Optica Applicata, Vol. XLVI, No. 2, 2016
DOI: 10.5277/oa160214

Laser frequency stability estimation and velocity measurement in the atmosphere with self-heterodyne

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We present a novel method of self-heterodyne for laser frequency stability estimation in the atmosphere. At the same time, it is used in the Doppler velocity measurement for moving targets in the atmosphere. The transmission range is 16.2 km (round-trip propagation with 8.1 km) in the horizontal atmospheric path, and the measuring results showed that the frequency stability of the whole laser detection system is 53.4 kHz with 20.7 kHz stand deviation. This frequency stability is the integrated influence with the whole detection system not just for the laser, but it can be considered as the estimation of the laser's frequency stability. It is much more practical for the heterodyne detection in the atmosphere. At last, the method of self-heterodyne was used for 16.2 km round trip velocity measurement. It is showed that the velocity absolute error is less than 1.5 cm/s in the turbulent atmosphere, and the relative error is not higher than 5%.

Keywords: frequency stability, atmospheric propagation, self-heterodyne, Doppler velocity.

1. Introduction

Laser heterodyne detection has the merit of high sensitivity and capacity of resisting disturbance due to the local oscillator, so the method has a wide variety of applications [1–3], and the laser frequency stability is the most significant influence factors for the SNR of the coherent (heterodyne) detection. On the other hand, the laser beam will be interfered through the turbulent atmosphere especially for the wave-front and the laser frequency. Therefore, some adaptive optics systems for wave-font correcting are used in the visible and near infrared laser equipments for improving the detection efficiency. Laser frequency stability is most conveniently evaluated from the beat note spectra of two lasers [4]. It can be determined by either frequency-domain (Fourier spectrum) or time-domain (Allan variance) analysis of the beat note spectra of the laser pairs. If the spectral purity (laser line-width) of one of the lasers (typically the local oscillator) is much better than that of the other laser, then the beat note spectrum may be entirely attributed to the worse of the two lasers. This method is a heterodyne detection, it can provide not only laser line-width data, but also optical power spectrum. In heterodyne detection, two lasers have to be used. One is the signal laser, the other one is a reference laser and referred to as the local oscillator (LO). The main difficulty in the application of the heterodyne detection is that two lasers must be used. And the line-width of the reference laser must be narrower than or at least comparable to that of the laser source to be measured in order to achieve reasonable measurement accuracy. For extremely narrow line-width measurements, the characterization of the reference laser itself is very difficult [5], therefore, much investigation about the laser line-width measurement has been brought forward in the last few decades [6–11]. The most popular method is a delayed self-heterodyne. The basic theory is the beat signal from the transmitted and the LO beams, but we need to point out that the frequency stability measurement is based on the beat signal. What is the difference between measuring line-width and short term frequency stability?

In 1980s, the most famous coherent laser radar system called "Firepond" has been established in MIT Lincoln Laboratory. The system stability (including turbulence disturbance) can be obtained through comparing the laser with its own output delayed by the round-trip time to and from the target. Hence, the effects due to disturbances with correlation times less than the round-trip time of the transmitted signal will be included in the measured beat note spectrum. As a result, the beat note spectrum can be considered as the estimation of the whole radar system not only the laser frequency stability, and this stability is more significant and applicative. This paper presents the field test of the coherent laser radar stability through 16.2 km atmosphere path with the self-heterodyne method, and this stability can be considered as the estimation of the laser. This method is the same as measuring the line-width with fiber delay in the laboratory. Therefore, maybe the line-width and the short term frequency stability for a high spectral purity laser has the same amplitude level.

2. Theory of the delayed self-heterodyne

The short term frequency stability of the laser can be estimated through the laser radar system with corresponding time delay, while the echo signal was reflected from the retro-reflector at different ranges. The beat note can be resolved with FFT and other spectrum analysis methods. This paper will focus on the measurement of stability through the delayed self-heterodyne, and the main theoretical description are as follows.

