields of science as a universal method of statistical classification. The essence of the diagram is that it indicates the most important relationships and similarities of the researched objects, and at the same time it presents detailed connections between them [Czekanowski 1913]. When performing the Czekanowski diagram, the procedure proposed by T. Panek and J. Zwierzchowski [2013, pp. 58-59] must be applied. Namely:

1. Defining the matrix of the distance between the objects  $D[d_{ij}]$ , by any metric.
2. The distance measures in the matrix D are divided into classes which are the parts of similarity of objects. The correct determination of the similarity scale significantly influences the final results of division of the group of the researched objects.
3. Each similarity classes of the objects are assigned to the relevant graphic symbols, obtaining the disordered Czekanowski diagram, which allows to visually assess ordering the objects.
4. Ordering the objects itself is performed by organizing the diagram, which consists of moving the rows and columns corresponding to the diagram so that the graphic symbols representing the smallest possible distance cantered along the main diagonal, and as it moves away from the main diagonal, the graphical symbols appear corresponding to the increasingly distances.
5. The order of the arrangement of the objects is determined by the order of the corresponding rows (columns).

The advantage of the taxonomic method of Czekanowski is that the entire distance matrix is taken into consideration while classifying. The objects are the most frequently included in one typological group only when in all or the majority of them there are similarities in the highest degree. The optimization of grouping of the objects ordered using the Czekanowski method were performed using the so-called grouping correctness meter [Nowak 1990, p. 64]. The basis for this meter structure is the assumption that in optimal grouping, each group should consist of the objects between which the so-called close connections occurs, and outside the groups, the so-called far connections. The grouping correct measure:

$$Q = \frac{n^{pb}}{n^w} \cdot \frac{n^{pd}}{n^z},$$

where  $n^w$ ,  $n^z$  is the number of connections respectively inside and outside the selected groups,  $n^{pb}$ ,  $n^{nd}$  is the number of connections respectively close inside the groups and far outside the selected groups.

### 2.3. The selected hierarchical agglomeration methods – the Ward Method

The basic hierarchical methods of grouping include: the nearest vicinity, the furthest vicinity, the median, the average group, the centre of gravity and the Ward. These methods differ in the way of determining the distance between the groups [Wishart 1969]. All of the above agglomeration procedures can be described using a single overall schema, which is considered to be the main agglomeration procedure [Nowak 1990, pp. 80-81]. This is based on the distance matrix between the researched objects. The general formula for transformation of the distance matrix while connecting the groups  $A_p$  and  $A_q$  in a new group  $A_r$  for the hierarchical agglomeration methods has the following form:

$$d_{ir} = a_p \cdot d_{ip} + a_q \cdot d_{iq} + b \cdot d_{pq} + c \cdot |d_{ip} - d_{iq}|,$$

where  $d_{ir}$  – distance between the  $A_i$  and  $A_q$ ,  $d_{ir}$  – distance between the  $A_i$  and  $A_p$ ,  $d_{iq}$  – distance between the  $A_i$  and  $A_q$ ,  $d_{pq}$  – distance between the  $A_p$  and  $A_q$ ,  $a_p$ ,  $a_q$ ,  $b$ ,  $c$  – transformation parameters characteristic of the groups different methods for creating the groups.

In order to perform the grouping, comparison and analysis of the investment funds of the shares, in Poland, the Ward method was applied, which was presented in the thesis of J.H. Ward [1963]. In this method, the distance matrix transformation formula parameters have the following values:

$$a_p = \frac{N_i + N_p}{N_i + N_r}, \quad a_q = \frac{N_i + N_q}{N_i + N_r}, \quad b = -\frac{N_i}{N_i + N_r}, \quad c = 0,$$

where  $N_r$ ,  $N_p$ ,  $N_q$  and  $N_i$  mean the number of the groups  $A_i$ ,  $A_p$ ,  $A_q$  and  $A_r$  [Nowak 1990, p. 81].

