Pseudo-phase mapping of speckle fields using 2D Hilbert transformation

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The use of a "window" 2D Hilbert transform for the reconstruction of the phase distribution of the intensity of a speckle field is proposed. It is shown that the advantage of this approach consists in the invariance of a phase map to a change of the position of the kernel of transformation and in the possibility to reconstruct the structure-forming elements of the skeleton of an optical field, including singular points and saddle points. We demonstrate the possibility to reconstruct the equi-phase lines within a narrow confidence interval, and introduce an additional algorithm for solving the phase problem for random 2D intensity distributions.

Keywords: speckle field, singular points, saddle points, phase problem.

1. Introduction

The phase problem in optics is well-known and has been the subject of many investigations. Phase retrieval consists in the determination of the transverse phase structure of an optical field, usually achieved by recording a series of intensity distributions at a series of distances after free space propagation. Progress in solving of the phase retrieval problem increases with the evolution of optical data processing techniques. The reconstruction of phase spectra is the base for solving many problems of image processing, pattern recognition, data coding, storage and communication. Conventionally, acquisition of the phase from a recorded speckle pattern [1–3] presumes the use of interference and holographic techniques by virtue of a reference wave. As it is known, the interference technique [4–7] is used for solving of a lot of optical tasks, including the phase problem retrieval [8].

At present, a topical problem consists in the reconstruction of phase distribution of remote objects, whose images can be distorted due to turbulence, aberration of imaging system or motion of a camera during exposure, *etc*. Besides, phase retrieval based on a recorded intensity distribution is urgent for non-invasive techniques of medical

diagnostics, assuming the use of light beams as probing instruments that exclude any mechanical disturbances of the studied sample. In these cases the use of a reference wave is impossible.

Speckle fields arising from scattering media such as biological tissue [9, 10] or turbulent atmosphere contain loci of minima, maxima and saddle points of intensity forming a field skeleton. The skeleton alone of an optical field being structure-invariant with respect to coordinate transformation provides comprehensive information on the field and the object. Motion of particles or media forming a field causes changes of a speckle field as well as its skeleton [11].

The idea of this study is:

– use of the reference points of a field, such as saddle points and amplitude zeroes, to reconstruct the phase map of a field;

– drawing the lines from these reference points corresponding to intensity gradients;

– finding the location of the lines of intensity gradients which coincides with the regions of smooth and slowly changing phase, *i.e.*, equi-phase lines.

Unlike an earlier published papers [12, 13], where the lines of intensity gradient were retrieved in a quasi-empirical way (as a result of computer processing of the recorded intensity distribution of a field), here we propose solving the phase problem with an algorithm based on the Hilbert transformation. This provides an increased accuracy of reconstructing the phase maps of a random speckle pattern, and accordingly obtaining comprehensive information on the studied optical field including the loci of phase singularities and saddle points of the intensity distribution. Generally, the Hilbert transform possesses important advantages in comparison with other transformations providing the visualization of optical inhomogeneities with an extremely high contrast [14].

Solving the phase problem using a 2D "window" Hilbert transform presumes reconstruction of the phase information contained in the recorded intensity distribution. For complete and reliable reconstruction of a phase map one must determine:

– distribution of equi-phase lines;

– loci of singular points and saddle points as the reference points of phase distribution, whose changes are connected with object motion;

– accuracy of the proposed algorithm for reconstruction of phase distribution, as well as investigate the invariance of the proposed algorithm to size and orientation of the scanning windows.

2. Statement of the problem

It is a common practice in physics and engineering to represent real-valued signals by the related complex-valued signals [15, 16]. For 1D signals, the concept of analytic signals based on the partial Hilbert transform was introduced to communication theory by Gabor in the 1940s. According to Gabor's theory, one derives a complex signal by suppressing all negative frequencies of the initial real-valued signal that can be represented as the sum of specified 1D signal and a pure imaginary component, *viz*.

the Hilbert transform of the initial signal. Several generalizations of a 1D Hilbert transform have been found, including 2D case.