Compared with heterodyne detection, delayed self-heterodyne detection provides a simpler method to perform laser line-width measurement without using a separate local oscillator. Instead, delayed self-heterodyne detection needs a large optical delay. The analysis is simplified and more explicitly presented here by using the complex amplitude representation of a quasi-monochromatic wave. The laser field $E(t)$ is modeled as a sinusoidal wave with constant amplitude A if the intensity noise is neglected,

$$E(t) = A\exp[j\varphi(t)] \tag{1}$$

The optical center angular frequency ω_0 has been ignored so $E(t)$ represents the complex amplitude of the laser field. From the Wiener–Khinchin theorem, the power spectral density can be obtained by the Fourier transform of the field autocorrelation function which is defined by

$$R_e(\tau) = \langle E(t)E^*(t-\tau)\rangle = A^2\langle \exp[j\Delta\varphi(t, \tau)]\rangle \tag{2}$$

where $*$ denotes the complex conjugate, the angle brackets denote the ensemble average and

$$\Delta\varphi(t, \tau) = \varphi(t) - \varphi(t-\tau) \tag{3}$$

is the phase jitter of the laser. Here we define that the phase-noise spectrum S_φ is the power spectrum of the phase fluctuations and the frequency noise spectrum S_f is the power spectrum of the frequency fluctuations. The mean square of phase fluctuations, $\langle\Delta\varphi^2(t, \tau)\rangle$ is related to the frequency noise spectrum by

$$\langle\Delta\varphi^2(t, \tau)\rangle = \frac{2}{\pi}\int_{-\infty}^{\infty}\sin^2\left(\frac{\omega\tau}{2}\right)S_f(\omega)\frac{\mathrm{d}\omega}{\omega^2} \tag{4}$$

The line shape of most lasers has a Lorentzian profile. The corresponding frequency noise of such lasers has a constant spectral density (white noise) given by

$$S_f = S_0 = 2\pi\Delta\nu \tag{5}$$

where $\Delta\nu$ is FWHM of the Lorentzian spectral line shape. Under these conditions, the phase jitter $\Delta\varphi(t, \tau)$ is a zero-mean, a stationary and Gaussian random process with variance

$$\langle\Delta\varphi^2(t, \tau)\rangle = \langle\Delta\varphi^2(\tau)\rangle = 2\pi\Delta\nu|\tau| \tag{6}$$

Equation (6) is obtained by substituting Eq. (5) into Eq. (4). Using the well-known condition

$$\langle \exp[\pm j\Delta\varphi(t, \tau)]\rangle = \exp\left[-\frac{\langle\Delta\varphi^2(t)\rangle}{2}\right] \tag{7}$$

in Eq. (6), and substituting Eq. (6) into Eq. (2), the Fourier transform of Eq. (2) leads to the Lorentzian line shape of the laser

$$S_E(f) = \left[\left(\frac{\Delta\nu}{2}\right)^2 + f^2\right]^{-1} \tag{8}$$

where f is the frequency deviated from the optical center frequency. The detected optical field is the sum of the laser beam and a time delayed and frequency shifted replica of itself

$$E_T(t) = E(t) + E(t - \tau_d)\exp(j\Omega t) \tag{9}$$

where τ_d and Ω are the time delay and angular frequency shift, respectively. For simplicity, we have assumed the two beams have equal amplitudes. The photocurrent $I(t)$ is proportional to the optical intensity because of the square law of the photodetector,

$$I(t) \propto E_T(t)E_T^*(t) \propto 2 + \exp\{-j[\varphi(t) - \varphi(t - \tau_d)]\}\exp(j\Omega t) + \text{c.c.} \tag{10}$$