The Ward method is to combine such agglomerates, which provide the minimum of the sum of the squares of the distances from the centre of gravity of the new agglomeration, which they form. As a result the group includes the objects which are the least varied due to the variables describing them [Panek, Zwierzchowski 2013]. The next stage of the research is to determine the optimal number of classes. In their theses, Milligan and Cooper [1985] tested dozens of the procedures for the determination of the number of classes based on the data of known structure. In the thesis, in order to select the optimal number of classes as well as to assess the quality of the obtained classification, the index of Silhouette was applied, which was proposed in the theses of Kaufman and Rousseeuw [1987-1990]. The value of the index of Silhouette for a fixed number of classes is determined with the following formula:

$$S(u) = \frac{1}{n} \sum_{i=1}^n \frac{b(i) - a(i)}{\max\{a(i); b(i)\}},$$

where  $u$  is the number of the classes,  $n$  is the number of the objects,  $i$  is the object number ( $i = 1, \dots, n$ ),  $a(i)$  the average distance  $i$  – of the object from all other objects belonging to the same class as the object  $i$  and  $b(i)$  the average distance  $i$  – of the object from the objects belonging to the nearest class of the object  $i$ .

The value of the considered index is in the range  $[-1, 1]$ . The argument maximising the value of the Silhouette indicator provides the optimum, in terms of the assessment of the quality of the classification, the number of the classes due to its compactness and separation abilities. In addition, this value allows for the subjective assessment of the quality of the performed classification.

### 3. Research results

The research included all the investment funds of the shares listed on the Warsaw Stock Exchange in the period from January 2009 to June 2015 (including 66 shares of investment funds: A1-A66<sup>1</sup>). The analysis was based on the weekly data from the considered period. All the analysed funds were characterised by the high level of indicator *SRRI* (*Synthetic Risk and Reward Indicator*), which is calculated based on the common methodology for the whole market and is presented in a simple formula – on a numerical scale from 1 to 7. The higher the level of the observed indicator, the greater the risk of volatility of the fund, but also the chance to achieve satisfactory profits. According to the *SRRI*, the analysed funds accounted for two of the unified group in terms of the risk (level 5-6). This is the indicator which next to the standard and classic deviation of the Sharpe indicator forms the basis of the assessment of the effectiveness and the risks of the investment funds by the managers. In order to verify whether this information is sufficient for the assessment of the funds in the first stage of the research, the values of the factors from the so-called portfolios imitating *SMB* and *HML* were determined. In order to form them, the monthly balance data from February 2009 to June 2015 were applied. The research includes the lists of all companies listed on the Warsaw Stock Exchange, the number of which during the period under consideration was 266 companies at the end of February 2009, and more than 400 in June 2015. The detailed structure of the factors was described in an earlier article by the authors [Homa, Mościbrodzka 2015]. The next step involved the generation of the relevant vector parameters for each investment ( $\alpha, \beta_1, \beta_2, \beta_{SMB}, \beta_{HML}$ ) i.e. an extended set of the factors for further analysis.

<sup>1</sup> The full names of the Investment Funds of the Shares were included Table 3 at the end hereof.

### 3.1. The result of the FI grouping with the Czekanowski method

The basis for the FI grouping of the shares with the Czekanowski method was the Euclidean distance matrix between the objects, based on the generated parameters ( $\alpha^j, \beta_1^j, \beta_2^j, \beta_{SMB}^j, \beta_{HML}^j$ ) determining the effectiveness and risk of the fund. The distance meters in this matrix are divided into five classes. Each similarity classes were assigned with the appropriate graphic symbols, forming the so-called unordered Czekanowski diagram shown in Figure 1.