The extension of the 1D Hilbert transform to 2D case was probably first reported by READ and TREITEL [17]. BOSE and PRABHU [18] derived another expression for 2D Hilbert transform in cotangent, sine and matrix forms. In the 2D case, the Hilbert transform is composed of two parts; one part acting on the component x and the other one on the component y. Thus the kernel of the transform can be written down as $h(x, y) = h_1(x) + h_2(y)$.

Let $f(x, y)$ be a function in the spatial domain describing a random intensity distribution of the recorded speckle pattern. Instead of the real signal representing real physical process, one introduces the analytic signal

$$V(x, y) = f(x, y) + i\hat{f}(x, y) \tag{1}$$

where $\hat{f}(x, y)$ is a 2D Hilbert transform of the initial signal, and $\hat{f}(x, y) = = f(x, y) * h(x, y)$, the asterisk denotes the convolution operator. Correspondingly, the reconstructed magnitude of a phase (the pseudo-phase [1]) of the analytic signal is found as

$$\varphi(x, y) = \mathrm{atan}\left(\frac{\hat{f}(x, y)}{f(x, y)}\right) \tag{2}$$

In this case, the reconstruction of the pseudo-phase of speckle patterns is generated using a 2D Hilbert transformation of the speckle patterns. This representation of the pseudo-phase is not unique, and an entirely different representation of the pseudo-phase can be generated if, for example, one decides to employ a 2D Fourier transformation in place of the Hilbert transformation. This is the classic phase-retrieval problem. Regardless of the numerical method used to generate the pseudo-phase, the operation simply exploits information already present in the signal without adding any new information. Once a 2D pseudo-phase representation is generated, the skeleton of the optical field can be determined.

3. Algorithm and results

Let a speckle pattern be registered using a charge coupled device (CCD) camera. Thus data processing consists in a pixel-by-pixel analysis of the recorded signal. As the obtained image is discrete, one uses a discrete Hilbert transformation (DHT) [17]. For mathematical modeling, as an analytical signal with subsequent reconstruction of a phase of the structure-forming elements, we use the so-called "window" Hilbert transformation, so that the kernel of transformation is convolved with a specified window with subsequent shift of the kernel to the next window.

The procedure consists of the following steps:

– Recorded speckle pattern is pixel-by-pixel transformed to a pattern with 256 grey levels.
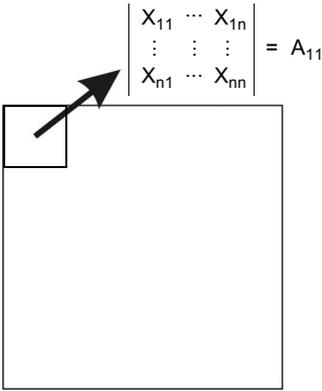
Fig. 1. Formation of the matrix A_{11} by cutting the $n{\times}n$ window.
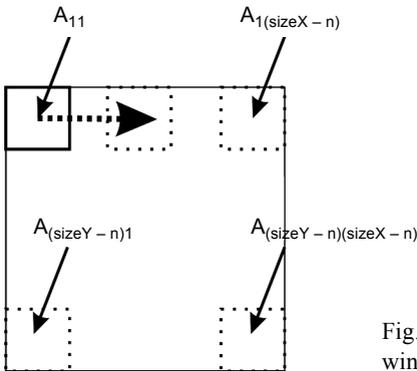
Fig. 2. Formation of the set of matrices by shifting the scanning window.

– This pattern is scanned for obtaining the matrices, as it is shown in Fig. 1. A window $n{\times}n$ is cut of the pattern, so that the magnitudes of intensity form the matrix elements, to say matrix A_{11}.

– Further, this window moves across the initial pattern with the one pixel-step horizontally and vertically that results in obtaining the set of matrices shown in Fig. 2.