where c.c. denotes the complex conjugate of the preceding term. In Equation (10), the constant coefficient related to the laser field intensity and the photodetector (PD) sensitivity has been ignored. The photocurrent contains a DC and a quasi-monochromatic signal centering at angular frequency Ω with a constant amplitude and random phase fluctuations equivalent to the laser field phase jitter. Here we are only interested in and consider the quasi-monochromatic term that contains the information about the laser phase noise. Similar to the case of the laser field, by ignoring the center angular frequency Ω, the complex amplitude of this term is given by

$$I_\Omega(t) = \exp\{-j[\varphi(t) - \varphi(t - \tau_d)]\} \tag{11}$$

The autocorrelation function of $I_\Omega(t)$ is thereby calculated by

$$\begin{aligned} R_\Omega(t) &= \langle I_\Omega(t) I_\Omega^*(t - \tau)\rangle = \\ &= \langle \exp\{-j[\varphi(t) - \varphi(t - \tau_d) - \varphi(t - \tau) + \varphi(t - \tau_d - \tau)]\}\rangle \end{aligned} \tag{12}$$

Using Eq. (7) in Eq. (12) and after some straightforward algebra, Eq. (12) can be expressed as a function of phase jitter variance

$$R_\Omega(\tau) = \exp[B(\tau, \tau_d)] \tag{13}$$

where

$$B(\tau, \tau_d) = -\langle\Delta\varphi^2(\tau_d)\rangle - \langle\Delta\varphi^2(\tau)\rangle + \frac{1}{2}\langle\Delta\varphi^2(\tau + \tau_d)\rangle + \frac{1}{2}\langle\Delta\varphi^2(\tau - \tau_d)\rangle \tag{14}$$

The spectrum of the corresponding photocurrent component is thus obtained by the Fourier transform of Eq. (13)

$$S_0(\omega, \tau_d, \Omega) = F[R_\Omega(\tau)] = F\{\exp[B(\tau, \tau_d)]\} \tag{15}$$

where F denotes the Fourier transform. In deriving Eq. (15), the carrier angle frequency Ω has been implied, thus $\omega = 2\pi f$ is defined as the angular frequency deviated from the carrier angular frequency. For the Lorentzian spectral line shape of the laser, Eq. (6) is satisfied and Eq. (15) therefore leads to

$$S(\omega) = \exp(-2\pi\Delta\nu\,\tau_d)\,\delta(f) + \\ + \frac{\Delta\nu}{\Delta\nu^2 + f^2}\left\{1 - \exp(-2\pi\Delta\nu\,\tau_d)\left[\cos(2\pi f\tau_d) + \frac{\Delta\nu}{f}\sin(2\pi f\tau_d)\right]\right\} \tag{16}$$

For the case of a Lorentzian-shaped laser field spectrum, the line shapes retain their form during conversion from the optical spectrum to the electrical domain through the delay self-heterodyne process, except that the electrical line shape has a line-width twice the actual optical line-width. When the optical delay line is infinitely long, the power spectral density can be written in a standard Lorentzian line

$$S(\omega) \propto \frac{\Delta\nu}{\Delta\nu^2 + f^2} \tag{17}$$

where $\Delta\nu = 1/(2\pi\tau_c)$ is the intrinsic line-width of the laser, and $f = \omega/(2\pi)$ is the measured frequency. Equation (16) indicates that as the delay time increases, the signal strength shifts from the delta function peak to the modified Lorentzian pedestal until the power spectrum becomes strictly Lorentzian. This true Lorentzian corresponds to the delay time when the phases of the optical field have become totally decorrelated.

This paper will adopt the method with line-width measurement for the stability estimation for the frequency-stable laser in the radar system, and the spectral width for the beat note will be regarded as the short term stability (can be over 10 ms time interval). This time delay, corresponding to 1500 km optical round trip path in the atmosphere, is long enough for many applications, and the whole system stability is more effective and significant for coherent laser radar performance evaluation.

3. Experiment of the laser frequency stability estimation

The experiment was operated in the outside field with the 16.2 km round trip distance in the horizontal atmosphere path, and the sketch map of the long range self-heterodyne is shown in Fig. 1.