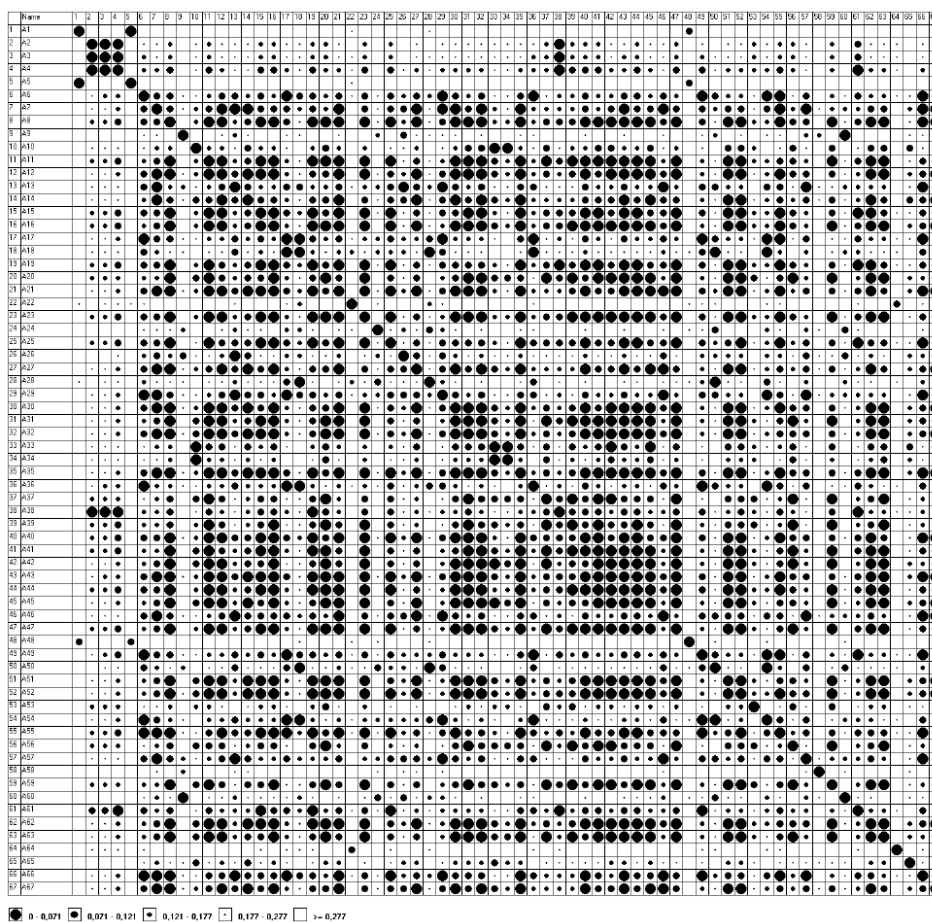


Fig. 1. Disordered Czekanowski's diagram

Source: own study.

In the next stage of the procedure, the funds are organized according to the connection strength. In the thesis, the close connection means the connection between

the objects which corresponds to a graphic symbol that represents the lowest class. The remaining connections between the groups are regarded as far connections. The results of the FI grouping represented by the ordered Czekanowski diagram are provided in Figure 2.

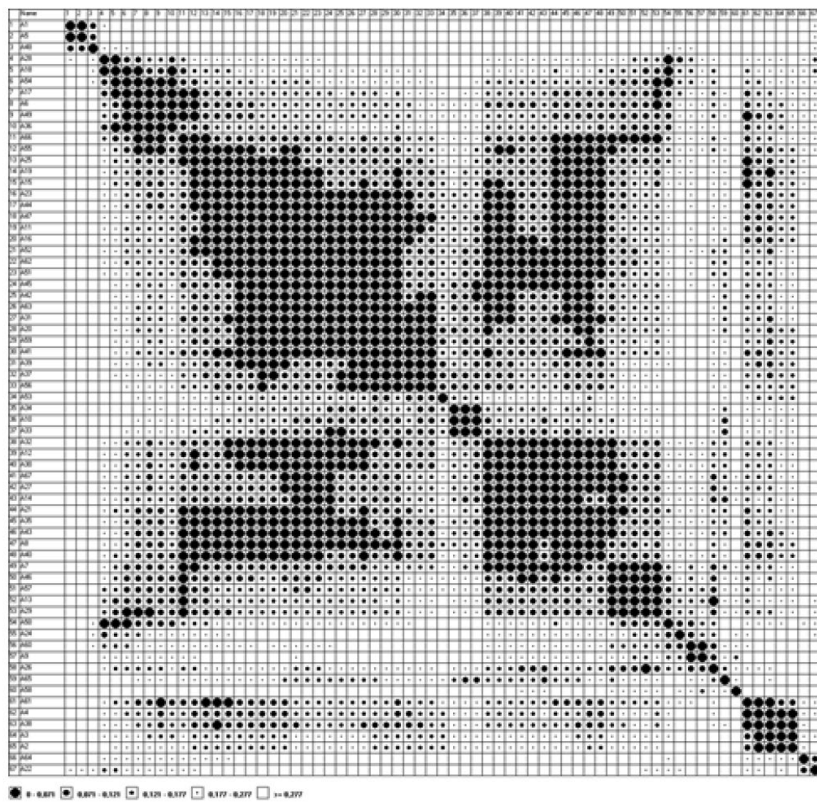


Fig. 2. Ordered Czekanowski's diagram

Source: own study.