– The kernel of the 2D Hilbert transform is chosen in the cotangent form [18] as

$$h(x, y) \;=\; \left[\cot\!\left(\frac{\pi x}{n}\right) + \cot\!\left(\frac{\pi y}{n}\right) \right]\frac{2}{n^2} \tag{3}$$

where $0 \leq x \leq (n-1)$ and $0 \leq y \leq (n-1)$. This equation defines the 2D Hilbert transform operator and may take the matrix form also. Thus, in the discrete case, which is analyzed in the given paper, the kernel is represented by the $n{\times}n$ matrix and has the complete width $n{\times}n$. It is convolved with the corresponding matrices obtained by scanning the initial image resulting in the 2D DHT.

– Using Eq. (2), one computes a pseudo-phase of the initial distribution.

Phase maps obtained by applying this algorithm are shown in Fig. 3. The size of the scanning window determines the accuracy of reconstruction of a phase distribution. The reconstruction accuracy is maximal when the size of a window equals the correlation length in the far field that corresponds to the average speckle size or equally,
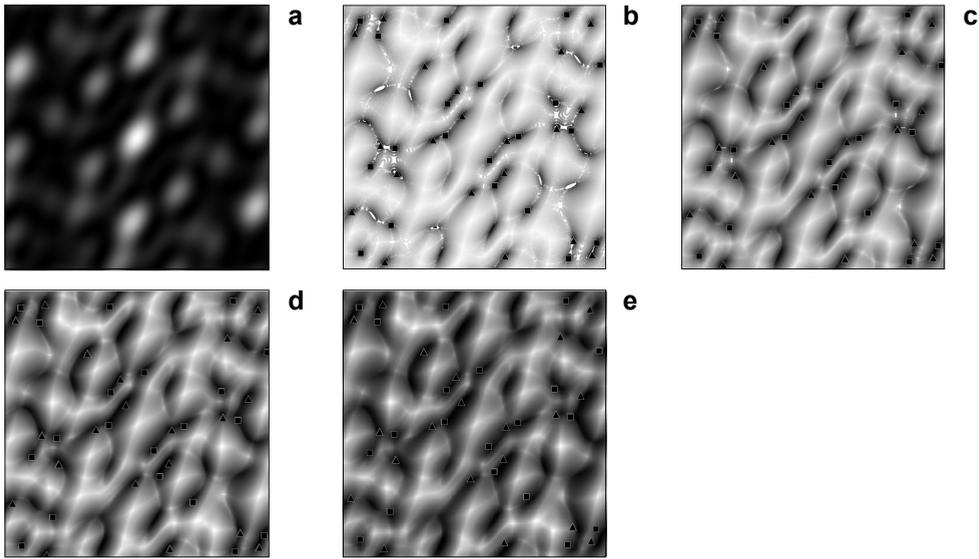
Fig. 3. Speckle pattern (**a**) and phase maps reconstructed using the Hilbert transformation for various sizes of the kernel: 5×5 pixels (**b**), 10×10 pixels (**c**), 15×15 pixels (**d**) and 20×20 pixels (**e**). Triangles and squares correspond to singularities of opposite signs.

the average distance between vortices (Fig. 3**b**, 5×5 pixels). Accuracy decreases as the window becomes larger due to smoothing of the phase pattern and decreasing the contrast of the equi-phase regions (Figs. 3**c**–3**e**). The original result obtained by the reconstruction of a phase distribution results in an invariance of the phase map for a turned kernel, as it is seen in Fig. 4 for simulation with rotation of a kernel by 45° (**a**) and 90° (**b**).

The reconstructed phase map is the same, irrespectively of choice of quadrant or selection of a part of the plane. This implementation of the algorithm for reconstruction of the spatial phase distribution provides considerable simplification for obtaining the phase maps of perturbed objects. Phase maps reconstructed with the kernel specified at various parts of a plane for a window whose width equals the correlation length of a field are shown in Fig. 4.
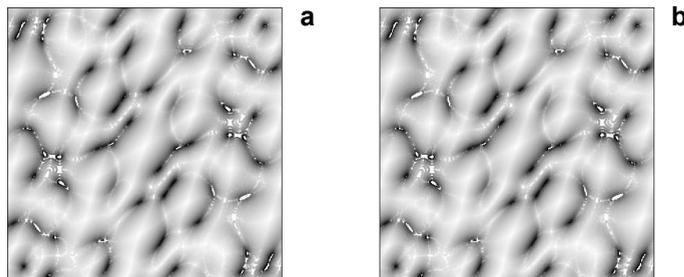
Fig. 4. Reconstructed phase map for kernel rotated by 45° (**a**) and 90° (**b**).