The narrow line-width laser beam was split into two sections with 5% reflectivity for the purpose of local oscillator (LO), and the reflected light incidents on the surface of the acoustic optics modulator (AOM) while the first order diffraction light beam was selected by the pinhole beyond the attenuator. The expander was used for beam spot matching for high heterodyne efficiency. On the other hand, the transmitted beam passes through the other expander also for beam matching with the LO beam. The polarized prism in the figure above was just for signal isolation. The 1/2 and 1/4 waveplates were selected for polarization matching. We can see that the transreceiver has focusing mechanism for a beam transform.

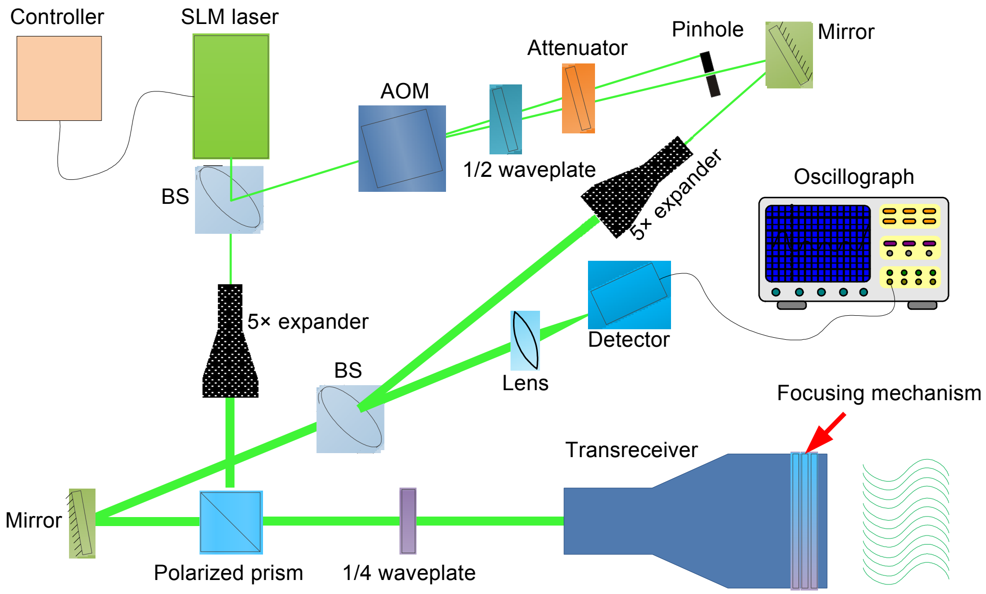

Fig. 1. Sketch map of the long range self-heterodyne detection. BS – beam splitter, SLM – single longitudinal mode, AOM – acoustic optics modulator.

The frequency stability of the narrow line-width laser was the main performance index of the remote laser heterodyne system, and the traditional method was using the delayed self-heterodyne in a long range fiber. This method will be restricted by the fiber length and the attenuation coefficient, especially for an ultra-narrow line-width laser. According to the theory of delayed self-heterodyne, when the optical delay line is infinitely long, the power spectral density of the beat signal can be written in a standard Lorentzian line, and the width of the power spectral density can denote the laser line-width (short term frequency stability) of the emitted laser. In fact, the delayed range cannot be infinitely long, and the delayed time should satisfy the relationship $\tau_d > 6\tau_c$. If the line-width is narrow to 1 kHz, this method cannot be realized in the experiment. Therefore, some researchers have given the approximation result for line -width measurements while delayed time $\tau_d \approx 6\tau_c$. In our experiment, the method for line-width measuring is used for the estimation of the short term frequency stability with the laser radar system.