As a result of grouping with the Czekanowski method, 18 groups of the funds similar in terms of effectiveness and investment risks were created. For such division of the funds, the correct measure is 0.71, and indicates a strong group structure, which also confirms the diversity of the researched instruments in terms of effectiveness and risk. Therefore it could be said that the management information about the uniformity of the funds groups based on a standard deviation, the *SRR* and the Sharpe ratio is insufficient and therefore can confuse the investors. As a consequence, the strategy adopted by it will not bring the expected income in the future and it will bear the negative consequences of the taken decisions. The uniform groups of the investment



funds of the shares with the parameters vector  $(\alpha^j, \beta_1^j, \beta_2^j, \beta_{SMB}^j, \beta_{HML}^j)$  for each group, selected with the Czekanowski method, are presented in Table 1.

**Table 1.** Groups of the funds FI including effectiveness and risk (grouping with the Czekanowski method)

	Funds	$\alpha$	$\beta_1$	$\beta_2$	$\beta_{SMB}$	$\beta_{HML}$
1	A1, A5	0.1635	0.4264	0.0336	-0.0043	-0.0060
2	A48	0.1469	0.5271	0.0287	0.0000	0.0000
3	A18, A28	0.0389	0.6984	0.0192	-0.0032	-0.0026
4	A6, A17, A36, A49, A54	-0.0025	0.7779	0.0093	0.0000	0.0000
5	A66, A55, A25, A19, A15, A23, A44, A47, A11, A16, A52, A62, A51, A45, A42, A63, A31, A20, A59, A41, A39, A37, A56	-0.0582	0.9148	0.0023	-0.0003	-0.0004
6	A53	-0.1786	0.9887	0.0000	0.0000	0.0000
7	A34, A10, A33	-0.0410	1.0353	0.0000	0.0000	0.0000
8	A32, A12, A30, A67, A27, A14, A21, A35, A43, A8, A40	-0.0082	0.9028	0.0026	0.0000	-0.0005
9	A7, A46, A57, A13, A29	0.0496	0.8448	0.0053	-0.0015	-0.0013
10	A50	0.0794	0.7384	0.0218	-0.0051	-0.0052
11	A24	-0.0523	0.8556	0.0076	0.0000	0.0000
12	A9, A60	0.2058	0.8048	0.0091	-0.0025	-0.0018
13	A26	0.1263	0.8668	0.0132	0.0000	0.0000
14	A65	0.0738	1.0362	0.0000	0.0000	0.0000
15	A58	0.2814	0.9543	0.0000	0.0000	0.0000
16	A2, A3, A4, A38, A61	-0.1457	0.8259	0.0083	-0.0012	-0.0010
17	A64	-0.1425	0.6118	0.0092	-0.0164	-0.0150
18	A22	-0.0305	0.5790	0.0165	0.0000	0.0000

Source: own study.

### 3.2. The selected hierarchical agglomeration methods – the Ward method

The basis for the FI grouping of the shares with the Ward method was the matrix of squares of Euclidean distances between the funds, based on the generated parameters vector  $(\alpha^j, \beta_1^j, \beta_2^j, \beta_{SMB}^j, \beta_{HML}^j)$  determining the investment effectiveness and risk of the fund. As a result of the application of the selected agglomeration method, a graphic illustration of grouping in the form of a dendrogram shown in Figure 3 was achieved.