The next step for obtaining the data on the optical field is the identification of phase singularities (optical vortices), *viz*. points with zero amplitude and thus undefined phase. The phase in the nearest vicinity of such points rotates through a full 2π radians along a circular path surrounding these points. In Figure 3, singular points are shown by triangles and squares, as positive and negative, corresponding to the topological charge computed as [19]

$$S \;=\; \frac{1}{2\pi} \oint_{c} \mathrm{d}\,\varphi(x, y) \tag{4}$$

where $\mathrm{d}\varphi(x, y)$ is the local phase gradient, and the contour integral is taken over a closed loop c around the singularity. Zero magnitude of S takes place at all points of a phase map excluding paths surrounding an equal number of positive and negative charged singularities. The sign "plus" or "minus" corresponds to increasing or decreasing the phase magnitude for counterclockwise circumference of singularity, respectively.

Figure 5 illustrates the reconstructed phase map with the saddle points of intensity obtained as the points located in regions with rapidly changing phase [12]. The analysis of the reconstructed phase for determination of the saddle points was performed for a 5×5 kernel of transformation ($n = 5$, Eq. (3)). Computation of a phase in the vicinity of the saddle points provides an estimation of the changing phase within each speckle, while the location of the saddle points of intensity correlates with the points of maximal intensity at the centres of the speckles. Circles or triangles show the saddle points in the vicinity of which the phase increases or decreases under clockwise circumference, respectively.

The application of the introduced approach for solving the inverse phase problem highlights the loci of singular points (Fig. 6, part 1) and the saddle points of intensity (Fig. 6, part 2) from the location of the phase lines obtained by simulation.

The reconstructed phase distribution facilitates the determination of the equi-phase lines, *cf*. Fig. 7, which are directly connected with the saddle points and form equi-phase
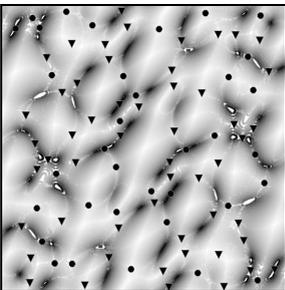
Fig. 5. Reconstructed phase map with the saddle points of intensity, in the vicinity of which the phase increases (circles) or decreases (triangles).
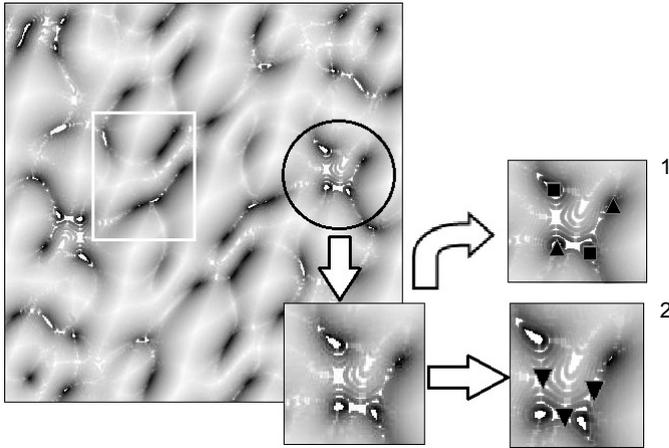
Fig. 6. Reconstructed phase map: insert shows the enlarged region of a map (ellipse), showing the distribution of the phase lines. Fragments 1 and 2 illustrate the loci of singular points and saddles, respectively.
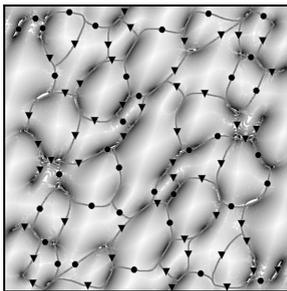
Fig. 7. Phase map with depicted regions of constant phase. Equi-phase lines are established as lines connecting the points of constant phase along the intensity gradient lines [12].

regions. We show by grey the equi-phase lines emerging from the saddle points and connecting the points of equal phase. As it has been shown in a recent paper [12], these lines trace the intensity gradients.