The analysis above has shown that the short term frequency stability of the laser radar system can be obtained from the delayed self-heterodyne, the emitted laser wavelength is 532 nm, and the optical path difference is 16.2 km corresponding to 5.4 μs transferring time delay. The atmospheric disturbance of the laser frequency was included in the whole path of the beam propagation, therefore, the measuring frequency spectrum can reflect the entire stability for the whole laser radar system. The delayed self-heterodyne frequency spectrum was shown in Fig. 2.

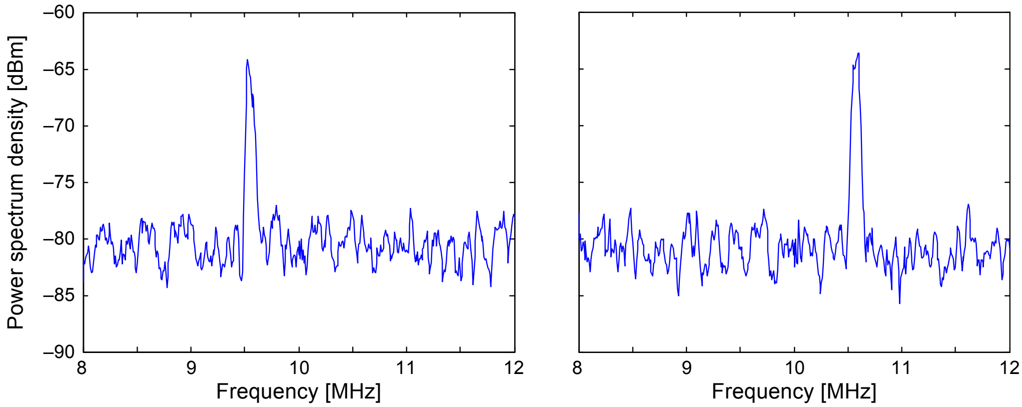

Fig. 2. Frequency spectrum of the beat signal with delayed self-heterodyne (left and right sub-graphs correspond to different Doppler frequency shifts).

Figure 2 shows that the Doppler frequency shift was around 10 MHz due to different moving velocity of the target. The center frequency of the AOM is 10 MHz in Fig. 1. Therefore, the 10 MHz is corresponding to the echo signal with stationary target. While the target is moving toward the receiver, the frequency is higher than 10 MHz. The target (corner cube) is moving randomly during the whole signal receiving course, and the waterfall plot of the spectrum is shown in Fig. 3.

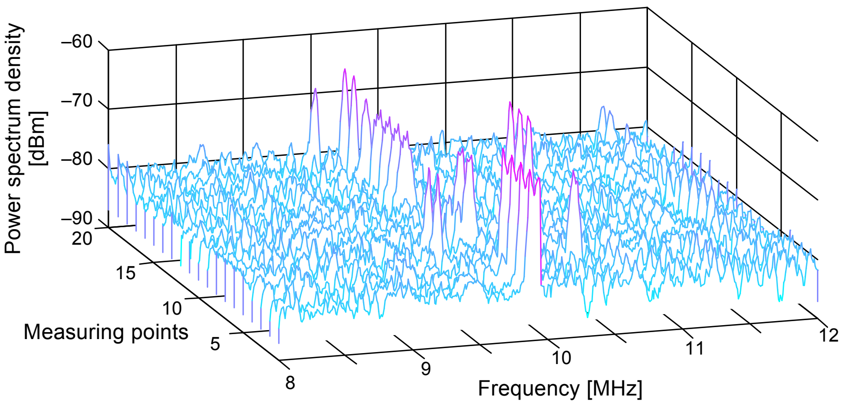

Fig. 3. The waterfall plot of the spectrum for the beat signal with delayed self-heterodyne.

The signal from the delayed self-heterodyne will be processed with some filtering methods, and the spectrum width can be defined with the 5 dB of the peak value. Therefore, the spectrum width of the measuring points is shown in Fig. 4**a**, and the statistical histogram is shown in Fig. 4**b**.