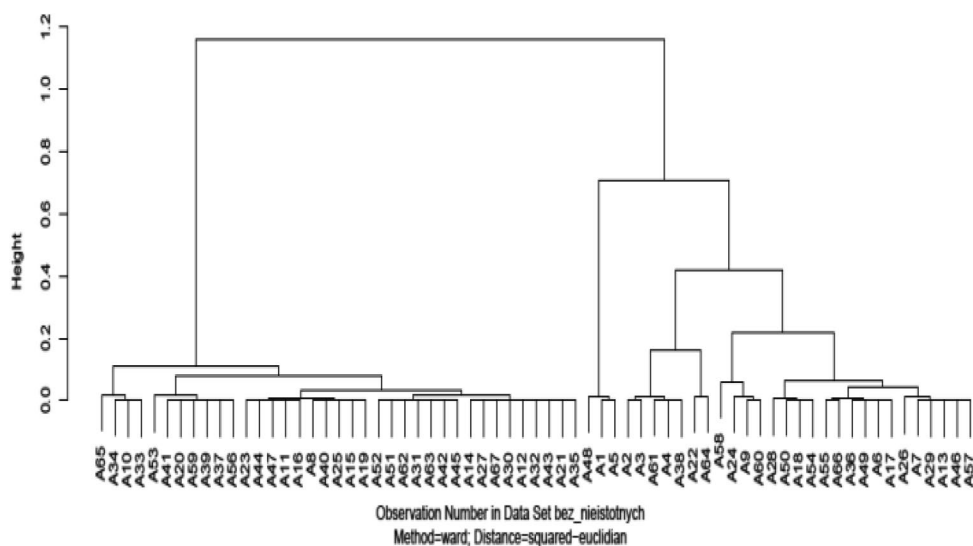


Fig. 3. The grouping with the Ward method

Source: own study.

Maximizing the value of the Silhouette measure (the value of 0.57 indicates the serious structure of the resulting classification) eight subgroups of the FI shares were obtained due the investment effectiveness and risk, the vector value of which  $(\alpha^j, \beta_1^j, \beta_2^j, \beta_{SMB}^j, \beta_{HML}^j)$  is presented in Table 2.

Table 2. Groups of the funds FI including effectiveness and risk (grouping with the Ward method)

	Funds	$\alpha$	$\beta_1$	$\beta_2$	$\beta_{SMB}$	$\beta_{HML}$
1	A1,A5,A48	0.1580	0.4600	0.0320	-0.0028	-0.0040
2	A2,A3,A4,A38,A61	-0.1457	0.8259	0.0083	-0.0012	-0.0010
3	A6,A7,A13,A17,A18,A26,A29,A36, A46,A49,A50,A54,A55,A57,A66	0.0308	0.7999	0.0096	-0.0012	-0.0012
4	A8,A11,A12,A14,A15,A16,A19, A21A23,A25,A27,A30,A31,A32, A35,A40,A42,A43,A44,A45,A47, A51,A52,A62,A63,A67	-0.0327	0.9084	0.0027	0.0000	-0.0001
5	A9,A24,A58,A60	0.2137	0.8190	0.0094	-0.0035	-0.0027
6	A10,A33,A34,A65	-0.0123	1.0355	0.0000	0.0000	0.0000
7	A20,A37,A39,A41,A53,A56,A59	-0.1048	0.9514	0.0009	-0.0008	-0.0013
8	A22,A64	-0.0865	0.5954	0.0129	-0.0082	-0.0075

Source: own study.

The obtained results indicate the application of the extended vector of the parameters determining the investment effectiveness and risk resulting in significant discrepancies in the assessment of their uniformity. This demonstrates the fact that the investors should take their decisions on the investment strategy based on the results of the obtained alternative grouping.

#### 4. Conclusion

The managers of the investment funds of the shares declare the level of risk by dividing them into uniform groups and measuring their effectiveness mainly with the average return rate of the investment-holding period, and the measure of the risk involves the standard or classic deviation and the Sharpe ratio or ratios. The investment funds of the shares, according to the information from the managers assessing their effectiveness and risk, are a group of the funds composed of two uniform subgroups with the SRRI index equal to 5 and 6.

Based on the results of the applied Ward method and the Czekanowski diagram, it was demonstrated that the application of the extended vector of the parameters relevant in assessing effectiveness and risks results in distinguishing the subgroups differing in terms of the investment risk and policy, from the point of view of the investment risk diversification, are of great importance. Therefore, the investor should have knowledge of the uniform subgroups of FI SHARES, which will allow to select the appropriate funds in accordance with its investment objective and the adopted strategy. Therefore while selecting the investment fund, taking into account the tendency to accept a certain level of risk and expectations for the return rates from the investments, the standard information about the investment funds provided by the managers should involve full knowledge of the diversity of the funds' groups.

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## Appendix 1

Groups of the funds FI including efficiency and investment risk (grouping the Ward method)

Funds	Symbols
AGIO Agresywny (AGIO SFIO)	A1
Alior Agresywny (Alior SFIO)	A2
Alior Stabilnych Spółek (Alior SFIO)	A3
Allianz Akcji (Allianz FIO)	A4
Allianz FIO Subfundusz Allianz Akcji Globalnych	A5
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