For comparing the results obtained by simulation and represented as the phase map in grades of grey (Fig. 7) with the reconstructed equi-phase lines (shown by grey in Fig. 7), the average phase along dark (relief) lines was derived. These are directly connected with the saddle points, *cf*. Figs. 5 and 6, *i.e.* the lines within the white square. The regions formed in such a way specify the regions of constant phase. Figure 8 illustrates a histogram of the deviation of the phase magnitude at the saddle point and at the points of these relief lines.

The deviation of the phase magnitude between the relief lines and the corresponding equi-phase lines is shown, in accordance with Fig. 7, in Fig. 9.
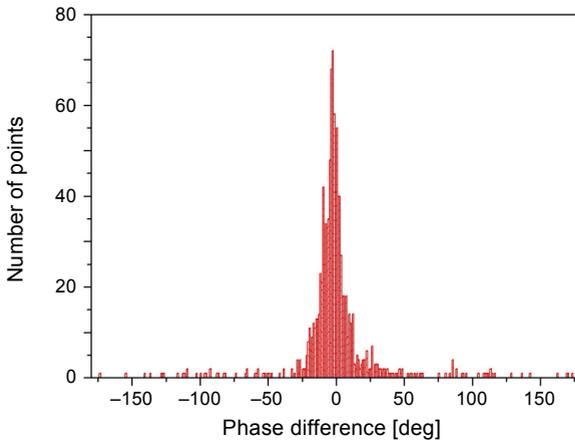
Fig. 8. Histogram of the deviation of the phase magnitude at the saddle point and along the relief lines emerging from this point: average is −0.95433°, confidence interval with probability 95%: (−2.68316°, 0.7745°).
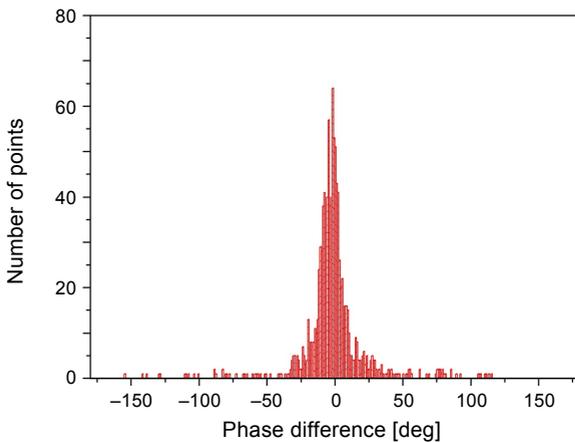
Fig. 9. Histogram of deviation of the phase magnitude between equi-phase lines marked by green in Fig. 6 and phase along dark (relief) lines: average is −1.42926°, confidence interval with probability 95%: (−2.91601°, 0.05748°).

As our computation shows, the proposed algorithm identifies a narrow confidence interval (about 3.5°) for the reconstruction of the phase lines that proves a high accuracy of the correspondence of the relief lines with the boundaries of the regions of constant phase. Thus, any changes of a remote object can be estimated from the changes of the phase distributions by applying a "window" Hilbert transformation avoiding any additional computing and simulation. Any change of the loci of the reference points – and connected with them the phase lines – is governed by the direction of movement thus following the location of the studied objects.

4. Conclusions

Within the framework of the approach based on the use of a discrete 2D "window" Hilbert transform, we have demonstrated the feasibility for reconstructing the phase of random 2D objects. We show the possibility to obtain comprehensive information on the structure of an optical field, including the field skeleton and the regions of constant phase. High accuracy of the reconstruction of equi-phase lines, singular points and saddle points, as well as invariance of the phase distributions following from direct simulation provides an additional algorithm for solving the inverse phase problem in optics.

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