The whole measuring data set contains 1689 data points, and the spectrum width average value is 53.4 kHz with 20.7 kHz (1σ) standard deviation. The analysis above

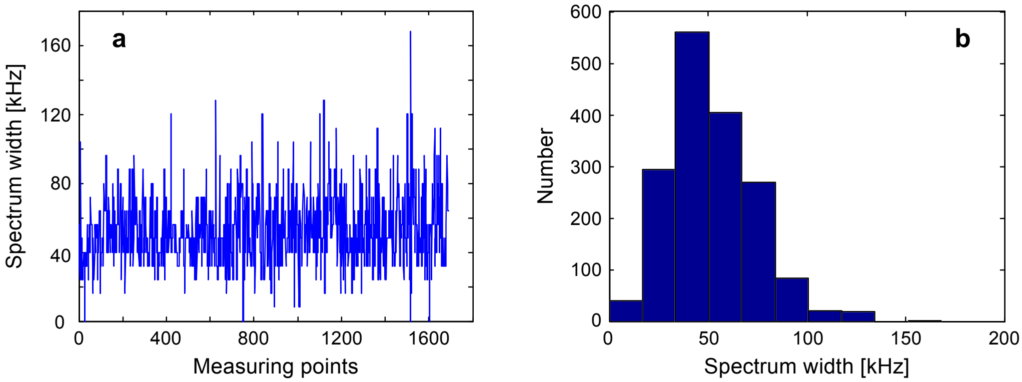

Fig. 4. The spectrum width of the measuring points (**a**) and the statistical histogram (**b**).

has shown that the beat signal from the echo of the retro-reflector and the local oscillator can reflect the short term frequency stability of the laser. In fact, it contains the atmospheric disturbance during the propagation. Therefore, the short term frequency stability of the laser used in the experiment was better than 53.4 kHz, and this can be recognized as the stability of the laser radar system, which is the most important index of the coherent detection radar system.

4. Experiment of the velocity measurement

The Doppler velocity measurement setup was the same as in Fig. 1, and the moving target was placed 8.1 km away (16.2 km optical path difference). The target was driven

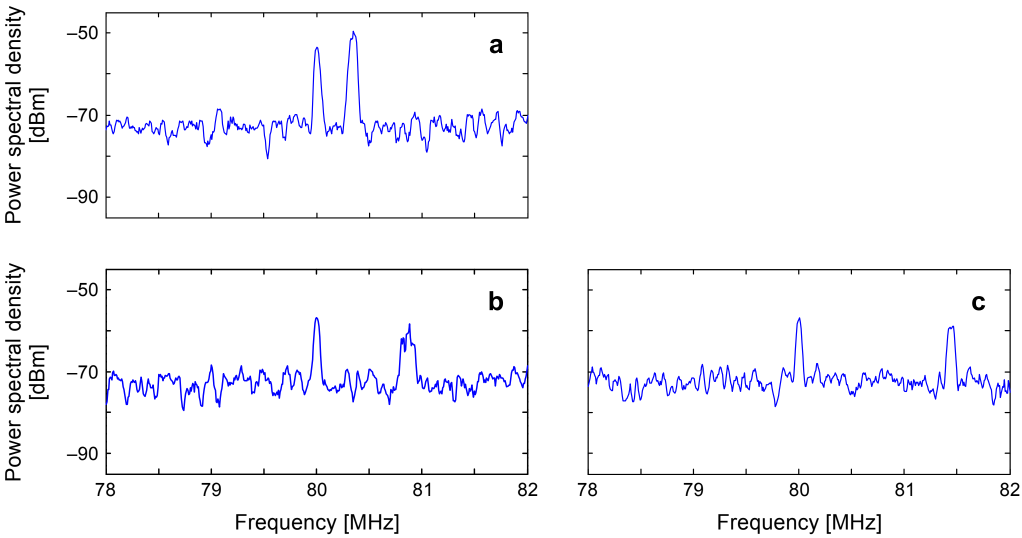

Fig. 5. The Doppler frequency shifts at different driving voltages: 0.1 V (**a**), 0.2 V (**b**), and 0.3 V (**c**).

by a DC motor with different driving voltages. Therefore, the velocity can be changed due to the adjustment of driving voltage, and in the experiment the voltages were fixed at 0.1, 0.2 and 0.3 V. The Doppler frequency shifts due to the target's relative motion at three different driving voltages above were shown in Fig. 5.

As shown in Fig. 5, the center frequency is 80 MHz; it is determined by the AOM with the system due to the amplitude modulation with LO beam. So the echo beam has no Doppler frequency shift if the remote target is stationary, and the beat frequency with LO beam will be 80 MHz. If the target moves toward the receiving detector, then the beat frequency will be higher than 80 MHz (see Fig. 5), and the frequency difference was determined by the moving velocity. The velocity measuring results with different driving voltages are shown in Fig. 6.

While the driving voltage of the target is 0.1 V, the average velocities are 10.45 and 10.56 cm/s, respectively, for two different date sets, and the standard deviations are 1.675 and 1.467 cm/s, respectively. We can obtain the average velocity and standard deviation for 0.2 and 0.3 V driving voltages with the same processing.

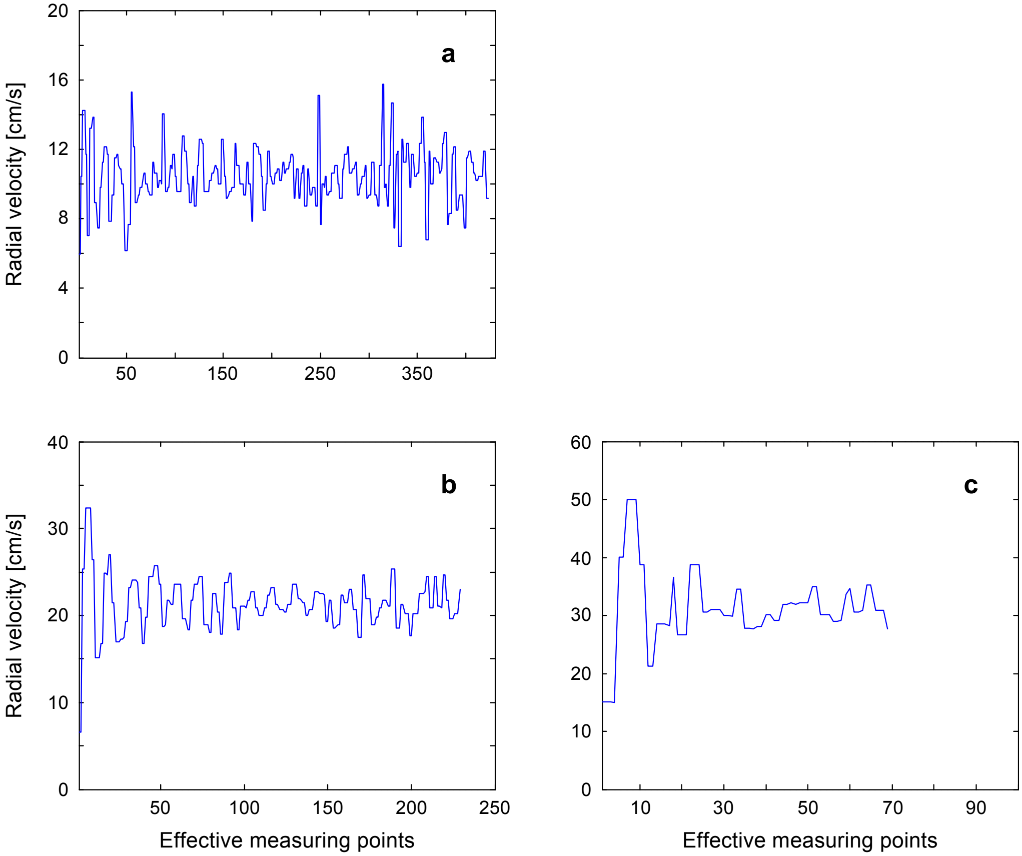

Fig. 6. The target (corner cube) radial velocity at different driving voltages: 0.1 V (**a**), 0.2 V (**b**), and 0.3 V (**c**).

Table 1. Velocity measurement with physical definition method.

Driving voltage [V]	Starting position [cm]	Final position [cm]	Time consuming [s]	Target velocity [m/s]	Average velocity [cm/s]
0.1	19.4	184.7	14.9	0.11093	10.87
	21.3	191.4	15.2	0.11191	
	22.5	190.7	15.7	0.10713	
	23.9	191.1	15.7	0.10649	
	24.4	193.3	15.8	0.10689	
0.2	20.2	192.0	8.0	0.21475	21.68
	19.5	192.8	8.1	0.21395	
	18.8	193.7	8.0	0.21863	
	23.1	191.4	7.5	0.22440	
	21.4	195.3	8.2	0.21207	
0.3	23.7	190.3	5.1	0.32667	32.60
	25.3	194.9	5.4	0.31407	
	19.7	194.2	5.3	0.32925	
	19.0	189.9	5.2	0.32865	
	23.2	192.3	5.1	0.33157	

In order to give the velocity value comparison, we need to measure the target velocity with different methods. The method of physical definition can be considered as the relative precise way. So the measuring results with the physical definition method are shown in Table 1.

The target moving velocities were 10.87, 21.68 and 32.60 cm/s with 0.1, 0.2 and 0.3 V driving voltages, and the comparison between these results and the Doppler (delayed self-heterodyne) method is shown in Table 2.

The experimental results show that the absolute error is lower than 1.5 cm/s, and the relative error is not higher than 5%, and they also show good consistency with each other. But we can see that the results of the Doppler method are always lower than the physical definition method. It is the system error due to the non-parallel relationship

Table 2. Velocity comparison between physical definition and Doppler methods.

Driving voltage [V]	Doppler method [cm/s]	Physical definition method [cm/s]	Absolute error [cm/s]	Relative error [%]
0.1	10.45	10.87	0.42	3.86
	10.56	10.87	0.31	2.85
0.2	21.13	21.68	0.53	2.53
	21.41	21.68	0.27	1.25
0.3	31.18	32.60	1.42	4.36
	31.99	32.60	0.61	1.87

between the laser direction and the target's moving direction. As it is known, the Doppler frequency shift is just determined by the velocity component parallel with the incident laser direction. Therefore, the results from the laser Doppler velocity method are always lower than the actual velocity, and results from the physical definition method will be much more close to the true value. In fact, we are not sure whether the direction of motion is absolutely parallel with the laser incident direction.

5. Conclusion

The laser frequency stability estimation in the atmosphere with the self-heterodyne method has been stated in this paper, and this parameter is the critical index of the coherent detection system. The beat signal from the echo and the local oscillator with 16.2 km optical path difference will be processed in the frequency domain through the FFT method. The 5 dB width of the frequency spectrum signal 53.4 kHz can be recognized as the short term frequency stability of the coherent laser radar system. It contains the atmosphere impact factor, so the frequency stability of the laser will be better than 53.4 kHz. This method can estimate the frequency stability of the laser radar system especially for the long range detection such as from the ground to a satellite. On the other hand, we use the same system for velocity measurement in the turbulent atmosphere at the distance of 16.2 km optical path difference with the delayed self-heterodyne method. The experimental results have shown that the velocity absolute error is less than 1.5 cm/s in the turbulent atmosphere, and the relative error is not higher than 5%.

Acknowledgements – This work was supported by the project of National Natural Science Foundation (No. 61205143).

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Received September 13, 2015
in revised form October 12, 2015